M3D Sawtooth Update

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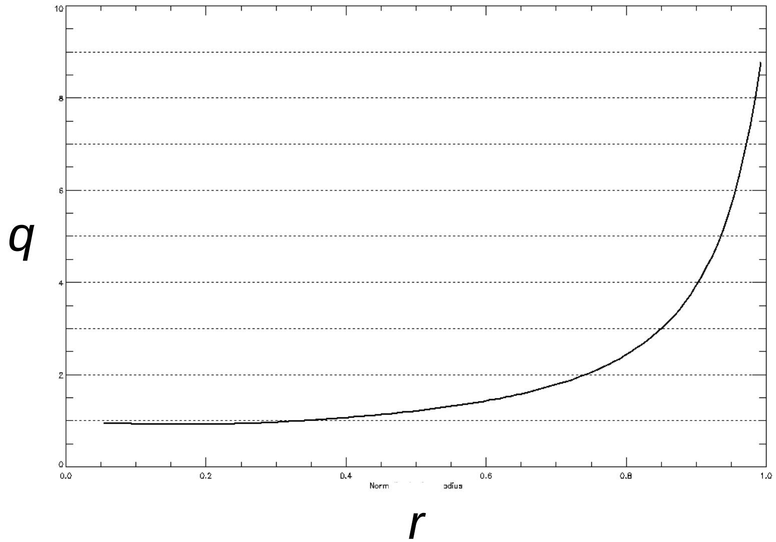
CEMM Meeting

Annapolis, MD

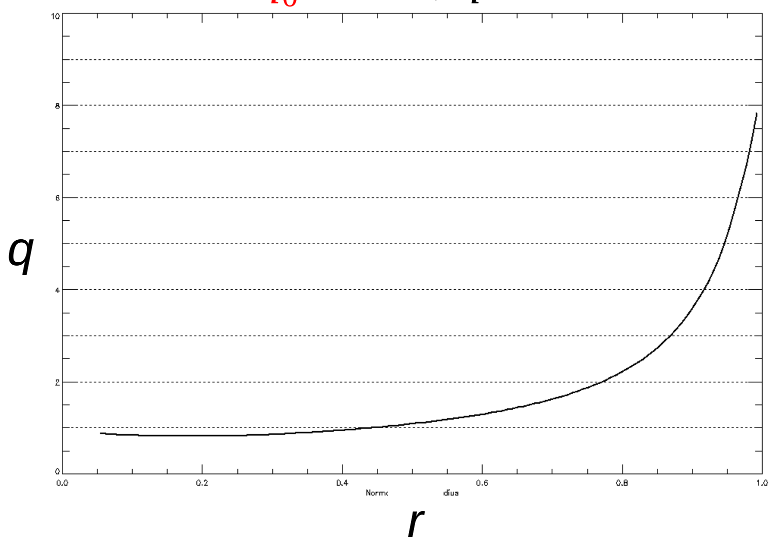
April 22, 2007

CDX Equilibria

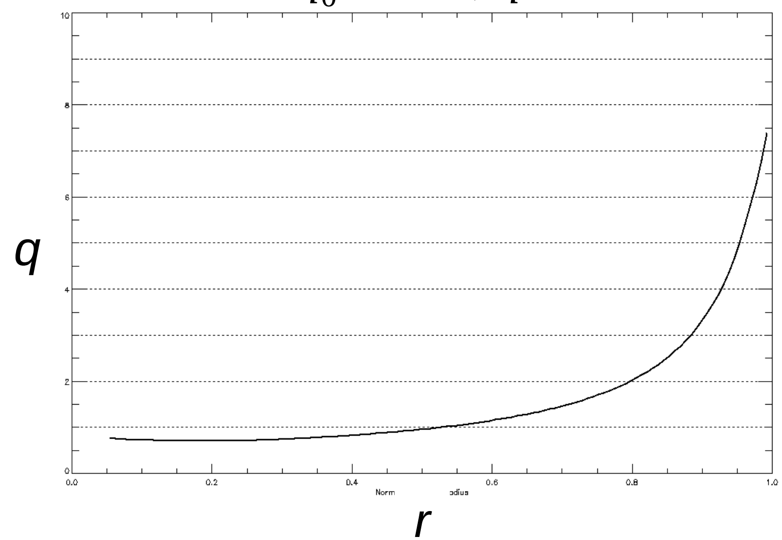
Original: time 11: q_0 = 0.92; q=1 at r=0.33



