Progress on Hybrid Kinetic-MHD simulations in NIMROD

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CEMM Meeting
Austin, Texas
May 1, 2011

Outline

1. Hybrid Kinetic-MHD in NIMROD
   - Overview
   - (1,1) benchmark with M3D
2. Components of $\dot{\delta f}$
   - 3 components
   - configuration space
   - velocity space
3. TAE simulations
   - Antenna Simulations
   - Energetic Particle Simulations

Hybrid Kinetic-MHD Equations

C.Z.Cheng, *JGR*, 1991

- $n_h \ll n_0$, $\beta_h \sim \beta_0$, quasi-neutrality $\Rightarrow n_e = n_i + n_h$
- momentum equation modified by hot particle pressure tensor:

$$\rho\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{U}\right) = \mathbf{J}\times\mathbf{B} - \nabla p_b - \nabla\cdot\underline{\mathbf{p}}_h$$

  - b, h denote bulk plasma and hot particles
  - $\rho, \mathbf{U}$ for **entire plasma**, both bulk and hot particle
- steady state equation $\mathbf{J}_0\times\mathbf{B}_0 = \nabla p_0 = \nabla p_{b0} + \nabla p_{h0}$
  - p_{b0} is scaled to accomodate hot particles
  - assumes equilibrium hot particle pressure is isotropic
- alternative $\mathbf{J}_h$ current coupling possible

Drift Kinetic $\delta \underline{\mathbf{p}}_h$ PIC Moment

- for drift kinetic equations, assume CGL-like

$$\delta \underline{\mathbf{p}}_h = \begin{pmatrix} \delta p_\perp & 0 & 0 \\ 0 & \delta p_\perp & 0 \\ 0 & 0 & \delta p_\parallel \end{pmatrix}$$

- evaluate pressure moment at $\mathbf{x}$

$$\delta \underline{\mathbf{p}}(\mathbf{x}) = \int m \langle \mathbf{v} - \mathbf{V}_h \rangle \langle \mathbf{v} - \mathbf{V}_h \rangle \delta f(\mathbf{x}, \mathbf{v}) d^3 v$$

δf is perturbed phase space density, m mass of particle, and V_h is COM velocity of particles

Drift Kinetic Equation of Motion

- follows gyrocenter in limit of zero Larmour radius
- reduces $6D$ to $4D+1$ $\left[\mathbf{x}(t), v_{\|}(t), \mu = \frac{\frac{1}{2} m v_{\perp}^2}{\|\mathbf{B}\|}\right]$
- drift kinetic equations of motion

$$
\begin{aligned}
\dot{\mathbf{x}} &= v_{\|}\hat{\mathbf{b}} + \mathbf{v}_D + \mathbf{v}_{E\times B} \\
& \quad \mathbf{v}_D = \frac{m}{eB^4}\left(v_{\|}^2 + \frac{v_{\perp}^2}{2}\right)\left(\mathbf{B} \times \nabla \frac{B^2}{2}\right) + \frac{\mu_0 m v_{\|}^2}{eB^2}\mathbf{J}_{\perp} \\
& \quad \mathbf{v}_{E\times B} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} \\
m\dot{v}_{\|} &= -\hat{\mathbf{b}} \cdot (\mu \nabla B - e\mathbf{E})
\end{aligned}
$$

Slowing Down Distribution for Hot Particles

- slowing down distribution function $f_{eq} = \dfrac{P_0 \exp(\frac{P_\zeta}{\psi_0})}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}$
- $P_\zeta = g\rho_\parallel - \psi$ canonical toroidal momentum, ε energy, ψ_p poloidal flux, ψ_0 gradient scale length, ε_c critical energy
- the evolution equation for δf

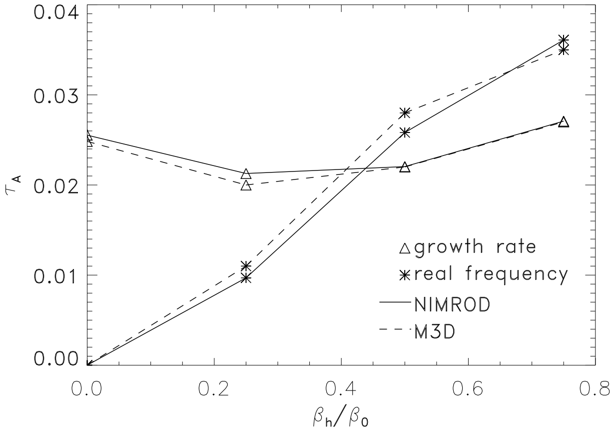
$$\begin{aligned}
\dot{\delta f} &= f_{eq}\left\{\frac{mg}{e\psi_0 B^3}\left[\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\delta\mathbf{B}\cdot\nabla B - \mu_0 v_\parallel \mathbf{J}\cdot\delta\mathbf{E}\right]\right. \\
&\quad \left. + \frac{\delta\mathbf{v}\cdot\nabla\psi_p}{\psi_0} + \frac{3}{2}\frac{e\varepsilon^{1/2}}{\varepsilon^{3/2}+\varepsilon_c^{3/2}}\mathbf{v}_D\cdot\delta\mathbf{E}\right\} \\
\mathbf{v}_D &= \frac{m}{eB^3}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)(\mathbf{B}\times\nabla B) + \frac{\mu_0 m v_\parallel^2}{eB^2}\mathbf{J}_\perp \\
\delta\mathbf{v} &= \frac{\delta\mathbf{E}\times\mathbf{B}}{B^2} + \mathbf{v}_\parallel\cdot\frac{\delta\mathbf{B}}{B}
\end{aligned}$$

