Preliminary

Benchmarking the NIMROD Code with Ideal MHD codes for Peeling-Ballooning Unstable Equilibria

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Motivation

- Linear, Ideal MHD (ELITE) has been successful in explaining many experimental observations of ELMs
- Using extended MHD codes is natural extension of ELITE

  Can study nonlinear processes such as:
    - onset
    - nonlinear evolution and heat deposition

  Can have additional physics
    - diffusivities: resistivity, viscosity, thermal diffusivities
    - two-fluid physics: Hall terms, gyroviscosity, electron stress tensor
    - closure physics: parallel heat flux, gyrokinetic, …

NIMROD Equations Are A Superset of the Ideal MHD Equations

- **Resistive MHD equations (neglect 2fluid for now):**

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \quad + \kappa_{divb} \nabla \nabla \cdot \mathbf{B}$$

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{J}$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

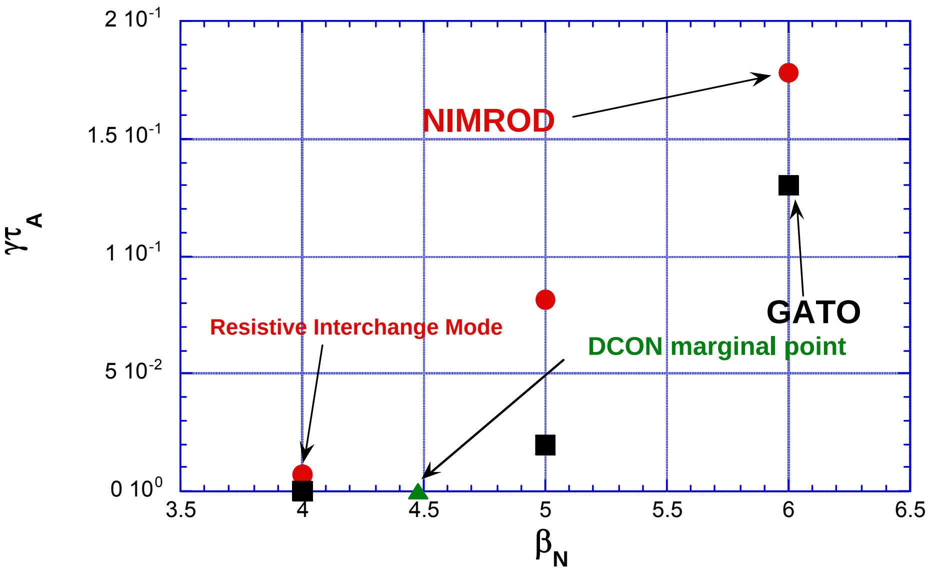
$$\frac{\partial n}{\partial t} + \nabla \cdot \left(n \mathbf{V} \right) = \nabla \cdot D \nabla n$$

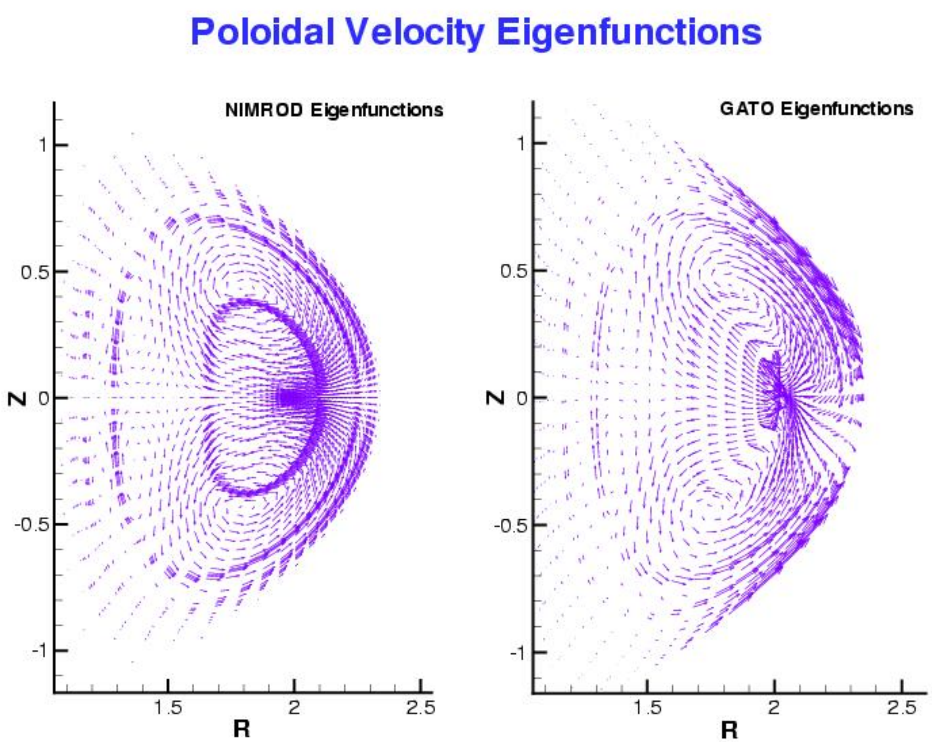
$$Mn \frac{d\mathbf{V}}{dt} = -\nabla p + \mathbf{J} \times \mathbf{B} - \nabla \cdot \Pi_{\parallel} - \nabla \cdot \Pi_{gv} \ \ (+ M \nabla \cdot n \mu \nabla \mathbf{V}) \ \ ,$$

$$\frac{n}{\gamma - 1} \left(\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -p \nabla \cdot \mathbf{V} + \nabla \cdot n \ \left[\left(\chi_{\parallel} - \chi_{\perp} \right) \ \hat{\mathbf{b}} \hat{\mathbf{b}} + \chi_{\perp} \mathbf{I} \right] \cdot \nabla T + Q$$

NIMROD Has Performed Previous Benchmarks With Ideal MHD Codes

- **Soloviev equilibria: 6% agreement**
- **Also done internal interchange with GATO as part of “disruption cases**





Many tearing benchmarks performed:

Sovinec, JCP (2004)

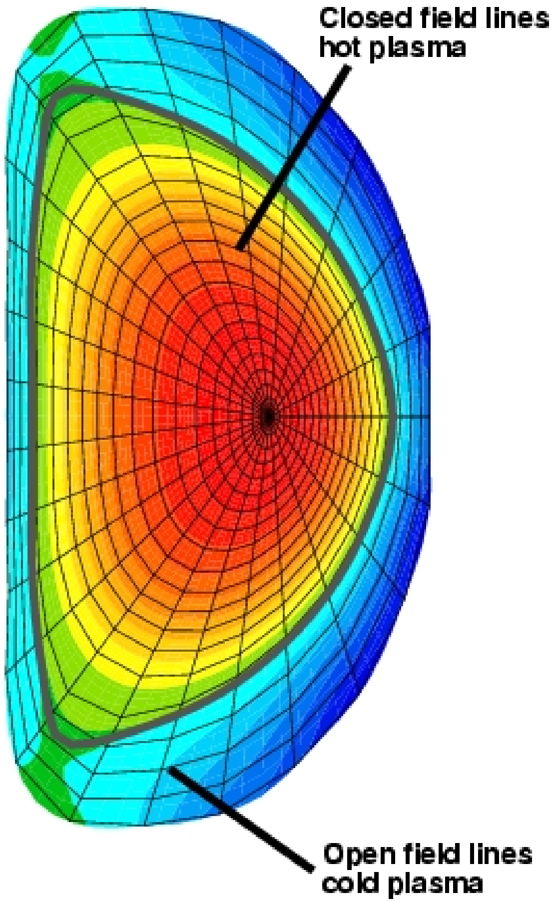
Free Boundary Modes Inherently Different For Benchmarking Codes

- **Linear, ideal MHD codes: Region beyond separatrix is a "vacuum"**
- **Extended MHD codes: Region beyond separatrix is "cold plasma"**