time 19: q_0 = 0.82; q=1 at r=0.44



time 29: q_0 = 0.71; q=1 at r=0.53



n=1 Eigenmodes

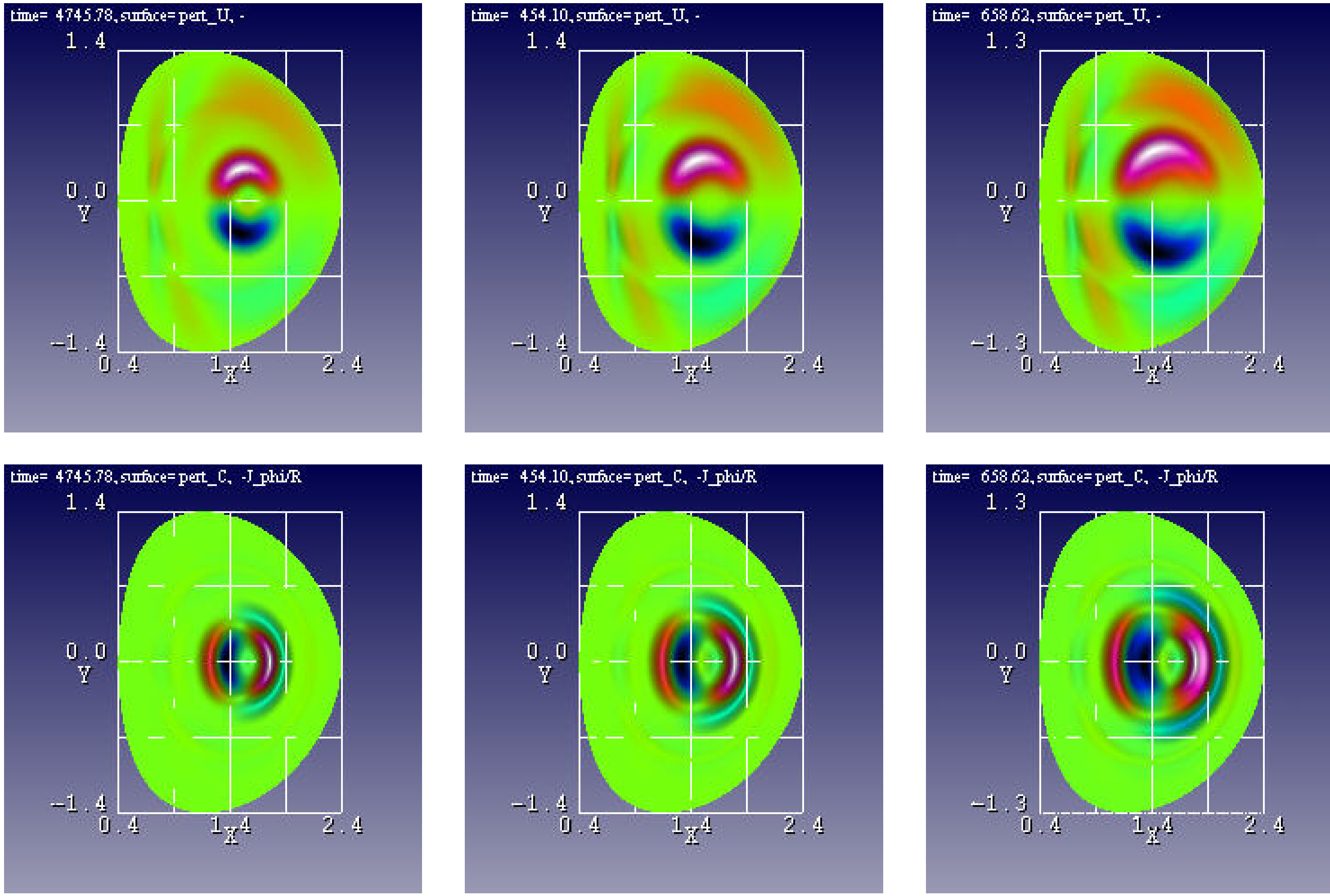
time 11 $q_0 = 0.92$

time 19 $q_0 = 0.82$

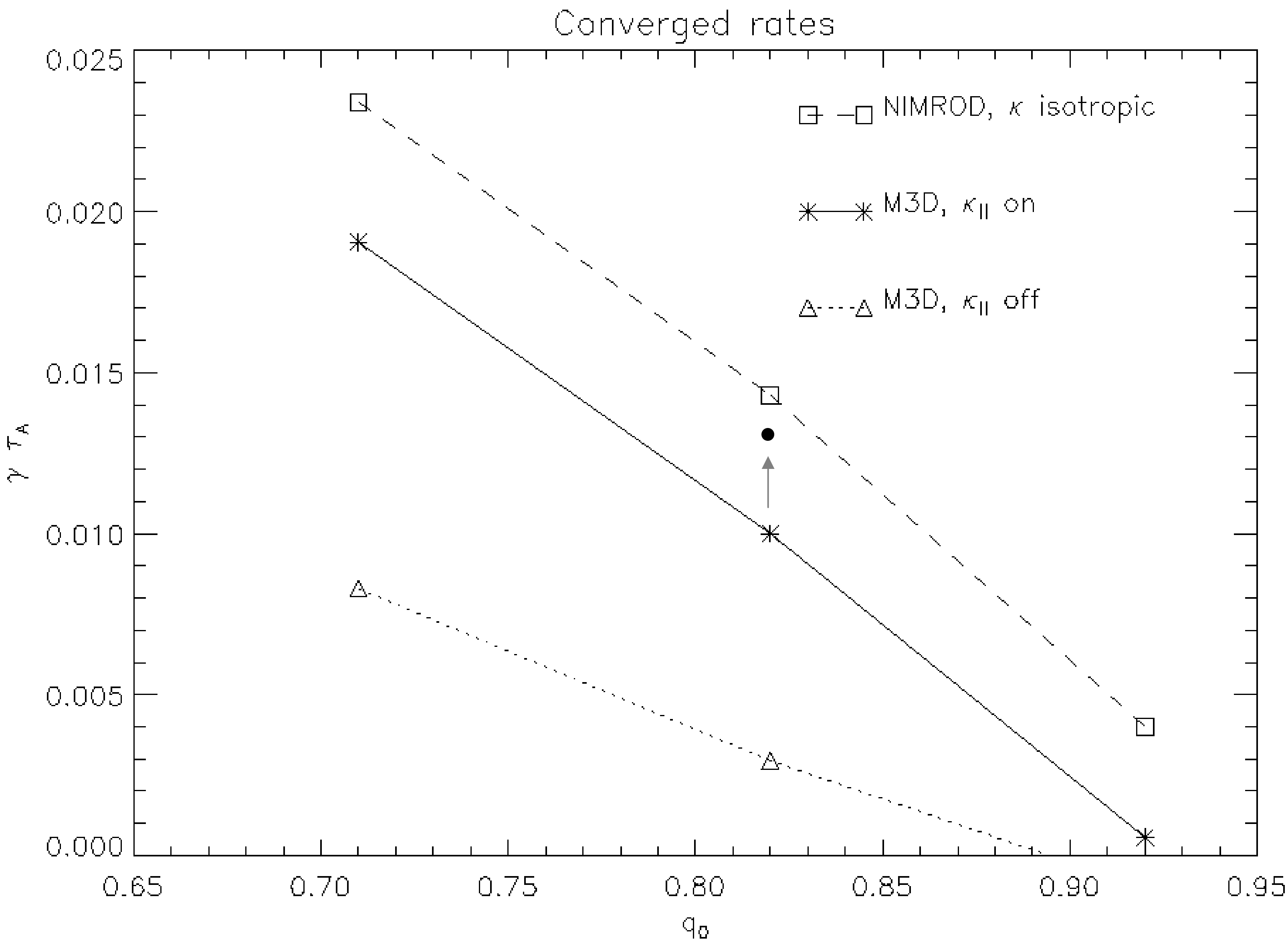
time 29 $q_0 = 0.71$

Poloidal velocity stream function

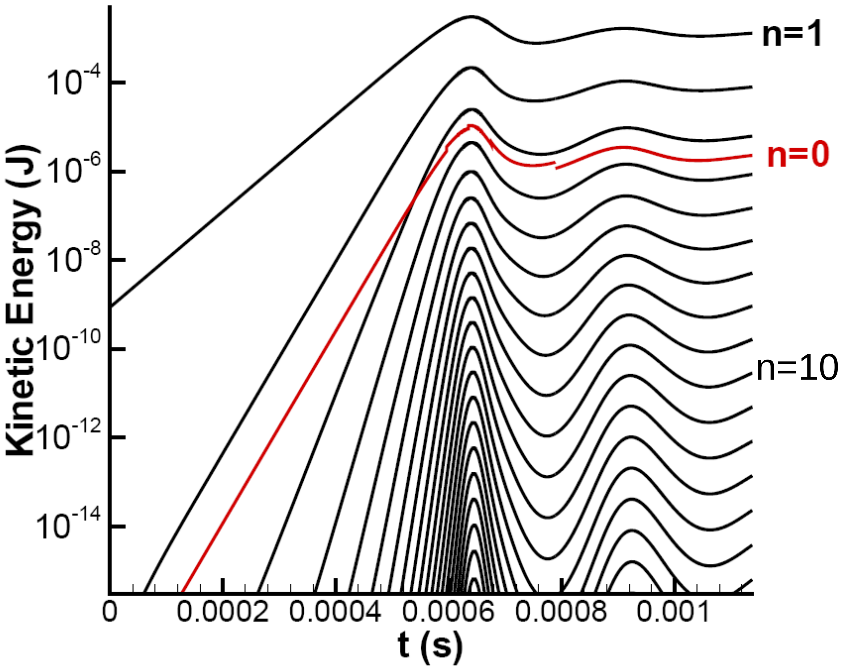
Toroidal current density



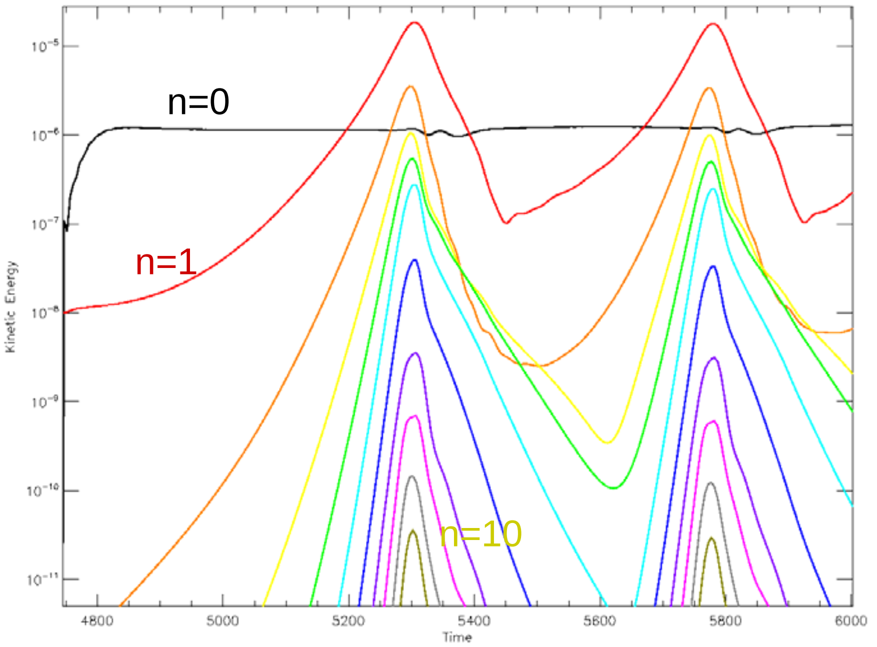
Linear n=1 Growth Rates



Differences Between Initial NIMROD and M3D CDX-U Nonlinear Results (time 11)