Summary of PIC Capabilities

- tracers, linear, (nonlinear)
- two equations of motion
  - drift kinetic ($v_{\|}, \mu$), Lorentz force ($\vec{\mathbf{v}}$)
- multiple spatial profiles - loading in **x**
  - proportional to MHD profile, uniform, peaked gaussian
- multiple distribution functions - loading in **v**
  - slowing down distribution, Maxwellian, monoenergetic
- room for growth
  - improved parallelization
  - efficient high order implementation
  - develop multispecies option, e.g. drift+Lorentz
  - full $f(\mathbf{z})$ PIC
  - numeric representation of $f_{eq}(\vec{\mathbf{x}}, \vec{\mathbf{v}})$
    - e.g. load experimental phase space profiles for evolution of δf
  - kinetic closure

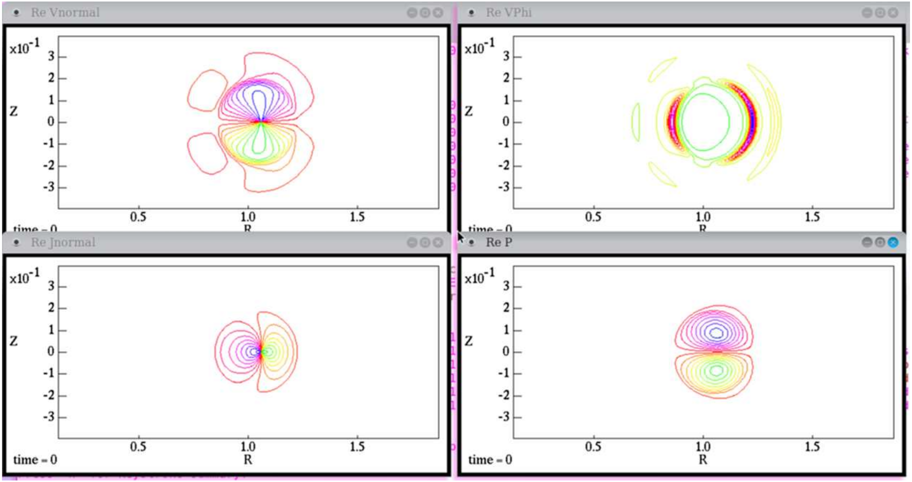
Benchmark of Drift Kinetic (1, 1) Kink With M3D

C. C. Kim, *PoP* **15** 072507 (2008)



circular,
monotonic q,
$q_0 = .6$, $q_a = 2.5$,
$\beta_0 = .08$,
$R/a = 2.76$,
$E_{hmax} = 10KeV$,
dt=1e-7, $\tau_A = 1.e6$

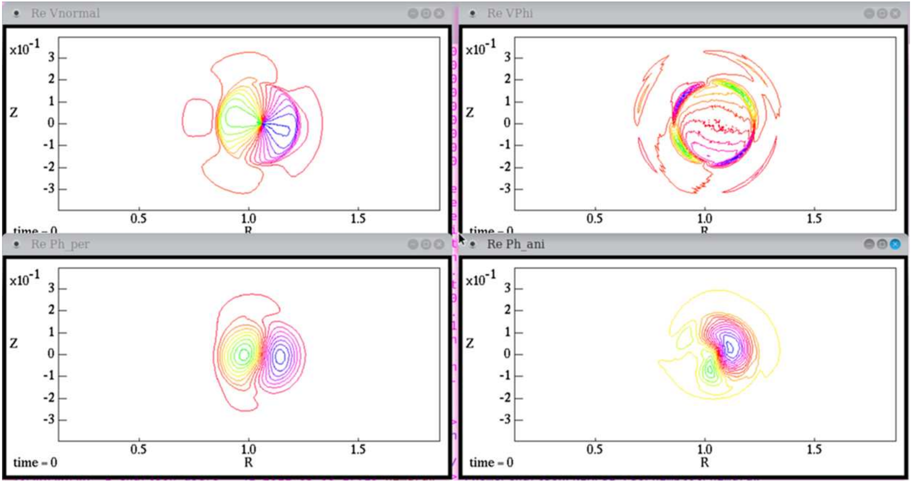
$n = 1$ Contour of Ideal Kink



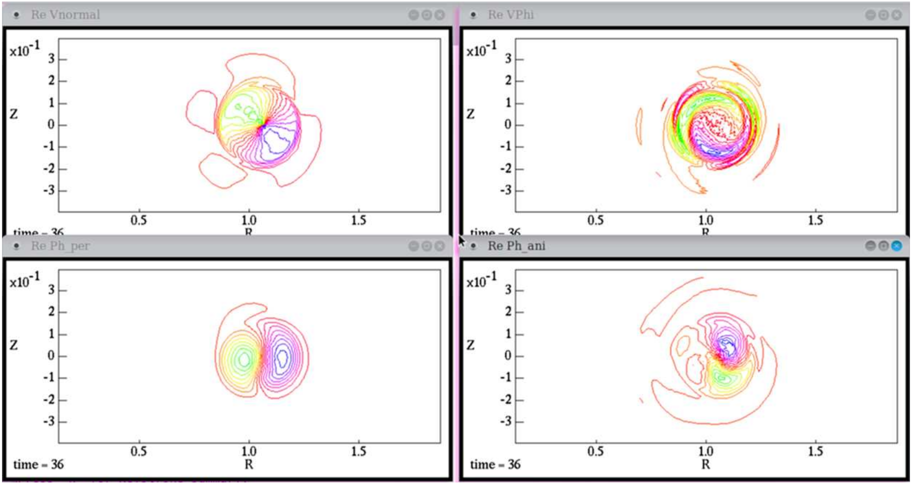
$n = 1$ Contour of Ideal Kink

- δV_{normal} shows (downward) tilt of $q \leq 1$ volume
- δV_{ϕ} unipolar flow (into screen) at $q = 1$ surface
    - tangential flow similar (m=1)
- $\delta J_{normal} \times B_{0\phi}$ consistent with δV_{normal}
    - $B_{0\phi}$ is out of screen
    - B_{0pol} counter clockwise
- δp build-up consistent with δV_{normal}

