A Physics-based Implicit Method for M3D

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An implicit method for M3D

- In the current version of M3D, only compressional Alfven waves are advanced implicitly. Thus, the time step size is limited by CFL condition due to shear Alfven waves.
- We have recently developed an implicit method which is valid for the full resistive MHD equations.

In (R, Z, φ) coordinates

2.1 Decompose v into (u, χ, v_φ), we have

$$\mathbf{v} = R^2 \epsilon \nabla u \times \nabla \varphi + \nabla_\perp \chi + v_\varphi \hat{\varphi}$$

2.2 Decompose B into (ψ, I), we have

$$\mathbf{B} = \nabla \psi \times \nabla \varphi + \frac{1}{R} \nabla_\perp F + R_0 I \nabla \varphi$$

2.3 F equation

$$\nabla_\perp^2 F = -\frac{a}{R} \tilde{I}', \ \text{with} F \equiv \frac{\partial f}{\partial \varphi}, I = \frac{R}{R_0} B_\varphi = 1 + \epsilon \tilde{I}.$$

$$\frac{\partial}{\partial t} w = R_0 B \cdot \nabla(\frac{c}{d}) + ...$$

$$\frac{\partial}{\partial t} c = \varepsilon R^2 B \cdot \nabla(w) + ...$$

Shear Alfven Waves

$$\frac{\partial}{\partial t} \tilde{I} = -(\frac{1}{\varepsilon} + \tilde{I})\Delta^* \chi + R^2 B_p \cdot \nabla(\frac{v_\phi}{R}) + ...$$

$$\frac{\partial}{\partial t}(\frac{\partial \chi}{\partial R}) = -\frac{1}{d}(\frac{1}{\varepsilon} + \tilde{I})\frac{\partial \tilde{I}}{\partial R} + ...$$

$$\frac{\partial}{\partial t}(\frac{\partial \chi}{\partial z}) = -\frac{1}{d}(\frac{1}{\varepsilon} + \tilde{I})\frac{\partial \tilde{I}}{\partial z} + ...$$

$$\frac{\partial}{\partial t} p = -\gamma p \Delta^+ \chi - \gamma p \frac{1}{R}\frac{\partial v_\phi}{\partial \phi} + ...$$

$$\frac{\partial}{\partial t} v_\phi = R B_p \cdot \nabla \tilde{I} - \varepsilon \frac{R}{d}\frac{\partial p}{\partial \phi} + ...$$

Compressional Alfven Waves

Implicit operator for shear Alfven waves

$$\frac{dw}{dt} = R_0 B \cdot \nabla(c/d) + other\ terms$$

$$\frac{dc}{dt} = \epsilon R^2 B \cdot \nabla w + other\ terms$$

Thus implicit equation for w can be written as

$$w^{n+1} - (\Delta t)^2 B \cdot \nabla(R^2/d) B \cdot \nabla w^{n+1} = RHS$$

Implicit operator for V_ϕ

$$V_\phi^{n+1} - (\Delta t)^2 (1/d) RB_p \cdot \nabla RB_p \cdot \nabla V_\phi^{n+1} = RHS$$

Main Results

- Initial results are very encouraging !
- Full MHD equations can be advanced stably for time step well over the shear Alfven CFL limit at zero resistivity and viscosity (for both linear and nonlinear runs).

Discussions and Future Plan

- Implicit method is necessary for high-resolution computation;
- The implicit method in this work is only partial. But it can serve an effective pre-conditioner for full implicit method;
- Future work will consider full linear and nonlinear implicit method such Newton-Krylov (JFNK) method.