Linear and nonlinear stability of a toroidal plasma and M3D results

L. Sugiyama
MIT

CEMM workshop
Sherwood Theory Meeting
Austin TX
April 30, 2011

Topics

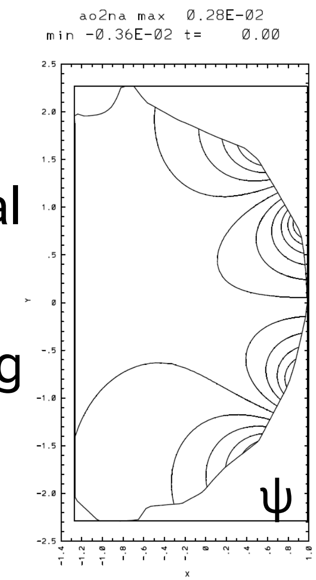
- Summary of current M3D simulation work
- Linear and nonlinear stability of ELM edge instability
  - Differences
  - Magnetic tangle?
- New projects/ideas

Summary of recent edge simulation work

- DIII-D Type I ELM (119690, 126006)
  - Nonlinear compared to linear stability and growth rates
  - New form for I-coil fields for M3D (B_ϕ and A_ϕ) with high toroidal resolution + other fields (C-coil, bus, error): D. Orlov, UC-SD
  - Full toroidal spectrum of n=3 RMP I-coil field – Fourier aliasing
  - TBD: Add all non-axisymmetric fields to ELM (126006)
  - Two-fluid + toroidal rotation effect on growth rates - important!
    - Combination has NL stabilizing effect, stronger than either alone
- NSTX lithium divertor ELM suppression
  - ELM seen in both pre-lithium and lithium 'stabilized' case (MHD, at high η. Numerical stability worse in ST geometry. Better grid, higher resolution.)
- CMOD EDA regime with QCM edge oscillation
  - Diamagnetic-profile equilibrium is MHD stable at 10x actual resistivity
  - Waiting for kinetic profile equilibrium, part of Joint Milestone 2011.

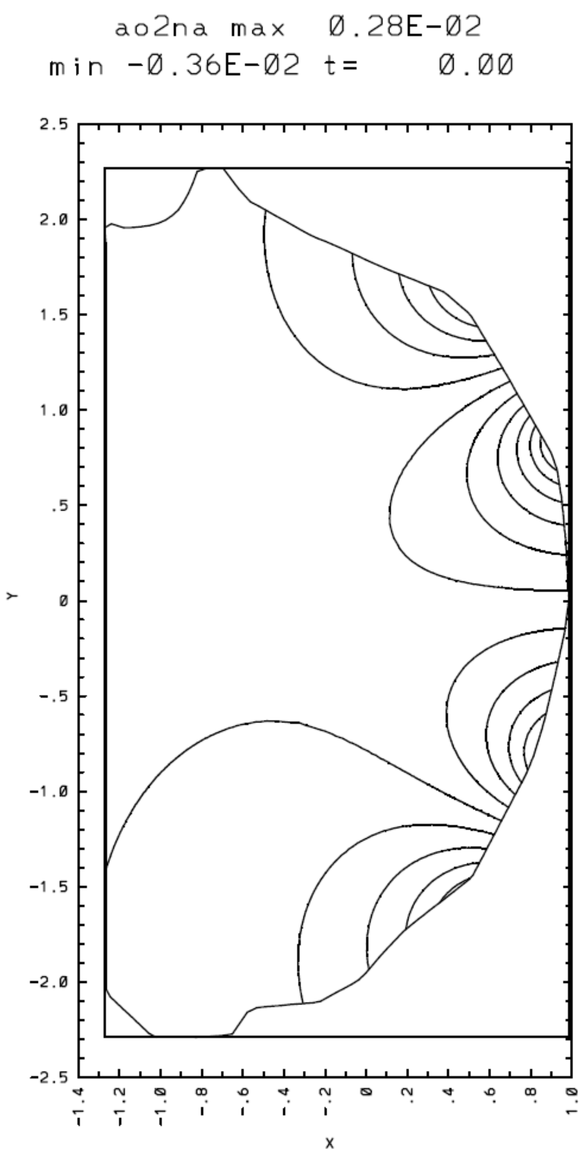
Summary of recent edge simulation work

- DIII-D Type I ELM (119690, 126006)
  - Nonlinear compared to linear stability and growth rates
  - New form for I-coil fields for M3D (B_ϕ and A_ϕ) with high toroidal resolution + other fields (C-coil, bus, error): D. Orlov, UC-SD
  - Full toroidal spectrum of n=3 RMP I-coil field – Fourier aliasing
  - TBD: Add all non-axisymmetric fields to ELM (126006)
  - Two-fluid + toroidal rotation effect on growth rates - important!
    - Combination has NL stabilizing effect, stronger than either alone