$n = 1$ Contour $\beta_{frac} = .25$

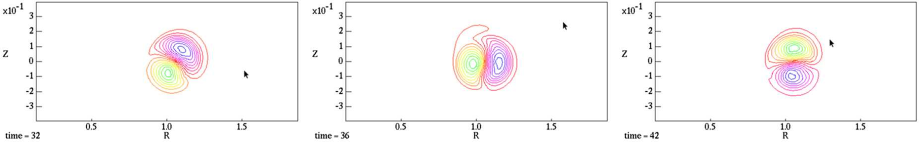


$n = 1$ Contour $\beta_{frac} = .75$

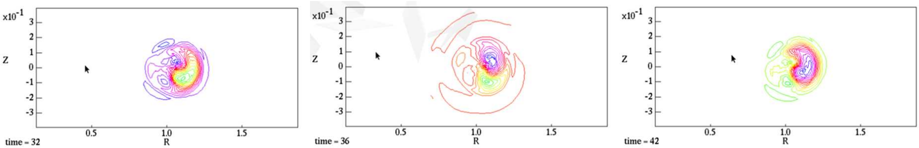


Evolution of $n = 1$ $\delta \mathbf{p}_{hot}$

$\delta p_{\perp hot}$ $t' = 0$ $\qquad$ $t' \simeq .2\omega_h$ $\qquad$ $t' \simeq .4\omega_h$

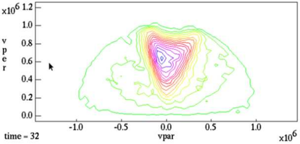


$\delta \Delta p_{hot}$ $t' = 0$ $\qquad$ $t' \simeq .2\omega_h$ $\qquad$ $t' \simeq .4\omega_h$

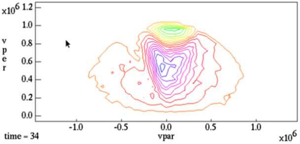


Evolution of $n = 1\ \delta f(v_\parallel, v_\perp)$

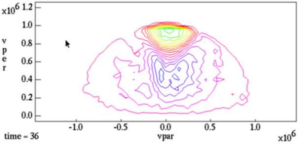
$t' = 0$



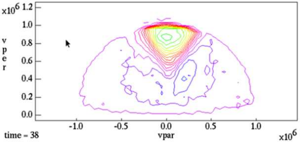
$t' \simeq .1\omega_h$



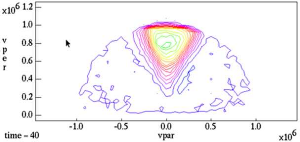
$t' \simeq .2\omega_h$



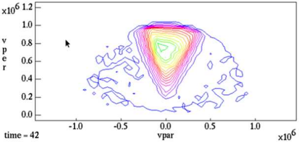
$t' \simeq .25\omega_h$



$t' \simeq .35\omega_h$



$t' \simeq .4\omega_h$



Observations of Moments of $\delta f_{n=1}$

- energetic particles cause real frequency response
- $\delta\Delta p$ concentrated on outboard side
- topology of toroidal flow changes
  - ? particular to δV_ϕ
  - ? signature of fishbone
- does not (dramatically) change topology of other field quantities
- does change phase relation, e.g. δV_{norm} and δp
- concentration of $\delta f_{n=1}(v_{||}, v_\perp)$ activity to trapped cone
  - ? what is origin and role of the asymmetry
  - ? what is role of trapped vs. passing particles
  - ? what is the structure in the trapped cone

3 Components of $\dot{\delta f}$

$$
\begin{aligned}
\frac{1}{f_{eq}}\dot{\delta f} &= \frac{mg}{e\psi_0 B^3}\left[\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\delta\mathbf{B}\cdot\nabla B - \mu_0 v_\parallel \mathbf{J}\cdot\delta\mathbf{E}\right] (1) \\
&+ \frac{\delta\mathbf{v}\cdot\nabla\psi_p}{\psi_0}(2) + \frac{3}{2}\frac{e\varepsilon^{1/2}}{\varepsilon^{3/2}+\varepsilon_c^{3/2}}\mathbf{v}_D\cdot\delta\mathbf{E}(3) \\
\mathbf{v}_D &= \frac{m}{eB^3}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)(\mathbf{B}\times\nabla B) + \frac{\mu_0 m v_\parallel^2}{eB^2}\mathbf{J}_\perp \\
\delta\mathbf{v} &= \frac{\delta\mathbf{E}\times\mathbf{B}}{B^2} + \mathbf{v}_\parallel\cdot\frac{\delta\mathbf{B}}{B}
\end{aligned}
$$

- $\dot{\delta f}$ has 3 components
  1. $g\rho_\parallel$ - kinetic term
  2. $v_{E\times B}\cdot\nabla\psi$ - radial particle flux
  3. $v_D\cdot\mathbf{E}$ - energy exchange
- examine δf moments of components, e.g. $\int(g\rho_\parallel)\delta f d\mathbf{v}$
- convolution is axisymmetric, i.e. $n=0$ and static
- $n=2$ *also exists but have not examined yet*

