Continuum drift kinetic calculations in NIMROD

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E. Held, A. Spencer, J.-Y. Ji, S. Kruger and NIMROD Team

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0.1 First-order DKE in the (ξ,s) velocity variables

Hazeltine's form for the drift kinetic equation (ϵ, μ) :

$$\partial_t f + (\mathbf{v}_{||} + v_D) \cdot \nabla f + \left(\mu \frac{\partial B}{\partial t} + e(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E}\right) \partial_\epsilon f = C.$$

Using pitch-angle,$\xi = v_{||}/v$, and normalized speed, $s = v/v_0$, yields

$$\begin{aligned} \partial_t f + (\mathbf{v}_{||} + v_D) \cdot \nabla f - +(\mathbf{v}_{||} + v_D) \cdot \left[\frac{1-\xi^2}{2\xi} \nabla \ln B \partial_\xi + s \nabla \ln v_0 \partial_s\right] f + \\ \left(\frac{e}{2e_0 s^2}(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E}\right)(s\partial_s f + 2g(\xi)\partial_\xi f) &= C \end{aligned}$$

with general form for drift

$$v_D = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{e_0 s^2}{eB}\left[\mathbf{b}\times\left(\left(1-\xi^2\right)\nabla \ln B + 2\xi^2 \kappa - \frac{v_0 s \xi}{e_0 B}\nabla \times \mathbf{E}\right) + (1-\xi^2)\frac{\mu_0 \mathbf{J}_{||}}{B}\right].$$

0.2 Simplify DKE for benchmark with NEO code.

Order $v_D << v_{||}$ and assume weak (relative to Dreicer) electric field:

$$\partial_t f_0 + \mathbf{v}_{||} \cdot \nabla f_0 - \mathbf{v}_{||} \cdot \left[\frac{1-\xi^2}{2\xi} \nabla \ln B \partial_\xi + s \nabla \ln v_0 \partial_s \right] f_0 \;=\; C(f_0)$$

which is satisfied by stationary Maxwellian with flux functions n and T.
To next order :

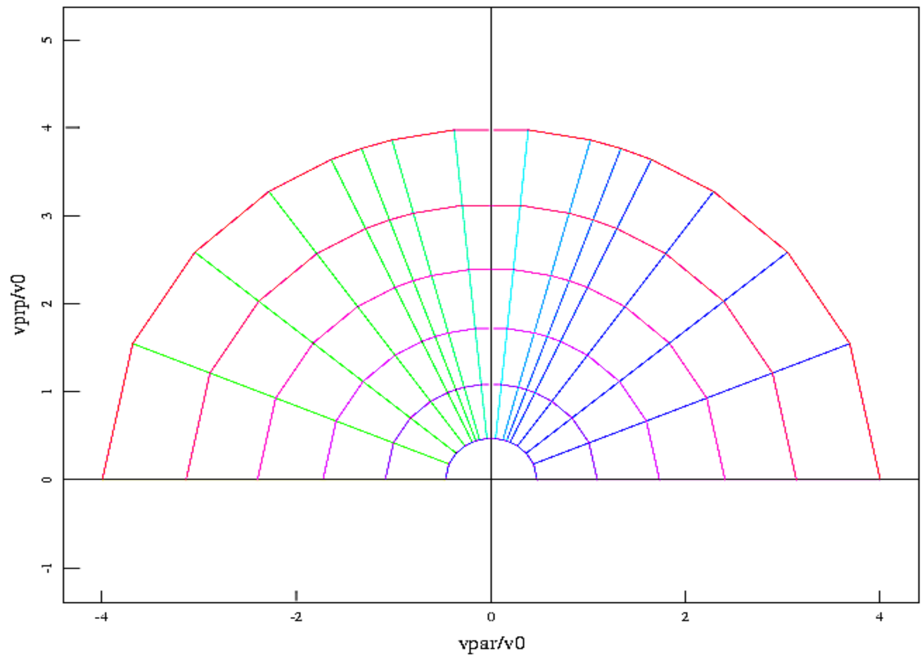
$$\begin{array}{c} \partial_t f_1 + \mathbf{v}_{||} \cdot \nabla f_1 - \left(\mathbf{v}_{||} \cdot \nabla \ln B \right) \dfrac{1-\xi^2}{2\xi} \partial_\xi f_1 \;=\; \\ -\mathbf{v}_D \cdot \nabla f_0 + s v_D \cdot \nabla \ln v_0 \partial_s f_0 - \dfrac{e}{2\epsilon_0 s} \mathbf{v}_{||} \cdot \left(\mathbf{E}^A - \nabla \phi_1 \right) \partial_s f_0 \;+\; C^{aa} + C^{ab} \end{array}$$