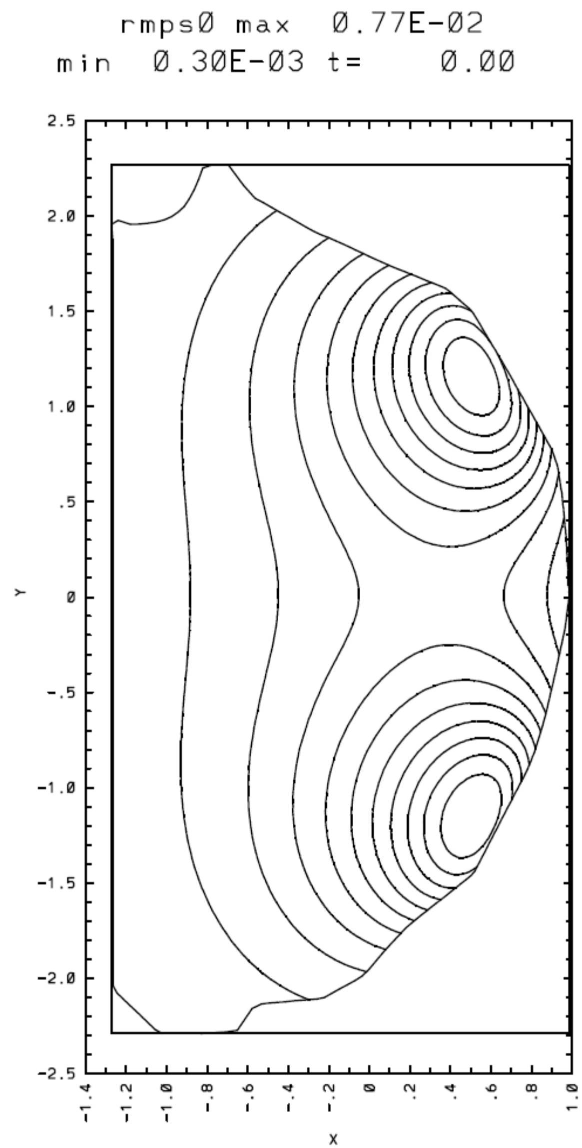


- NSTX lithium divertor ELM suppression
  - ELM seen in both pre-lithium and lithium 'stabilized' case (MHD, at high η. Numerical stability worse in ST geometry. Better grid, higher resolution.)
- CMOD EDA regime with QCM edge oscillation
  - Diamagnetic-profile equilibrium is MHD stable at 10x actual resistivity
  - Waiting for kinetic profile equilibrium, part of Joint Milestone 2011.

RMP (I-coil) field



$(R/R_o)\ \psi$



RB_ϕ

Summary of edge simulations -2-

- DIII-D EHO 128542
  - Ran original equilibrium in MHD/ MHD+rot/ 2F/ 2F+rot – similar type of instability with different growth rates; low n=2,3 as in experiment
  - Running new equilibrium reconstruction with $q_o > 1.0$; strange.
  - TBD: add error-field and correction (I-coil).
- MAST ELM – new case; testing.
- New wall-load and divertor diagnostic (VisIt), including fluxes $v_n \cdot X$
  - ELM divertor traces qualitatively similar to experimental measurements
  - Harmonics to compare to experiment
- Still missing cases for V&V:
  - Pre/post RMP comparison for density pumpout in H-mode. (Can't use 126006. DIII-D cases identified, waiting for good data.)
  - ELM crash with fast time data for detailed comparison to expt.
- NERSC Cray XT-6 now allows NL sim at $(2x)^3$ resolution of 2009/10 with good turn around!