NIMROD



M3D

- M3D n=1 growth rate quickly exceeds linear due to rapid drop in q_0.
- M3D has much higher *n*=0 energy compared to other modes.
- Periods between crashes differ: ~800 τ_A for NIMROD vs. 480 τ_A for M3D.
- 2nd crash energy is diminished more in NIMROD than in M3D.

Understanding the M3D Result

- Linear growth rate of n=1 increases in nonlinear run because q_0 is steadily dropping.

- Rate of change of q_0 is directly proportional to average velocity of n=0 flow, which shows up as large n=0 kinetic energy. It varies inversely with dt.

- n=0 flow arises from discretization error; converges to zero as poloidal resolution increases.

- Culprit: interaction of flow with toroidal current density (C) source term exposes poor conservation properties of existing formulation of C equation.

Original C Equation

Poloidal flux (aeqn, numerically unstable, boundary problems):

$$\frac{\partial \psi}{\partial t}=\frac{R}{R_0}[U,\psi]+\frac{R}{R_0}(U,F)-(\chi,\psi)+[\chi,F]+\eta C+\frac{\partial \Phi}{\partial \phi}. \tag{1}$$

Toroidal current density (ceqn, poor conservation properties):

$C_a \equiv \Delta^* \psi$:

$$\begin{aligned}
\frac{\partial C_a}{\partial t} &= \frac{R}{R_0}\left\{[U,C_a]+\left[\Delta^\dagger U,\psi\right]+2\left[\frac{\partial U}{\partial R},\frac{\partial \psi}{\partial R}\right]+2\left[\frac{\partial U}{\partial z},\frac{\partial \psi}{\partial z}\right]\right\}+\frac{2}{R_0}\left[U,\frac{\partial \psi}{\partial R}\right]+\frac{2}{R_0 R}\frac{\partial U}{\partial z}\frac{\partial \psi}{\partial R} \\
&+\frac{R}{R_0}\left\{\left(U,\nabla_\perp^2 F\right)+\left(\Delta^\dagger U,F\right)+2\left(\frac{\partial U}{\partial R},\frac{\partial F}{\partial R}\right)+2\left(\frac{\partial U}{\partial z},\frac{\partial F}{\partial z}\right)\right\}+\frac{1}{R_0}\left(\frac{\partial F}{\partial R},U\right)-\frac{1}{R_0 R}\frac{\partial F}{\partial z}\frac{\partial U}{\partial z} \\
&-\left\{\left(\psi,\nabla_\perp^2 \chi\right)+(C_a,\chi)+2\left(\frac{\partial \psi}{\partial R},\frac{\partial \chi}{\partial R}\right)+2\left(\frac{\partial \psi}{\partial z},\frac{\partial \chi}{\partial z}\right)\right\}+\frac{1}{R}\left(\frac{\partial \chi}{\partial R},\psi\right)+\frac{1}{R^2}\frac{\partial \psi}{\partial R}\frac{\partial \chi}{\partial R} \\
&+\left\{\left[\nabla_\perp^2 \chi,F\right]+\left[\chi,\nabla_\perp^2 F\right]+2\left[\frac{\partial \chi}{\partial R},\frac{\partial F}{\partial R}\right]+2\left[\frac{\partial \chi}{\partial z},\frac{\partial F}{\partial z}\right]\right\}-\frac{1}{R}\left\{\left[\frac{\partial \chi}{\partial R},F\right]+\left[\chi,\frac{\partial F}{\partial R}\right]\right\} \\
&+\frac{\partial}{\partial \phi}\left(\nabla_\perp^2 \Phi\right)-\frac{1}{R}\frac{\partial^2 \Phi}{\partial \phi \partial R}
\end{aligned} \tag{1'}$$

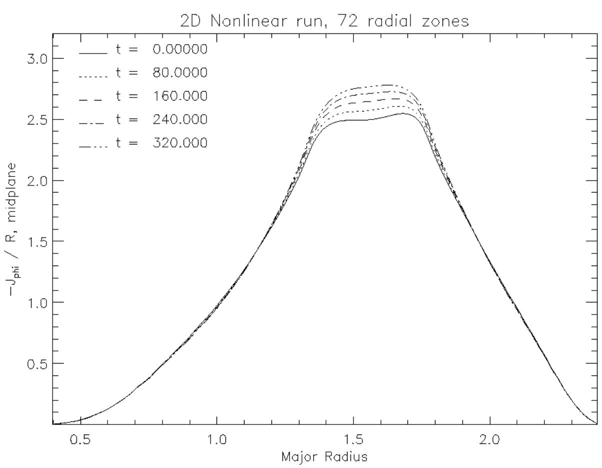
Conservative C Equation

If $\dot{\psi} = b,$ then $\lambda_i \nabla_\perp \cdot \left(\frac{1}{R^2} \nabla \dot{\psi} \right) = \frac{\lambda_i}{R^2} \dot{C}_a = \lambda_i \nabla_\perp \cdot \left(\frac{1}{R^2} \nabla b \right)$

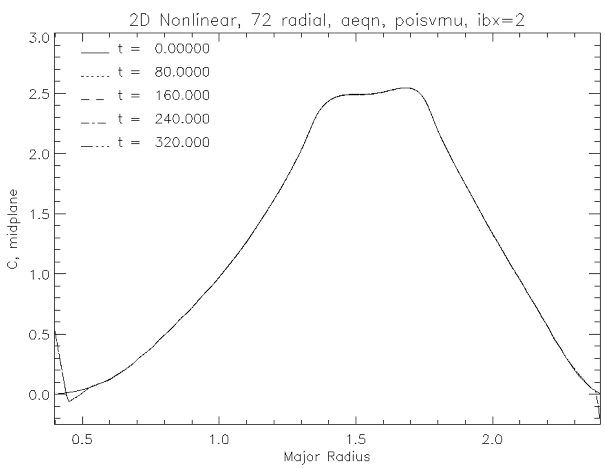
Integrating over the domain results in $\overleftrightarrow{\mathbf{M}} \cdot \left(\frac{\dot{C}_a}{R^2} \right) = \overleftrightarrow{\mathbf{S}} \cdot \left(\frac{b}{R^2} \right),$

or approximately $\overleftrightarrow{\mathbf{M}} \cdot \dot{C}_a = \overleftrightarrow{\mathbf{S}} \cdot b.$