Using $g = f_1 - (e\phi_1/T_0) f_0$ yields (compare with Eq. 23 of Belli and Candy, 51 PPCF 2009):

$$\begin{array}{rl} \partial_t g + \mathbf{v}_{||} \cdot \nabla g - \mathbf{v}_{||} \cdot \nabla \ln B \partial_\xi g & = \\ -\mathbf{v}_D \cdot \nabla f_0 + s v_D \cdot \nabla \ln v_0 \partial_s f_0 & + \\ C^{aa} + C^{ab} - \dfrac{e}{2\epsilon_0 s} \mathbf{v}_{||} \cdot \mathbf{E}^A \partial_s f_0 & +(e f_0/T_0) \partial_t \phi_1 \end{array}$$

0.3 Use efficient (ξ, s) velocity grid.

- Use 1D FE grid in pitch angle. In each element $f(\mathbf{r}, t, \xi, s) = \sum_i f_i(\mathbf{r}, t, s)\phi_i(x)$.
- Pitch-angle coefficients, f_i, computed on speed grid determined by quadrature for s part of velocity-space integrals.
- Sample grid: 3 cells in ξ with 5th-order polynomials, 6 speed points = 96 unknowns.



0.4 Solve Spitzer problem to test speed grid.

Solve for perturbed distribution functions, $f_{1e}$and f_{1i}:

$$\partial_t f_{1e} + C(f_{1e}, f_{0e}) + C(f_{0e}, f_{1e}) + C(f_{1e}, f_{0i}) + C(f_{0e}, f_{1i}) = v_{||}(q_e E_{||}/T_0)\, f_{0e}.$$

$$\partial_t f_{1i} + C(f_{1i}, f_{0i}) + C(f_{0i}, f_{1i}) + C(f_{1i}, f_{0e}) + C(f_{0i}, f_{1e}) = v_{||}(q_i E_{||}/T_0)\, f_{0i}.$$

Use full, linearized Coulomb collision operators for electrons and ions :

$$C(f_{1a}, f_{0b}) + C(f_{0a}, f_{1b}) = \sum_k \frac{f_{0a}}{\sigma_k^1} P_1(v_{||}/v) M_{||a}^{1k}(\mathbf{r}, t) \left[\nu_{ab}^{1k,0}(s_a) + \nu_{ab}^{0,1k}(s_a)\right].$$

Test momentum conservation for various speed grids, $s_a = v/v_{Ta}$.

0.5 Speed grid determined by nonclassical quadrature scheme.

Quadrature in $s_a$to compute moments of $f_{1a} = F_a(s_a)P_1(v_{||}/v)$:

$$\begin{aligned} M_{||a}^{1k} &= \frac{4\pi}{3n_a} v_{Ta}^3 \int_0^\infty ds s^3 L_k^{1+1/2}(s^2) F_a(s) \\ &= \frac{4\pi}{3n_a} v_{Ta}^3 \sum_j w_j s_j^3 L_k^{1+1/2}(s_j^2) F_a(s_j) \end{aligned}$$

