Absence of Complete Finite-Larmor-Radius Stabilization in Extended MHD

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Previous Theory

Extended MHD models the dominant finite Larmor radius (FLR) effects.

FLR fully stabilizes g-mode when $\omega_* \geq 2\Gamma_{\mathrm{MHD}}$, where $\omega_* \propto k_\perp$ (Roberts and Taylor [62]).

New Results

FLR stabilization by gyroviscosity or 2-fluid effects alone in extend MHD may not be ubiquitous.

Abstract

It is well known that the kinetic effects due to finite Larmor radius (FLR) are able to stabilize the pure interchange mode in a weakly unstable plasma under gravity [1]. The dominant FLR stabilization effects on the interchange instability can be retained by taking into account the ion gyroviscosity or the generalized Ohm's law in an extended MHD model [2-4]. However, recent simulations and theoretical calculations indicate that the complete FLR stabilization of the pure interchange mode may not be attainable by the ion gyroviscosity or the two-fluid effect alone in the framework of extended MHD [5,6]. For a class of plasma equilibria in certain finite-β or non-isentropic regimes, the critical wavenumber for the complete FLR stabilization tends toward infinity, and the FLR stabilization effects are eliminated.

[1] M. N. Rosenbluth, N. A. Krall, and N. Rostoker, *Nucl. Fusion* Suppl. Pt. **1**, 143 (1962).

[2] K. V. Roberts and J. B. Taylor, *Phys. Rev. Lett.* **8**, 197 (1962).

[3] J. D. Huba, *Phys. Plasmas* **3**, 2523 (1996).

[4] N. M. Ferraro and S. C. Jardin, *Phys. Plasmas* **13**, 092101 (2006).

[5] D. D. Schnack and S. E. Kruger, private communication (2007).

[6] P. Zhu, D. D. Schnack, F. Ebrahimi, E. G. Zweibel, M. Suzuki, C. C. Hegna, and C. R. Sovinec, Report UW-CPTC 07-8 (2007).

Outline

1. Introduction
2. FLR due to gyroviscosity
3. FLR due to 2-fluid effects
4. FLR due to both effects
5. Summary

Major Updates since Last APS

1. Major difference from others' work [e.g. Ferraro and Jardin, 2006]
2. Relevance to physically valid regime of extended MHD ($k_y d_i \ll 1$)
3. Relevance to low β, fusion plasma regime

Motivation (Schnack and Kruger [07])

From: Dalton Schnack <schnack@wisc.edu>
Sent: Wednesday, July 25, 2007 5:28 pm
To: Nimrod Developer announcements <nimrod-devel@nimrodteam.org>
Cc: Steve Jardin <jardin@pppl.gov>
Subject: [Nimrod-devel] GV benchmarking with 3.2.4