Linear vs Nonlinear ELM instability

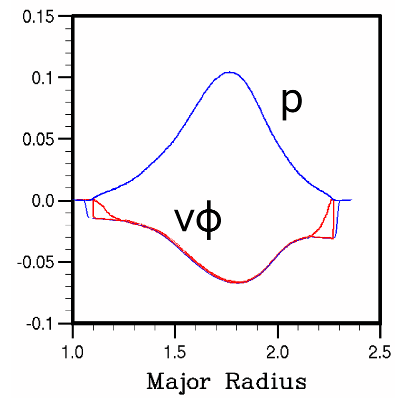
- What role does the magnetic tangle play in stability?
- Linear perturbation theory excludes full magnetic tangle
  - Small magnitude of perturbation, $|\tilde{p}| \ll |p_o|$
  - Single toroidal harmonic ↔ linearization drops nominally smaller terms
- In theory, a magnetic tangle results from any small enough transverse perturbation; should be biggest near X-point, away from ballooning-type instability driving term at midplane
  - Not linearized, not single harmonic; stochastic
  - Asymptotic field line splitting in different directions; Field splitting can be obtained by linear superposition of equilibrium + single-n perturbing field
  - Not flux tube boundary conditions
  - X-point system does not preserve energy since X-lines intersect domain boundary (Only small exterior effect for simulation?)
- Other nonlinear effects are important in ELM
  - NL harmonic interaction leads to low mode-number and n=1 effects

Linear vs Nonlinear: Growth rates for DIII-D ELM

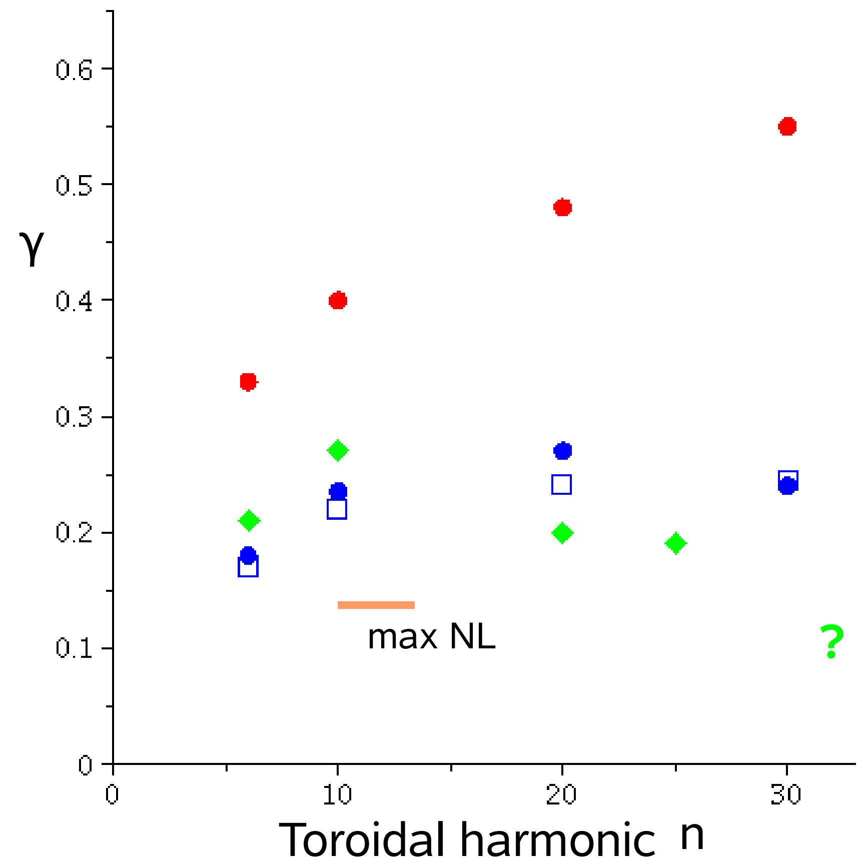
- Compute linear growth rates for the DIII-D Type I ELM 126006 case
  - Match nonlinear simulation conditions, for comparison
- Linear and nonlinear growth rates are different in MHD. Strong nonlinear effects affect ELM at finite, but nonlinearly small, size.
  - Linear rates show expected MHD ballooning dependence; inc. with n
  - Nonlinear, dominant harmonics are moderate n=10,13; m
  - Maximum NL γ is smaller (0.13 vs. 0.5+ for n=23 or 0.35 for n=10).
  - Linear γ reduced by toroidal rotation, maximum NL γ increased.
- Not exactly same models
  - Linear pert has no $\partial n/\partial t$; NL evolves density.
- More accurate linear perturbation results should use higher resolution, especially higher harmonics with strong rotation shear.