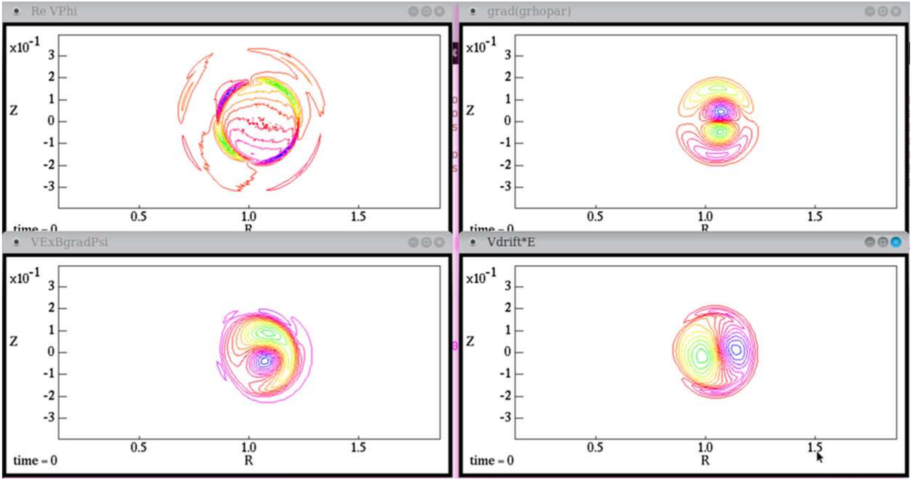
0D Comparison

- normalized amplitude (wrt $v_{E\times B}\cdot\nabla\psi$ - dominant component)
- %(passing/trapped) contribution

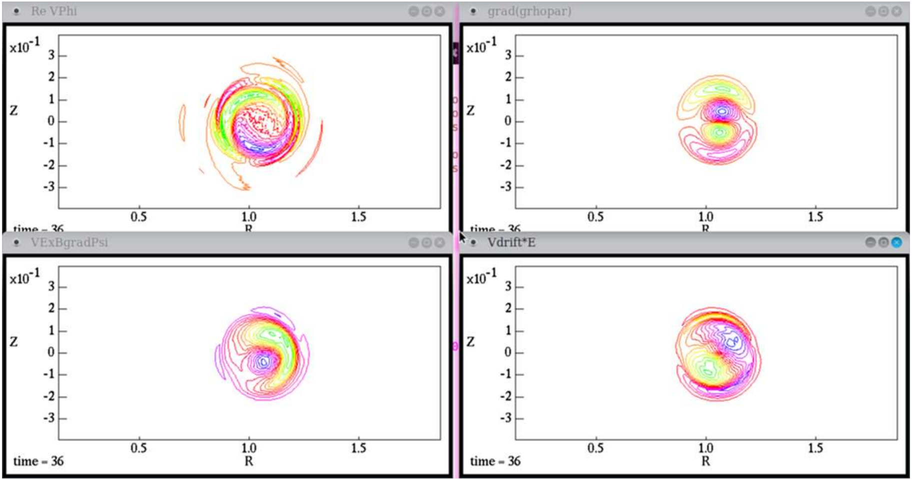
β_{frac}	$g\rho_{\parallel}$	$v_{E\times B}\cdot\nabla\psi$	$v_D\cdot\mathbf{E}$
.25	-0.031(-13/113)	(69/31)	-0.010 (45/55)
.50	-0.055(ε/100)	(58/42)	-0.015 (27/73)
.75	-0.057(0/100)	(52/48)	-0.020 (18/82)

- $v_{E\times B}\cdot\nabla\psi$ equal passing and trapped (*destabilizing*)
- $g\rho_{\parallel}$&$v_D\cdot\mathbf{E}$ small, negative, and mostly trapped (*stabilizing*)
  - $g\rho_{\parallel}$ term interesting at lower β_{frac} (*destabilizing?*)

Configuration Space $\beta_{frac} = .25$



Configuration Space $\beta_{frac} = .75$



Configuration Comparison

β_{frac}	$g\rho_{\|\|}$	$v_{E\times B}\cdot\nabla\psi$	$v_D\cdot\mathbf{E}$
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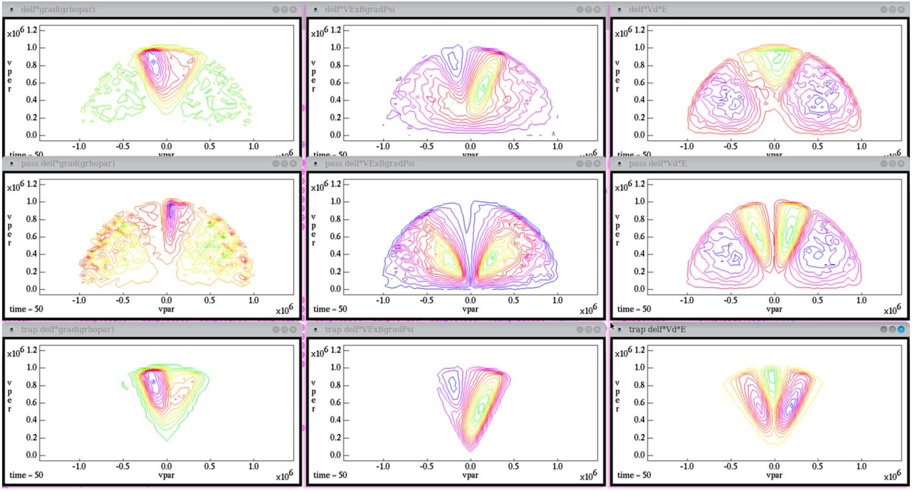
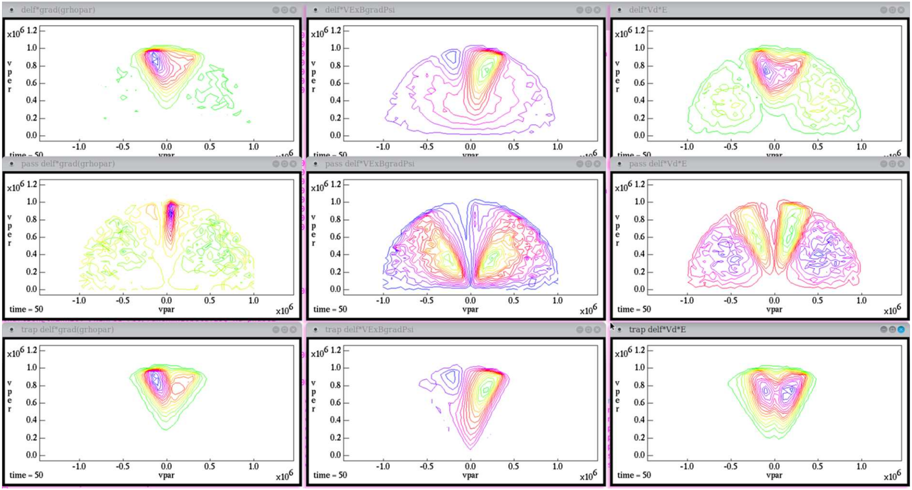
- $v_D\cdot\mathbf{E}$ increases tilt
- $v_{E\times B}\cdot\nabla\psi$ ‘tail’ grows
- subtle differences in $g\rho_{\|\|}$
  - peak at $q\sim 1$ surface shifts outboard
  - all trapped particle for $\beta_{frac} > .25$
- worth examining passing vs. trapped population

