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NIMROD Simulations of SSPX Spheromak Physics

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LLNL

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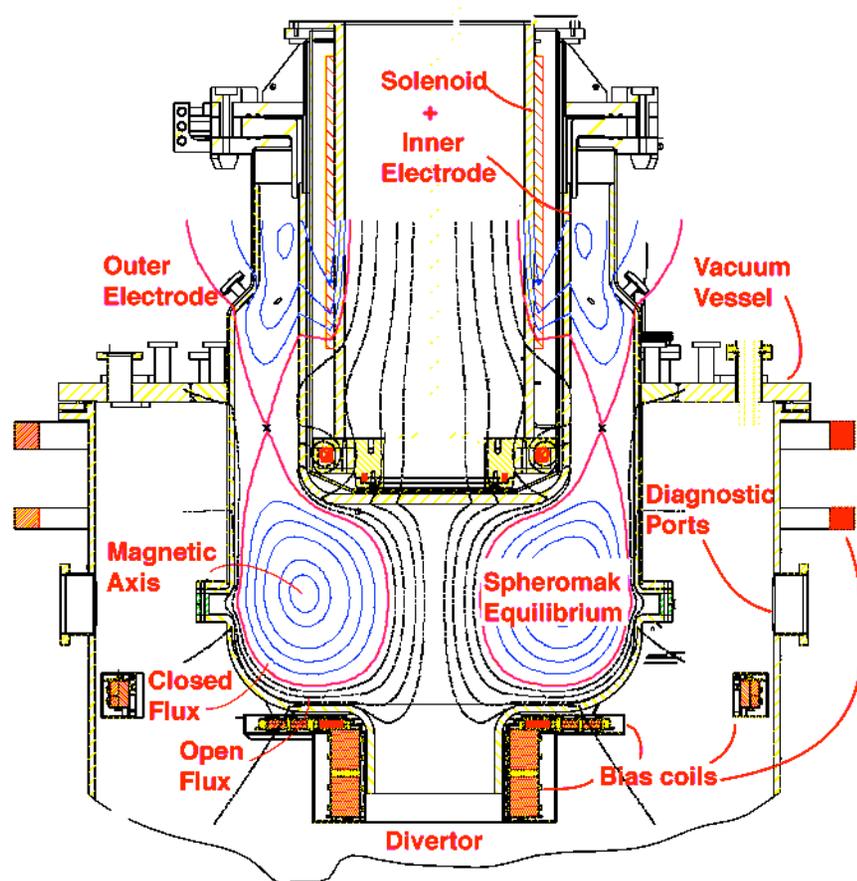
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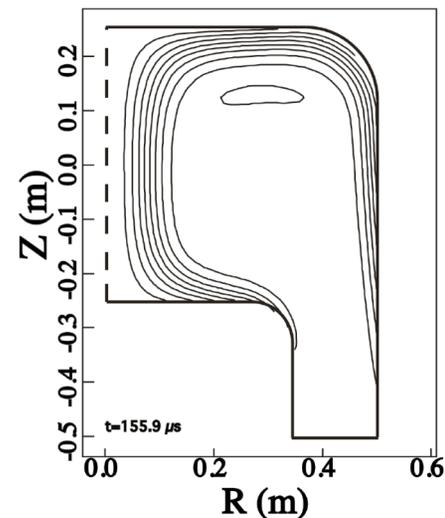
EXPERIMENTAL RESULTS

SSPX is a gun driven spheromak — a strong arc is established in bias flux between a cathode and flux conserver

The magnetic pressure due to the current drives a bias magnetic flux into the flux conserver – generating toroidal magnetic field and flux



- The plasma pinches and non-axisymmetric modes grow generating local poloidal field but no net poloidal flux
- When the mode amplitudes become large nonlinear coupling generates an axisymmetric field
- Reconnection is required to generate net poloidal flux

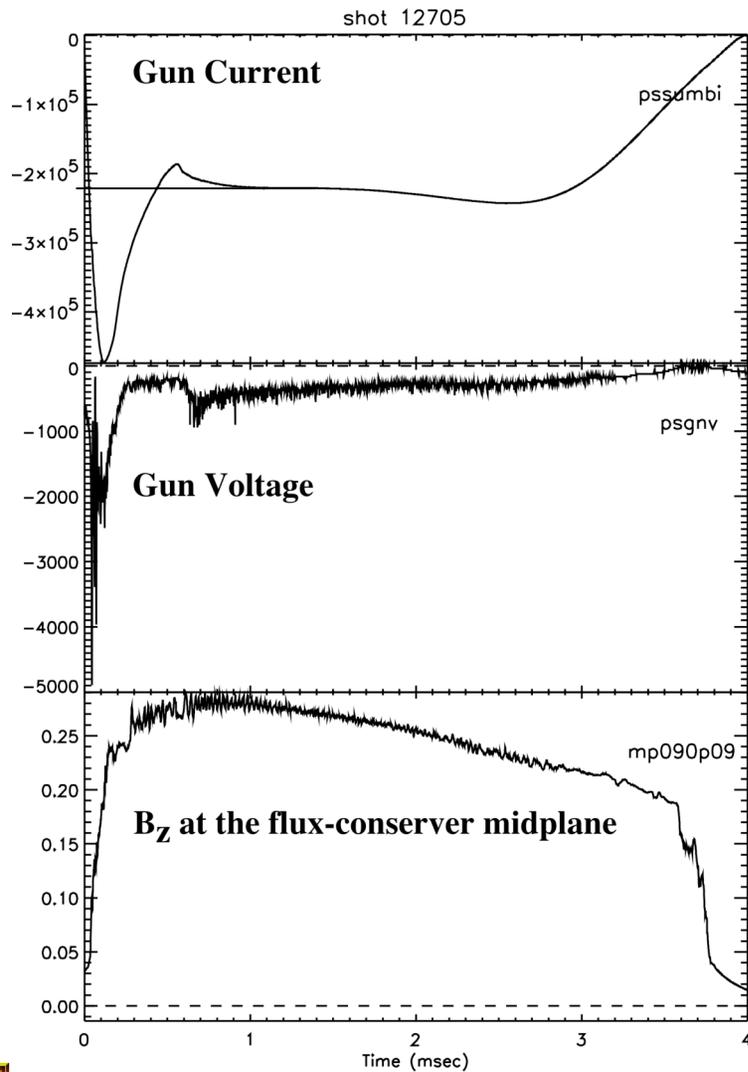


Flux “bubble” in NIMROD

Note: Nimrod is up-side-down relative to SSPX



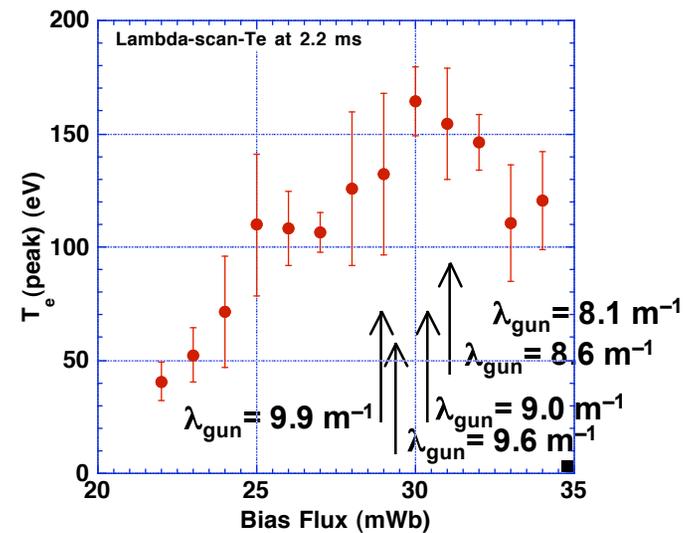
Spheromak characteristics are very sensitive to the ratio of $\lambda_{\text{gun}} = \mu_0 I_{\text{gun}} / \Psi_{\text{gun}}$ to the eigenvalue of $\nabla \times \mathbf{B} = \lambda_{\text{fc}} \mathbf{B}$ in the flux conserver geometry



This experimental run kept the current and current time history fixed and varied bias flux and thus λ_{gun}

Bias flux configuration was “modified flux”

- The electron temperature showed a clear maximum at about 30 mWb
- Magnetic fluctuations, rate of decay of the magnetic field, etc. also were quite sensitive to the value of λ_{gun}



Arrows mark NIMROD runs

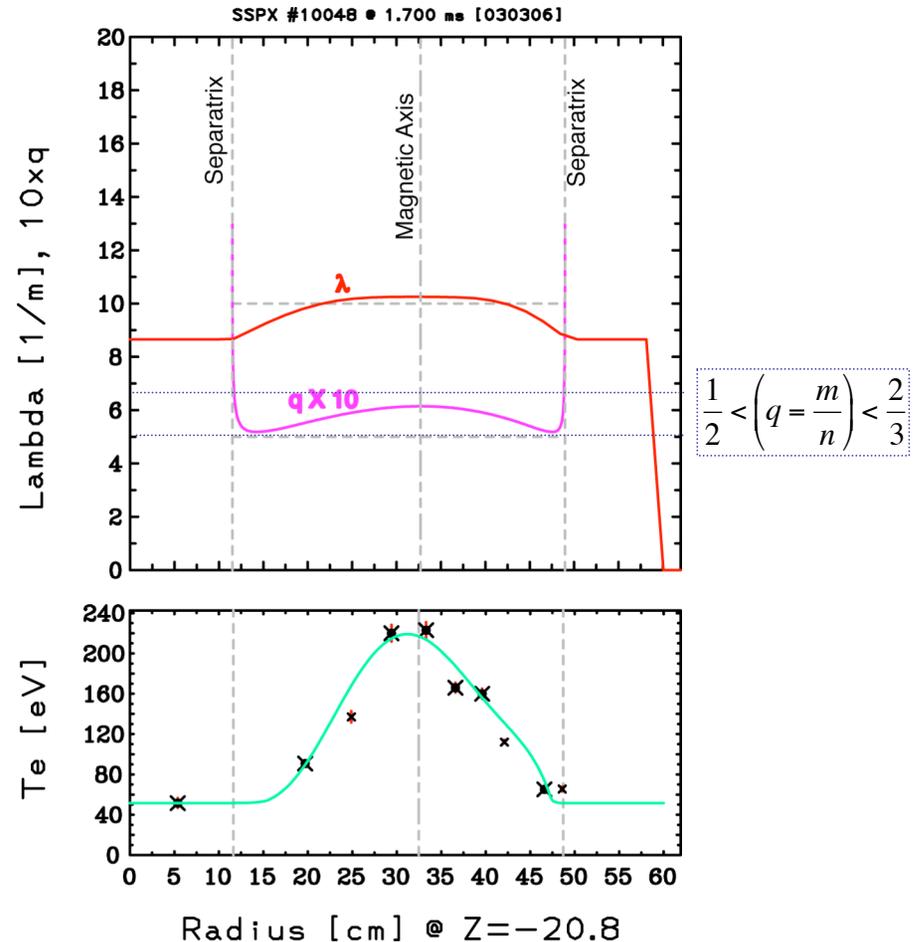


High temperatures with “parabolic” profiles correlate with flat q-profiles in the range 1/2 to 2/3

The q-profiles from fitting the solution of the Grad-Shafranov equation to magnetic probe measurements

- Observed magnetic modes correlate well with the calculated q-profiles
 - Provides confidence the fits are a good approximation to the experimental profiles

Note that the fit assumes axisymmetry — a good approximation for the equilibrium



When $q=m/n$ falls between resonant surfaces, plasma is quiescent



MODELING USING THE EXPERIMENTAL GUN-CURRENT TIME HISTORY

What determines the q-profile and the resultant resonant modes?