- **Typical DIII-D Parameters:**
  T_{core}~10 keV T_{sep}~1-10 eV
  n_{core}~5x10^{19} m^{-3} n_{sep}~ 10^{18} m^{-3}

- **Spitzer resistivity: $\eta \sim T^{-3/2}$**
  - **Suppresses currents on open field lines**
  - **No current on open field lines => vacuum**

- **Limit of resistive codes to linear MHD codes then is infinite resistivity in the vacuum region**



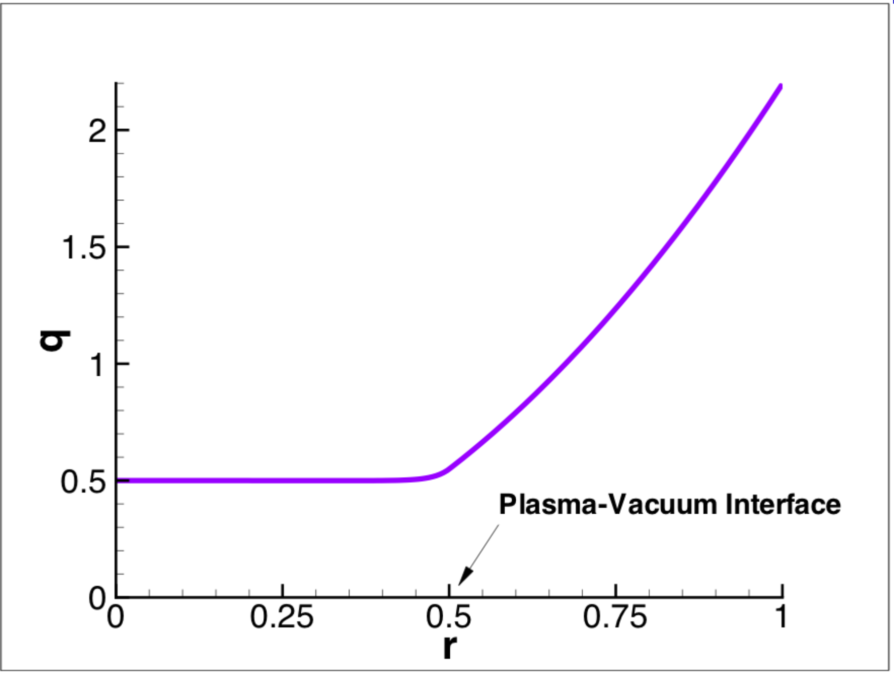
□□□Laundry List of Things Nonlinear Codes Need To Do To Benchmark with Ideal MHD Codes

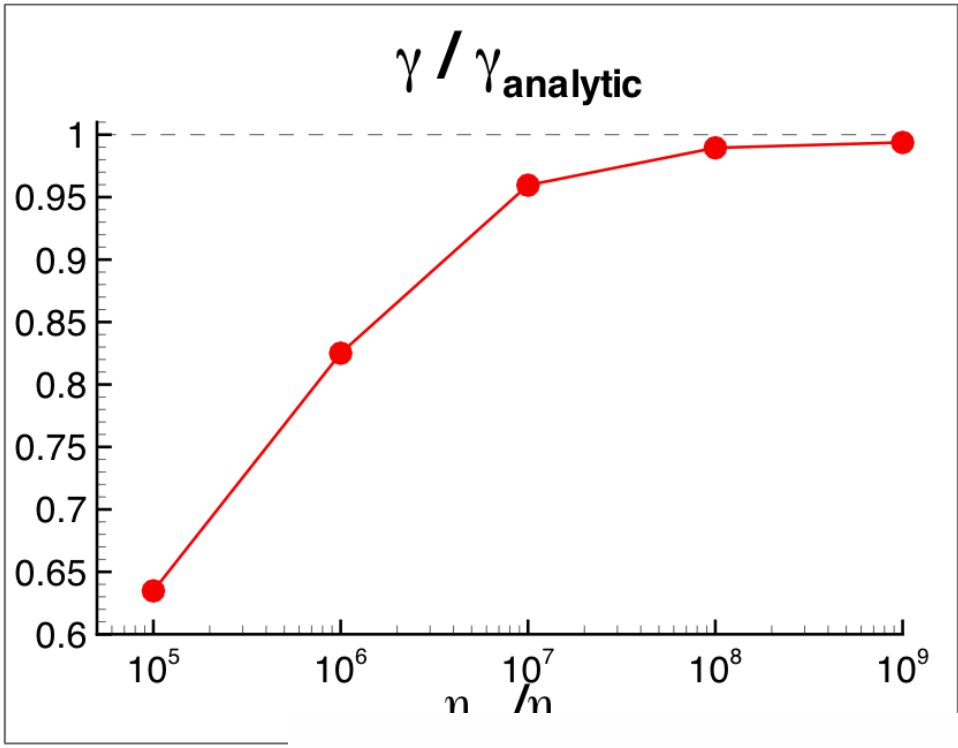
- To get growth rate:
  Evolve equations, take slope of the log of energies
  When flat, divide by 2 to get most unstable eigenvalue
- To compare, need to converge on:
  - Discretization:
    - Spatial Grid
    - □Δt
  - Physics parameters
    - □μ (–> 0)
    - □η_{core} (–> 0)
    - □η_{vac} ˜(□Inf
    - □Δw_η ˜(□0

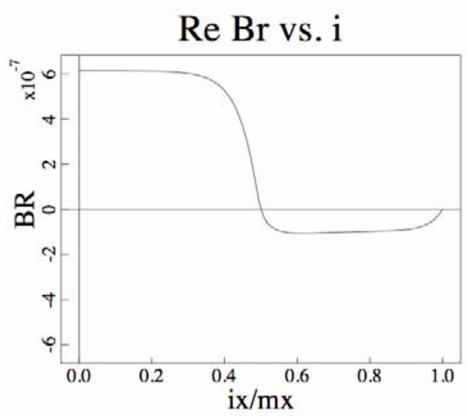
Nonlinear code specific.
Rule of thumb:
$\gamma\Delta t<10^{-3}$

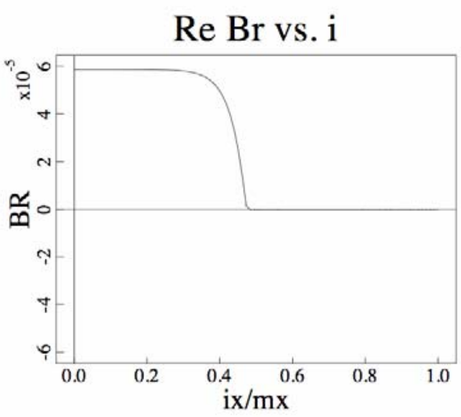
Hard for ELMs!