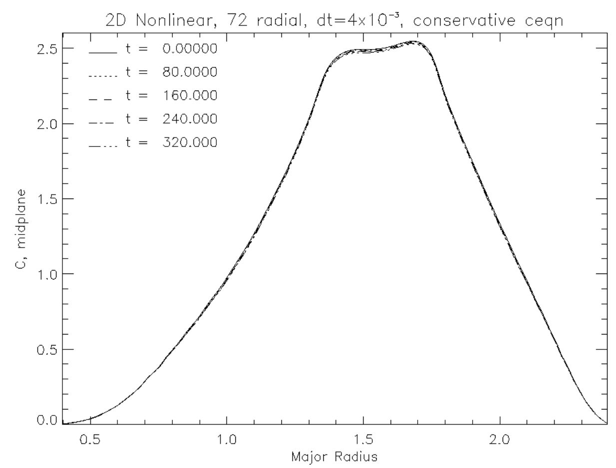
Original ceqn



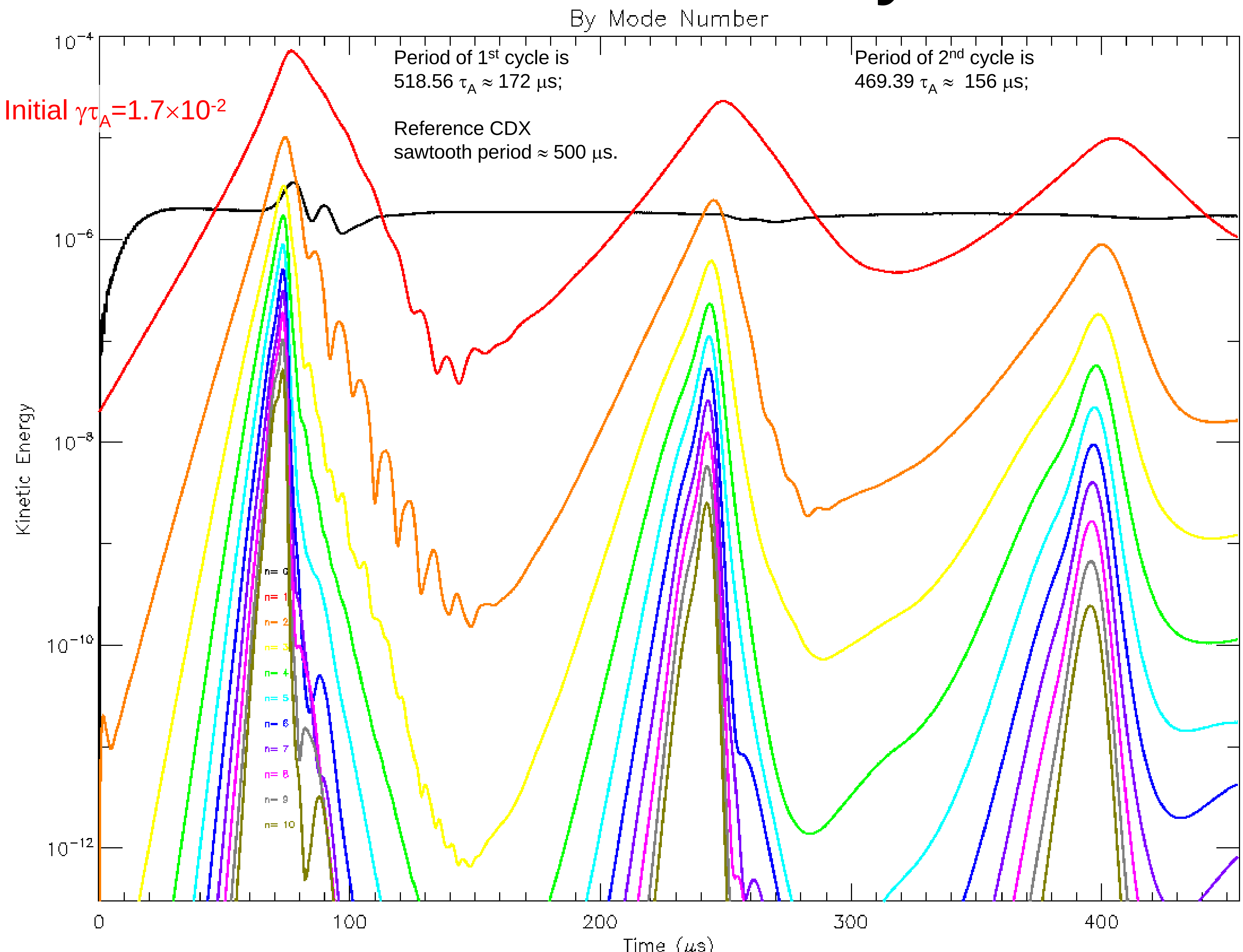
Original aeqn



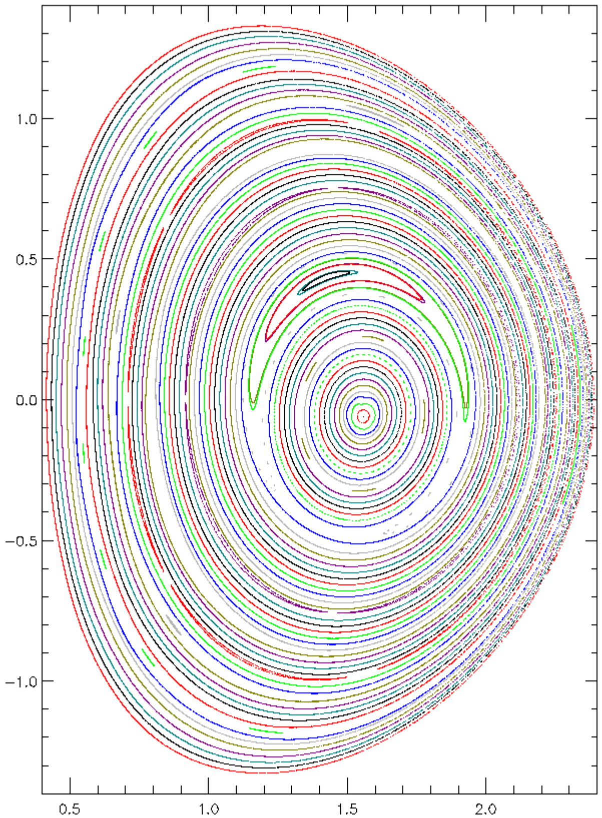
Conservative ceqn



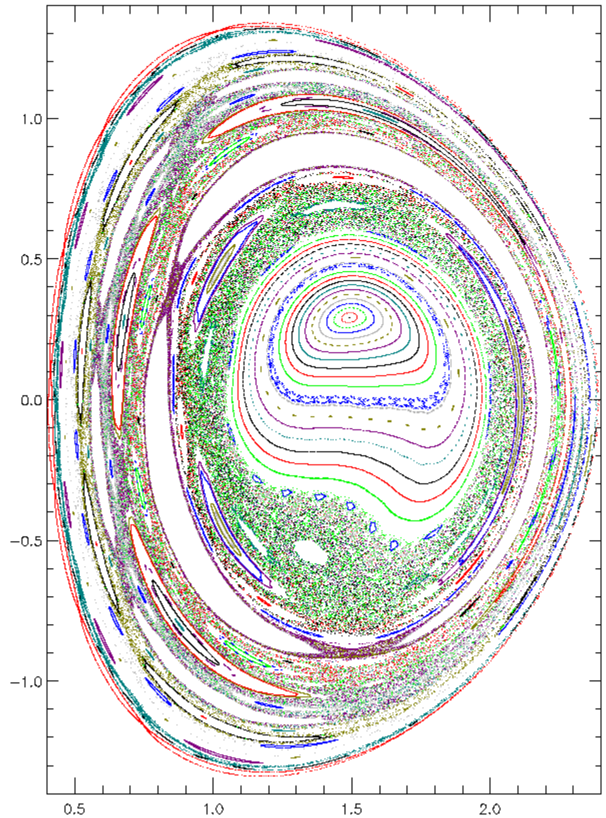
Mode History