Two NIMROD runs duplicated the time-history of the gun current in SSPX

- Comparing lam06 and lam07:
 - These had exactly the same λ_{gun} except for the “glitch” early in lam06 — correcting this caused a gun voltage spike and lowered the toroidal current
 - lam06: The q-profile was flat following this correction
 - lam07: The q-profile was peaked on the magnetic axis

Controlling profiles is more complex than matching λ_{gun} to λ_{FC}

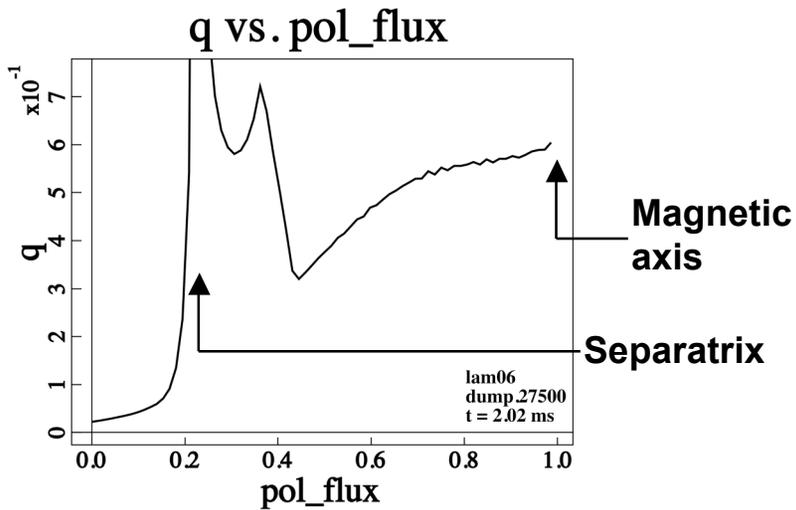
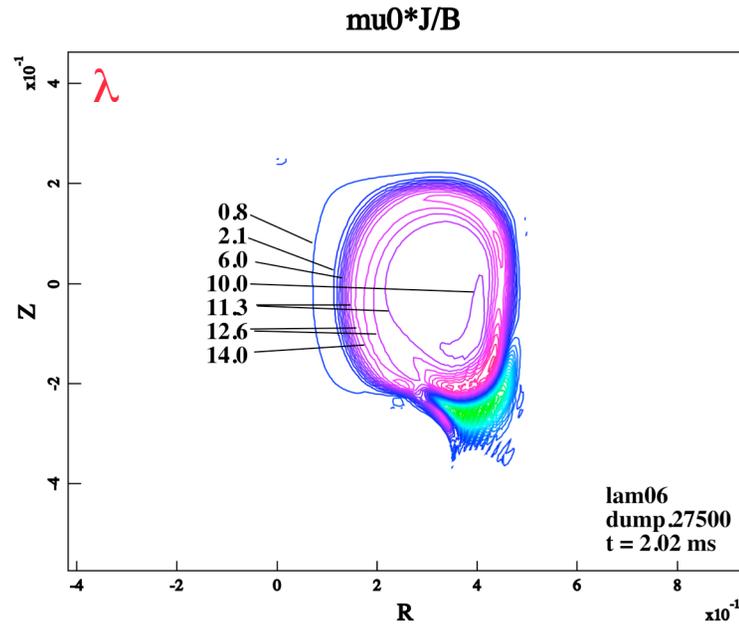
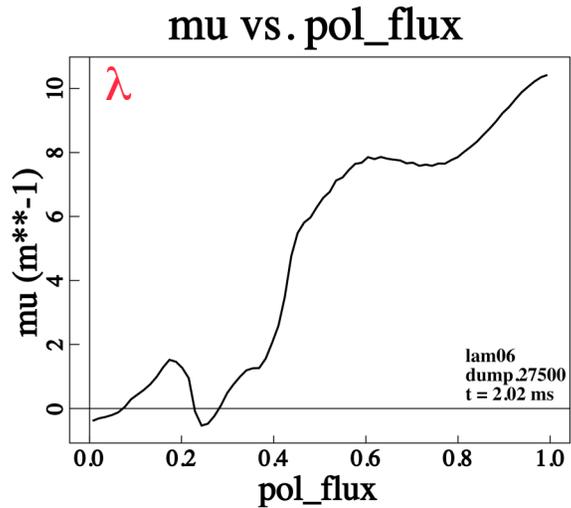
There were several differences between the two runs:

- The initial mode amplitudes in lam07 are greater than in lam06, resulting in a larger flux- and current-amplification
- Viscosity was less in lam07 than in lam06
- The voltage spike in lam06 was absent in lam07

The consequences of the differences are not fully clear



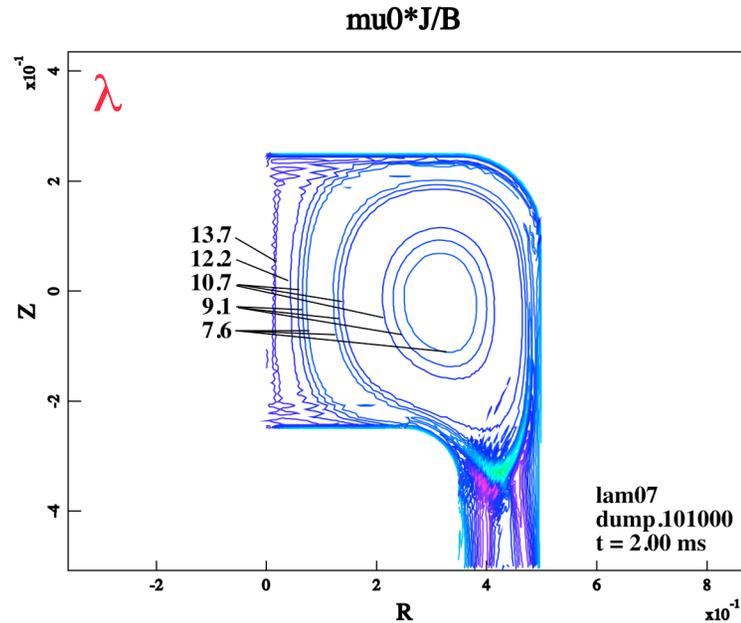
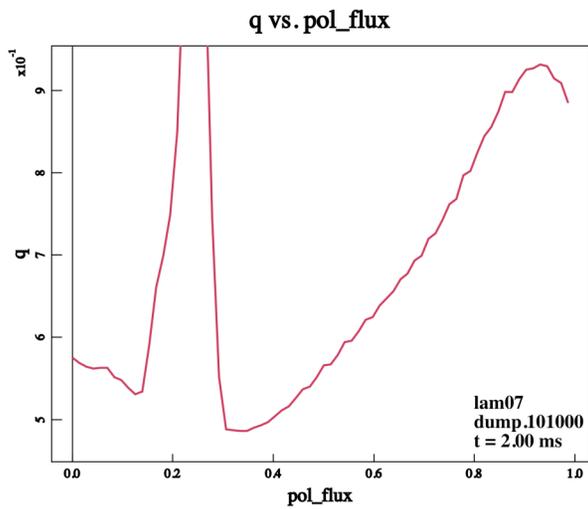
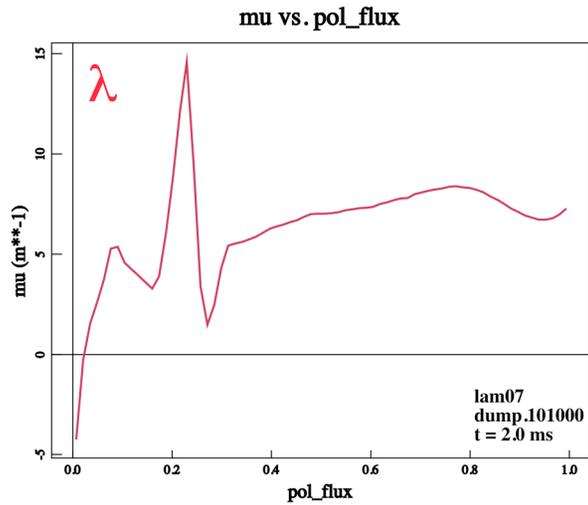
λ - and q-profiles in lam06 – q varies from < 0.4 to $< 2/3$



λ is small on the geometric axis. The consequence is a "large" λ inside the separatrix which results in a relatively low value of q on the magnetic axis



λ - and q-profiles in lam07 – q varies from $< 1/2$ to > 0.9



λ is large on the geometric axis. The consequence is a “small” λ inside the separatrix which results in a relatively high value of q on the magnetic axis



The λ -profile determines the q-profile inside the separatrix (azimuthal averages)

The value of q on the magnetic axis is $q_0 = \frac{2}{\lambda_0 R_0}$

where the nearby surfaces have been taken to be circular

Assuming $\lambda_0 = 10 \text{ m}^{-1}$, $R_0 = 0.33 \text{ m} \rightarrow q_0 = 0.61$

To get the value of q_0 in the range of 0.5 - 0.6 we want to increase the value of λ on the magnetic axis to $> \lambda_{\text{flux conserver}}$

The lam06 run, which had a temperature time history more similar to experiment than the lam07 runs, had $\lambda_0 > 10 \text{ m}^{-1}$

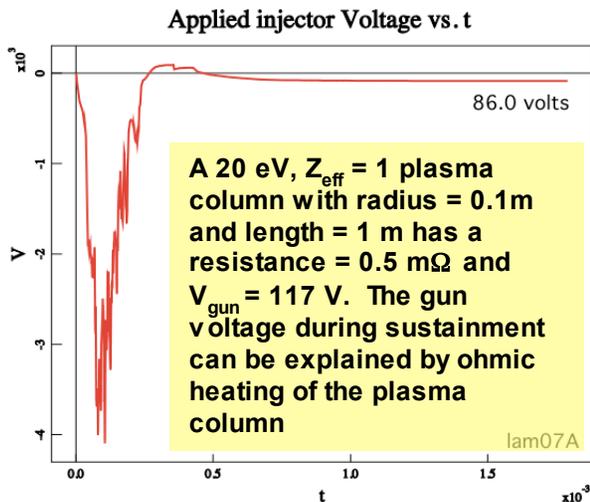
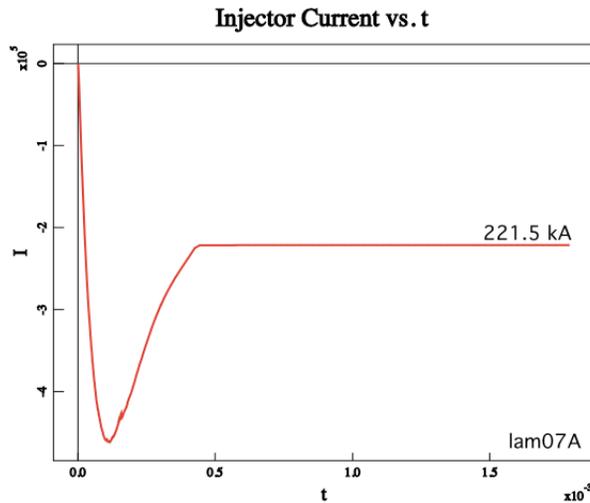
We will see in further NIMROD runs (below) that the behavior is more complex than this simple picture — Low-order modes often become strong and the q-profile may lock onto the corresponding rational values



MODELING TO UNDERSTAND THE SENSITIVITY TO THE GUN λ

Effects of the gun sustainment current in lam07

Replace the experimental time history with a fixed current – vary the current to examine effects on flux surfaces, field topology, magnetic fluctuations and energy confinement