Free-boundary capability has been benchmarked with Shafranov equilibria

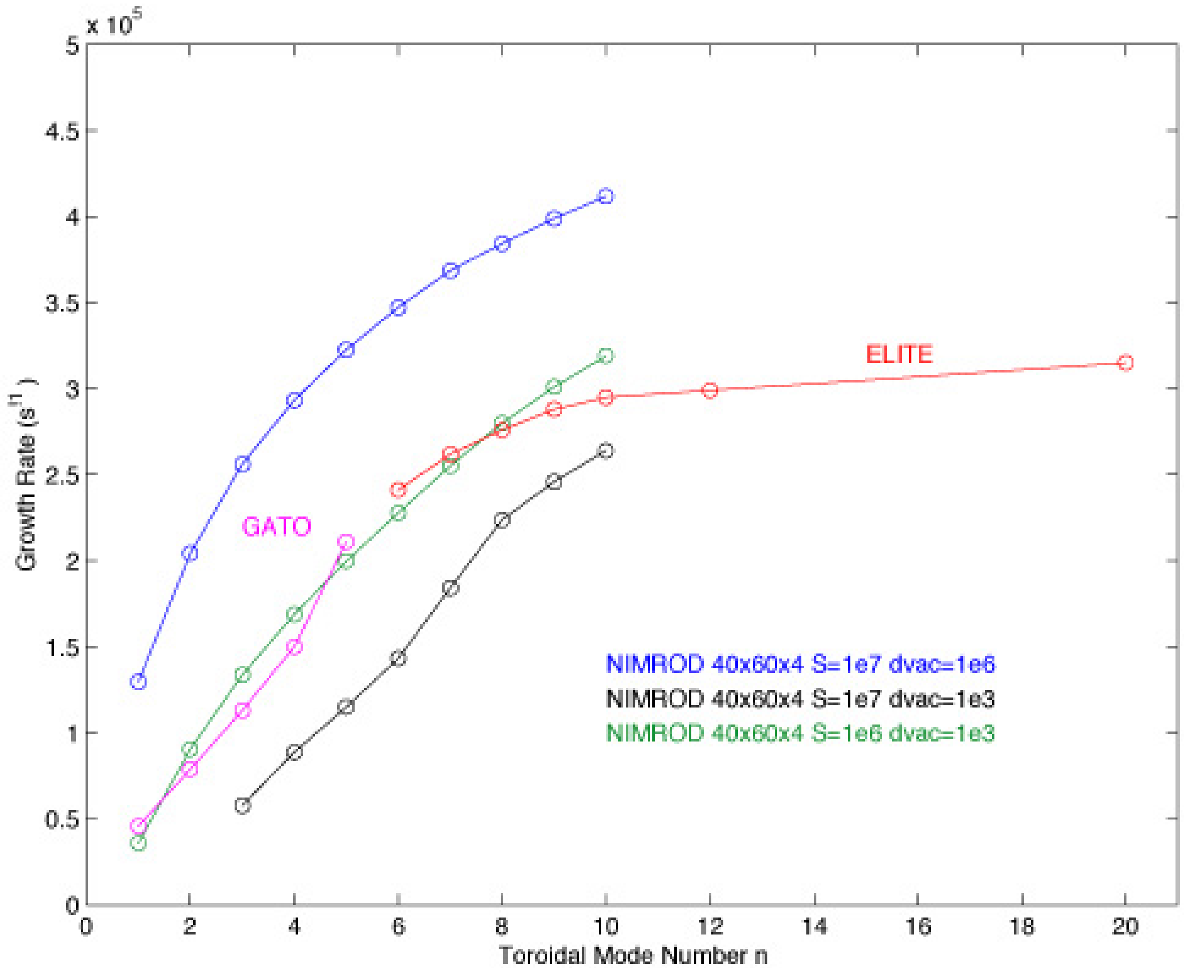








Growth Rates From Ideal Codes Bracketed by NIMROD depending on resistivity in edge region

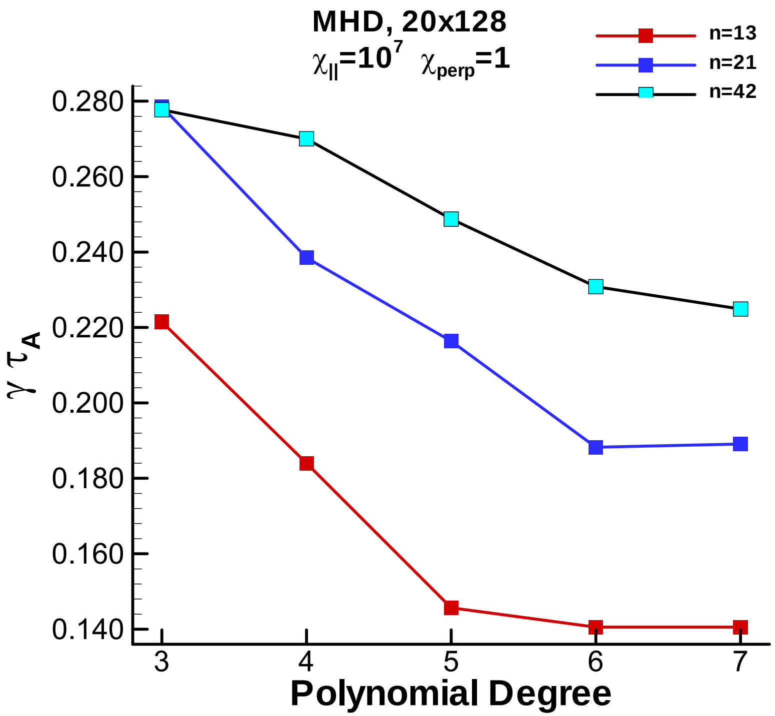


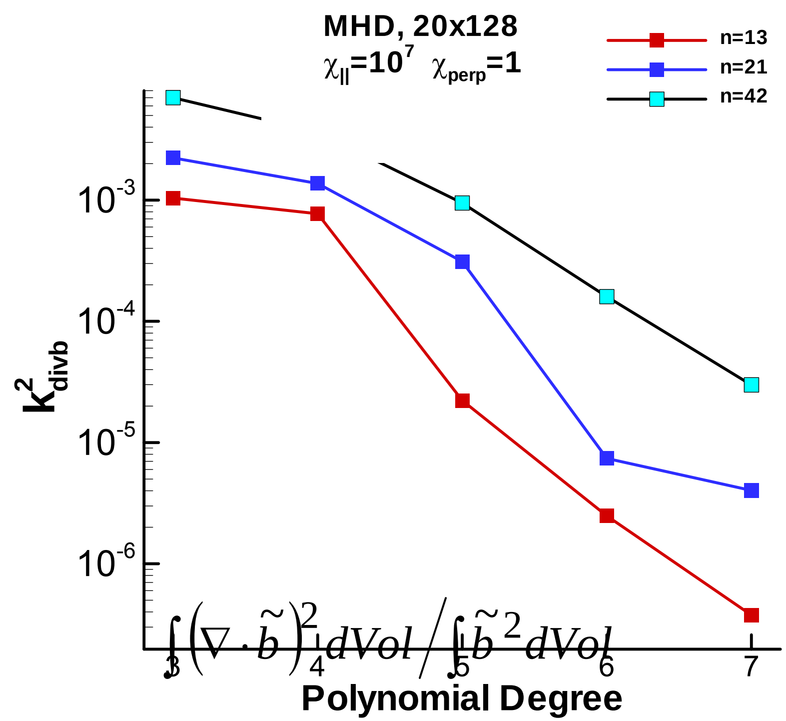
Linear Convergence Study Shows Significant Resolution Needed

- **Convergence demonstrated for renormalized Spitzer temperature-dependent resistivity profiles 100x experiment**

Parameters:

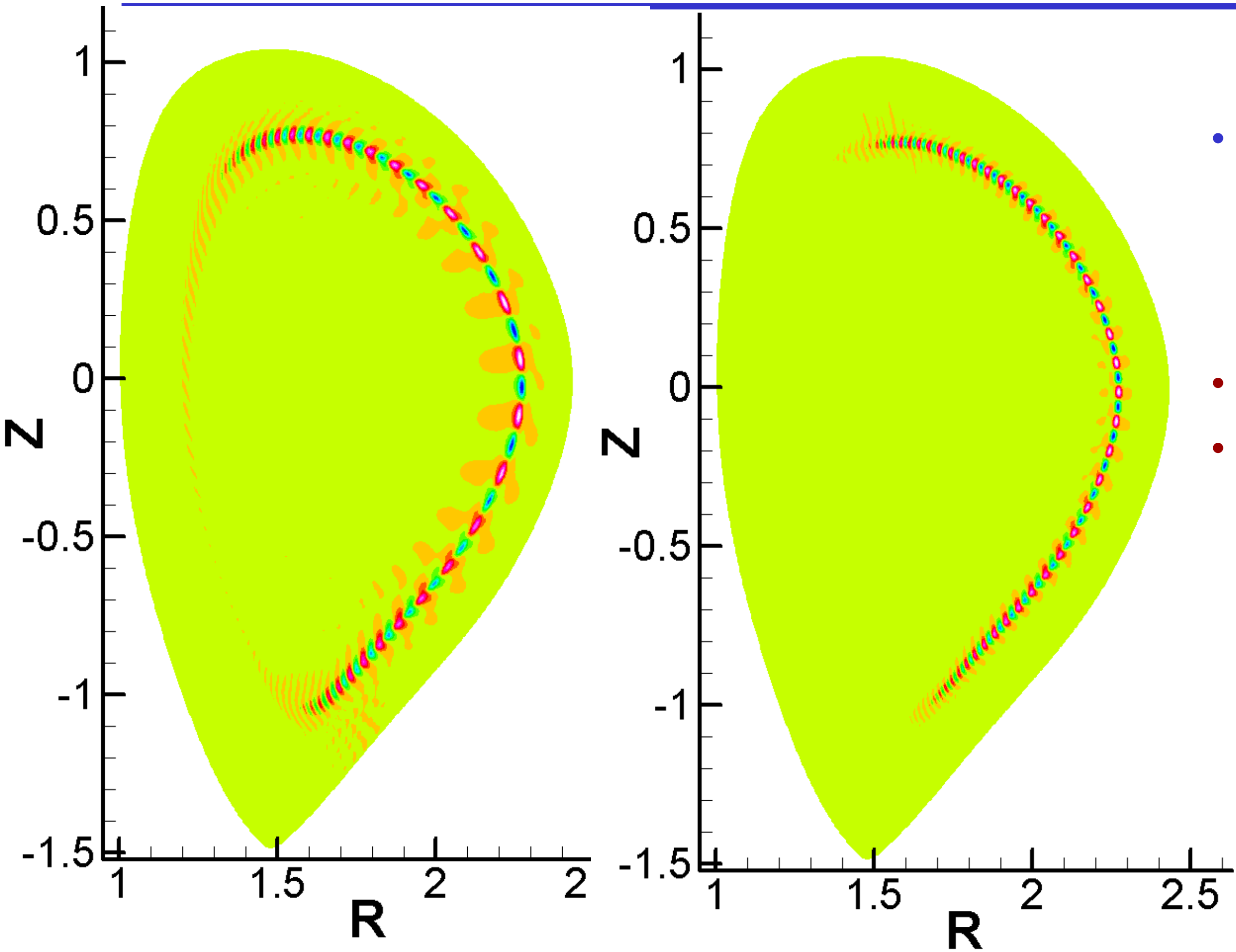
resistivity = 7 m²/s at the top of the pedestal, $\chi_{||}=1.5\times10^7$ m²/s, $\chi_\perp=1.5$ m²/s, $\nu=25$ m²/s, artificial particle diffusivity of $D_n=2.5$ m²/s





Growth-rate and divergence error for three modes in the MHD spectrum with anisotropic thermal conduction

Eigenfunctions for n=21, n=42 With MHD Model Show Fine-Scale Structure



- **Larger n-value modes have larger growth rates (ballooning-like character not in agreement with ideal-MHD results)**
- **Resistive effects?**
- **Further work needed**

Toroidal component of flow velocity from the resistive MHD eigenmodes of the n=21 mode and n=42 mode, computed with polynomial basis functions of degree 6

Equilibrium representation an important issue in performing code comparison

Few definitions first:

- Direct Equilibria: Solve for $\psi = \psi(R,Z)$
- Inverse Equilibria: Solve for $R(\psi,\theta)$, $Z(\psi,\theta)$

All MHD codes: Flux-aligned in core

NIMROD and M3D: Not flux-aligned in vacuum (in general)

- Generally need to use direct equilibria to create grid

For benchmarking:

- Prefer inverse equilibria to avoid errors associated with creating the flux-aligned grid (mapping problem)

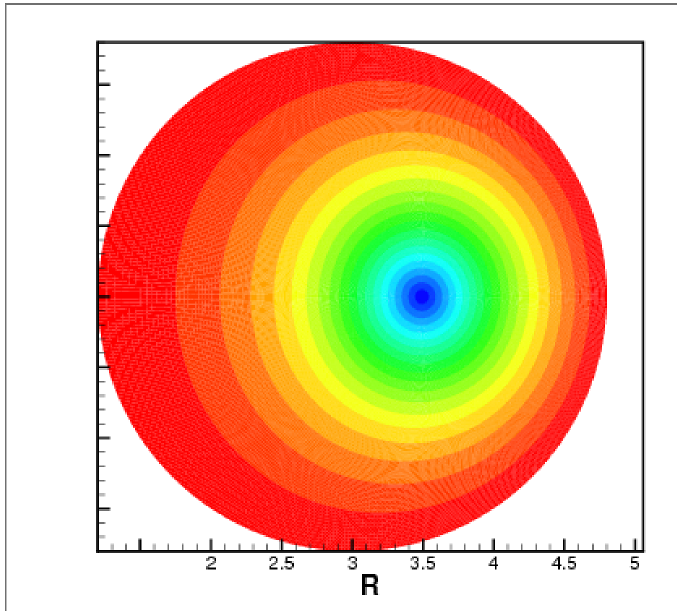
How to use inverse equilibria AND get equilibrium vacuum fields?

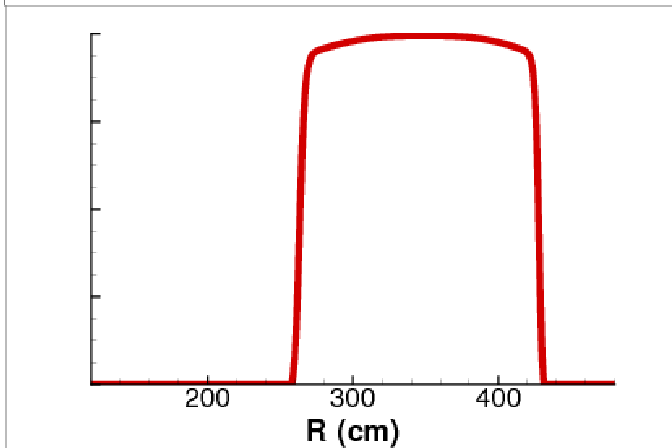
Two approaches:

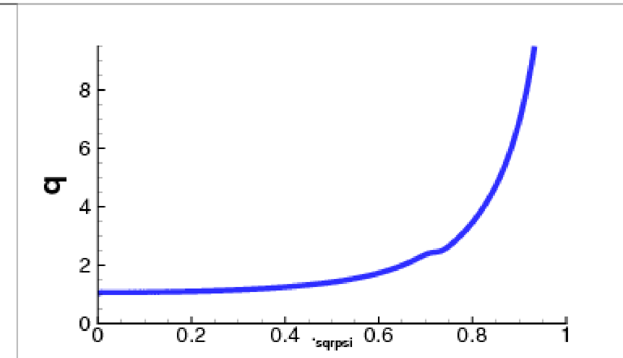
1) Use Green's functions to calculate those fields
2) Modify inverse equilibria code to solve "free-boundary equilibria"

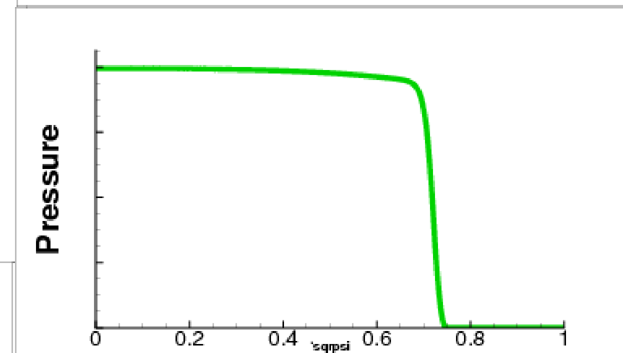
By modifying TOQ, suitable equilibria was created