Colleagues,

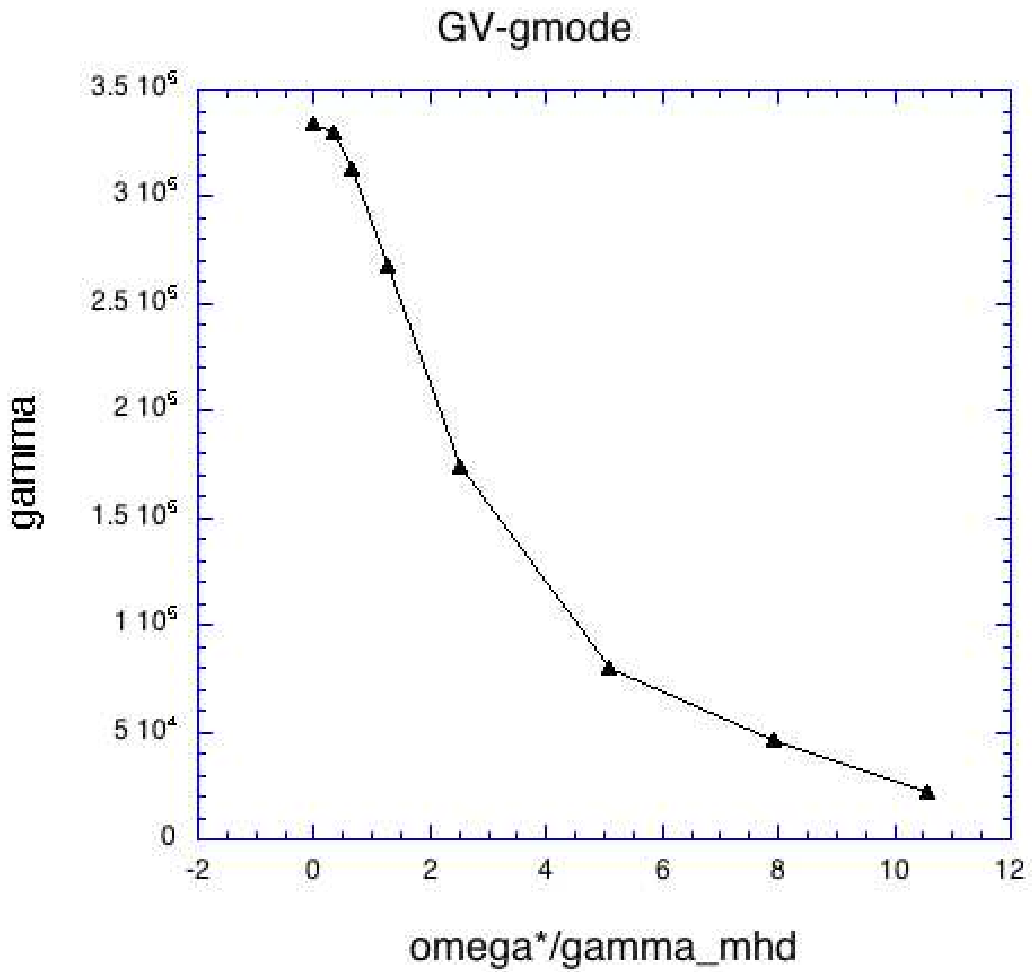
As a result of my recent visit to Boulder and collaboration with Scott K., I have concluded that my previous validation tests on the g- mode with gyro-viscosity (NOT Hall) were buggy and should be completely discarded. Scott and I found errors in the equilibrium specification for these cases. This has been fixed and the cases have been repeated with nimrod3.2.4. The results are appended. Previously stabilization occurred at $\omega_*/\Gamma_{\mathrm{MHD}} \sim 1.67$. Now, as you can see, the mode is never completely stabilized with GV alone. I think Scott confirmed that identical results for a single case were obtained with the latest version of nimuw. As part of our debugging, Scott and I went over the GV coding with a fine tooth comb and, to be best of our knowledge, it is coded correctly.

Dalton

Growth rate remains nonzero when ω_* well above $2\Gamma_{\mathrm{MHD}}$

(Schnack and Kruger [07])

$$B = 6.0$$
$$g = 10^{12}$$
$$n = 2.0 \times 10^{20}$$
$$\beta = 2\mu_0 p/B^2 = 1.0$$
$$p = 1.4323944 \times 10^6$$
$$c_s = 5.974138 \times 10^6$$
$$\Omega_i = 2.87507603 \times 10^8$$
$$k_{2\mathrm{fl}} = 353$$
$$k_{\mathrm{gyr}} = 203$$
$$k_y = 2094$$
$$k_y L_n = 20943$$
$$k_y d_i = 30$$
$$d_i/L = 1.47 \times 10^{-3}$$
$$\Gamma_{\mathrm{MHD}} = 3.35761 \times 10^5$$
$$V_A = 6.544340 \times 10^6$$



A Revisit of g-mode Dispersion in Extended MHD

- Extended MHD: gyroviscosity $\boldsymbol{\pi}$ and 2-fluid Ohm's law:

(1) $$\frac{d\mathbf{u}}{dt} = -\nabla p + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g} - \nabla \cdot \boldsymbol{\pi}_i$$

(2) $$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \frac{1}{ne} (\mathbf{J} \times \mathbf{B} - \nabla p_e)$$

(3) $$(\pi_i)_{xx} = -(\pi_i)_{yy} = -\frac{p_i}{2\Omega} \left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right)$$

(4) $$(\pi_i)_{xy} = (\pi_i)_{yx} = \frac{p_i}{2\Omega} \left(\frac{\partial u_x}{\partial x} - \frac{\partial u_y}{\partial y} \right)$$

- Equilibrium: $d[p(x) + B(x)^2/2]/dx = \rho(x) g$
- Pure interchange perturbation: $\mathbf{u} = [u_x(x)\mathbf{e}_x + u_y(x)\mathbf{e}_y] e^{ik_y y - i\omega t}$
- Local approximation orderings: $k_y L_x \sim \epsilon$, $k_y d_i \sim \delta$, $u_y \sim \epsilon u_x$, $\epsilon \ll 1$, where $L_x = (d \ln /dx)^{-1}$, $d_i = v_{Ti}/\Omega$.