Five cases:

lam07B $I_{\text{gun}} = 200.0$ kA $\lambda_{\text{gun}} = 8.1$ m $^{-1}$

lam07E $I_{\text{gun}} = 212.0$ kA $\lambda_{\text{gun}} = 8.6$ m $^{-1}$

lam07A $I_{\text{gun}} = 221.5$ kA $\lambda_{\text{gun}} = 9.0$ m $^{-1}$

lam07D $I_{\text{gun}} = 237.0$ kA $\lambda_{\text{gun}} = 9.6$ m $^{-1}$

lam07C $I_{\text{gun}} = 243.8$ kA $\lambda_{\text{gun}} = 9.9$ m $^{-1}$

λ_{gun} is based on a nominal bias flux of 31 mWb

$\lambda_{\text{flux conserver}} = 9.6$ m $^{-1}$

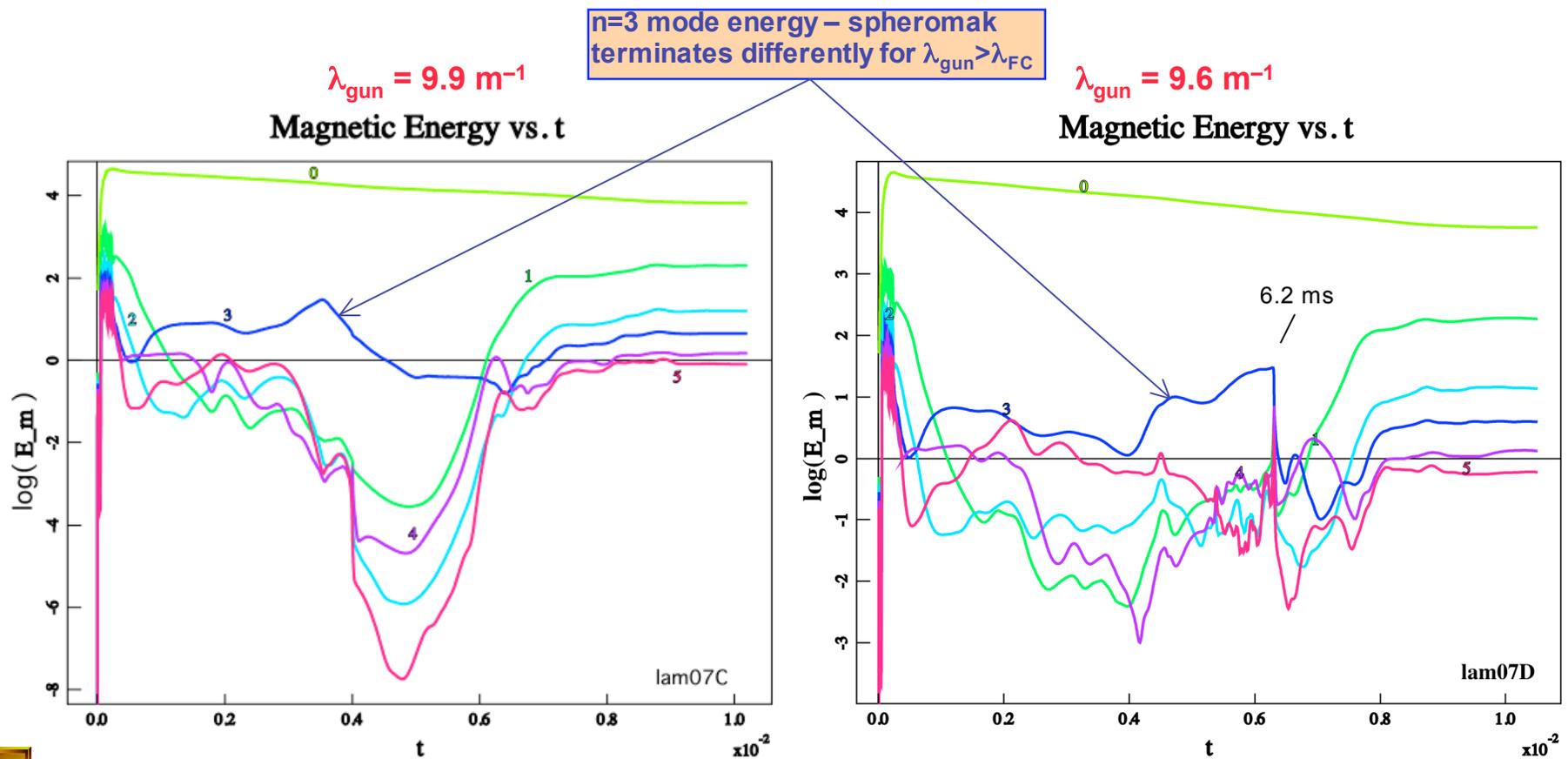


Changing sustainment current

The spheromak slowly decays for all λ_{gun} values examined here

The termination of the discharge is strongly correlated with the growth of the $n=3$ ($m=2$) mode

The spheromak is lost — and a pinched, unstable column formed — after the $n=3$ mode energy grows to within about 10^{-4} of the axisymmetric ($n=0$) energy

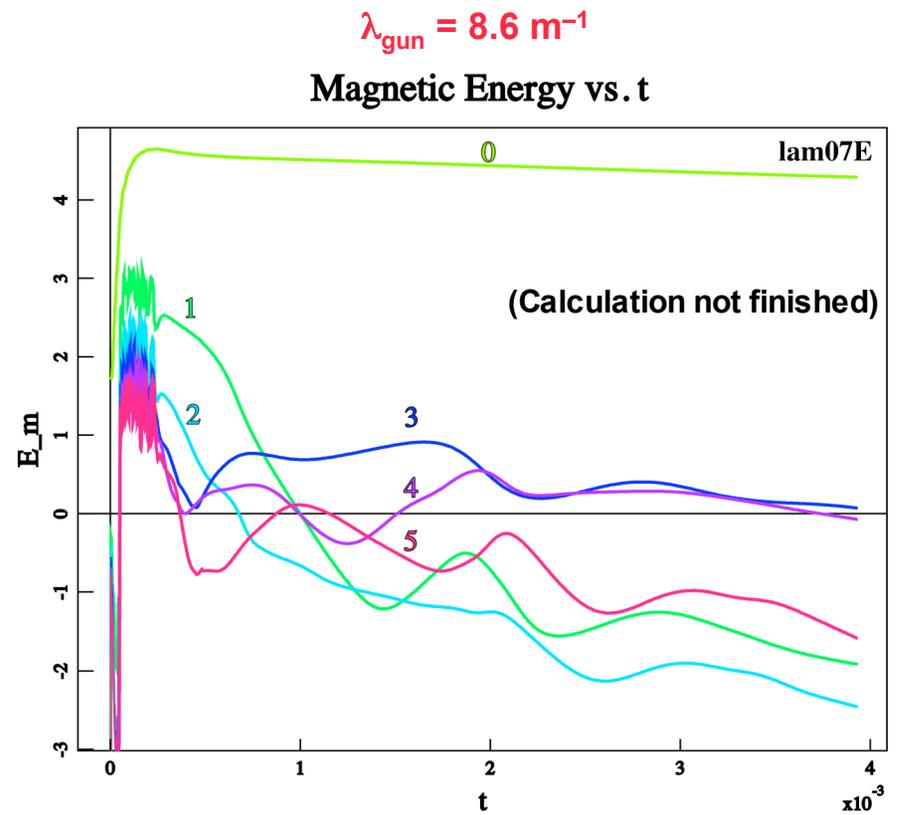
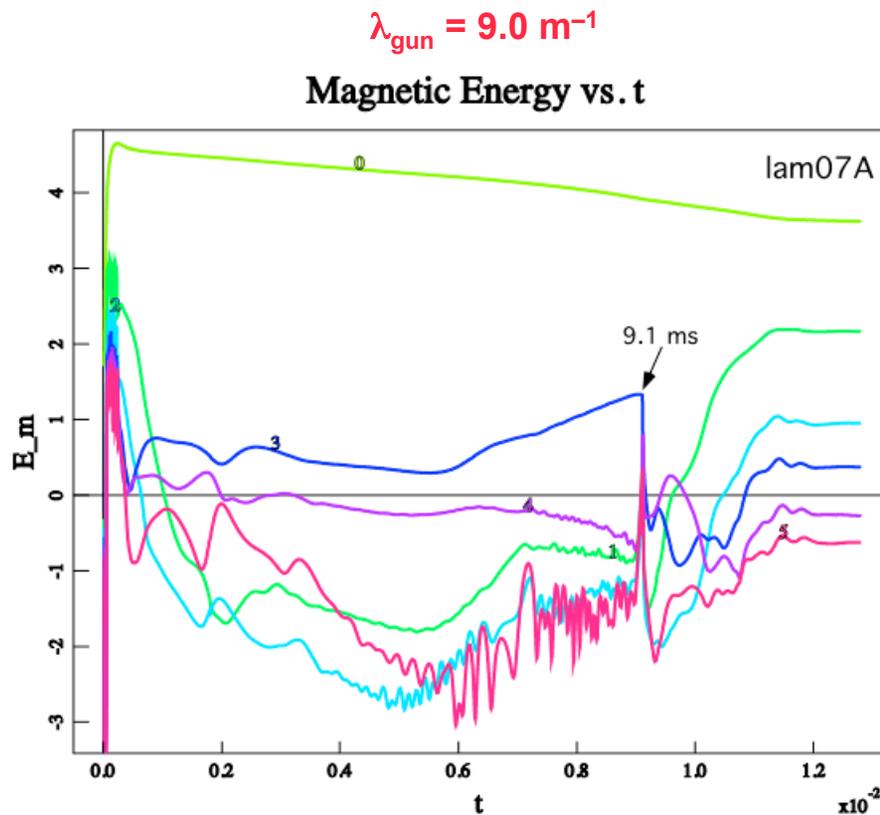


Azimuthal mode numbers labeled on the figures

As λ_{gun} decreases below the eigenvalue, the plasma duration increases

The n=3 mode continues to dominate the MHD spectrum at 9.0 m^{-1} , but by 8.6 m^{-1} the n=4 mode reaches a comparable amplitude

Mode amplitudes are less after 2 ms for 8.6 m^{-1}

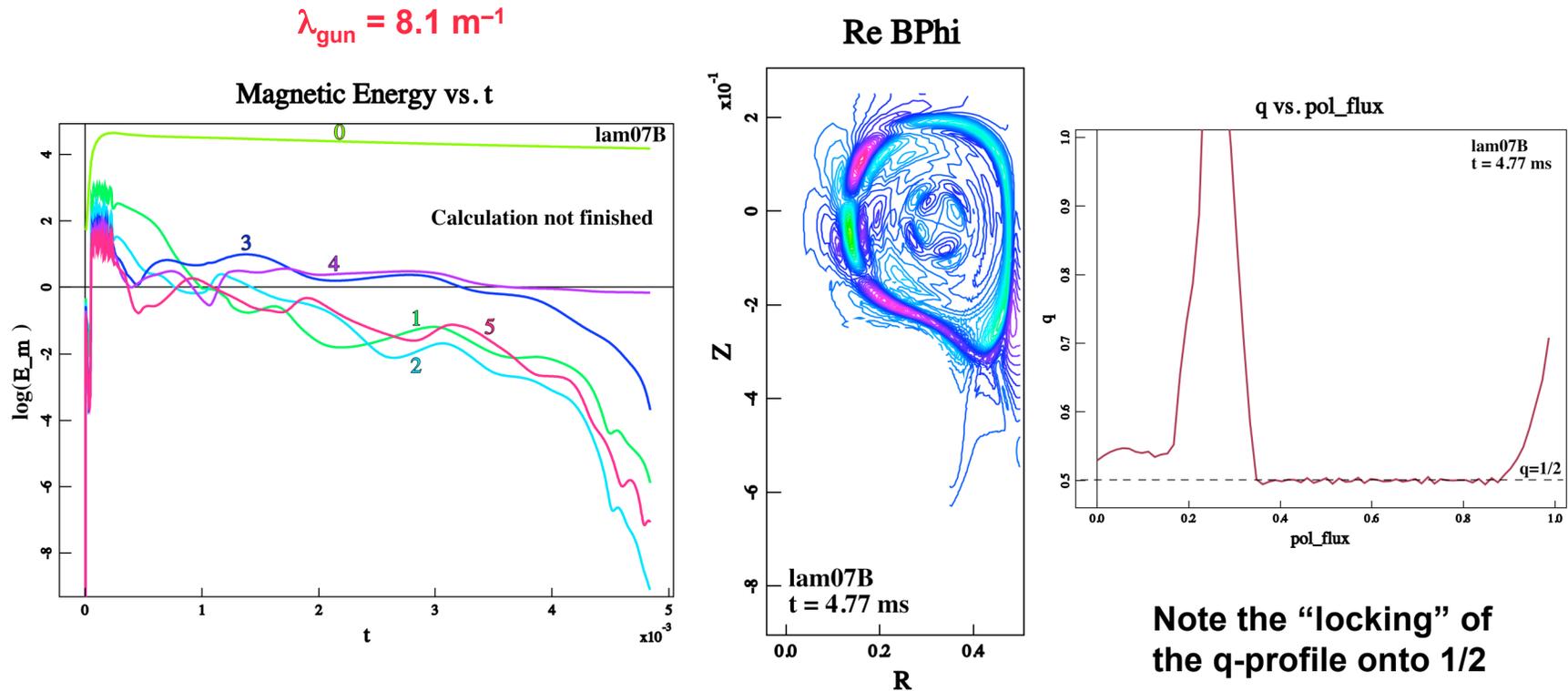


Azimuthal mode numbers labeled on the figures

As λ_{gun} decreases further, n=4 becomes dominant

The n=3 mode continues to dominate the MHD spectrum at 9.0 m^{-1} , but at 8.6 m^{-1} the n=4 becomes higher amplitude than 3