Velocity Space $\beta_{frac} = .25$



Velocity Space $\beta_{frac} = .75$



Velocity Space Comparison

β_{frac}	$g\rho_{\parallel}$	$v_{E\times B} \cdot \nabla\psi$	$v_D \cdot \mathbf{E}$
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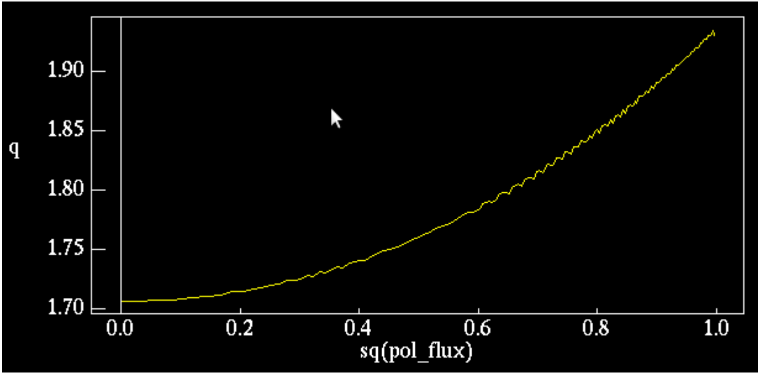
- $v_D \cdot \mathbf{E}$ most dramatic change as trapped particles dominate
- peak in $v_{E\times B} \cdot \nabla\psi$ shifts to larger v
  - as expected from increasing $\omega_{(1,1)}$
- interesting peaks in passing particles at $v_{\parallel} \simeq 0$
  - barely passing $\rightarrow$ potatoe orbits
- strong asymmetries are evident
- particles have orbits in both **x** and **v** space
  - trapped particles dominate outboard
  - passing particles dominate inboard
- plot in alternate coordinates, e.g. (P_ζ, μ)

ITPA TAE benchmark

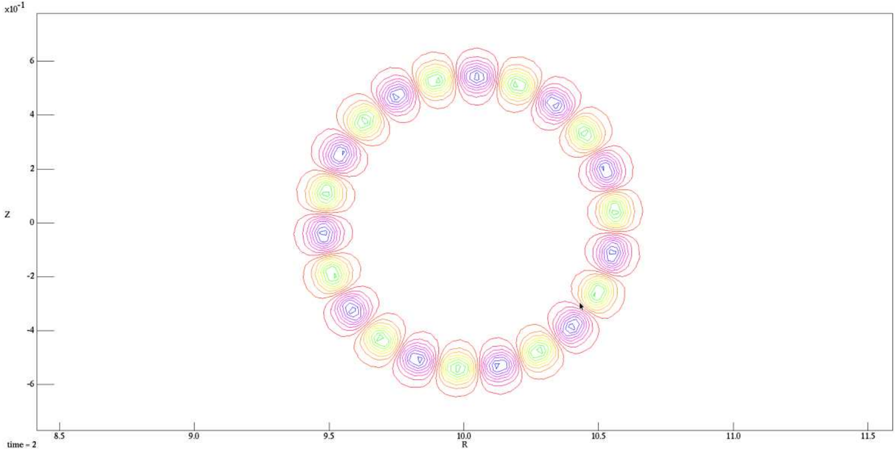
- $q = 1.71 + .16\left(\frac{r}{a}\right)^2$
- $R = 10\text{m},\ a = 1\text{m},\ B_0 = 3\text{T},\ n = 2 \times 10^{19}\text{H}^+/\text{m}^3$
  - $\tau_A = .67\mu\text{s}$
- $n_h = n_{0f} \exp\left[-\frac{\Delta}{L_f}\tanh\left(\frac{s - s_f}{\Delta}\right)\right]$
  - $n_{0f} = .75 \times 10^{17}\text{D}^+/\text{m}^3,\ \Delta = .2,\ L_f = .3,\ s_f = .5$
- $f_h(v)$ is Maxwellian, $T_h = 100 - 800\text{KeV}$
- TAE gap at $s = .5\text{m},\ q = 1.75$
  - $n = 6,\ m = [10, 11]$ dominant modes
  - expect $\omega \simeq 4 \times 10^5\text{rad/s},\ \gamma = \mathcal{O}(10^5)\text{s}^{-1}$

