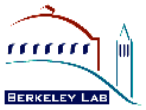
## SAP – factorization based preconditioner and hybrid solver




❖ Sherry Li, Ichitaro Yamazaki , LBNL
Meiyue Shao , student visitor, Fudan Univ., China

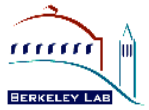
❖ New ILU routines in SuperLU (serial); Maintain stability, efficiency
- **Use dual dropping rules: primary rule is value-based with threshold, secondary dropping to control memory growth**
- **Several versions of MILU (Modified ILU): compensate for the discarded elements by adding them to diagonal**
- **Retain partial pivoting**
- **Retain supernode (dense matrix) structure while dropping, so to maintain good time efficiency**

❖ Competitive with inverse-based multilevel ILU (e.g., ILUPACK, Bollhofer & Saad)
- **Tested 50+ matrices, we can solve more problems**

❖ SuperLU_4.0 to release June 2009
- **xGSISX (ILU driver) + GMRES**

❖ Parallel ILU in the plan

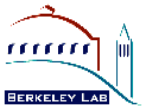
# ILU results (serial)




- M3D-C1 matrices; will test NIMROD matrices once complex version is ready
- Opteron 2.2 GHz (jacquard at NERSC), one processor
- ILU parameters:
  - **drop_tol = 1e-4, gamma = 10, diag_thresh = 0.1**

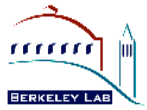
| Problems | order | Nonzeros (millions) | ILU time | ILU fill-ratio | GMRES time | GMRES iters | SuperLU time | SuperLU fill-ratio |
|---|---|---|---|---|---|---|---|---|
| matrix31 | 17,298 | 2.7 m | 8.2 | 2.7 | 0.6 | 9 | 33.3 | 13.1 |
| matrix41 | 30,258 | 4.7 m | 18.6 | 2.9 | 1.4 | 11 | 111.1 | 17.5 |
| matrix61 | 66,978 | 10.6 m | 54.3 | 3.0 | 7.3 | 20 | 612.5 | 26.3 |
| matrix121 | 263,538 | 42.5 m | 145.2 | 1.7 | 47.8 | 45 | fail | - |
| matrix181 | 589,698 | 95.2 m | 415.0 | 1.7 | 716.0 | 289 | fail | - |

## Hybrid solver (parallel)




- Based on Domain Decomposition: interior domains are solved directly, the interface (Schur complement) solved iteratively, with ILU preconditioner
- Implementation: extension of the state-of-the-art software
  - **Use graph partitioning packages (e.g., ParMetis, PT-Scotch) to obtain a hierarchical interface decomposition**
  - **Modify SuperLU_DIST to do parallel ILU**
  - **Purely algebraic, widely applicable, tested for both Fusion and Accelerator problems**
- Numerical properties are independent of number of processors
  - **Earlier work HIPS (Henon & Saad, 2008): number of domains needs to increase with number of processors, leads to large Schur complement and deteriorating convergence**
  - **We are working on exploiting two levels of parallelism: maintain small number of domains, use multiple procs for each domain, multiple procs for interface**
  - **Find balance point for best efficiency of direct and iterative solvers**

## Preliminary results of hybrid solver (parallel)




- Matrix211, M3D-C1, Order 801,378, Nonzeros 129 millions
- Cray XT4 (franklin at NERSC), quad-core Opteron 2.3 GHz
- Phases of the hybrid solver
  - **Stage 1 – domain solves + form Schur complement (S)**
  - **Stage 2 – ILU of S**
  - **Stage 3 – preconditioned iterative solve with PETSc**

| Domains (= Procs) | Stage 1 time | Stage 2 time | GMRES (S) order | time | iters | Hybrid time | Hybrid fill | SuperLU Fill | time |
|---|---|---|---|---|---|---|---|---|---|
| 8 | 179.7 | 50.2 | 11 k | 22. | 23 | 253.2 | 544 m | 1590 m | 209. |
| 32 | 24.2 | 20.6 | 28 k | 7.2 | 25 | 52.2 | 530 m | 1751 m | 74.5 |
| 127 (128) | 7.4 | 12.1 | 59 k | 5.4 | 22 | 25.0 | 568 m | 1957 m | 34.5 |
| 503 (512) | 1.1 | 14.1 | 129 k | 6.4 | 22 | 22.1 | 653 m | 2010 m | 33.7 |

- Currently improve Stage 2: limit numbers of domains, allow multiple procs for each domain, could have smaller, and easier S