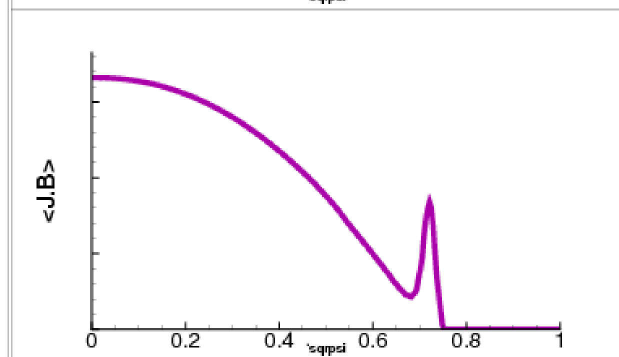
- Quick modification to set currents in region to zero.
- Pedestal width twice the experimental value to simplify needed resistivity transition
- Minor radius~85 cm ~40 cm vacuum region on outboard midplane
- Good for benchmark:
  - Minimized mapping/grid-alignment issues
  - Less resolution needed
  - Control over profiles





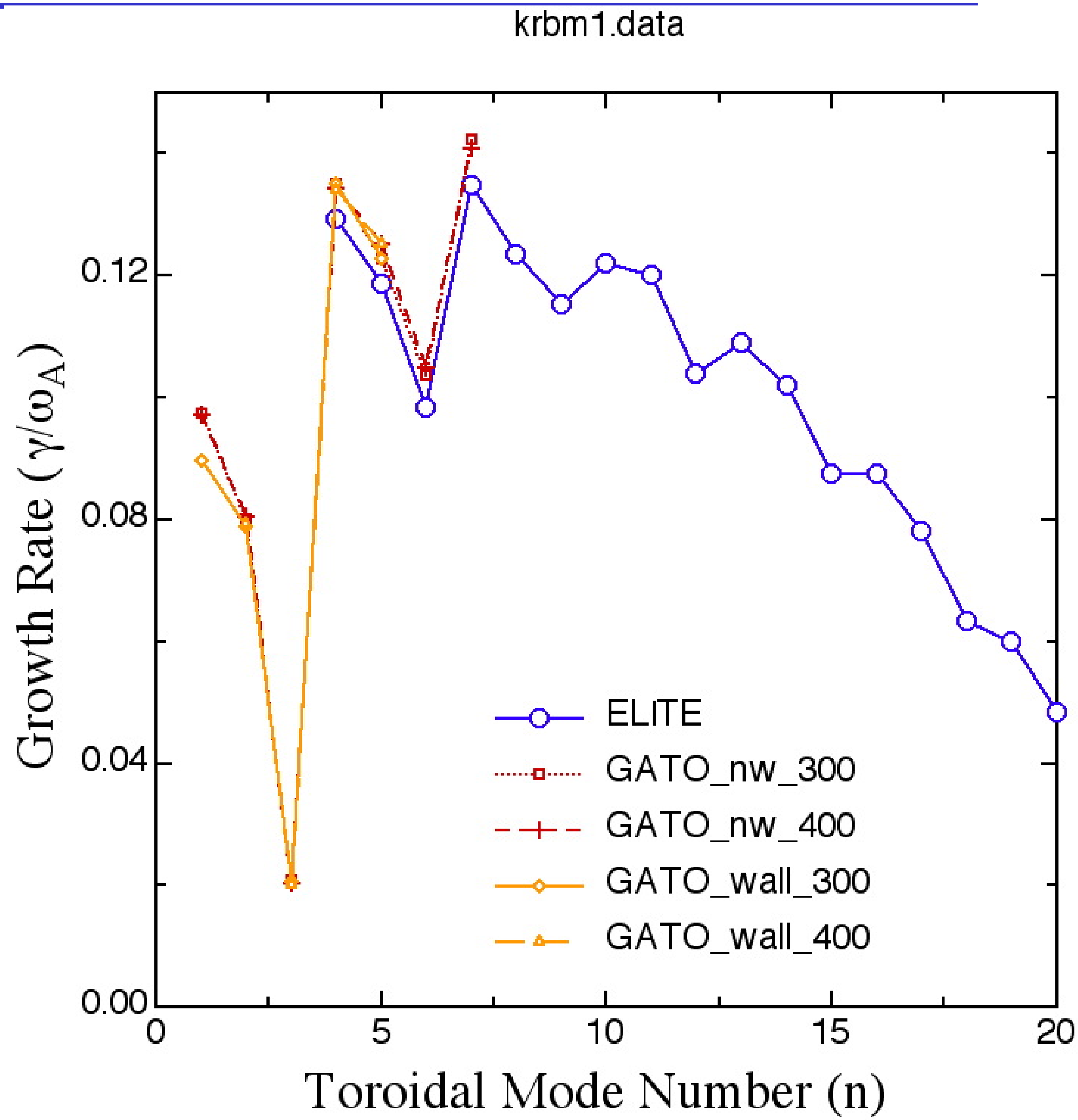






ELITE Shows Interesting Spectrum

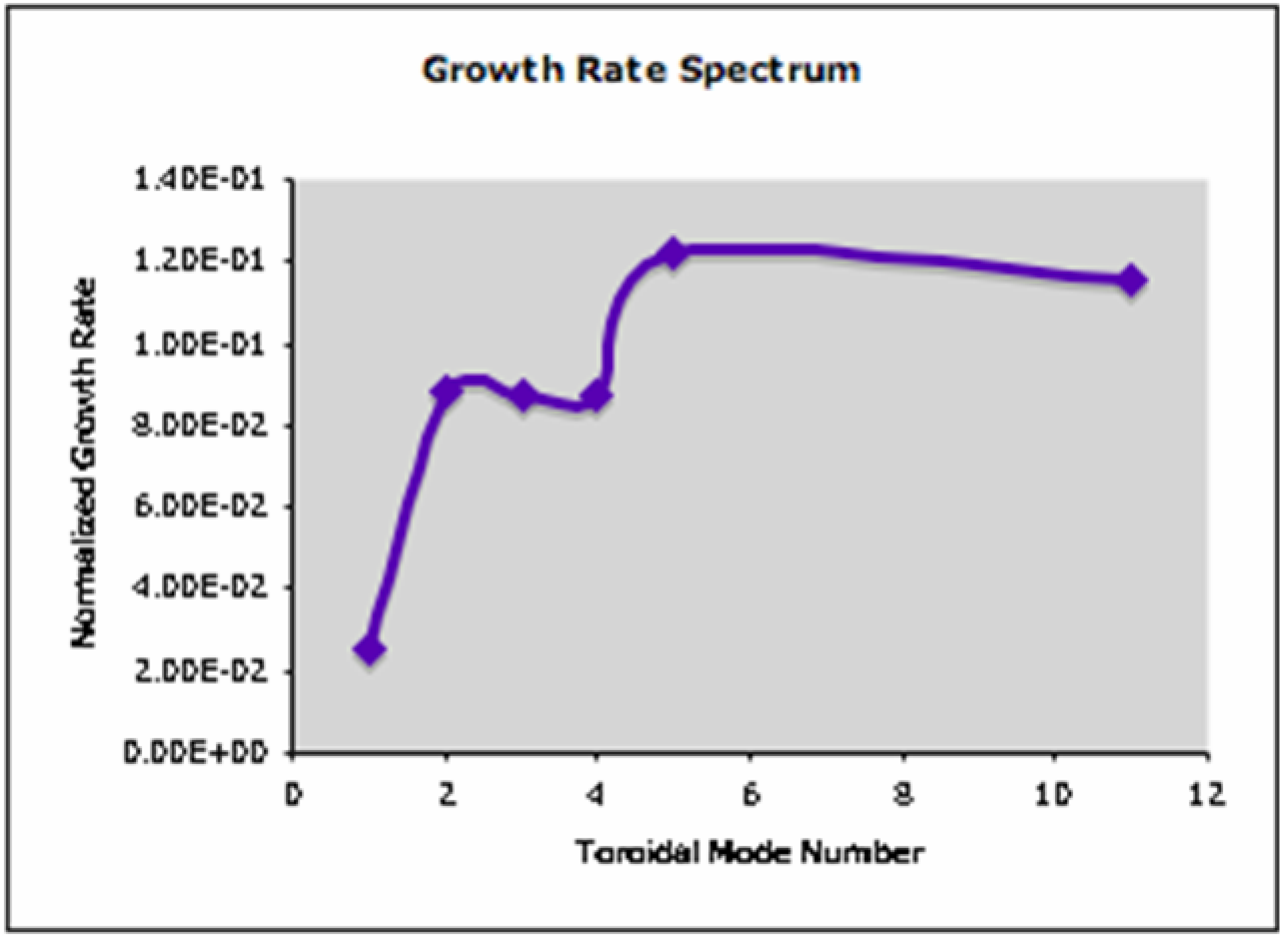
- Spectrum behavior due to proximity of resonant surfaces to vacuum