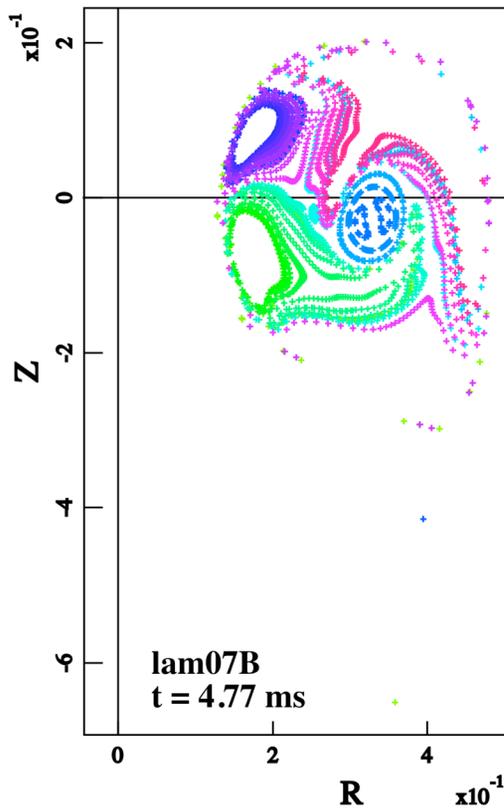
The n=4 mode is predominantly m=2 — with a large amplitude near the separatrix and lower amplitude in the plasma interior



Azimuthal mode numbers labeled on the figures

Fieldline tracing shows a complex island structure at $\lambda_{\text{gun}}=8.1 \text{ m}^{-1}$

The internal field structure ($n=4, m=2$) is reflected in a Poincaré puncture plot



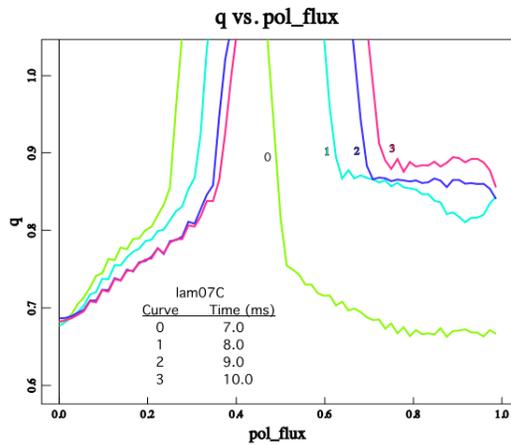
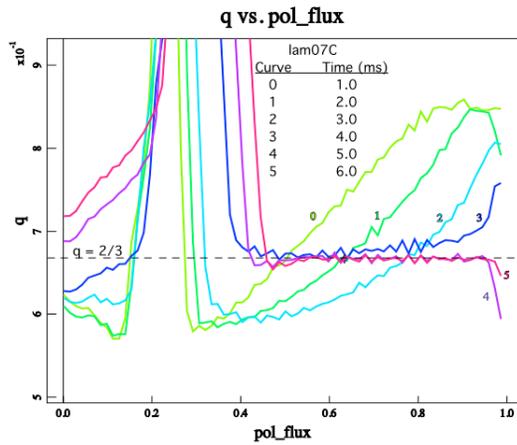
For the example shown:

- Starting points were on two horizontal lines:
R = 0.10 \rightarrow 0.31, Z = 0.12
R = 0.05 \rightarrow 0.31
- All fieldlines within the bounding surface seen surrounding the spheromak structure were confined for $> 200 \text{ m}$ (presumably ∞)
- No fieldlines reached the “open” areas within the boundary



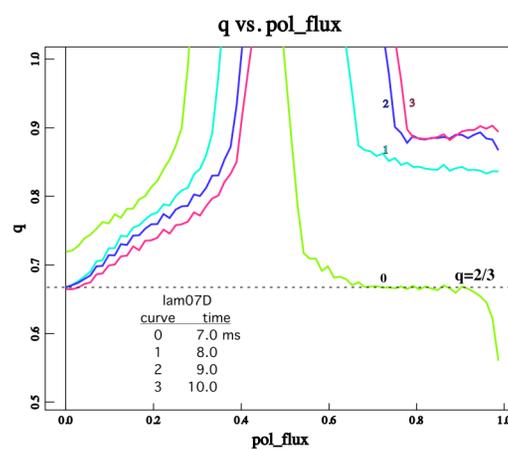
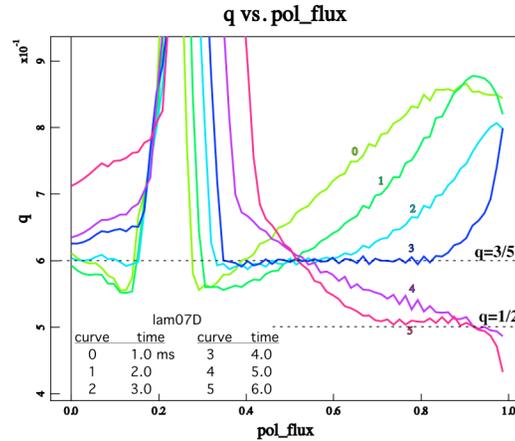
q-profile evolution — The interior boundary of the separatrix region moves towards the geometric axis as the plasma decays

$\lambda_{\text{gun}} = 9.9 \text{ m}^{-1}$



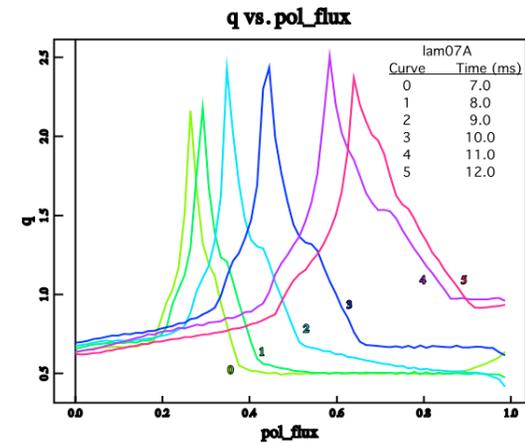
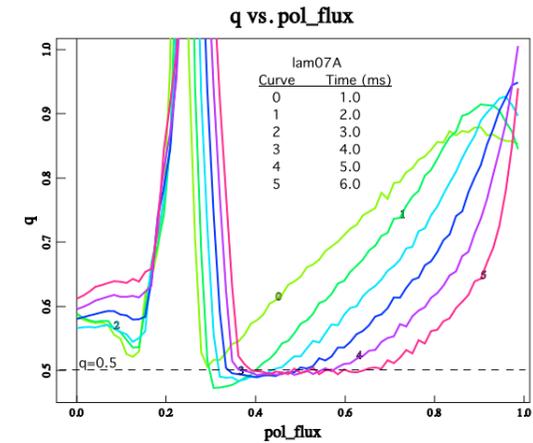
Spheromak collapses at ~4-5 ms

$\lambda_{\text{gun}} = 9.6 \text{ m}^{-1}$



Spheromak collapses at 6.3 ms

$\lambda_{\text{gun}} = 9.0 \text{ m}^{-1}$

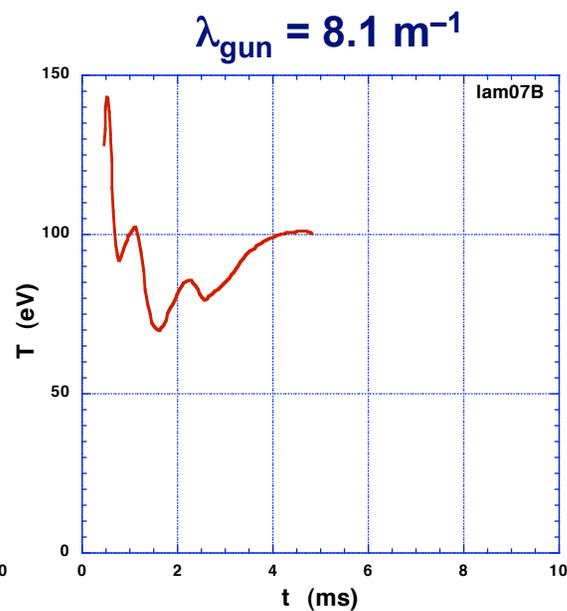
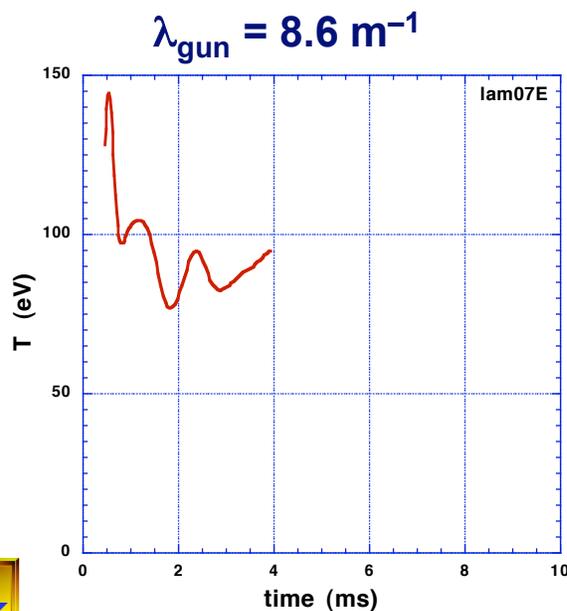
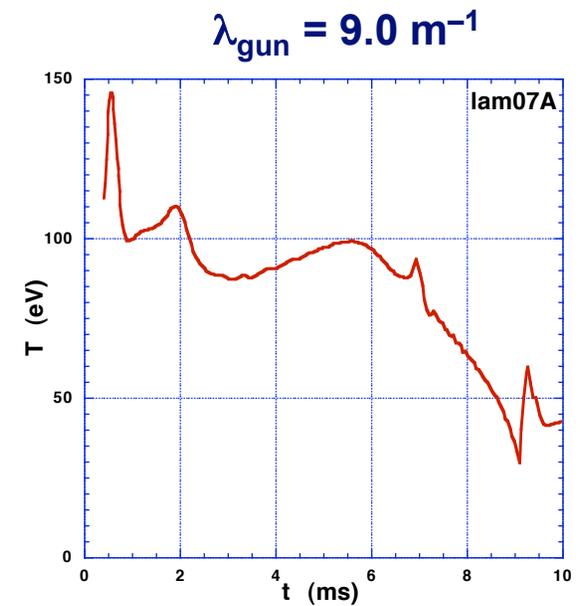
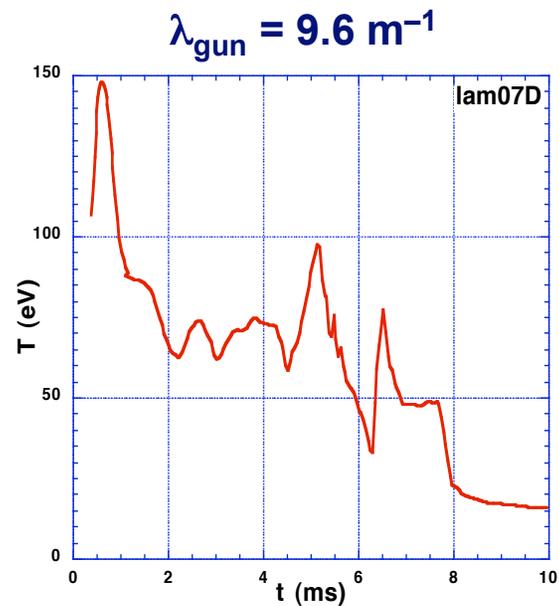
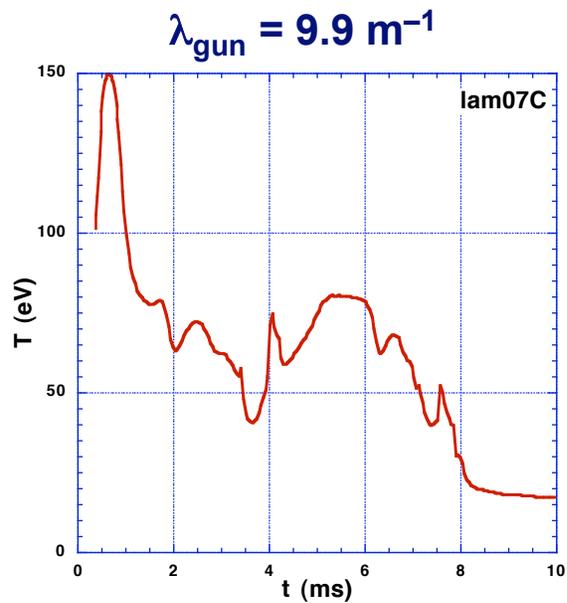