q profile



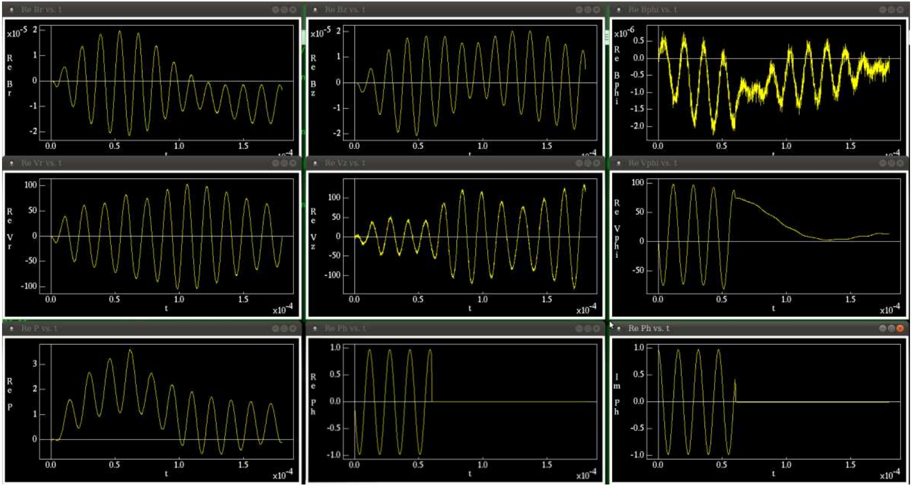
- not exactly ITPA benchmark profile
- erroneously thought I had better control of q profile
- q_{wall} a little large
- uncertain of curvature
- will need better q-profile control

Antenna drive looks promising



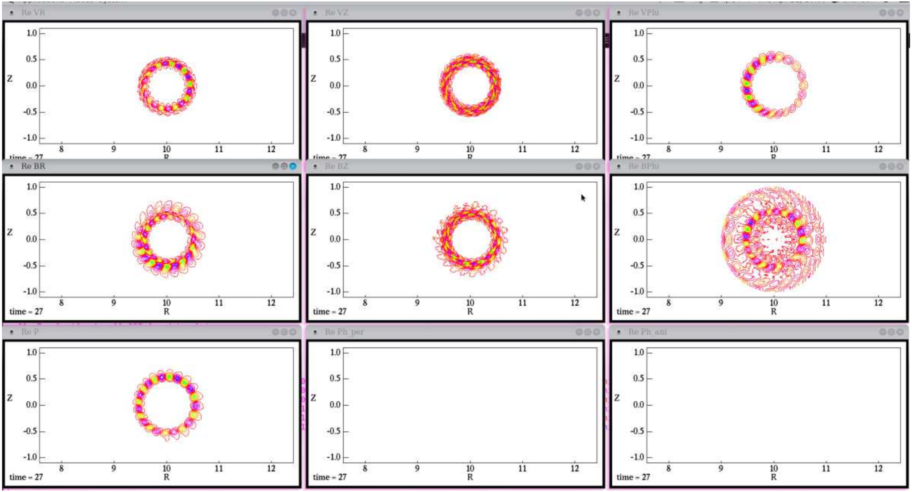
- drive with imposed external source in δp_{hot}
- $\omega = 4 \times 10^5 \text{rad/s}$, $m = 11$, $r = .55$, $\Delta = .05$

Antenna drive time trace



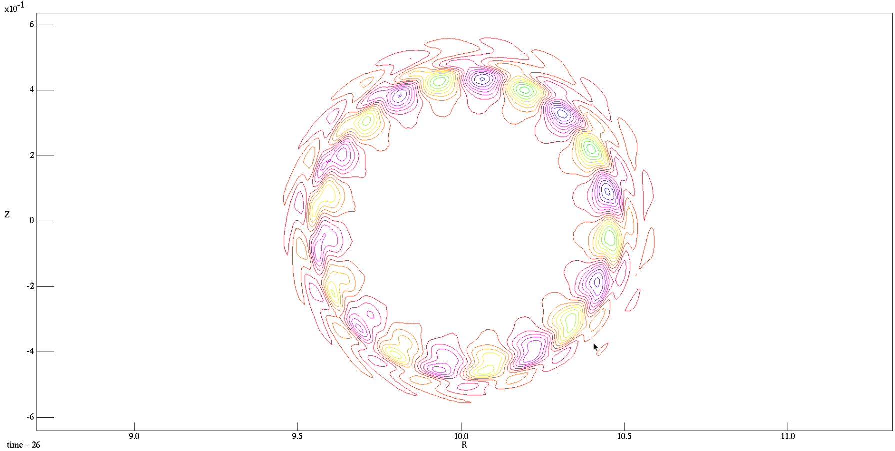
- response $\omega \simeq 4.1 \times 10^5 \text{rad/s}$ after shutoff