FLR stabilization due to gyroviscosity alone

(5)
$$\omega^2 + \omega_* \omega + \Gamma_{\rm GYR}^2 = 0$$

where

(6)
$$\omega_* = \frac{\dfrac{k_y \delta}{\Omega}\left[(1+\beta)\dfrac{p'}{\rho} - \dfrac{2+\gamma\beta}{1+\gamma\beta} g\beta\right]}{1 + \dfrac{k_y^2 \delta^2}{4\Omega^2}\dfrac{p}{\rho}\dfrac{\beta}{1+\gamma\beta}}$$

(7)
$$\Gamma_{\rm GYR}^2 = \frac{\Gamma_{\rm MHD}^2}{1 + \dfrac{k_y^2 \delta^2}{4\Omega^2}\dfrac{p}{\rho}\dfrac{\beta}{1+\gamma\beta}}$$

(8)
$$\Gamma_{\rm MHD}^2 = \frac{g^2}{u_A^2(1+\gamma\beta)} - \frac{\rho'}{\rho} g.$$

Here, $\Omega = eB/m_i$, $\beta = \mu_0 p/B^2$, $u_A^2 = B^2/\mu_0 \rho$, γ is the adiabatic index, and $\delta = p_i/p$. Reduces to [RT62] when $\beta \to 0$.

FLR stabilization could be absent in certain finite β regime

In the case of constant magnetic field B, $dp/dx = \rho g$, so that

$$\omega_* = \frac{\frac{k_y \delta g}{\Omega}\left(1 - \frac{\beta}{1+\gamma\beta}\right)}{1 + \frac{k_y^2\delta^2}{4\Omega^2}\frac{p}{\rho}\frac{\beta}{1+\gamma\beta}}$$

FLR stabilization requires

$$\omega_*^2 > 4\Gamma_{\mathrm{GYR}}^2, \tag{9}$$

or

$$\frac{k_y^2\delta^2}{\Omega^2} \geq \frac{4\Gamma_{\mathrm{MHD}}^2}{\left[g^2\left(1-\frac{\beta}{1+\gamma\beta}\right)^2 - \frac{p}{\rho}\frac{\beta}{1+\gamma\beta}\Gamma_{\mathrm{MHD}}^2\right]} \tag{10}$$

As it turns out, in the case studied by Dalton and Scott in NIMROD simulation, the stabilization criterion can not be satisfied for any real k_y when $\beta \geq 0.445857$. The equilibrium in that simulation has a $\beta \sim 0.5$.

FLR stabilization is dependent on the equilibrium type

For isothermal equilibrium ($\nabla T = 0$) [Ferraro and Jardin, 03]

(11) $$\left(\frac{k_c^{\mathrm{FJ}}\tau\beta}{\Omega}\right)^2 = \frac{4\Gamma^2_{\mathrm{MHD}}}{\left[\frac{u_{\mathrm{A}}^2}{L_\rho}(1+\beta) + \frac{2+\gamma\beta}{1+\gamma\beta}g\right]^2 - \frac{u_{\mathrm{A}}^2}{1+\gamma\beta}\Gamma^2_{\mathrm{MHD}}} > 0, \quad \forall\beta$$