Spheromak collapses at 9.1 ms



Note the "locking" of the q-profile to a low-order mode rational surface

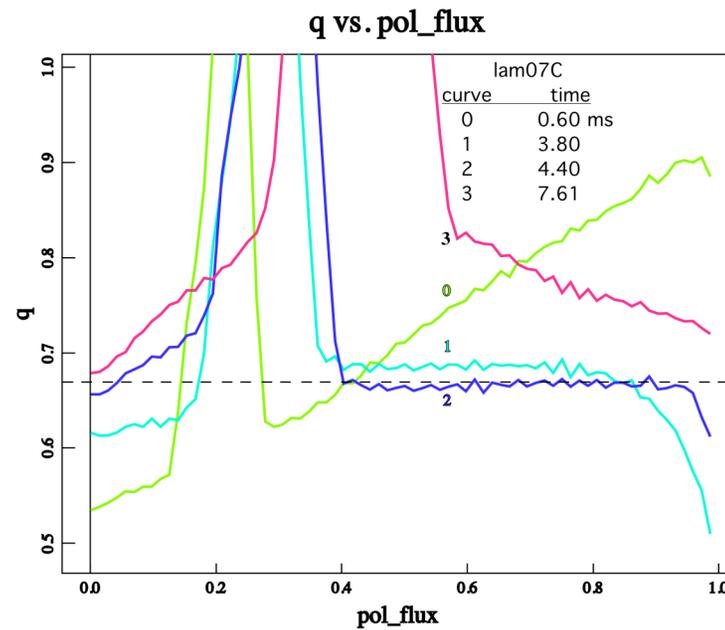
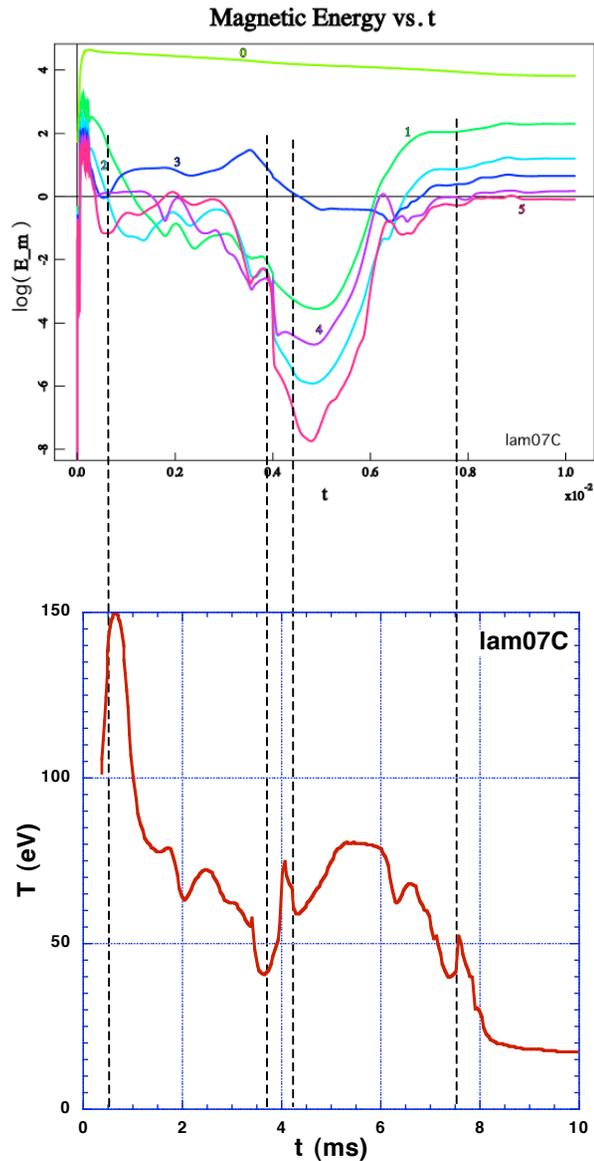
Peak temperature evolution is sensitive to λ_{gun}



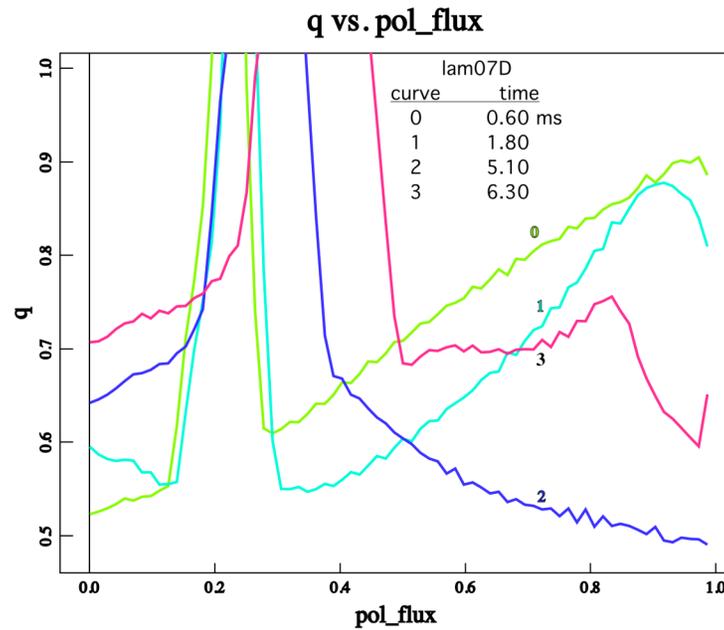
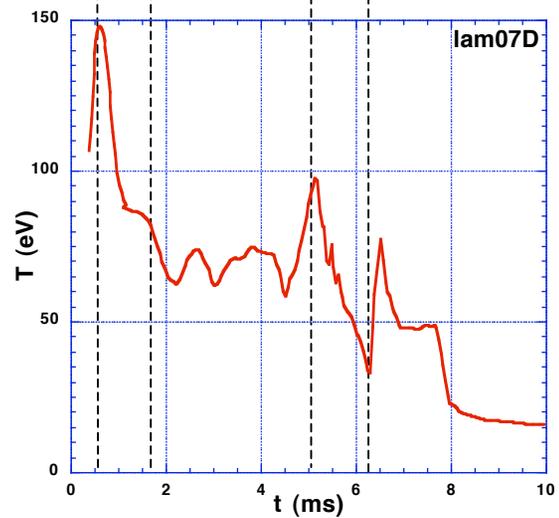
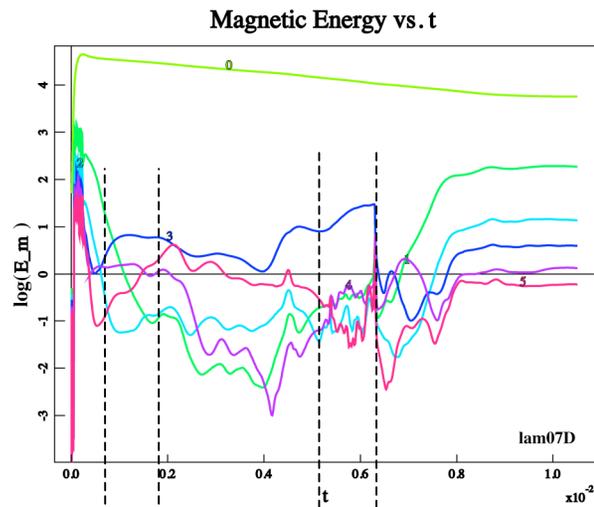
In the next set of viewgraphs we compare this with the magnetic fluctuation spectrum, fieldline quality, etc.



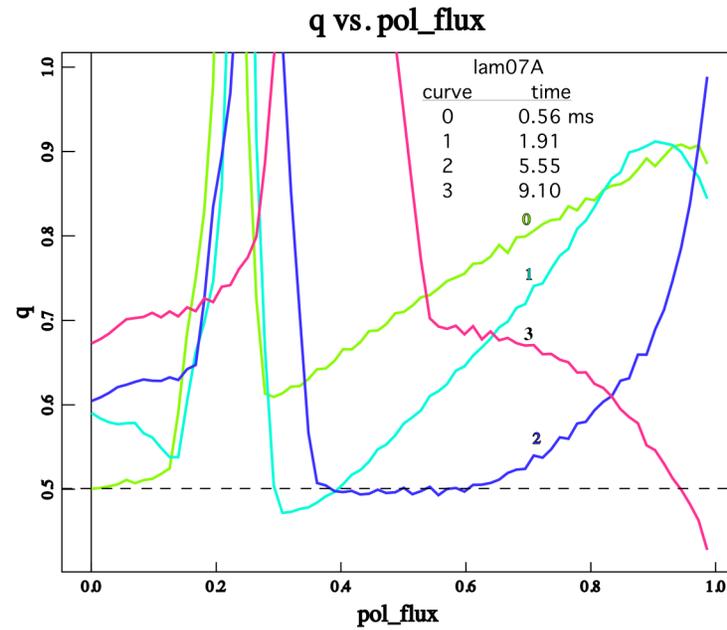
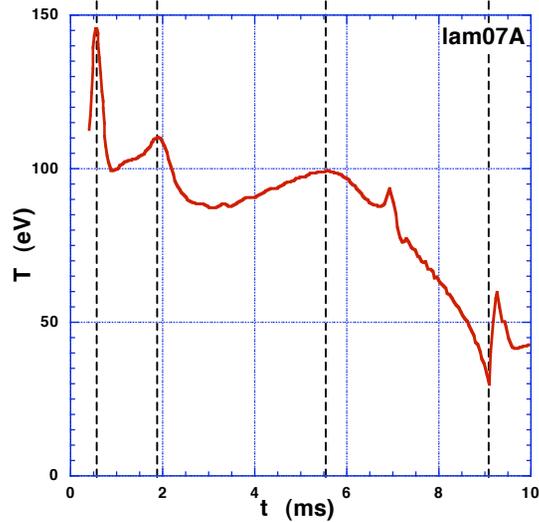
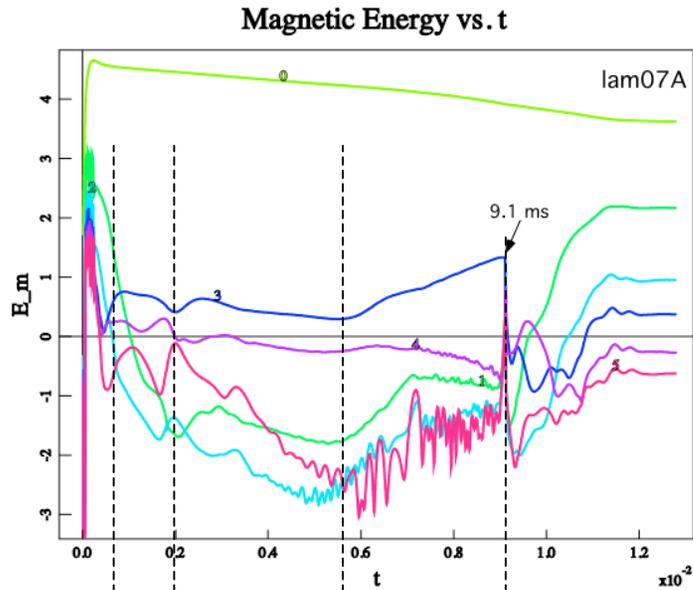
lam07C — $\lambda_{\text{gun}} = 9.9 \text{ m}^{-1}$ — The structure in the temperature correlates with the magnetic mode evolution and q-profile



lam07D — $\lambda_{\text{gun}} = 9.6 \text{ m}^{-1}$



lam07A — $\lambda_{\text{gun}} = 9.0 \text{ m}^{-1}$



For the lowest-order spheromak mode:

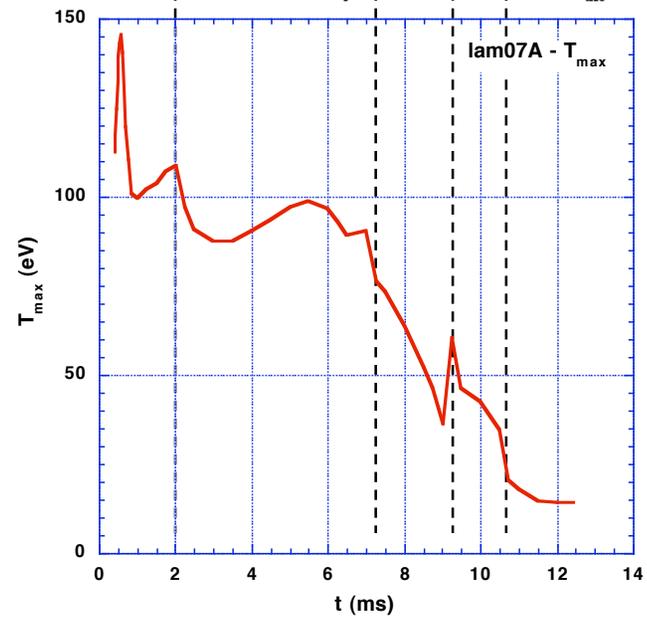
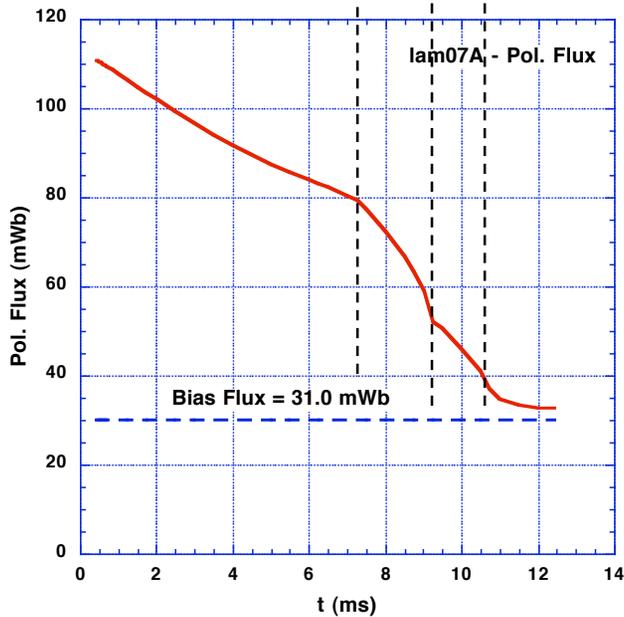
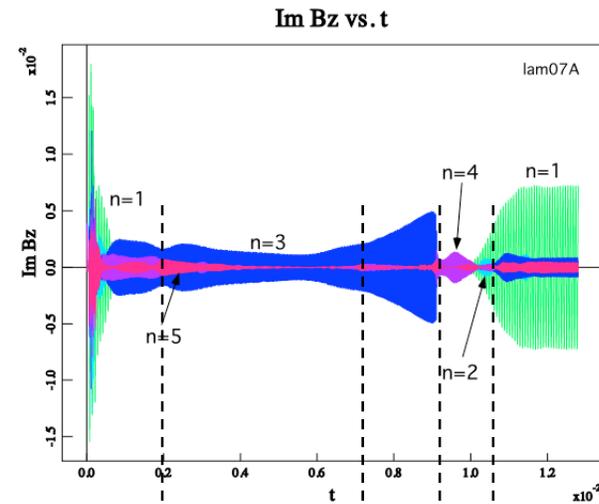
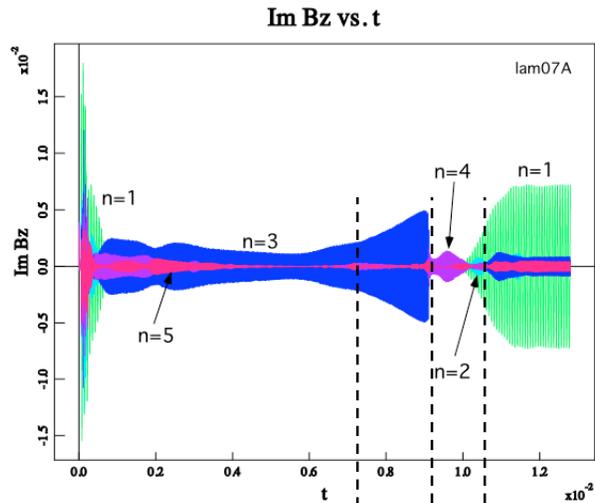
$$\tau_B = \frac{\mu_0}{\eta_l} \frac{1}{\lambda_{fc}^2}$$

$$= 6.5 \text{ ms at } 50 \text{ eV}$$

consistent with the decay of the q-profile

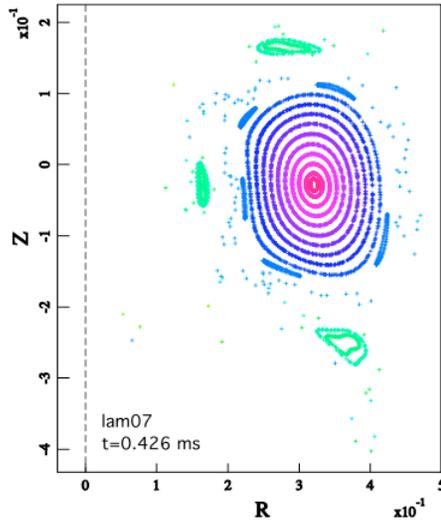


lam07A – Changes in flux decay rate and temperature both correlate with the fluctuation spectrum

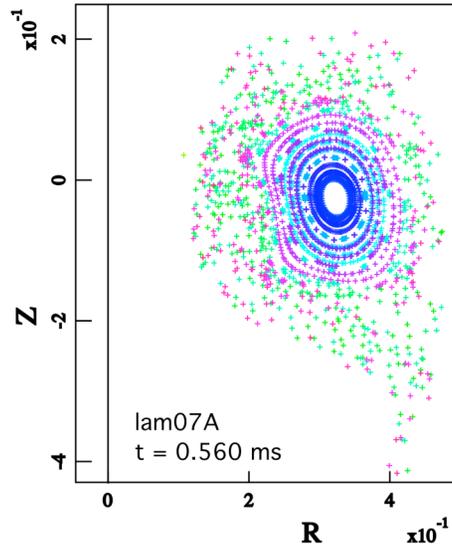


lam07A — $\lambda_{\text{gun}} = 9.0 \text{ m}^{-1}$ — Temperature tracks fieldline quality