Preliminary Results Do Not Show Similar Spectrum

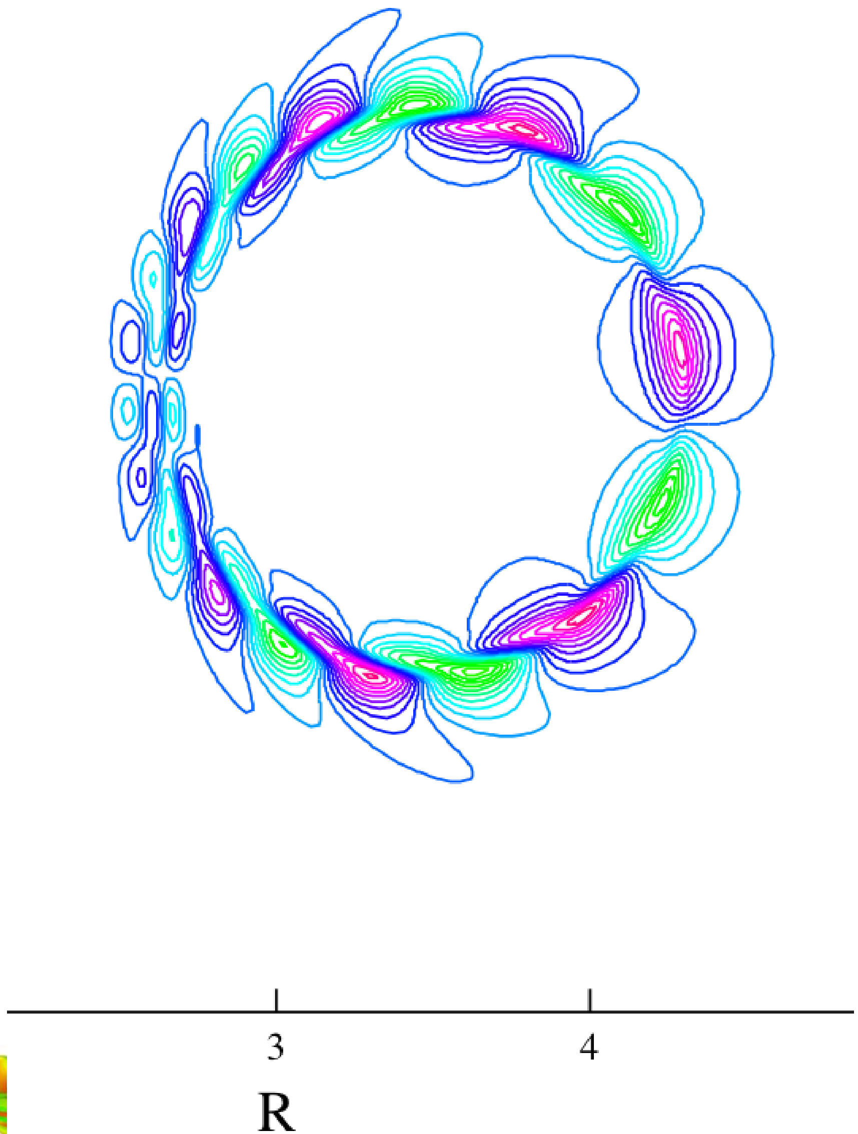
MANY Caveats

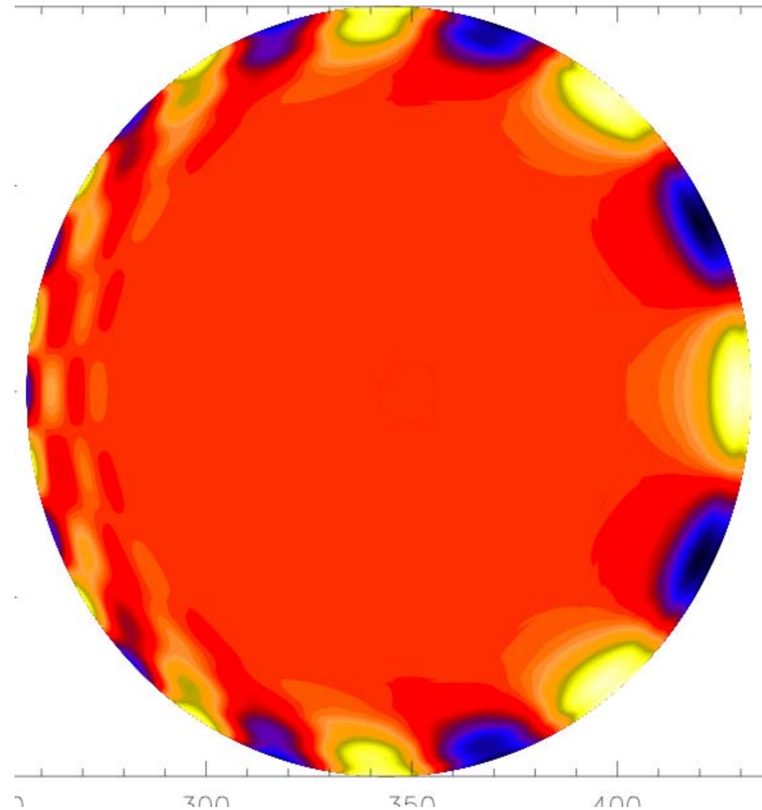
- $S_{core}=10^5$
- $S_{vac}=10^8$
- $\Delta w_\eta=0.2$ (using tanh function)
- $x_\eta=0.78$ (compared to vacuum location of $x_{vac=}0.755$)