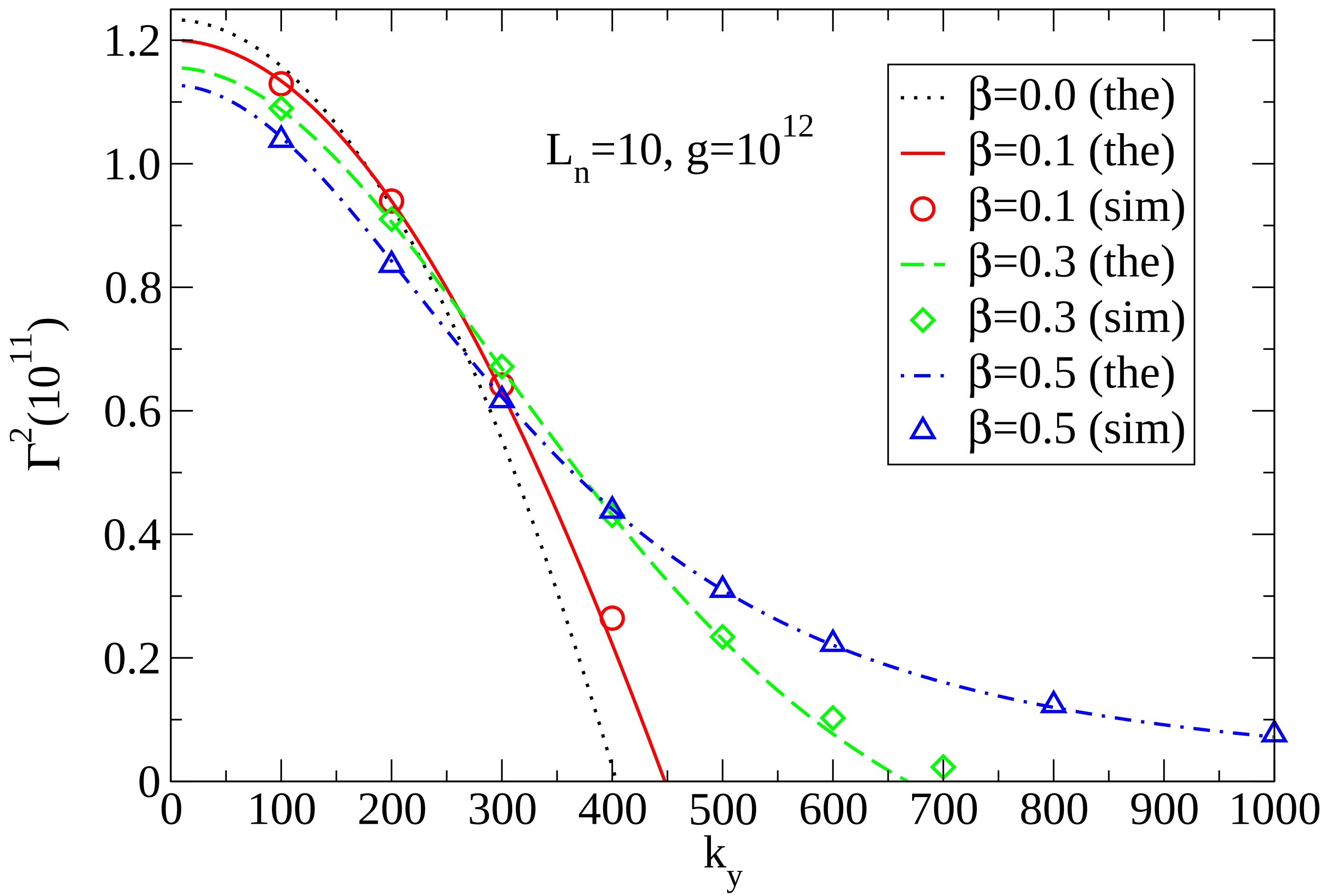
For uniform $\mathbf{B}$ equilibrium ($\nabla\mathbf{B} = 0$)

(12) $$\left(\frac{k_c^{\mathrm{SK}}\tau}{\Omega}\right)^2 = \frac{4(1+\gamma\beta)\Gamma^2_{\mathrm{MHD}}}{u_{\mathrm{A}}^2\frac{g}{L_\rho}(\beta_- - \beta)(\beta_+ + \beta)} < 0, \quad \text{for} \quad \beta > \beta_- > 0$$