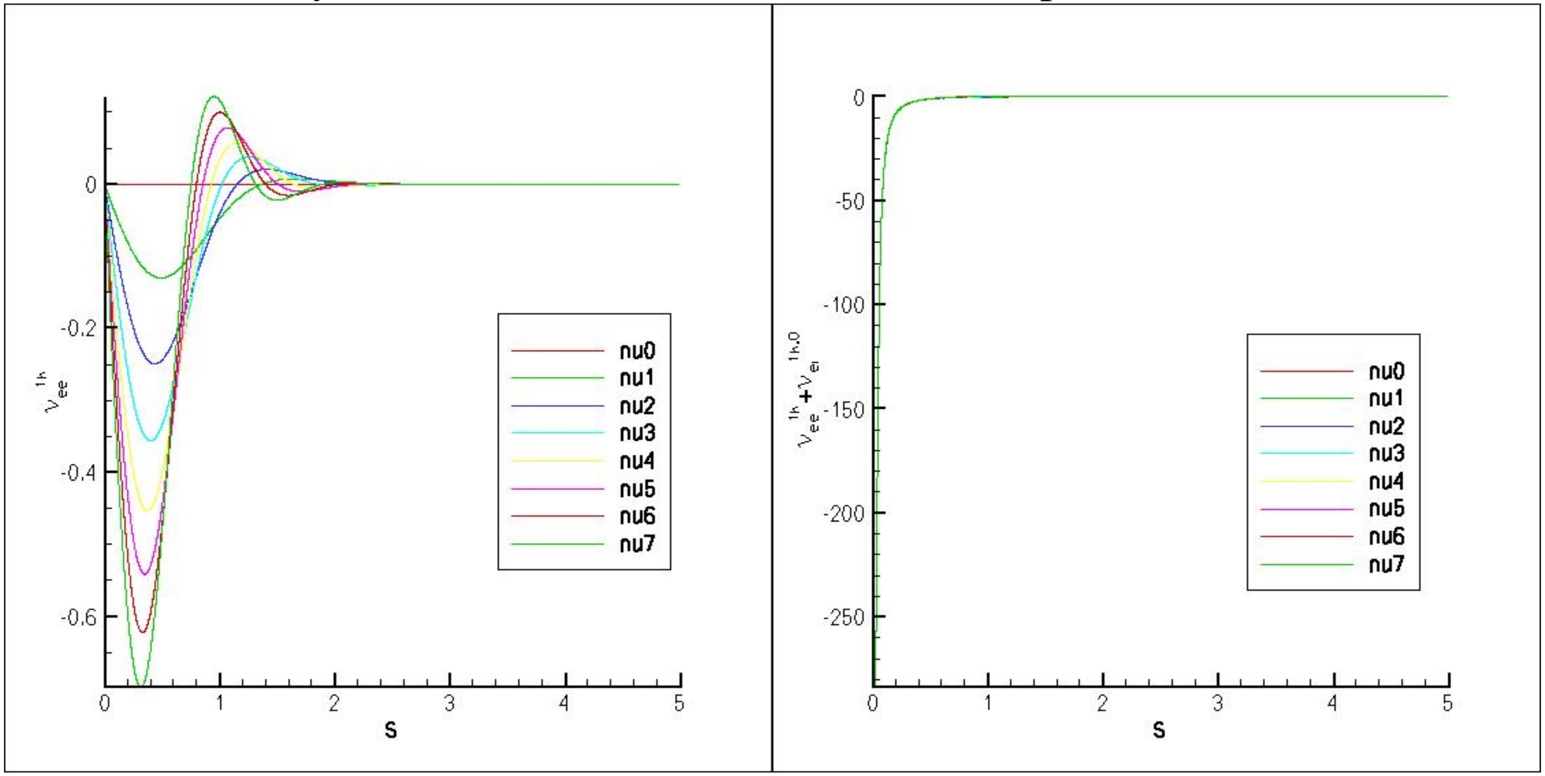
For cases with thermodynamic drives proportional to Maxwellian: $w(s) = s^\alpha exp(-s^2)$ on $s\epsilon[0, \infty)$.

For slowing down distributions of hot particles: $w(s) = s^\alpha/(1+s^3)$ on $s\epsilon[0, s_{max}]$.

0.6 Resolve ν_{ab}'s for momentum conservation.

$C(f_{1a}, f_{0b}) + C(f_{0a}, f_{1b}) = \sum_k \frac{f_{0a}}{\sigma_k^1} P_1(v_{||}/v) M_{||a}^{1k}(\mathbf{r}, t) \left[\nu_{ab}^{1k,0}(s_a) + \nu_{ab}^{0,1k}(s_a)\right].$

e/e only   e/e + e/i test particle



0.7 Results for Spitzer problem

Evolve kinetic equations for 100 electron collision times.

	conductivity	electron momentum	ion momentum
exact	1.9623169	-2.2256440E-4	2.2256440E-4
32 nodes	1.9623200	-2.2256482E-4	2.2244207E-4
8 nodes	1.9624281	-2.2257996E-4	2.1716122E-4
4 nodes	1.9933479	-2.2613467E-4	1.3289914E-4

Conductivity and electron momentum accurately reproduced.

Ion momentum conservation requires more quadrature points.

Introduce momentum restoring concepts into this discrete approach.

0.8 Future Work

1. Experiment with speed grids on Spitzer conductivity and thermalization problems.

2. Improve discrete scheme to exactly conserve density, momentum and energy.

3. Resume NEO and hot particle benchmark calculations.

4. Related work (at Utah State) on continuum solutions of kinetic equations:
   (a) Andy Spencer is developing a Fokker-Planck code using NIMROD's 2D finite-element/Fourier representation for velocity space (will submit JCP paper soon).

   (b) Jeong-Young Ji's higher-order (21 or 29 or higher?) moment equations to be implemented in NIMROD in conjunction with PSI-Center efforts.