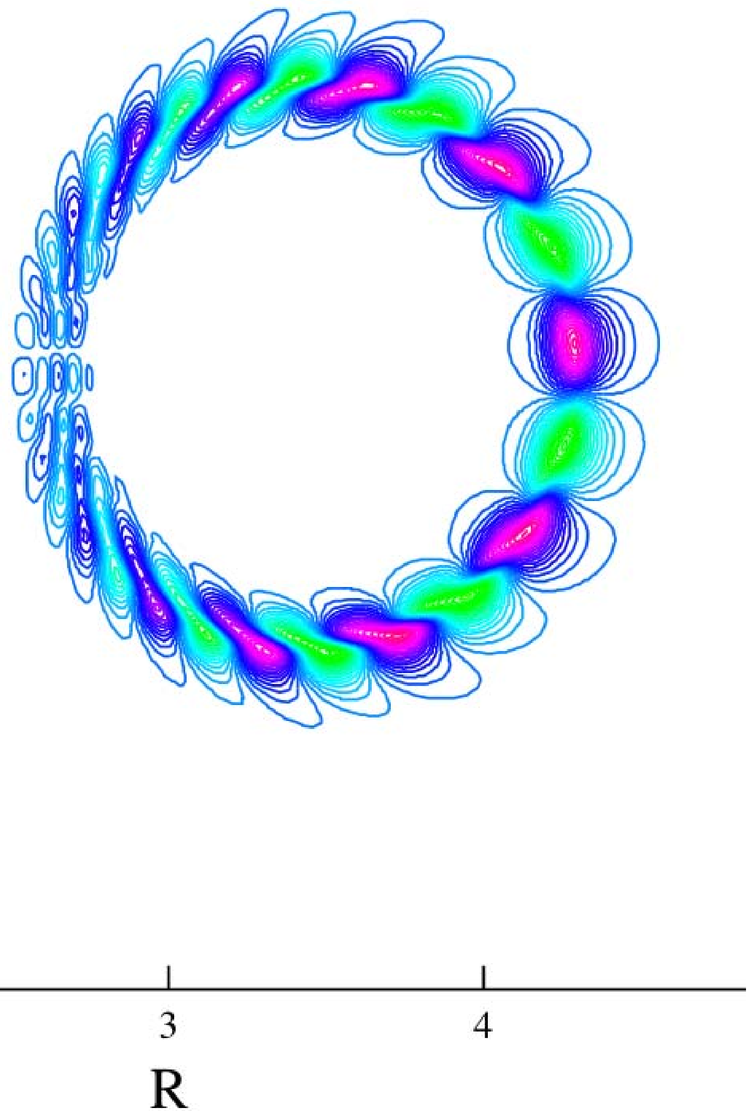
mx=50, my=40-60, pd=5

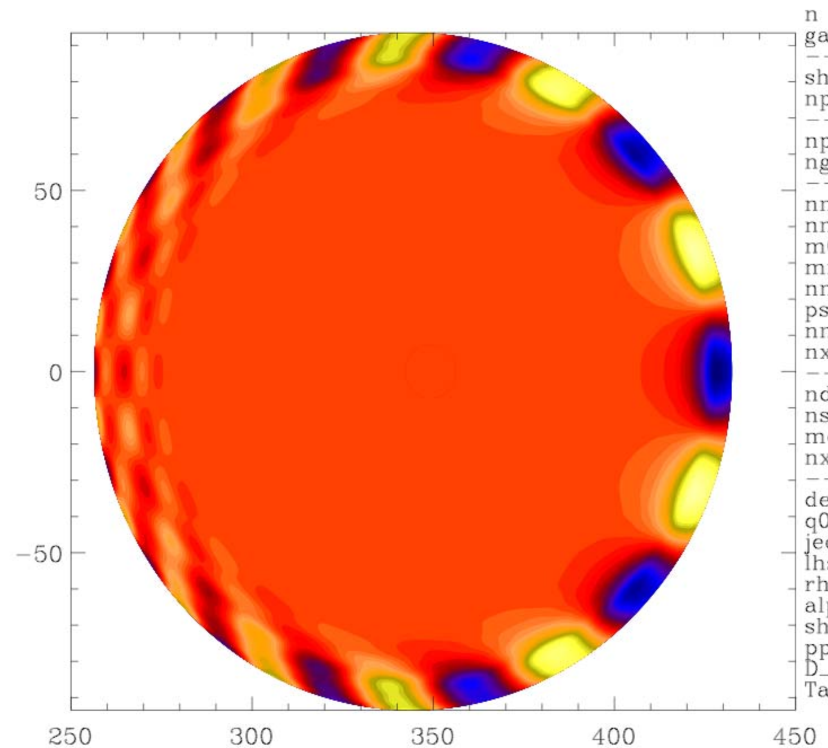
Comparison of Eigenfunctions: n=4





Comparison of Eigenfunctions: n=5





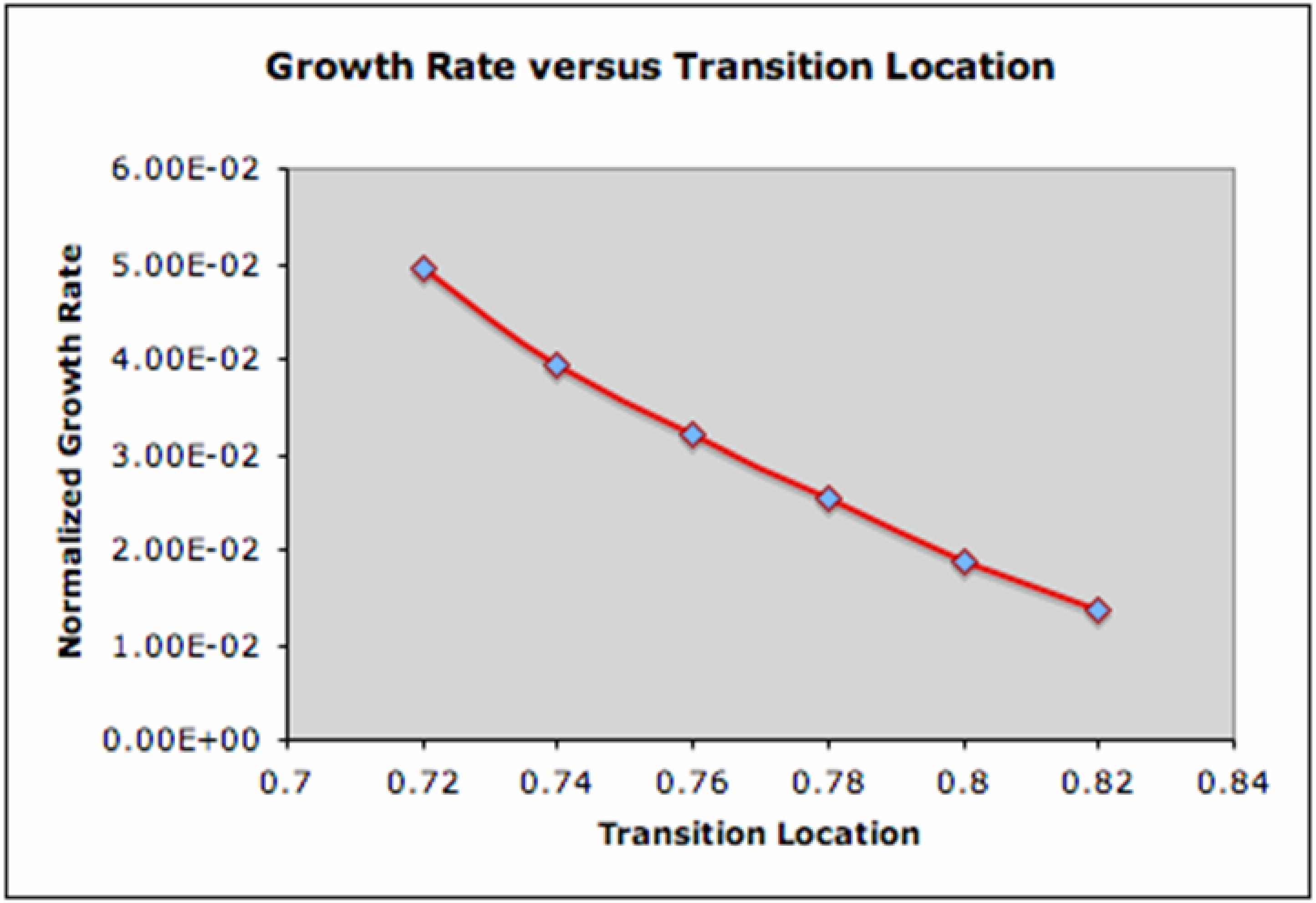
Sensitivity to Location of Resistivity Transition Region