Antenna response contours



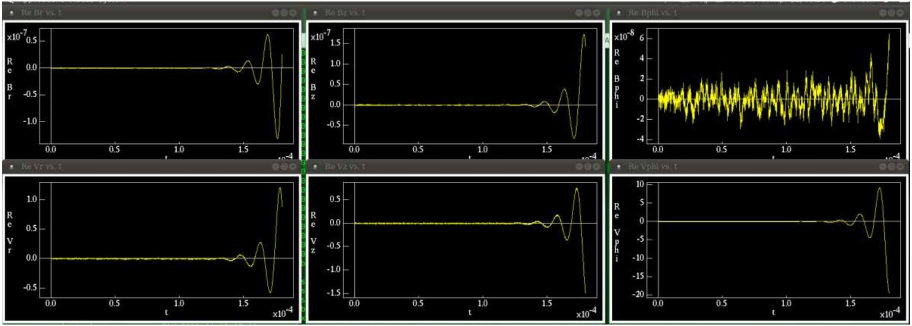
- mix of $m = 10, 11$

Vnorm response contours



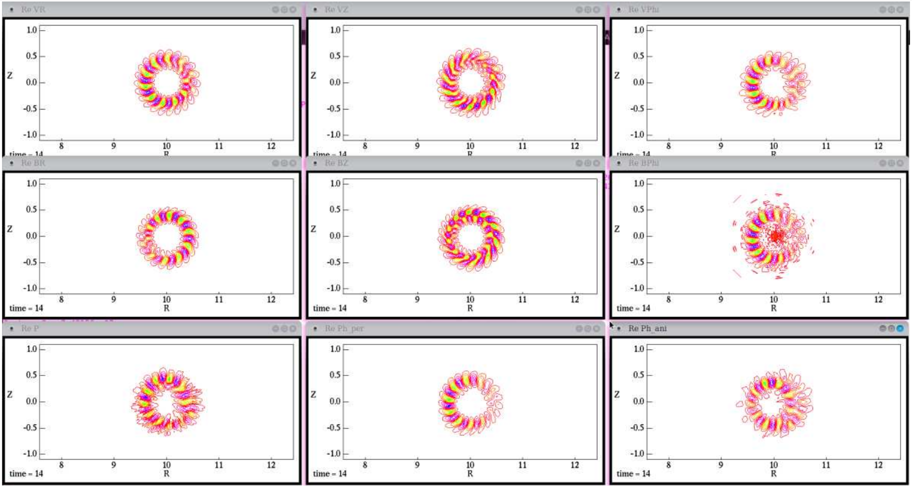
- dominant $m = 10$, $r = .45$

Timetrace of TAE shows real frequency



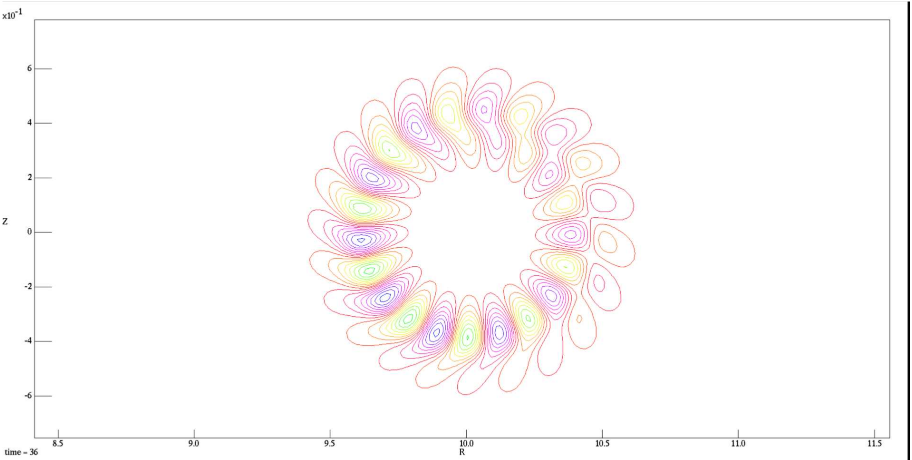
- $\omega = 4 \times 10^5$rad/s, $m = 10$, $r = .45$, $\gamma \simeq 9.5 \times 10^5$
- rough agreement with ITPA benchmark
- V_ϕ amplitude larger than expected!

Energetic Particles drive TAE



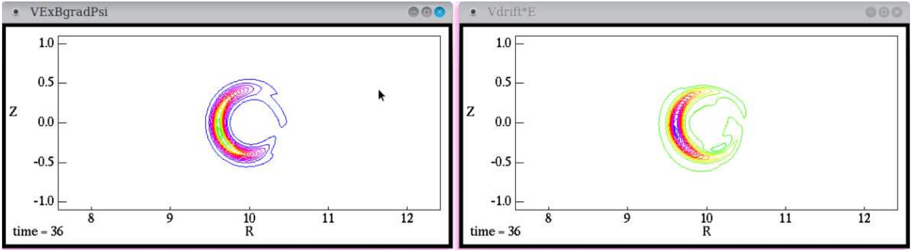
- $n_h = n_{0f} \exp\left[-\frac{\Delta}{L_f}\tanh\left(\frac{s - s_f}{\Delta}\right)\right]$
    - $n_{0f} = 3 \times 10^{17} \mathrm{D}^+/\mathrm{m}^3$, $\Delta = .2$, $L_f = .3$, $s_f = .5$

Contour of Vnorm



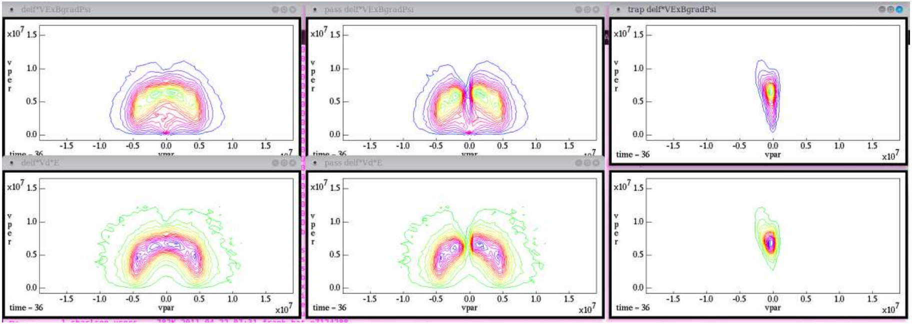
- poloidal mode coupling
- anti-ballooning

δf diagnostic shows expected symmetry in response



- original motivation for implementation of diagnostic
- $v_D \cdot \mathbf{E}$ about -2% of $v_{E\times B} \cdot \nabla\psi$

Banded Structure in δf diagnostic

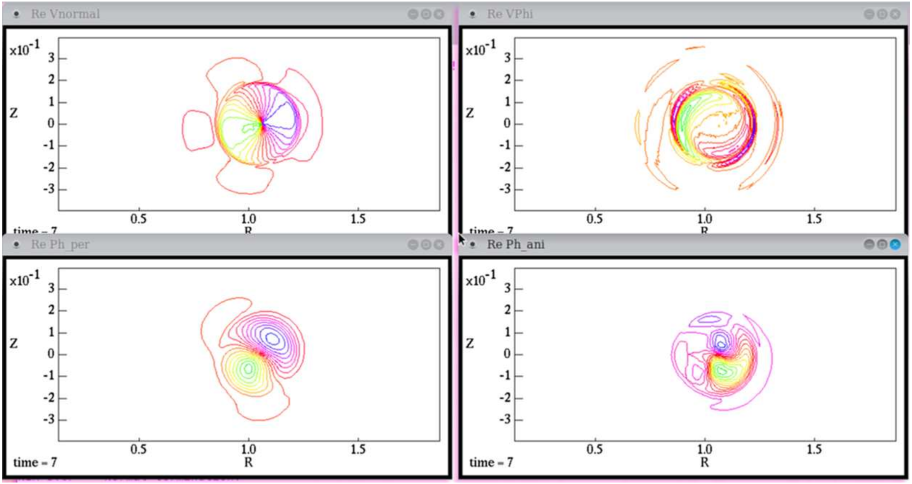


- banded structure indicates TAE is resonant wrt ε not $\mathbf{v}$
- not sure if nodule structure is real

