Two-Fluid Toroidal Steady States with Flow

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Goals

- We are using M3D-C^1 to calculate axisymmetric toroidal steady-states of a comprehensive two-fluid model.
- These steady-states are steady on all timescales and are the self-consistent solutions including two-fluid MHD, gyroviscosity, flow, and anisotropic transport.
- In particular, we would like to understand the effects of two-fluid terms and gyroviscosity on the steady-states.
- These steady-states may be used as accurate equilibria for three-dimensional stability studies.

Physical Model

$$
\begin{aligned}
\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{u} &= \sigma + D\nabla^2 n, \\
n\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) &= \mathbf{J}\times\mathbf{B} - \left(\nabla p + \nabla\cdot\Pi\right) - \mathbf{u}\sigma, \\
\frac{1}{\Gamma-1}\left(\frac{\partial p}{\partial t} + \nabla\cdot p\mathbf{u}\right) &= -p\nabla\cdot\mathbf{u} + \frac{d_i}{\Gamma-1}\frac{\mathbf{J}}{n}\cdot\left(\nabla p_e - \Gamma\frac{p_e}{n}\nabla n\right) \\
&\quad - \nabla\cdot\mathbf{q} - \Pi : \nabla\mathbf{u} + d_i\Pi_e : \nabla\frac{\mathbf{J}}{n} + \frac{1}{2}u^2\sigma, \\
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla\times\mathbf{E}, \\
\mathbf{J} &= \nabla\times\mathbf{B}, \\
\mathbf{E} + \mathbf{u}\times\mathbf{B} &= \eta\mathbf{J} + \frac{d_i}{n}\left(\mathbf{J}\times\mathbf{B} - \nabla p_e - \nabla\cdot\Pi_e\right), \\
\Pi &= \Pi_\circ + \Pi_\wedge + \Pi_\parallel, \\
\Pi_e &= \lambda\nabla\mathbf{J}, \\
\mathbf{q} &= -\kappa_\circ\nabla T - \kappa_\parallel\mathbf{B}\mathbf{B}\cdot\nabla T
\end{aligned}
$$

Method

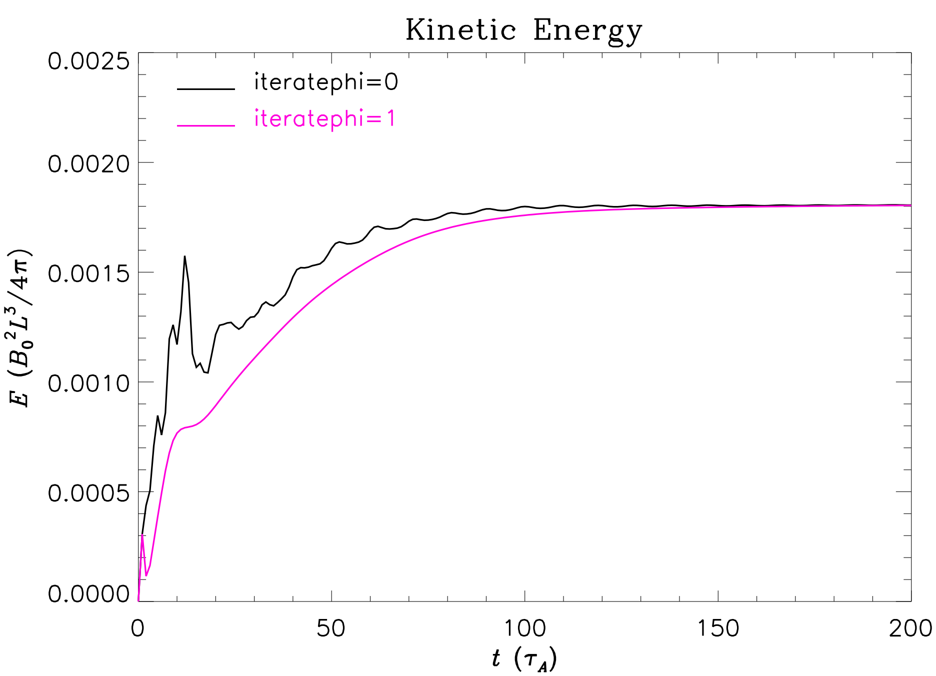
- The simulation is initialized with a solution to the Grad-Shafranov equation.
- A loop voltage is applied by changing the flux at the boundary of the simulation domain at a constant rate $\dot{\psi} = V_L/2\pi$.
- A localized density source in included to offset diffusive flux out of the simulation domain.
- The simulation is run until a steady state in all hydrodynamic quantities is reached.
- The resistivity is proportional to $T^{-3/2}$. The vacuum region is simply a low temperature region outside the separatrix.
- Viscosity smoothly becomes vary large at the boundary to damp flows in the vacuum region.

Difficulties in Low Aspect-Ratio Simulations

- Nonlinear numerical instabilities (NNI) occur when there exist flows, highly anisotropic heat flux, and ohmic heating in a high-S core.
  - This is solved by re-calculating the resistivity after the pressure advance, then re-doing the pressure advance.
- Initial transient dynamics lead to rapid transient flows and lead to NNI.
  - This is solved by initially using a large viscosity, and ramping it down while approaching steady-state.
- Large flows near the boundary lead to NNI (especially in NSTX simulations).
  - This is solved by keeping the viscosity large near the boundary.
- Profiles with sharp gradients in resistivity near the LCFS never reach steady state.
  - Not yet solved...