For n=1 mode

- $S_{core}=10^5$
- $S_{vac}=10^8$
- $\Delta w_\eta=0.2$

vacuum location: $x_{vac=}0.755$

2% change in location gives 33% change in growth rate



mx=50, my=40-60, pd=5

Conclusions

- Efforts to better understand approach of resistive MHD to the linear ideal MHD limits is underway

- Equilibrium has been created that satisfies several important criterion for benchmarking with linear ideal MHD codes:
  - Contains vacuum region
  - Has wide pedestal width (~8% of minor radius)
  - Inverse equilibrium to reduce mapping/grid-alignment errors

- Preliminary results show we get the modes, but more work is needed

Future Directions

- **Finish the parameter scan over the full range of modes**
- **Revisit previous studied equilibria (original nimbm and the 2 Osborne equilibria)**
  - **Equilibria used were underresolved based on ideal benchmark work by Snyder**

- **This work has motivated development of *LINROD* to make these developments easier**
  - **Scan over parameters**
  - **Simplify determining convergence**
  - **Possibly turn into full eigenvalue code?**
    (Work by Werner and Cary. Can it be generalized?)

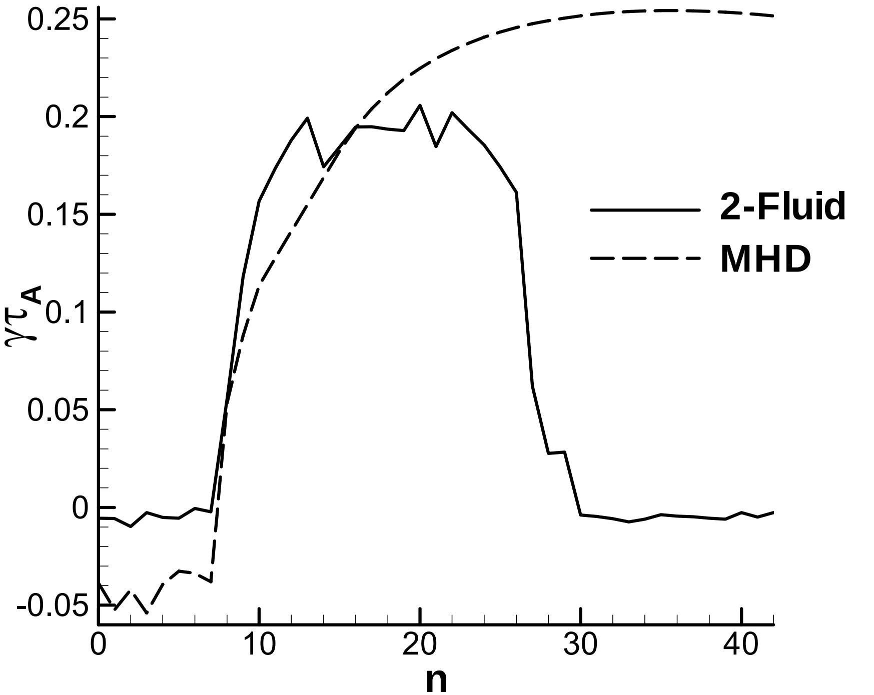
Conclusions From Workshop

- **Strauss: Most important thing to benchmark is the threshold boundary.**
- **Bateman: Mode widths important as well**
  - **General agreement with this**
- **Still some uncertainty on what physics will stabilize the low n modes at marginal stability**

- **Action items:**
  - **Variations of krbm cases will created to map out boundary (Snyder and Kruger)**

Two-fluid and FLR Effects

- **Resolution requirements with two-fluid electric field and gyroviscosity more stringent**
- **Increasing particle diffusivity to 5 m^2/s facilitates convergence**



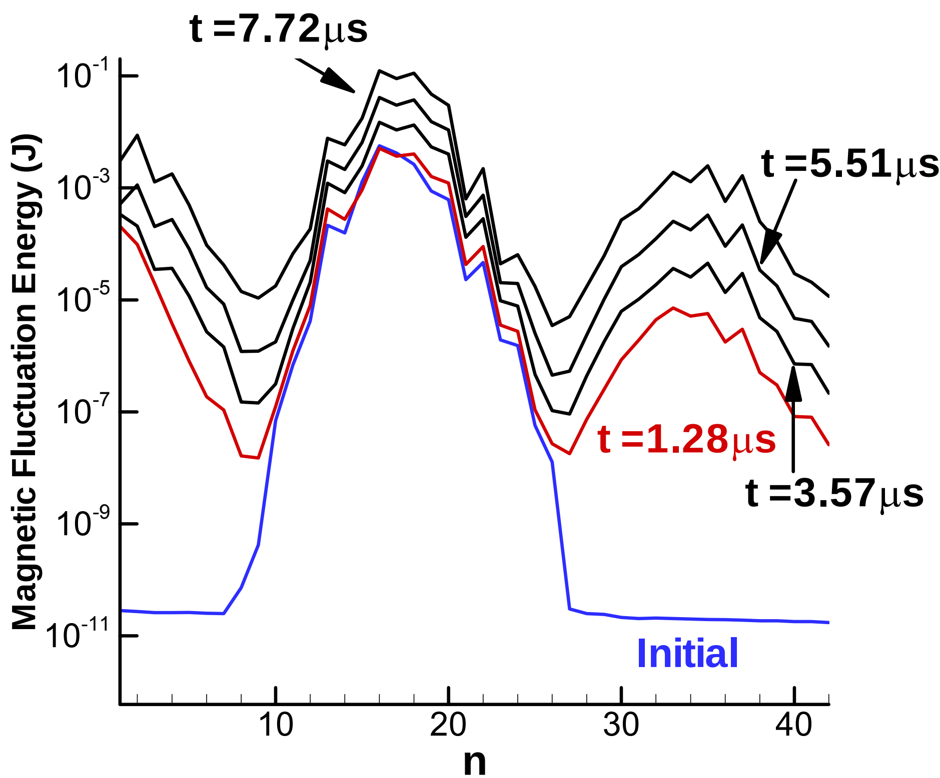
Two-fluid and single-fluid linear growth-rate spectrum for toroidal wavenumbers $0 \leq n \leq 42$ obtained with the 20×120 mesh of biquintic elements

Most unstable modes have n between 10 and 20

Modes with $n \geq 30$ are stabilized

Nonlinear Evolution of ELMs

- Linear results used as initial conditions in non-linear NIMROD computations
- Evolution of kinetic fluctuation energies over first nonlinear time-steps:
  - two-wave coupling produces high-n harmonic of the spectrum peak and coupling to n-values below 10



Dissipation coefficients of $\chi_{\|}=1.5\times10^{7}$ m^2/s, $\chi_{\perp}=1.5$ m^2/s, and $\nu=25$ m^2/s

Poloidal mesh of 20×120 with biquintic finite elements

□□□NIMROD Has Unique Features For Calculating Growth Rates

- **NIMROD separates fields. Allos nonlinear terms to be turned off**

$$E = -\mathbf{V}_{ss} \times \mathbf{B} - \mathbf{V} \times \mathbf{B}_{ss} - \mathbf{V} \times \mathbf{B} + \eta_{ss}\mathbf{J} + \eta\mathbf{J}_{ss} + \eta\mathbf{J} \quad ,$$

$$Mn\frac{d\tilde{\mathbf{V}}}{dt} = -\nabla\tilde{p} + \tilde{\mathbf{J}} \times \mathbf{B}_{ss} + \mathbf{J}_{ss} \times \tilde{\mathbf{B}} + \tilde{\mathbf{J}} \times \tilde{\mathbf{B}} - \nabla p_{ss} + \mathbf{J}_{ss} \times \mathbf{B}_{ss} + S$$