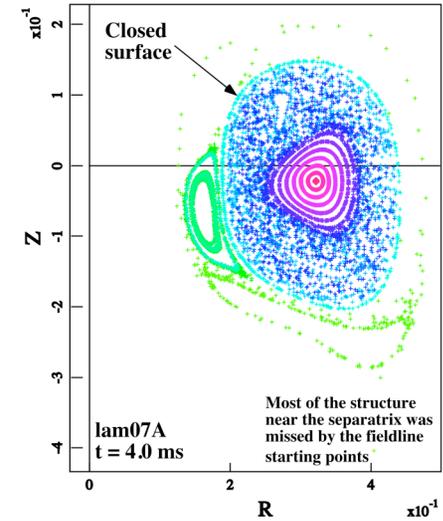
t = 0.426
Surface of Section



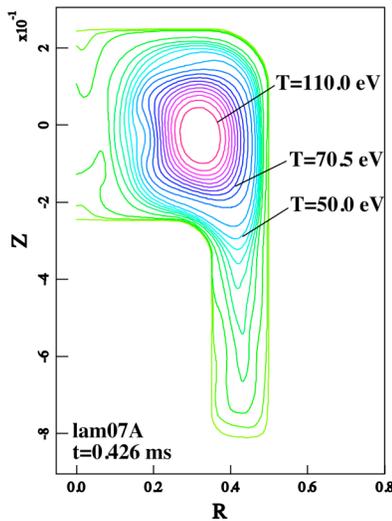
t = 0.560
Surface of Section



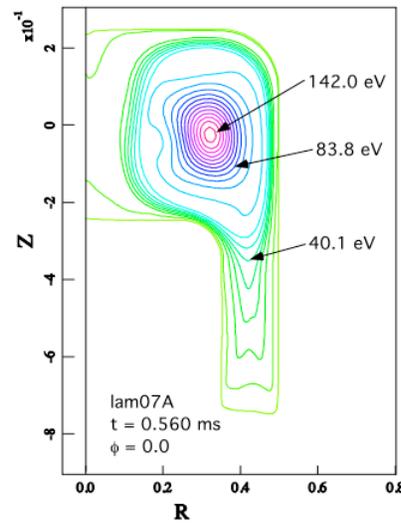
t = 4.0
Surface of Section



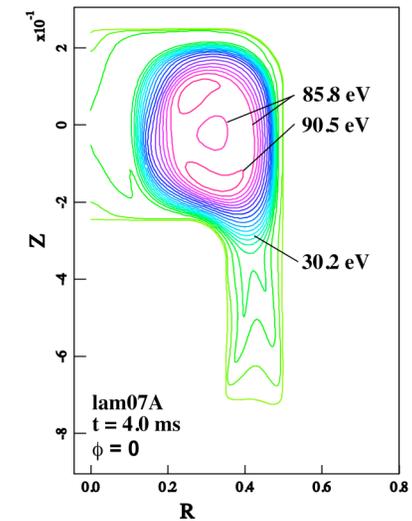
Re Tele



Re Tele

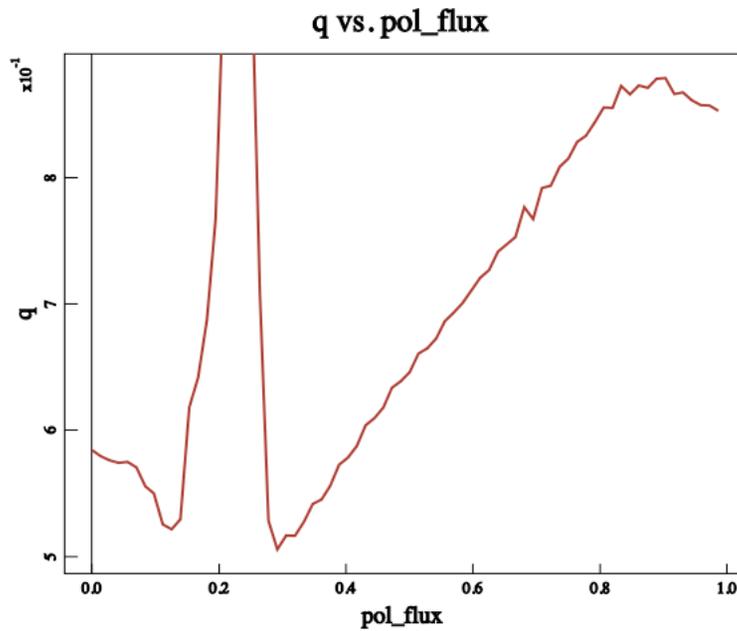


Re Tele

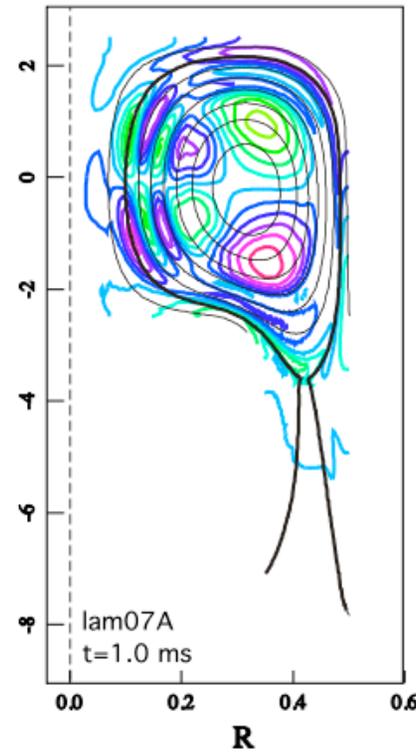


m/n=2/3 at 1.0 ms is primarily an internal mode with structure both in the plasma interior and near the separatrix

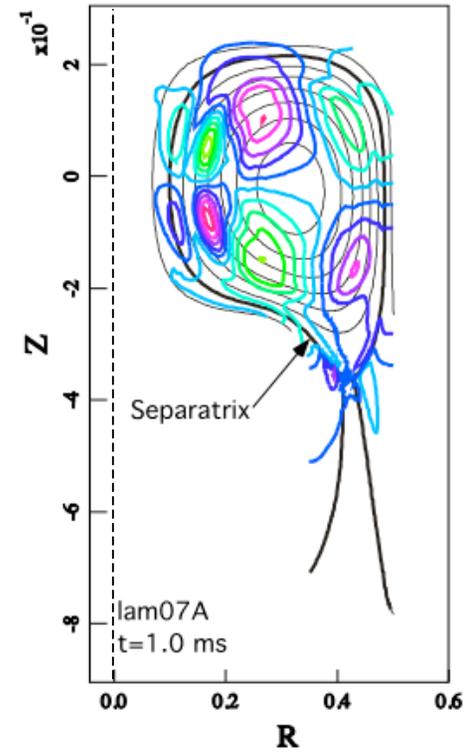
- Note the radial mode structure near the mean-field separatrix



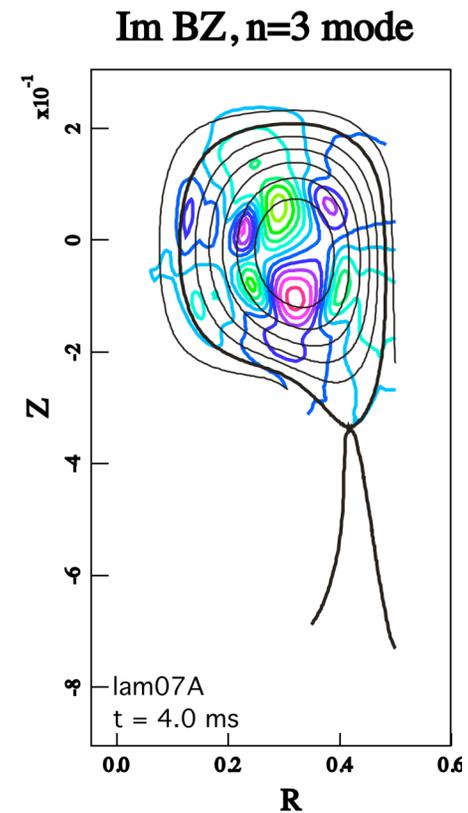
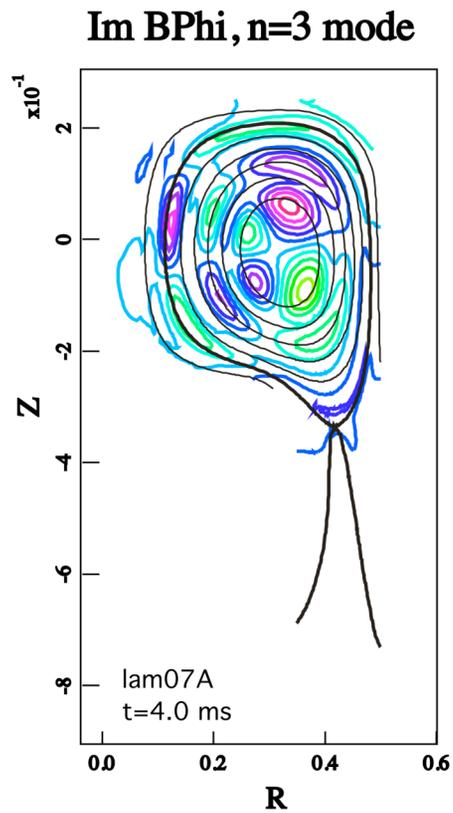
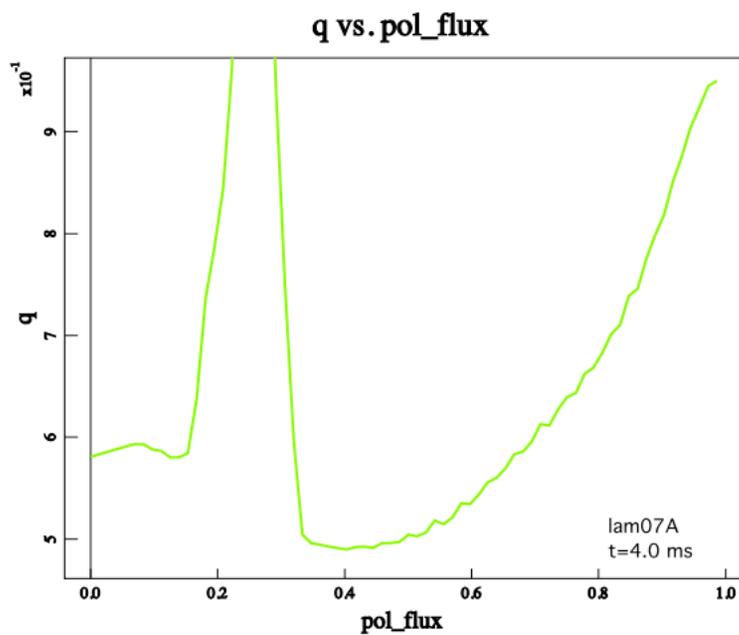
Re BPhi n=3 mode



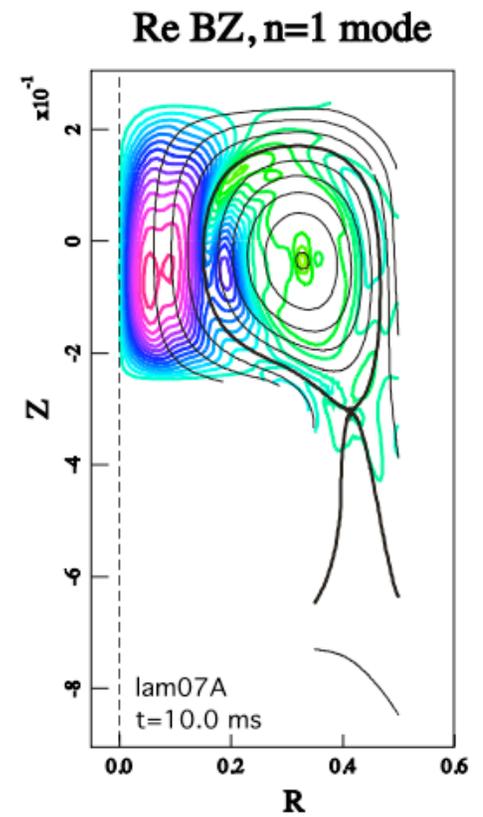
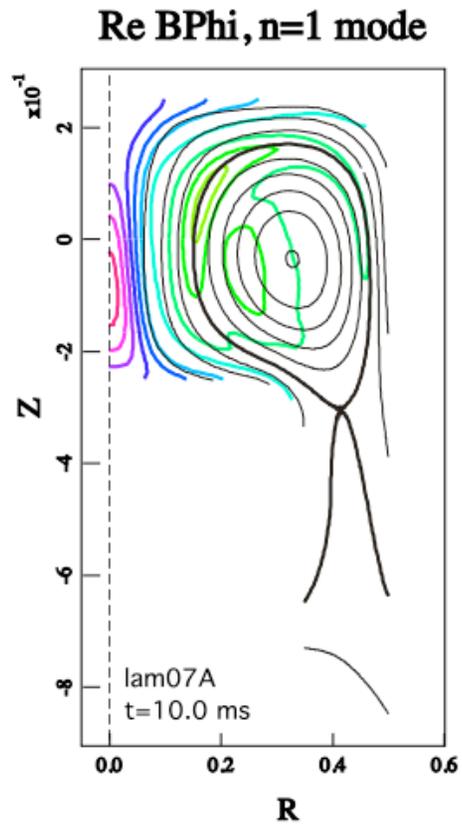
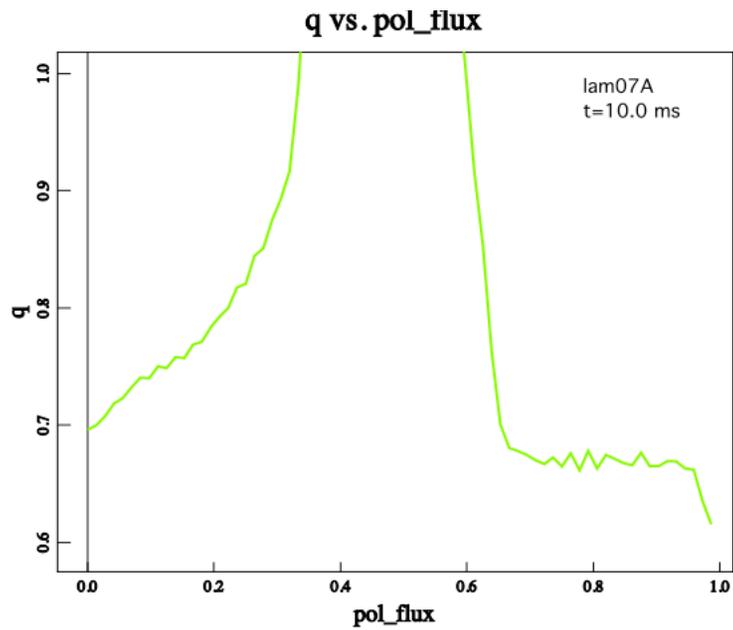
Re BZ, n=3 mode



m/n=2/3 at 4.0 ms



m/n=1/1 at 10.0 ms is primarily a column mode



Summary of the effects of λ_{gun} in NIMROD runs

The model shows many characteristics similar to the experiment

- The temperature, magnetic fluctuation spectrum, etc., are sensitive to λ_{gun}
 - Plasma quality (e.g. temperature) deteriorates above and below an optimum
 - The optimum λ_{gun} is close to, but below λ_{FC}
- The magnetic spectrum is consistent with the q-profile