Field/η Iteration

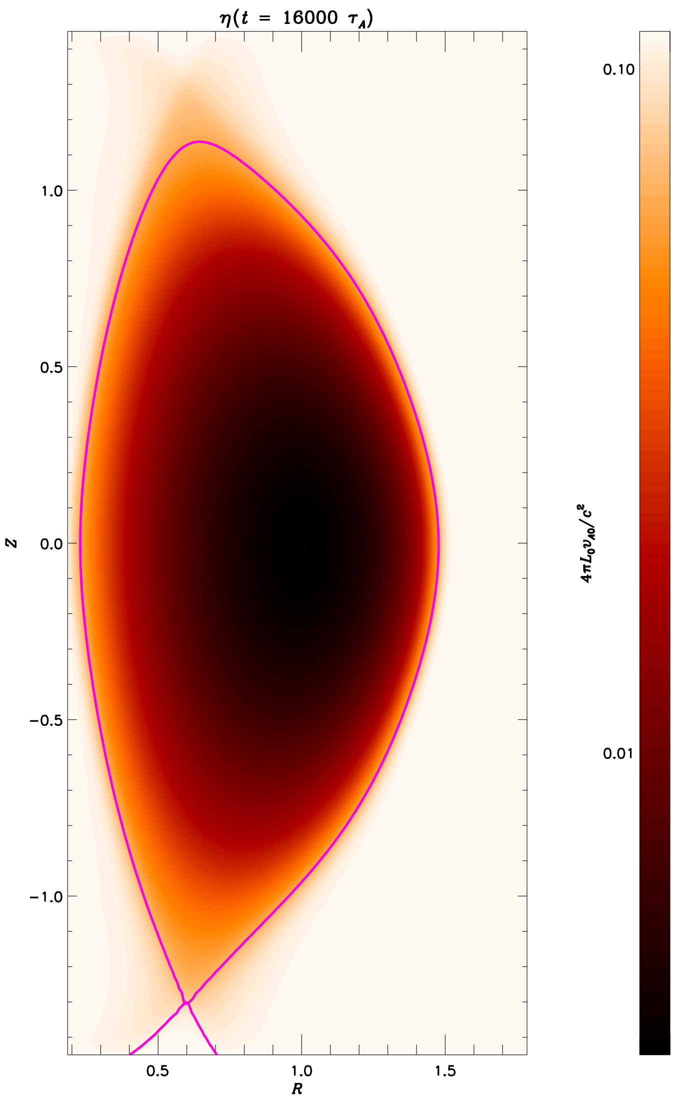
- Some problems when simulation includes all of: low η, fast temperature convection, strongly anisotropic heat flux, and ohmic heating.
  - δt-scale oscillations; T may go negative in core.
- Can be mitigated by increasing spatial resolution, but not by decreasing time step
- Or, solve $\mathbf{v}^{n+1}$, n^{n+1}, $(\mathbf{B}, p)^{n+1}$, η^{n+1}, $(\mathbf{B}, p)^{n+1}$.

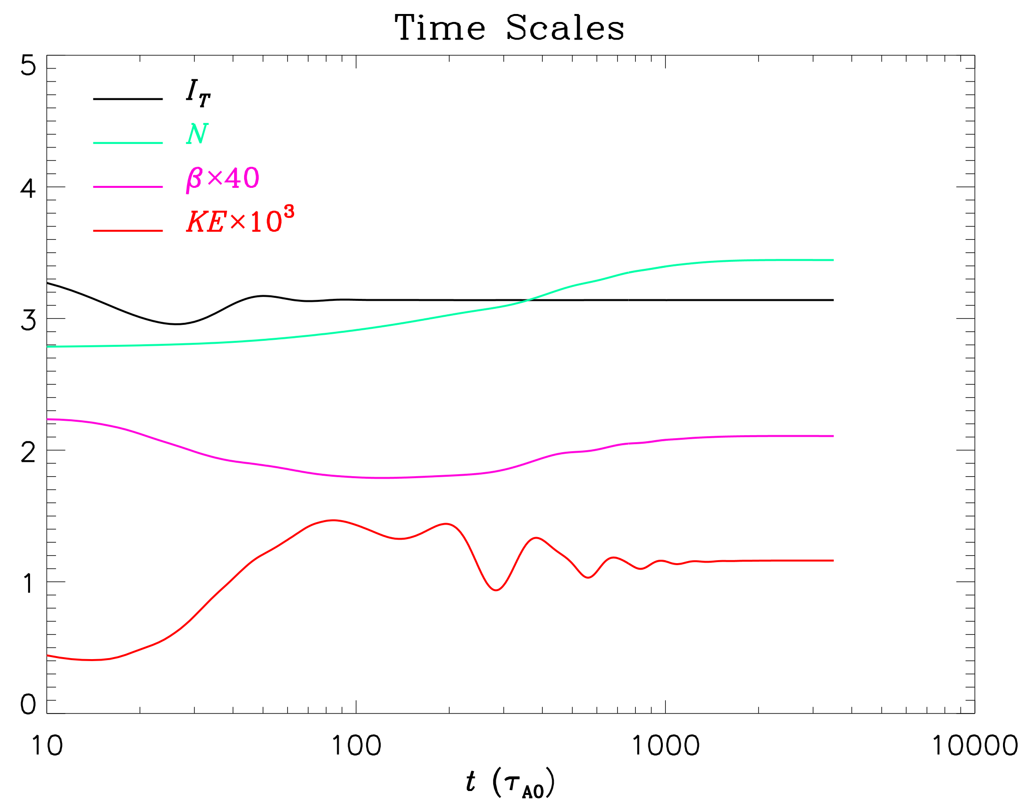


Low-S Case

- The "low-S" cases were run with the following parameters:

$$S_0 \sim 300, \quad S_e \sim 10, \quad \langle\beta\rangle \sim 20\%, \quad Re \sim 10^5$$