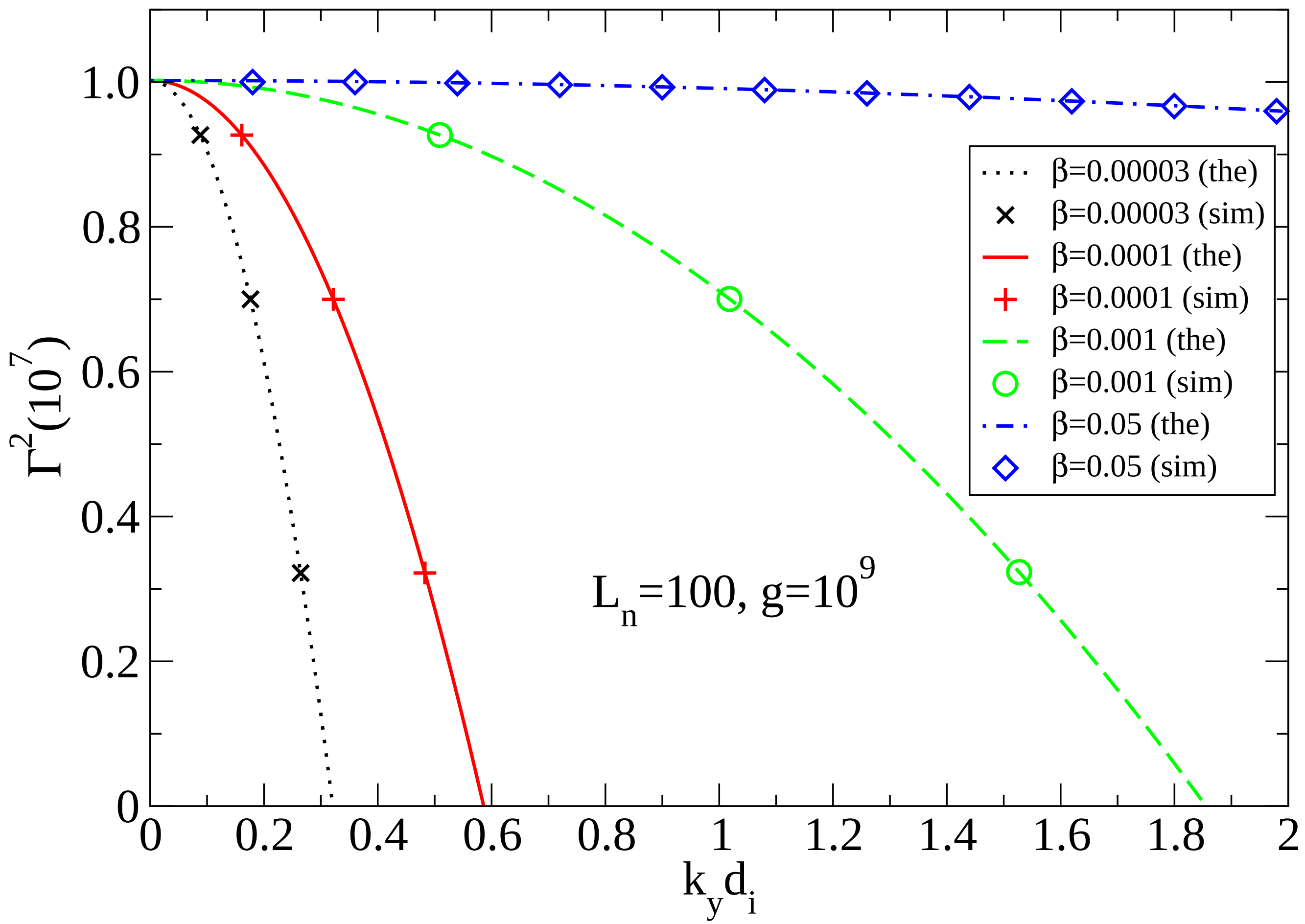
where

(13) $$\beta_\pm = \frac{\sqrt{(2-\gamma)^2 g^4 + \frac{4u_{\mathrm{A}}^2 g^3}{L_\rho}} \pm (2-\gamma)g^2}{2u_{\mathrm{A}}^2\frac{g}{L_\rho}}$$

Comparison between NIMROD simulation and theory ($k_y d_i \gtrsim 1$)



In physically valid regime of extended MHD ($k_y d_i \ll 1$)