Some issues are clarified by the model

- There are three types of modes:
 - o Peaked in the central column (“flux core”)
 - o Peaked around the mean-field separatrix
 - o Peaked in the plasma interior
- The mode amplitude at the flux conserver is sensitive to the mode structure and thus may not track the actual mode energy

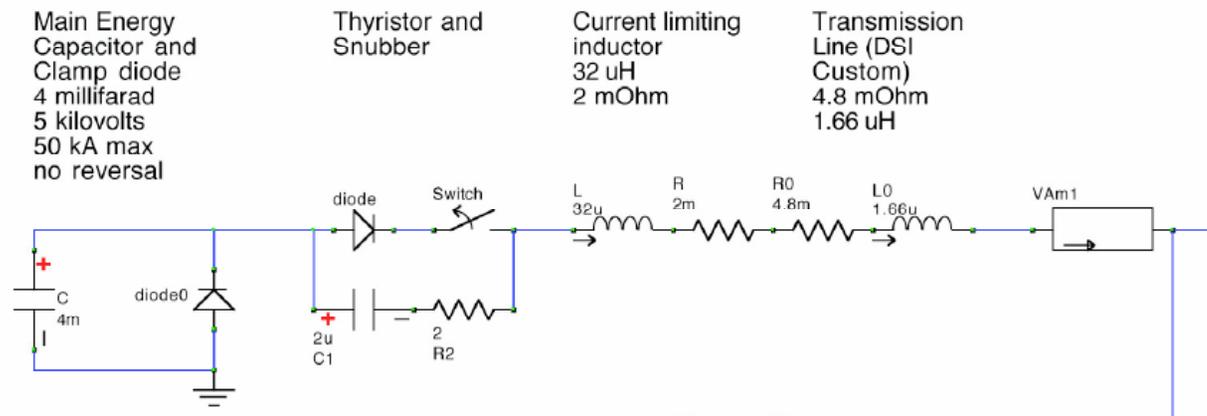
The experiment appears to be less sensitive to λ_{gun} than the model

- We have seen “good” T_e -profiles in SSPX with peaks > 300 eV — these have not been found yet in the modeling



MODELING A NEW CAPACITOR BANK FOR SSPX

NIMROD Has Been Augmented with External Circuit Equations to Simulate the New SSPX Capacitor Bank



Single capacitor bank module shown

- 32 modules (total) in parallel across the spheromak
- Existing formation bank can be used to start the discharge or for other pulses

The diodes constrain $V_C(t) \geq 0$ and $I(t) \geq 0$. Furthermore, $I = 0, V_C \leq V_{sp}$
 We then integrate the coupled 1st order ODEs for each of the 32 modules in the new capacitor bank with input parameters defining when the modules turn on, the initial voltages, and all of the inductances, resistances and capacitances.

$$V_C(t) = L \frac{dI}{dt} + RI + V_{sp}(t)$$

$$\frac{dV_C}{dt} = -\frac{1}{C}I$$

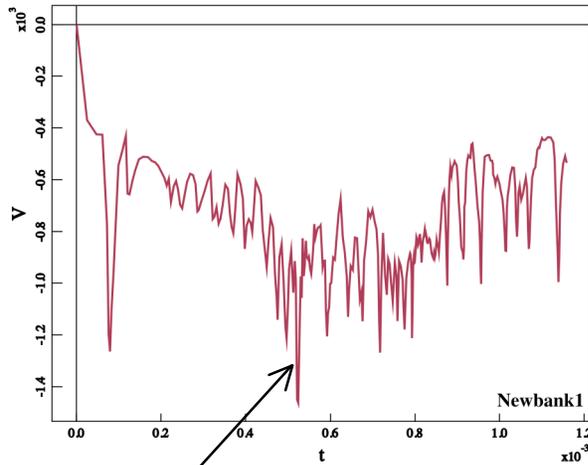


Finally we sum the output current which then goes the gun.

A high gun voltage develops and persists as long as the gun current is high

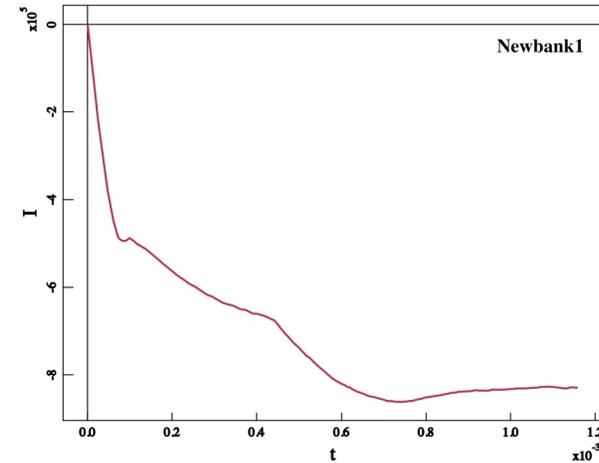
Gun voltage

Applied injector Voltage vs. t



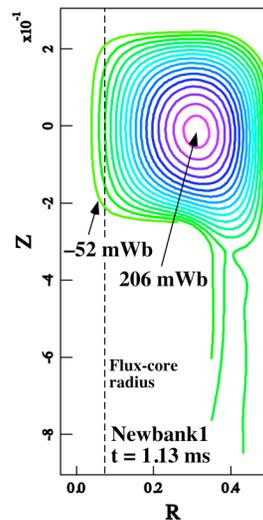
Gun current

Injector Current vs. t



Voltage spikes are generated by magnetic reconnection events

Poloidal flux



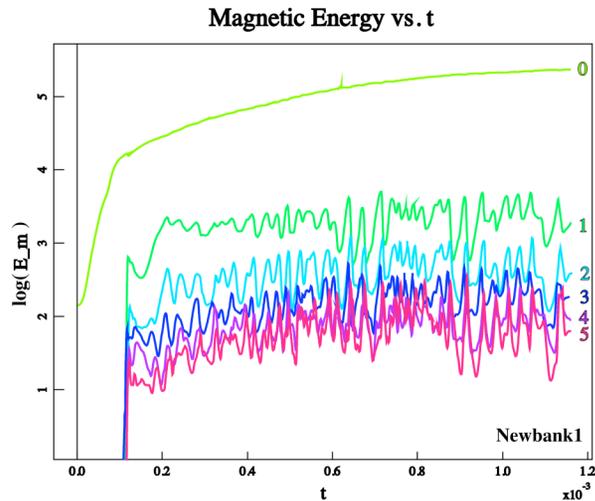
Azimuthally-averaged poloidal flux

Azimuthally-averaged poloidal flux is developed during this process, and a good mean-field spheromak has been produced

(Poloidal flux shown at $t = 1.13$ ms)



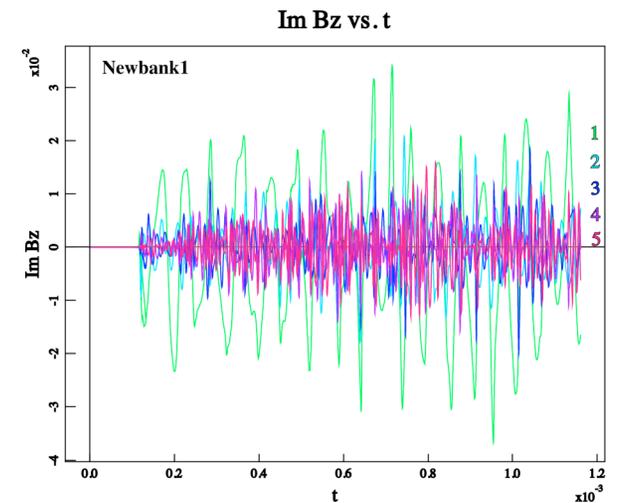
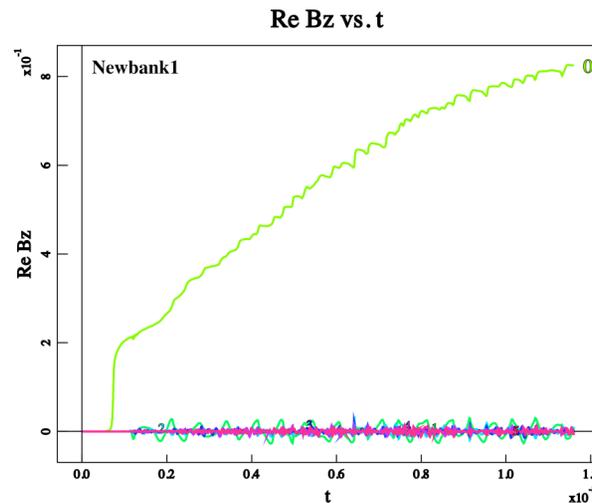
Magnetic energy and fields continue to evolve even when the gun current saturates



The axisymmetric ($n=0$) part of the magnetic energy grows as the energy in the non-axisymmetric nodes “bursts”

- dropping when toroidal flux is converted into poloidal flux
- voltage spikes occur when flux conversion occurs due to reconnection

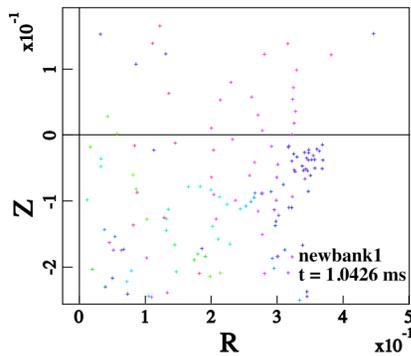
Poloidal field at the wall



Fieldline topology and electron temperature are very sensitive to the state of the reconnection events

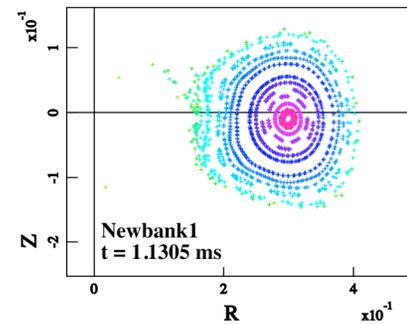
t = 1.0426 ms

Surface of Section

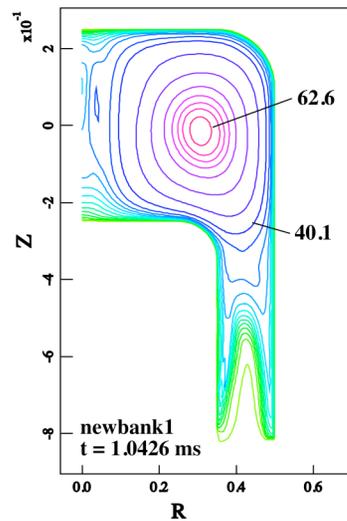


t = 1.1305 ms

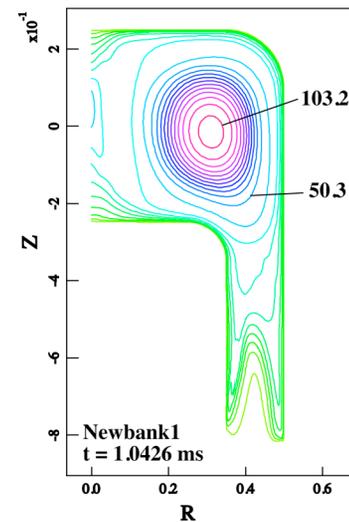
Surface of Section



Re Tele



Re Tele



Summary of new capacitor bank for SSPX

- NIMROD predicts significantly higher magnetic field than observed with present capacitor banks
 - Present = 0.35 T (max)
 - Prediction = 0.8 T
- Field continues to build after peak in gun current (still with $\lambda_g > \lambda_{FC}$)
- Fieldlines chaotic during reconnection events; heal between events
 - Electron temperature responds rapidly to the fieldline topology due to rapid thermal conductivity along B

Modeling with the capacitor bank is not finished

- Three modules have not turned on yet
- The coasting phase (when peak temperatures are expected) has not started