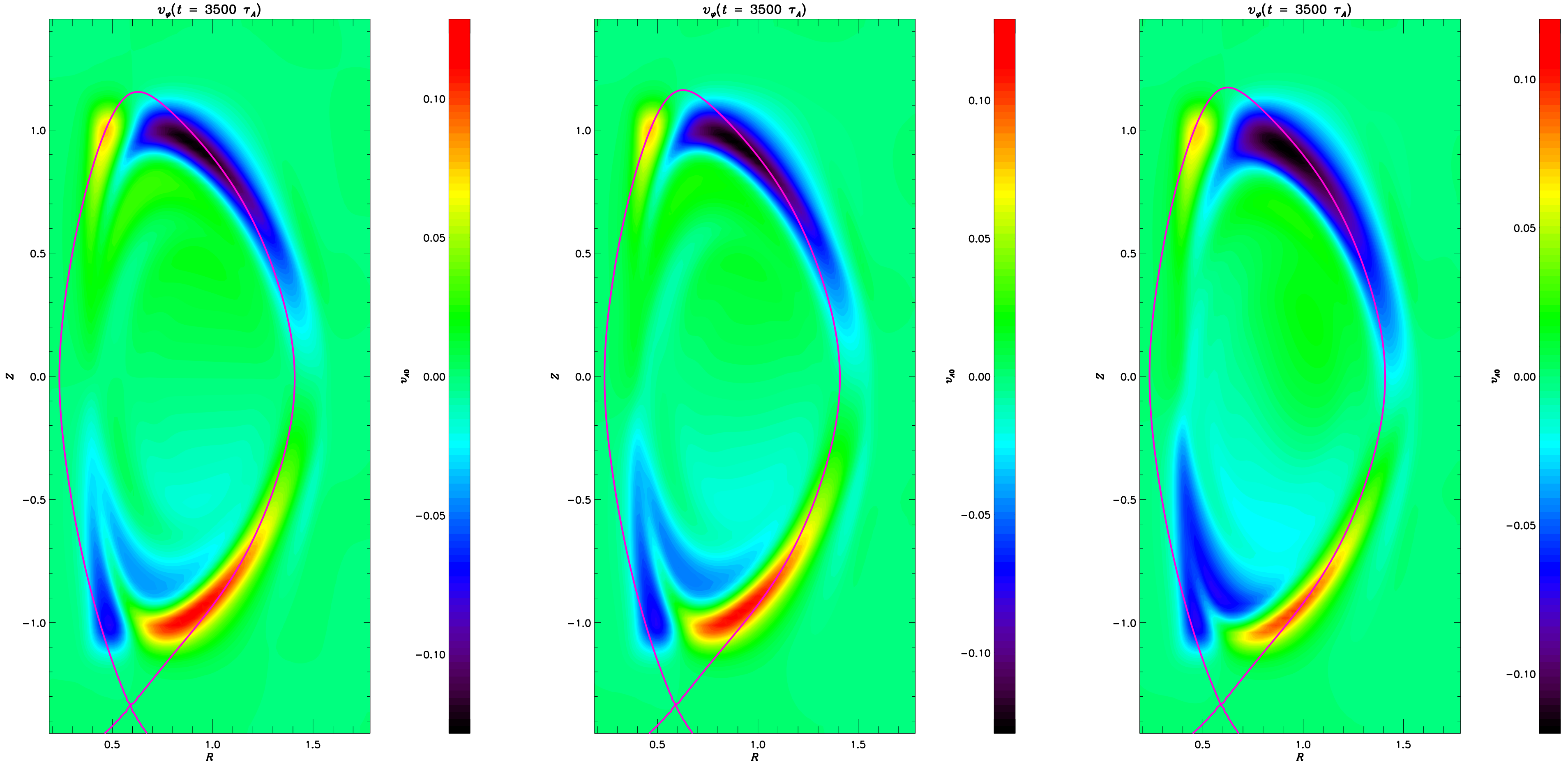




$$\delta t = 5\tau_A$$

Low-S Case: Toroidal velocity

- Flows are extremely strong ($\sim$ 100 km/s)
- Two-fluid/gyroviscous effects do not make much difference in this case