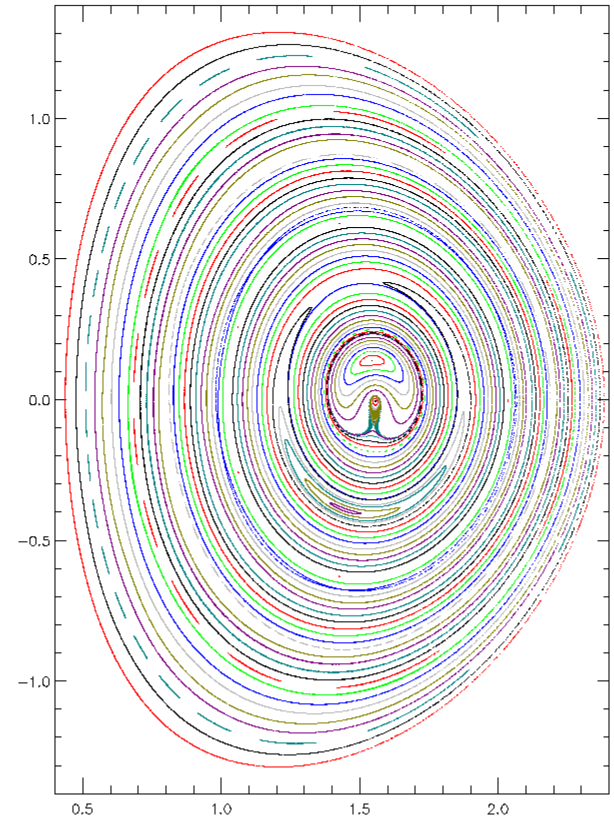
Poincaré Plots



t = 38 μs; φ=π/2
(compare to NIMROD t=129 μs)

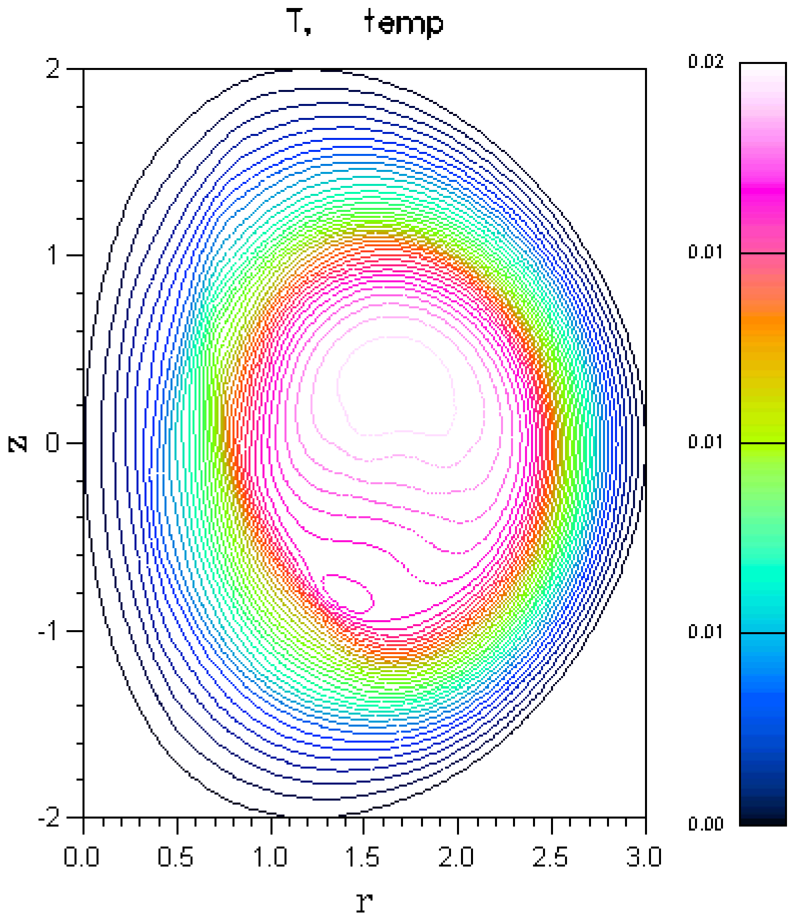


t = 74.5 μs; φ=π/2
(compare to NIMROD t=163 μs)



t = 115.3 μs; φ=π/2
(compare to NIMROD t=200 μs)