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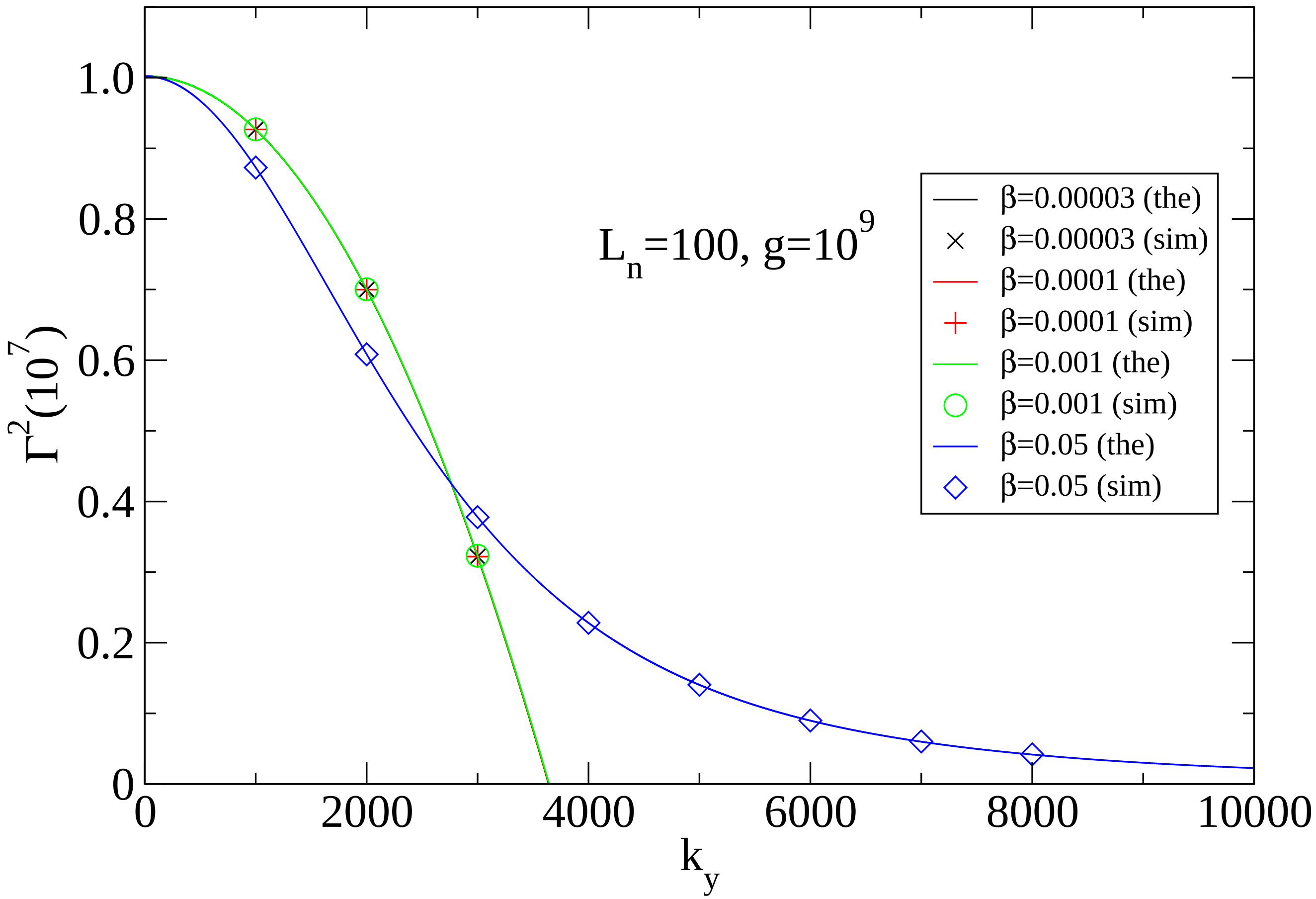

FLR stabilization due to 2-fluid Ohm's law only

(14) $$\omega(\omega^2 + \omega_* \omega + \Gamma^2_{\mathrm{MHD}}) + D = 0$$

where

(15) $$\omega_* = -\frac{k_y \lambda}{\Omega} \frac{1}{1+\gamma\beta} \left[g - \tau \frac{p}{\rho} \left(\ln \frac{p}{\rho^\gamma} \right)' \right]$$

(16) $$D = -\frac{k_y \lambda}{\Omega} \frac{\frac{\rho'}{\rho} g}{1+\gamma\beta} \tau \frac{p}{\rho} \left(\ln \frac{p}{\rho^\gamma} \right)'$$

and $c_s^2 = \gamma p/\rho$, $\tau = p_i/p$, and λ is a tracer multiplier. Reduce to [RT62] in isentropic case when $d \ln (p/\rho^\gamma)/dx = 0$.

When $D \neq 0$, there are 3 eigenmodes. When D is not small, there are situations when there are 2 complex conjugate roots so that there's alway one growing mode for any k_y. In that case, FLR stabilization could be lost.