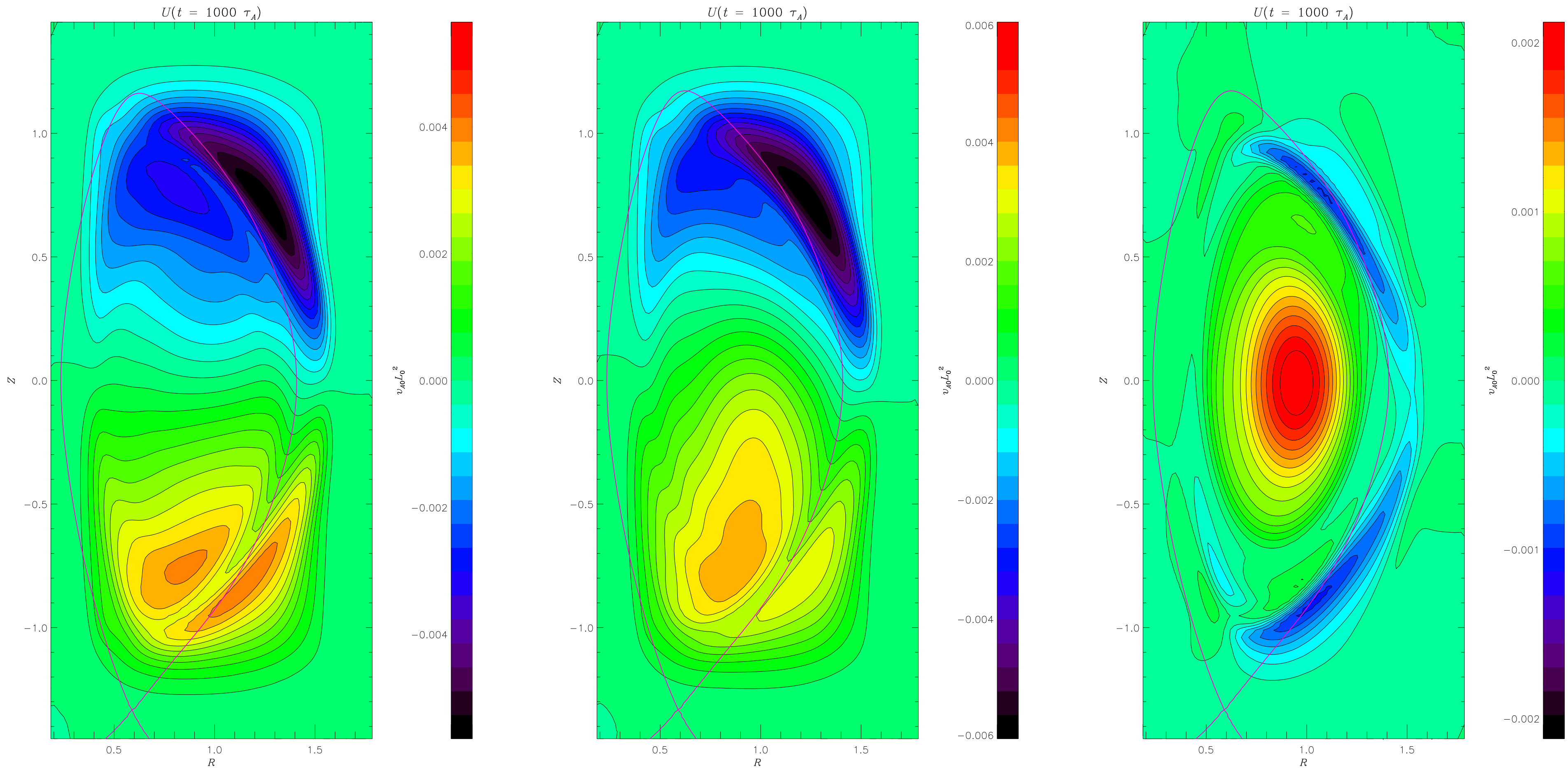


One fluid

Two fluid; no GV

Two fluid + GV

Low-S Case: Poloidal velocity



Two fluid; no GV   Two fluid + GV   Difference

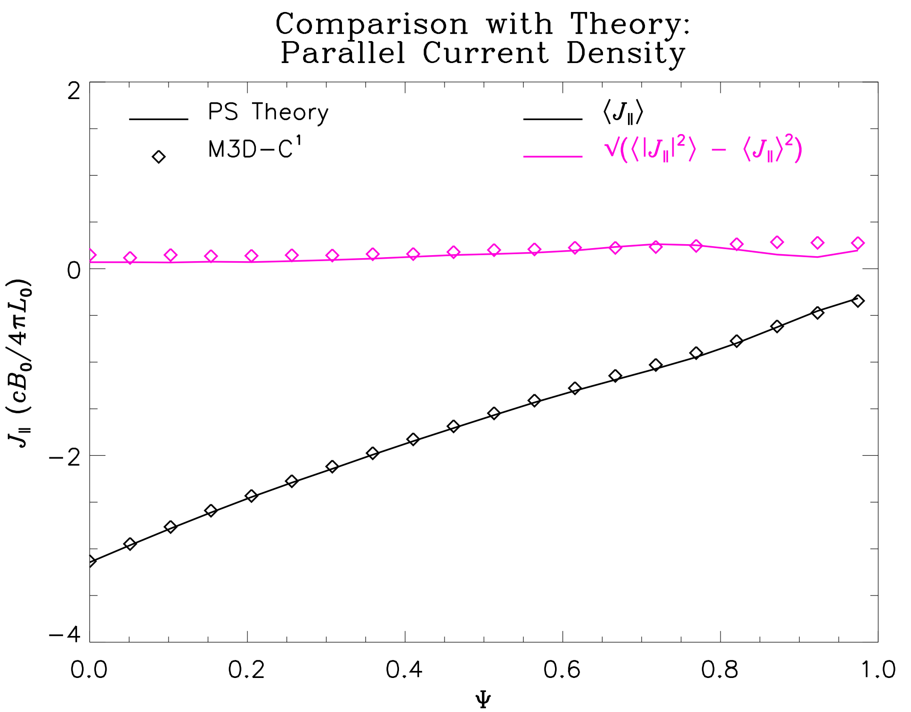
- Poloidal flows in core $\sim 5 \times 10^{-4}\ v_A$.

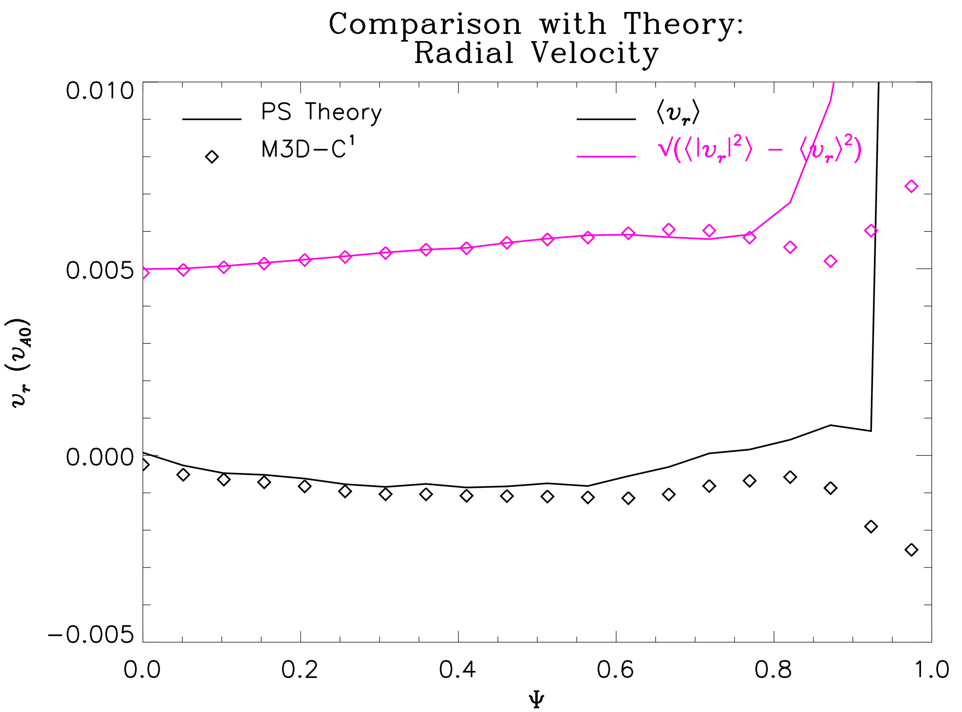
Comparison with Theory

- A steady-state satisfying $\nabla p = \mathbf{J} \times \mathbf{B}$ to lowest order will have:

$$J_\parallel = -\frac{I}{\langle B^2 \rangle} \left[\frac{V_L}{2\pi\eta} \left\langle \frac{1}{R^2} \right\rangle + p' \left(1 - \frac{\langle B^2 \rangle}{\langle B^2 \rangle} \right) \right]$$