Temperature Contours



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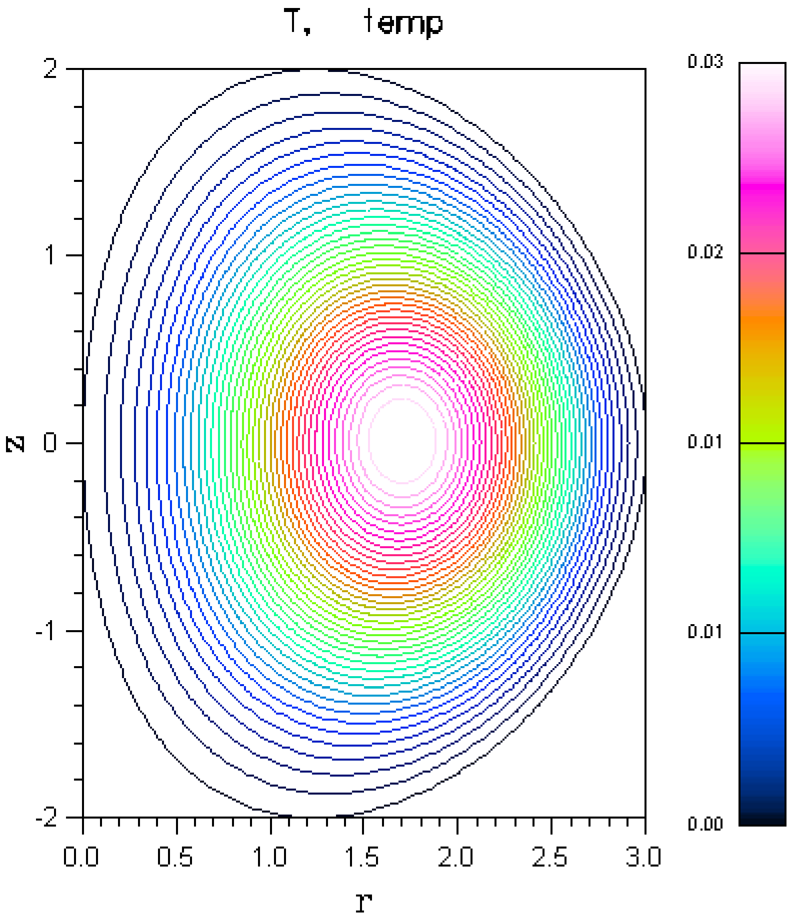
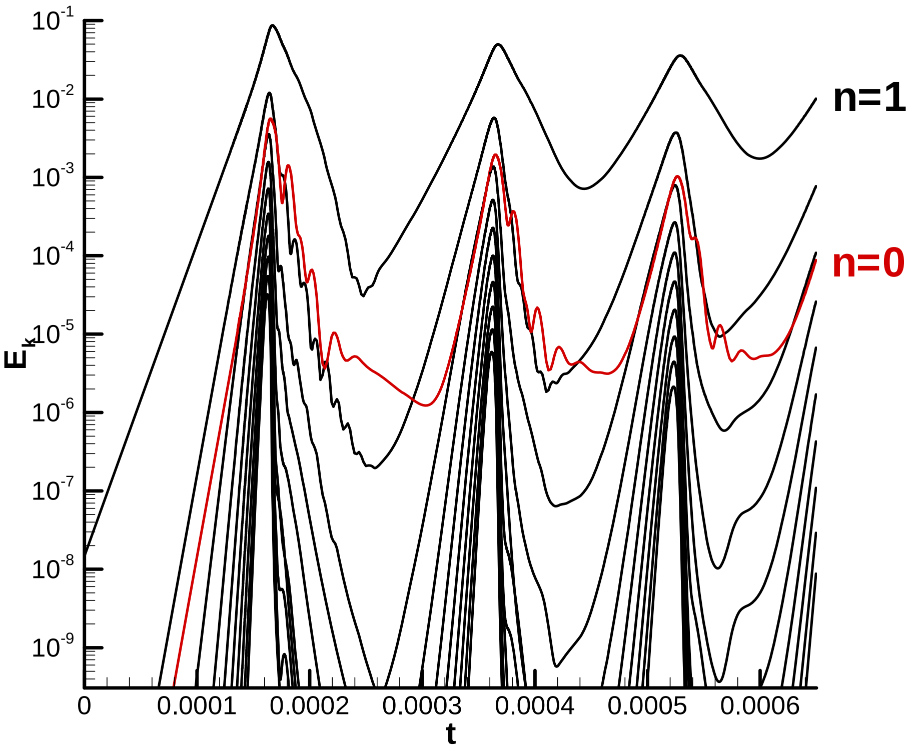
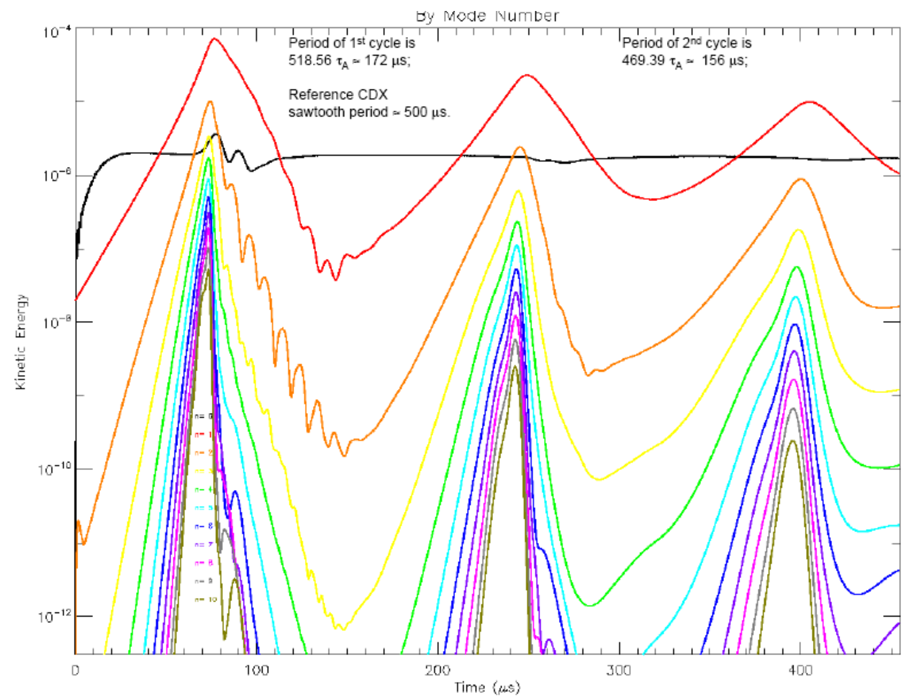


t = 115.3 μs; φ=π/2
(compare to NIMROD t=200 μs)