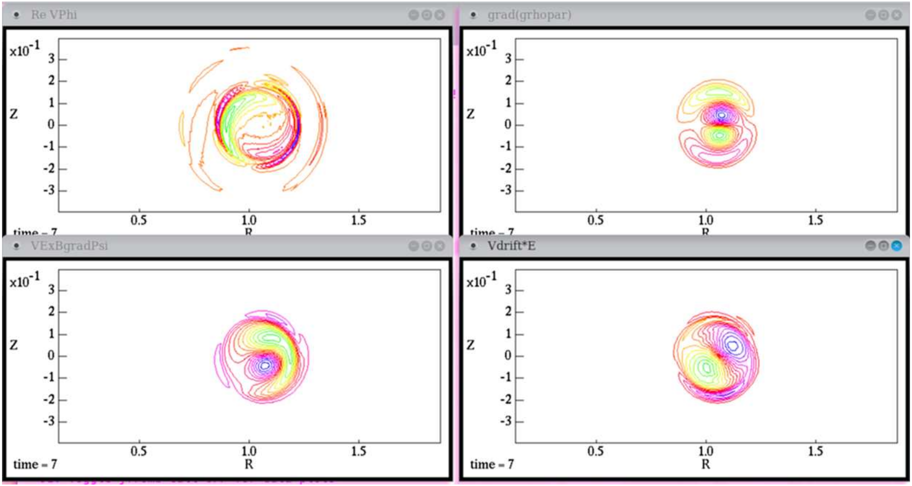
Summary

- improved accuracy and performance of hybrid kinetic-MHD
- implemented new diagnostics
  - identify interesting phenomenological features
  - both trapped and passing particles is important
    - trapped particles dominate outboard
    - (barely) passing particles dominate inboard
  - from phenomenology, move to quantification, then theory
- TAE simulations are promising
  - large V_ϕ amplitude troubling
  - δf diagnostic shows anti-symmetry between $v_D \cdot \mathbf{E}$ and $v_{E \times B} \cdot \nabla\psi$
  - work continues on energetic particle driven TAE
  - will need better q-profile control

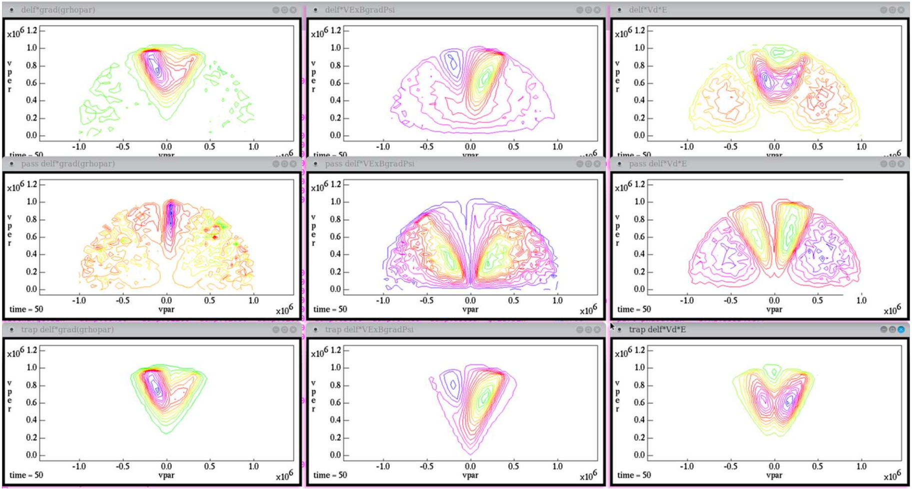
$n = 1$ Contour $\beta_{frac} = .50$



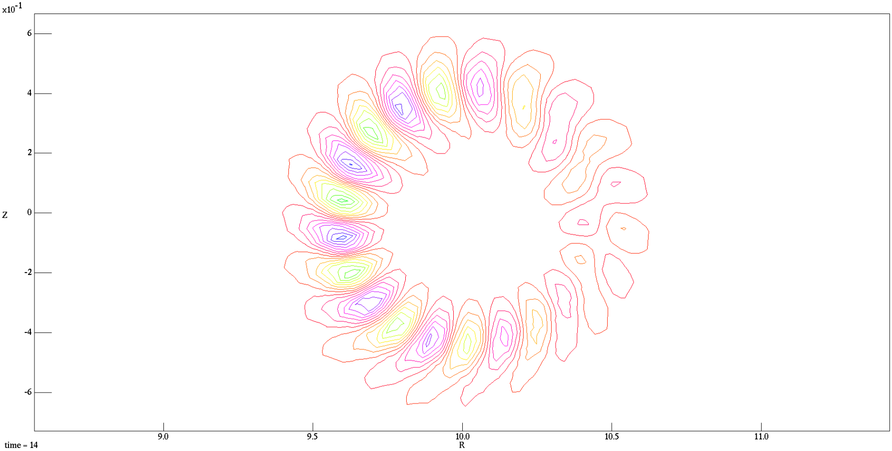
Configuration Space $\beta_{frac} = .50$



Velocity Space $\beta_{frac} = .50$



Contour of Phot



- peaks inboard!