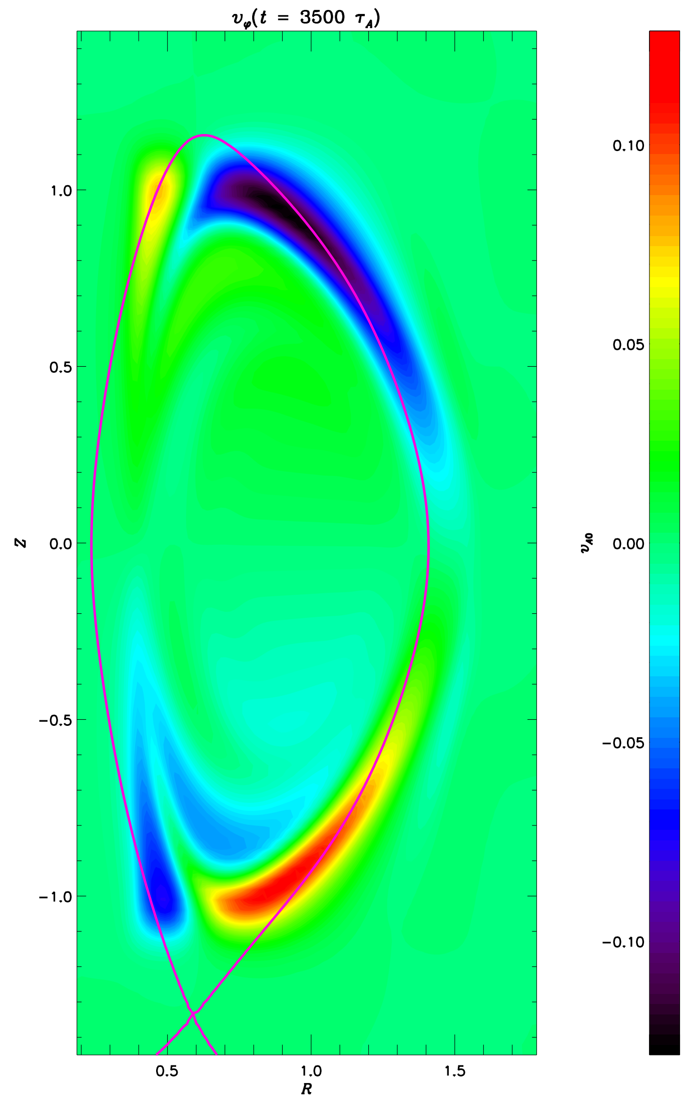
$$\mathbf{v} \cdot \nabla\psi = -\frac{V_L}{2\pi} \left(1 - \frac{\langle B_\varphi^2 \rangle}{\langle B^2 \rangle} \right) - \eta p' R^2 \left(1 - \frac{B_\varphi^2}{\langle B^2 \rangle} \right)$$





Comparison with Aydemir [6]

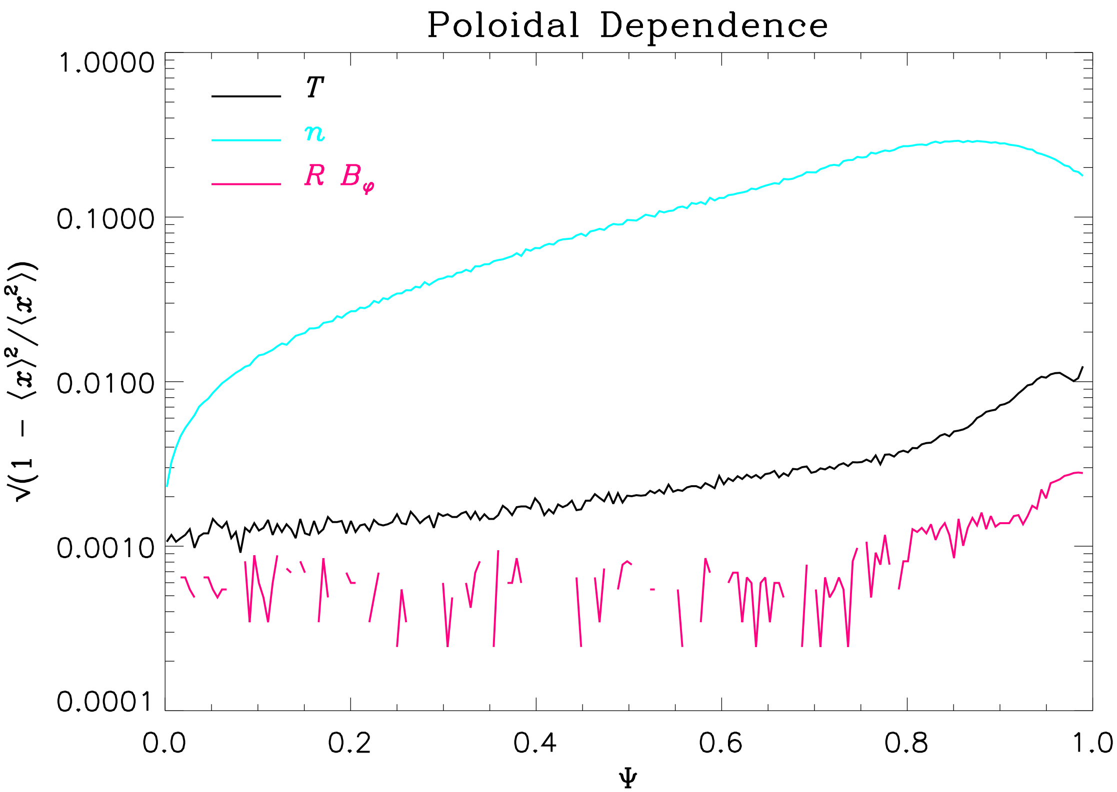
- Our results agree with Aydemir's observations that:
  - Toroidal flows are greatest near the x-points, reaching ~ 10 km/s.
  - Flipping the sign of B_φ flips the sign of the toroidal velocity, but not poloidal velocity.
  - Total viscous torque oppositely directed to toroidal flow at the divertor x-point.