2-fluid FLR stabilization in weakly unstable regime

In weakly unstable regime $(g/L_\rho)/(\omega\Omega) \ll 1$, $D/\omega \sim 0$, so that
For isothermal equilibrium,

(17)
$$\frac{k_c^2}{\Omega^2} = \frac{4(1+\gamma\beta)^2\Gamma_{\mathrm{MHD}}^2}{\left[\frac{\tau(\gamma-1)u_{\mathrm{A}}^2}{L_\rho}(\beta-\beta_{\mathrm{crit}})\right]^2}$$

where $\beta_{\mathrm{crit}} = gL_\rho/[\tau(\gamma-1)u_{\mathrm{A}}^2]$.
For uniform-B equilibrium,

(18)
$$\frac{k_c^2}{\Omega^2} = \frac{4(1+\gamma\beta)^2\Gamma_{\mathrm{MHD}}^2}{\left[\frac{\tau\gamma u_{\mathrm{A}}^2}{L_\rho}(\beta-\beta_{\mathrm{crit}})\right]^2}$$

where $\beta_{\mathrm{crit}} = (1-\tau)gL_\rho/(\tau\gamma u_{\mathrm{A}}^2)$.

FLR stabilization due to both gyroviscosity and 2-fluid effects

(19) $$\omega(\omega^2 + \omega_* \omega + \Gamma^2_{\mathrm{FLR}}) + D = 0,$$ where

$$\omega_* = \frac{k_y}{\Omega} \frac{\delta \left[(1+\gamma\beta)(1+\beta)\frac{p'}{\rho} - (2+\gamma\beta) g\beta\right] - \lambda \left[g - \tau \frac{p}{\rho}\left(\ln \frac{p}{\rho^\gamma}\right)' + \frac{k_y^2 \delta^2}{4\Omega^2}\frac{p^2}{\rho^2}\frac{\rho'}{\rho}\right]}{(1+\gamma\beta)\left(1 + \frac{k_y^2\delta^2}{4\Omega^2}\frac{p}{\rho}\frac{\beta}{1+\gamma\beta}\right)}$$

$$\Gamma^2_{\mathrm{FLR}} = \Gamma^2_{\mathrm{GYR}} + \frac{k_y^2 \lambda\delta}{\Omega^2}\frac{p}{\rho} \frac{(1+\beta)\left(\tau\frac{p'}{\rho} - g\right)\frac{p'}{p} + \left[(1+\gamma\beta\tau)g - (1+\beta)\gamma\tau\frac{p'}{\rho}\right]\frac{\rho'}{\rho} + \left(\frac{\rho g}{p} - \tau\frac{p'}{p}\right) g\beta}{(1+\gamma\beta)\left(1 + \frac{k_y^2\delta^2}{4\Omega^2}\frac{p}{\rho}\frac{\beta}{1+\gamma\beta}\right)}$$

$$D = -\frac{k_y \lambda}{\Omega} \frac{\frac{\rho'}{\rho} g\tau \frac{p}{\rho}\left(\ln\frac{p}{\rho^\gamma}\right)'}{(1+\gamma\beta)\left(1 + \frac{k_y^2\delta^2}{4\Omega^2}\frac{p}{\rho}\frac{\beta}{1+\gamma\beta}\right)}$$

Comparison with previous extended MHD theories

- [Roberts and Taylor, 1962]: First showed FLR stabilization by ion-gyroviscosity and/or 2-fluid effects in low β, incompressible regime.
- [Huba, 1996]: Revisited FLR stabilization by ion-gyroviscosity in low β, incompressible regime; focused on non-local and nonlinear effects.
- [Ferraro and Jardin, 2003]: First extended FLR stabilization by ion-gyroviscosity and/or 2-fluid effects to finite β, compressible regime; mostly focused on isothermal equilibrium.
- [This work]: Demonstrated the absence of the FLR stabilization due to ion-gyroviscosity or 2-fluid effects alone in certain equilibria in finite β, compressible regime.

Summary

- Recent theory calculation explained the absence of complete FLR stabilization by ion-gyroviscosity alone first found in extended NIMROD simulations.
- Previous theory on complete FLR stabilization of pure interchange g-mode [RT62] by gyroviscosity or 2-fluid effects, strictly applies only in low β or isentropic regime.
- In finite β or non-isentropic regime, complete FLR stabilization of pure interchange g-mode may not be attainable by gyroviscosity or 2-fluid effects alone, respectively.
- Finite-β effects on FLR stabilization may not be negligible either for other interchange type of modes, such as the localized interchange mode in sheared configuration, or the ballooning instability in ELMs.