Comparison with NIMROD Results

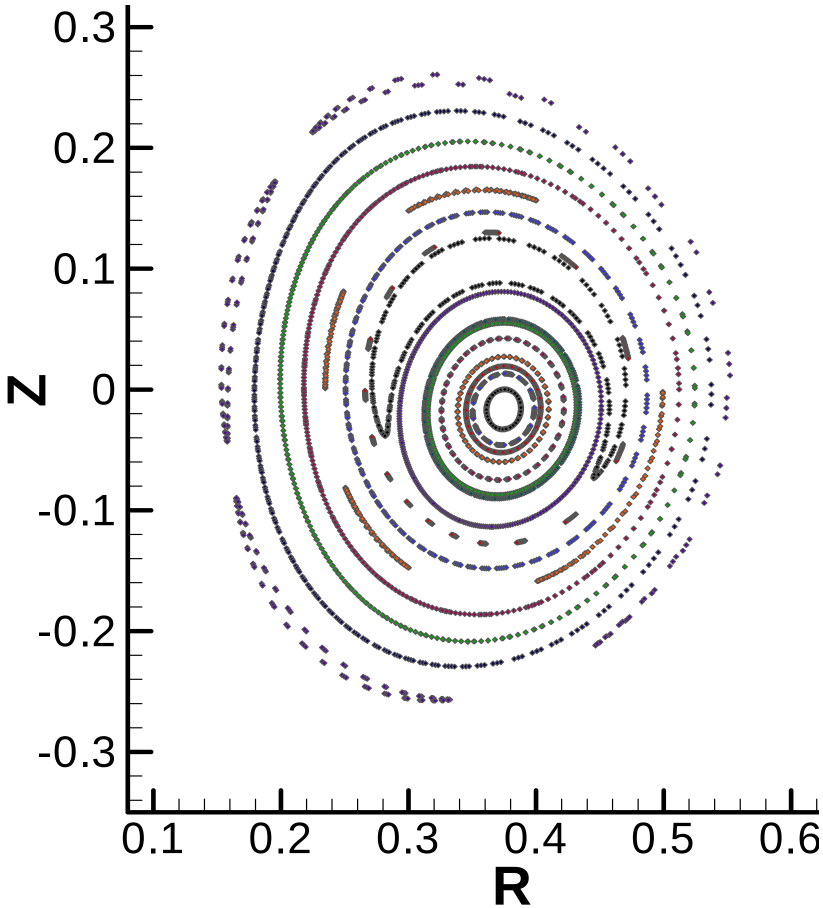




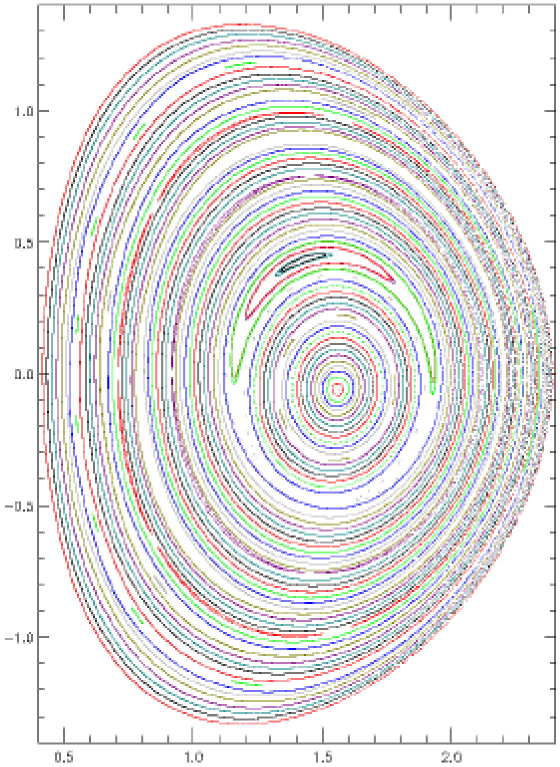
NIMROD Kinetic energy in first 10 modes

M3D Kinetic energy in first 10 modes

Poincaré Plots

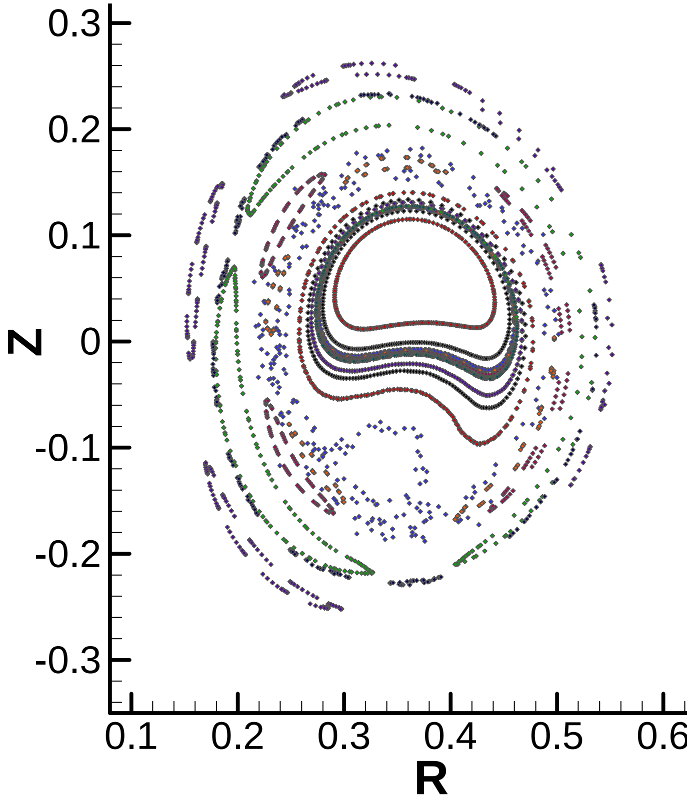


NIMROD 129 μs

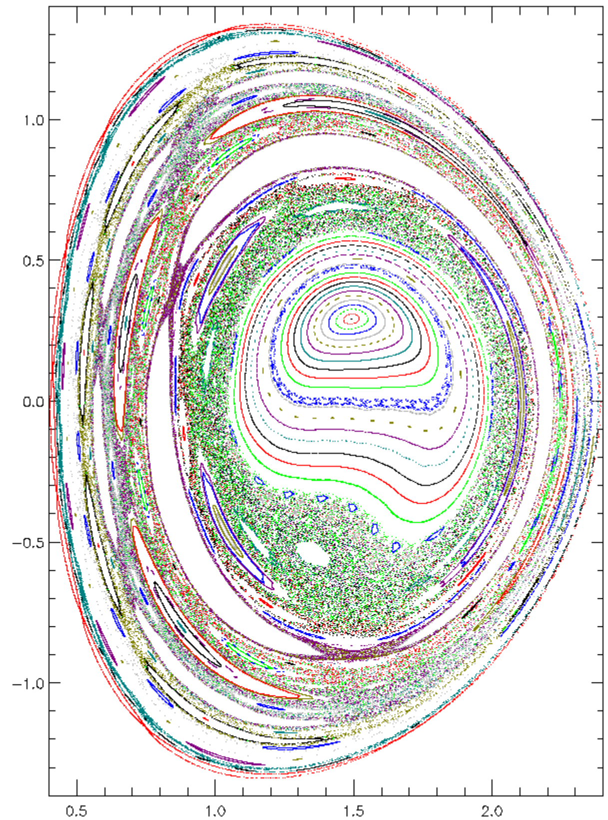


M3D 38 μs

Poincaré Plots

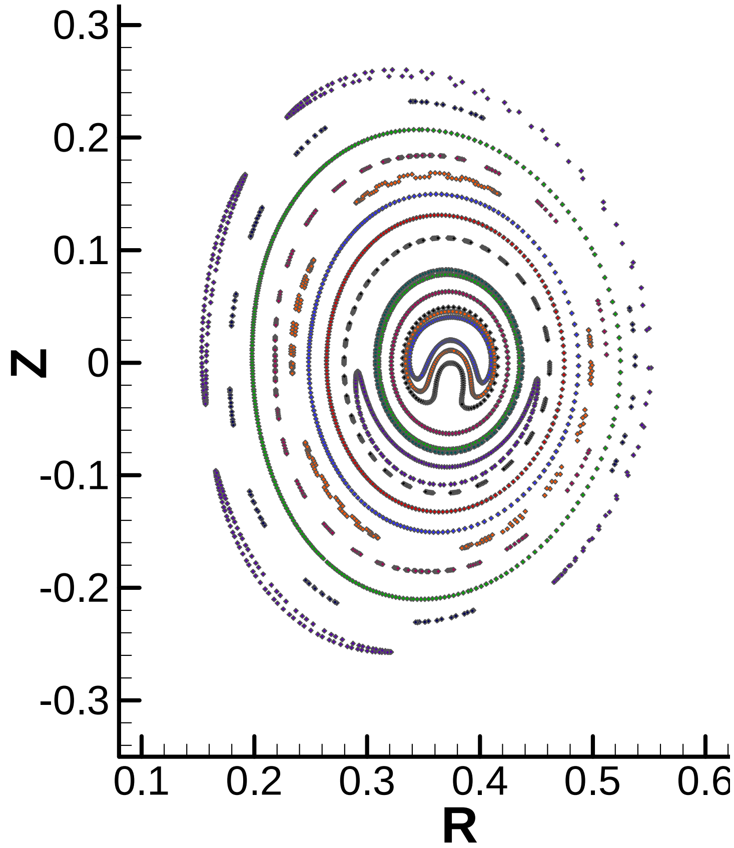


NIMROD 163 μs

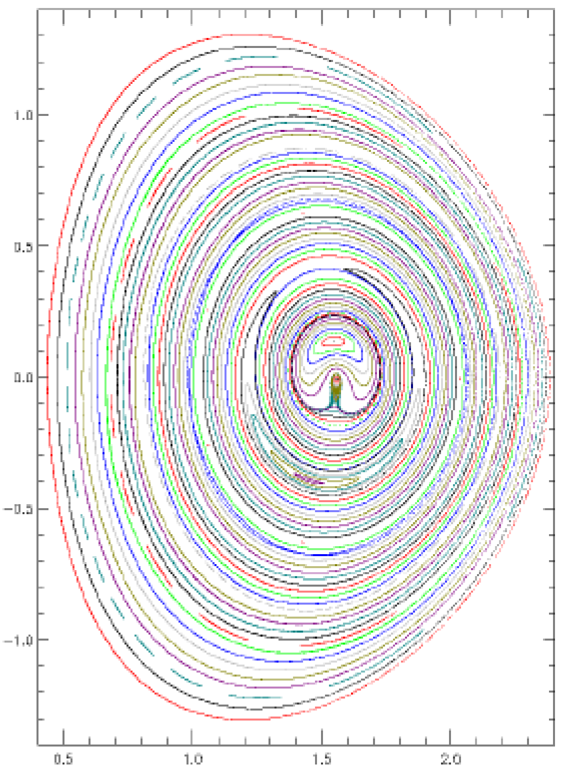


M3D 74.5 μs

Poincaré Plots

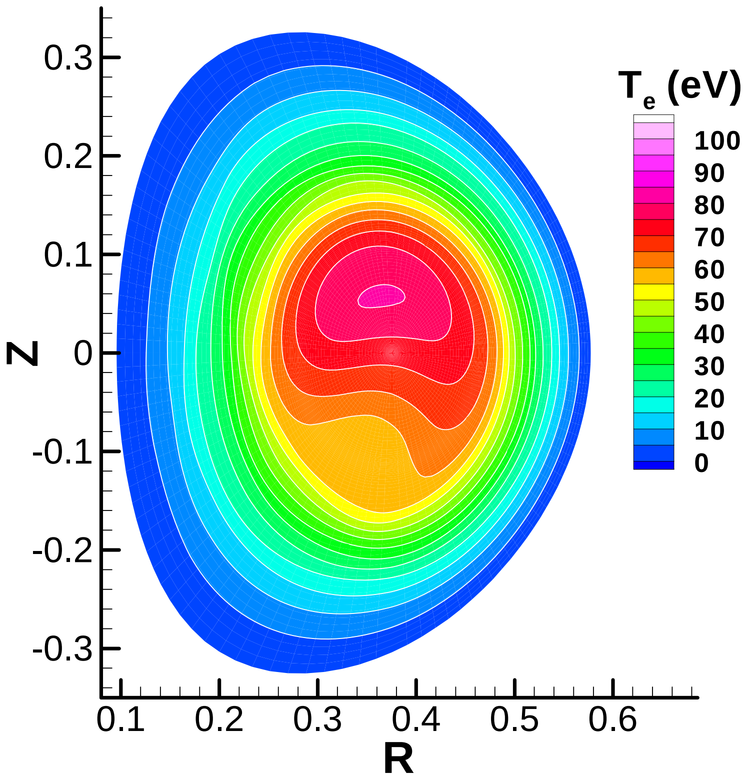