- These facts do not change at high β.
- Bootstrap current is not necessary to achieve these flows.

Poloidal Dependence

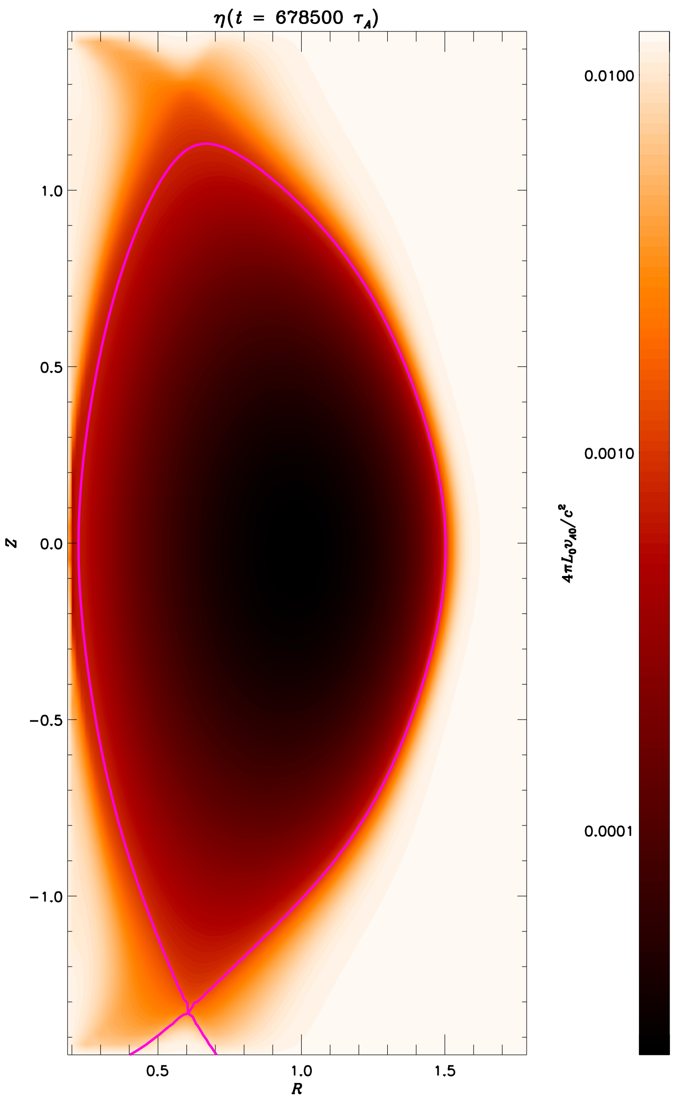
- T and RB_φ are good flux quantities; n is not.
- p has essentially the same poloidal dependence as n.
- The parallel velocity (Φ) and angular momentum (Ω) functions of Guazzotto *et al.* [7] are not found to be approximately constant on flux surfaces.

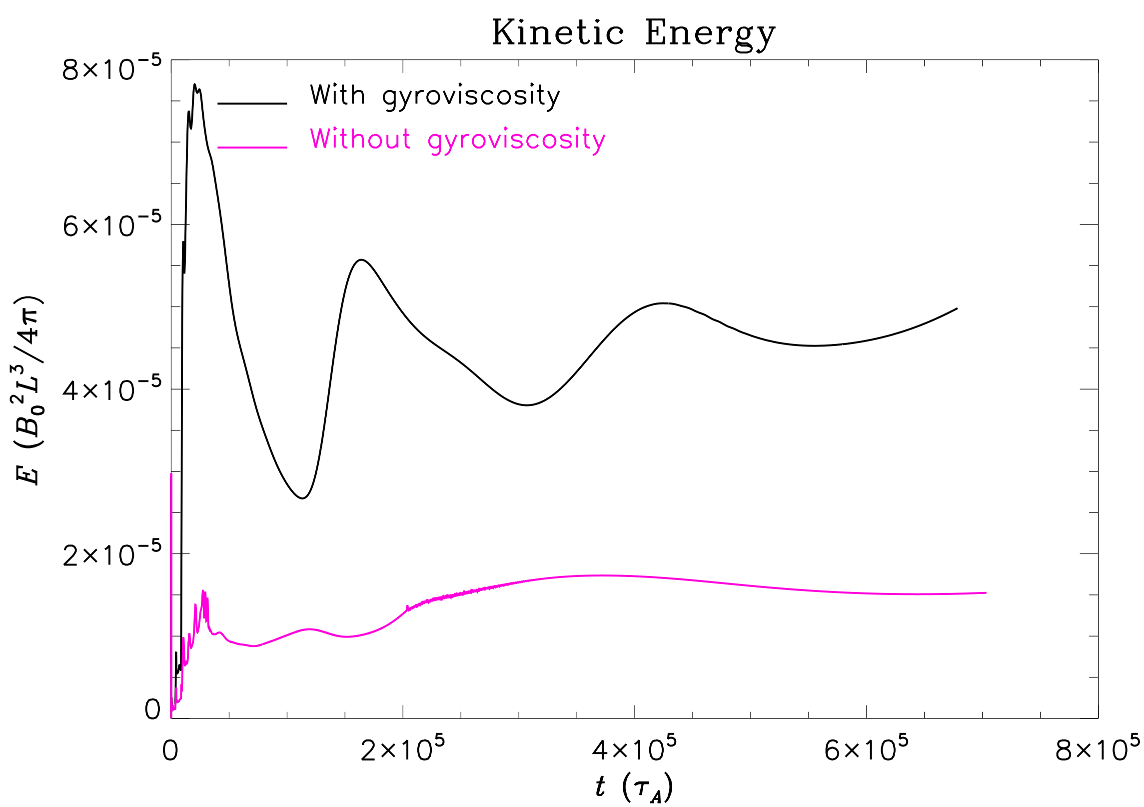


High-S Case: No Steady State (yet)

- The "high-S" cases were run with the following parameters:

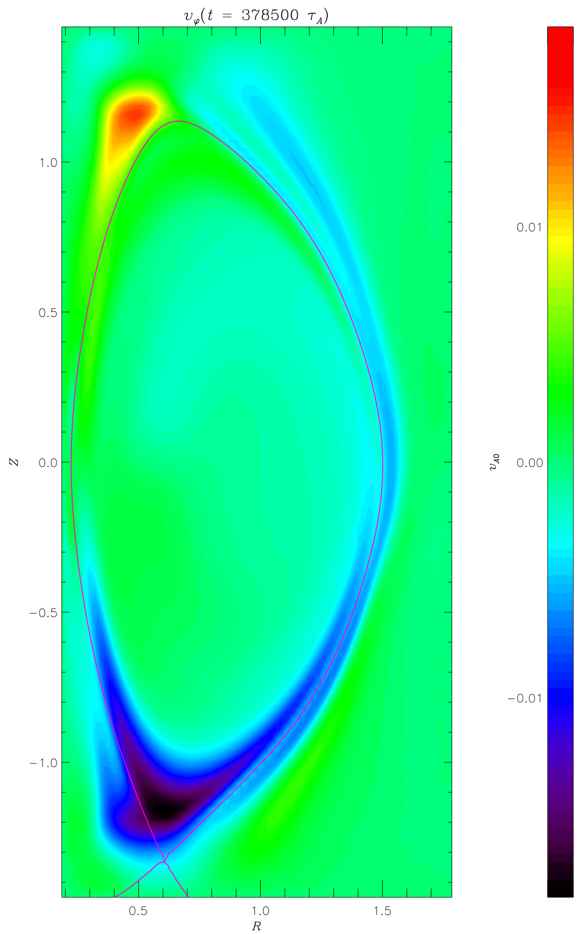
$$S_0 \sim 10^5, \quad S_e \sim 10^2, \quad \beta_0 \sim 13\%, \quad Re \sim 10^5$$