NIMROD 200 μs

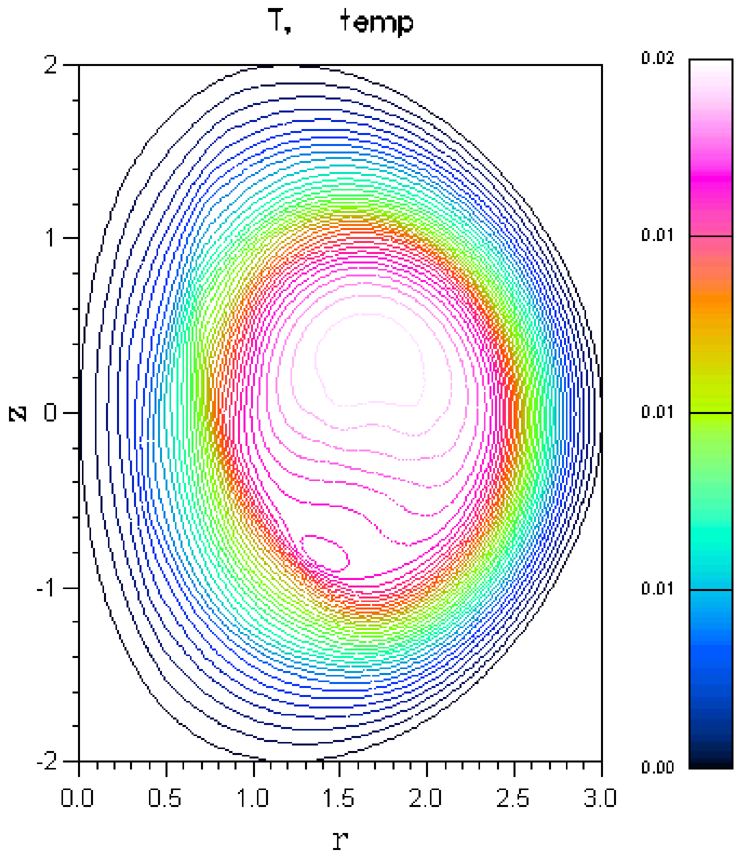


M3D 115.3 μs

Temperature Contours

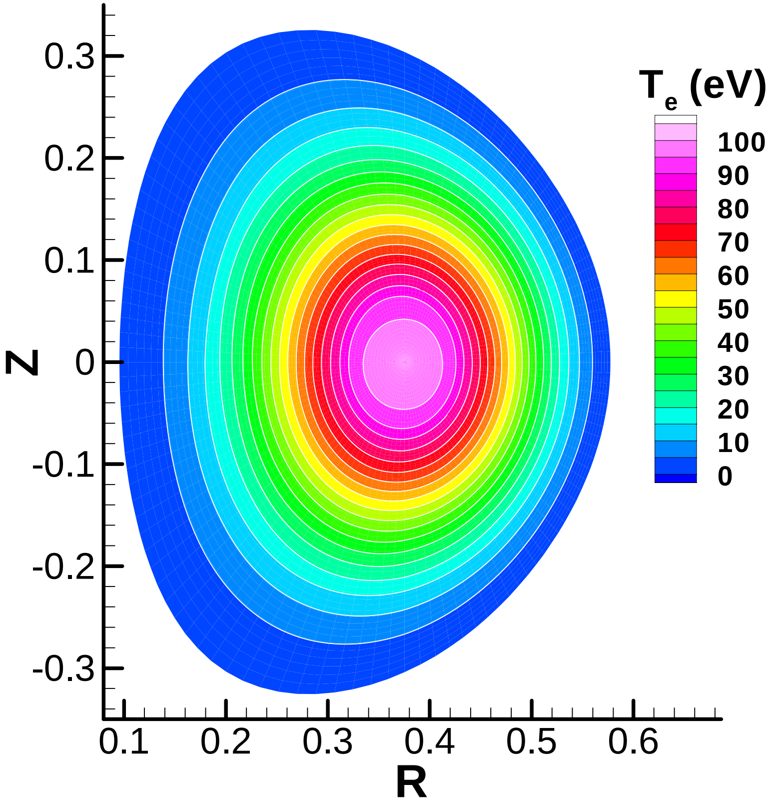


NIMROD 163 μs

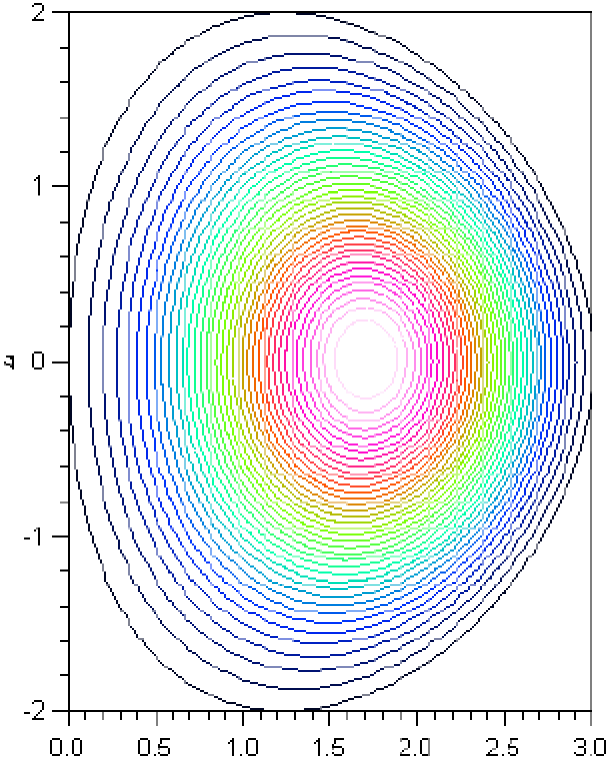


M3D 74.5 μs

Temperature Contours



NIMROD 200 μs



M3D 115.3 μs

Conclusions

- M3D and NIMROD are now in substantial agreement on the nonlinear CDX sawtooth benchmark.

- The next important step is validation: running a more physically accurate test case to try to achieve better agreement of both codes with experimental results.