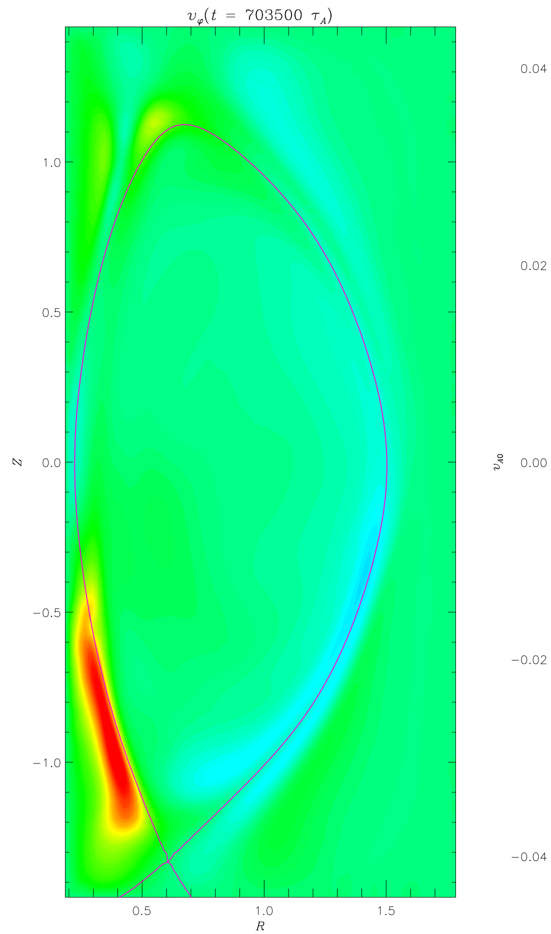


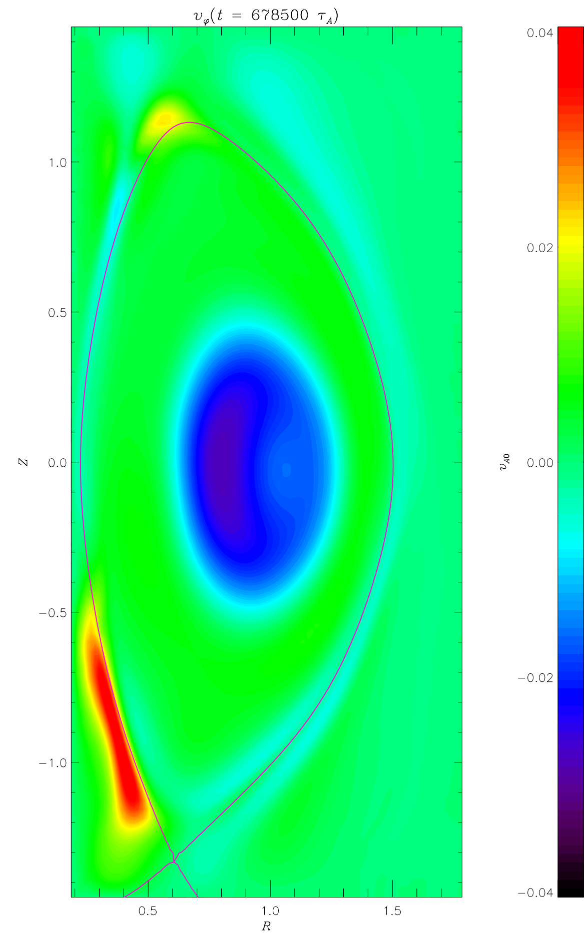


$$\delta t = 200\tau_A$$

High-S Case: Toroidal Velocity







One fluid

$v_\varphi \approx 0.02 v_A \approx 20\mathrm{km/s}$

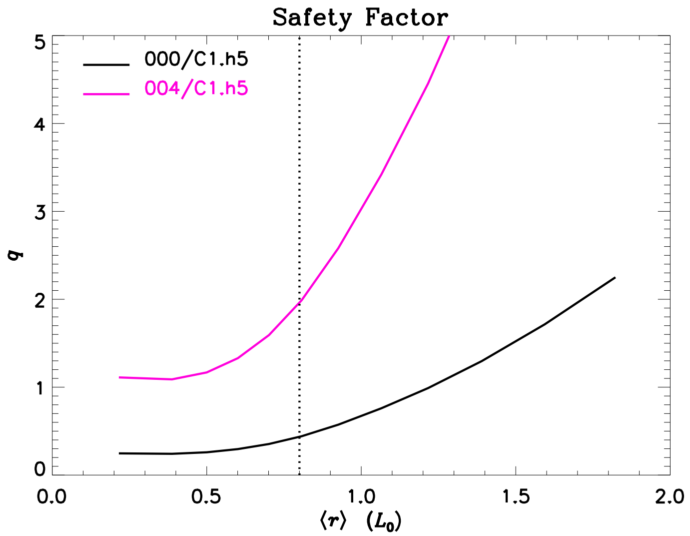
Two fluid; no gyro

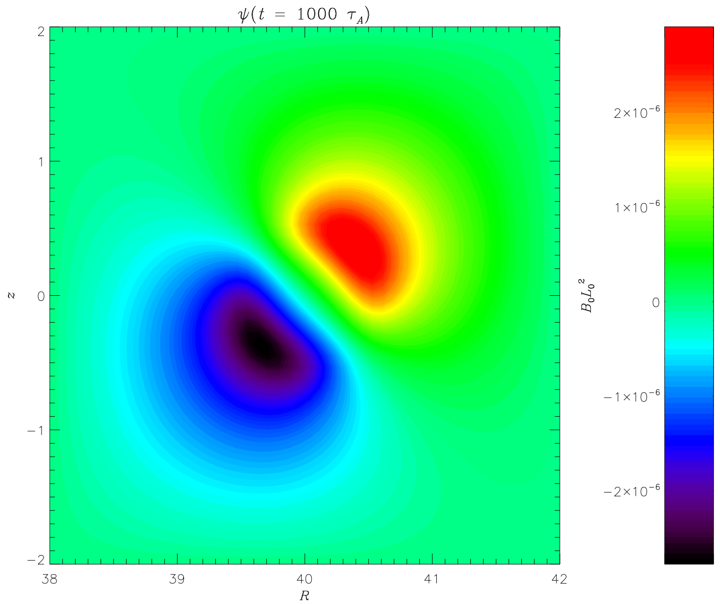
Two fluid + gyro

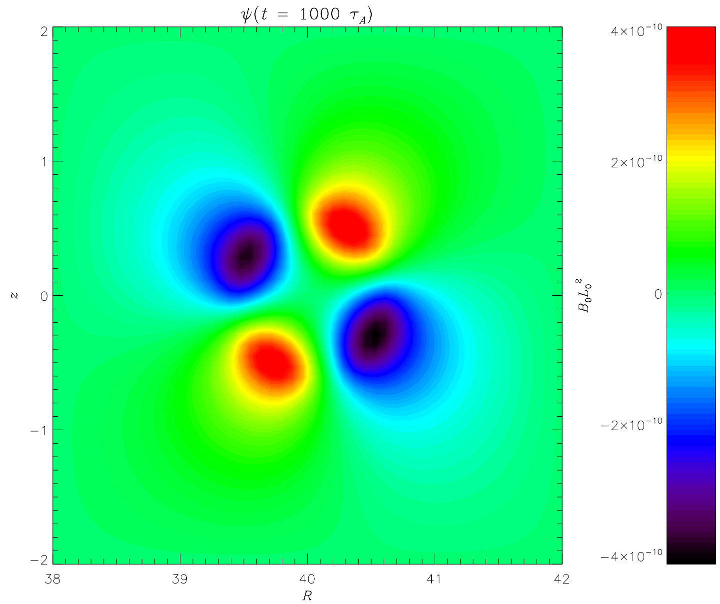
$v_\varphi \approx 0.03 v_A \approx 30\mathrm{km/s}$

Stability

- We are adding the capability for linear nonaxisymmetric stability calculations.
- This capability is implemented for the two-field model.







Conclusions

- We have been able to obtain self-consistent steady-states of the extended-MHD equations for realistic plasma configurations with free boundaries.
- The flows observed in the steady-states are in relatively good agreement with Pfirsch-Schlüter theory.
- Gyroviscosity leads to parallel flows in the core.
- The strong flows near the x-points observed by Aydemir dominate in low-S case, but two-fluid effects and gyroviscosity dominate in high-S case.

Future Work

- To get to steady-state with both realistic resistivity and viscosity using this method will require more work (more spatial resolution? different time step?). There is no guarantee that a steady state exists.
- We need better modeling of edge/SOL quantities for realistic simulations.
  - Density sink; realistic boundary shapes
  - Pedestal modeling for H-mode
- Need some model for neoclassical parallel viscosity.
- Coupling to realistic transport models
- We are moving forward with linear 3D capability; thinking about nonlinear 3D capability.

References

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