Linear growth rates reduced by toroidal rotation

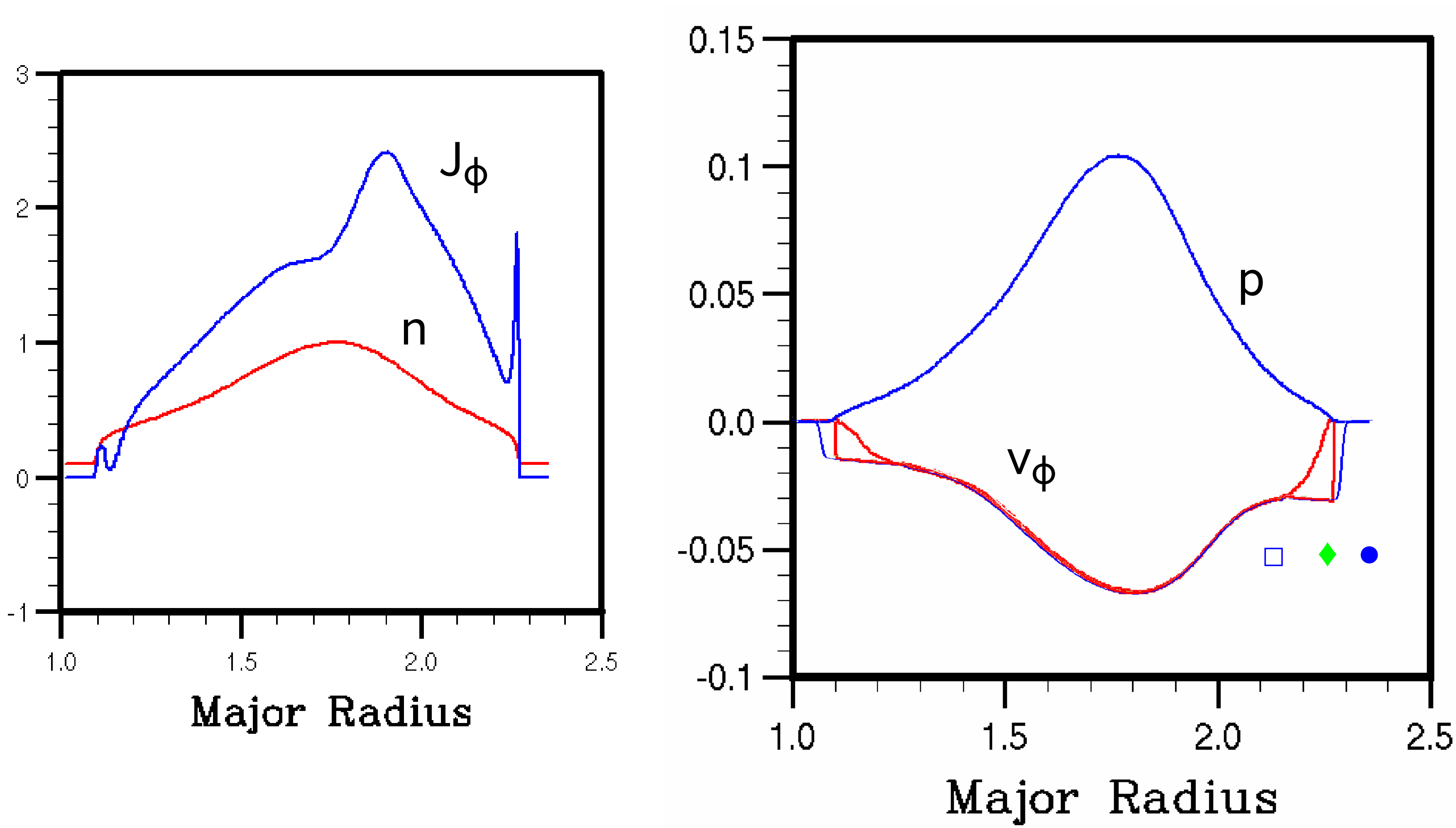
- DIII-D ELM 126006
- MHD without rotation (●) has expected ballooning behavior
- MHD with toroidal rotation, varying edge rotational shear
  - Experimental profile (□)
  - Modified to constant Ω over 0.7<ψ<1 near plasma edge, chopped to Ω=0 at separatrix (♦) (n=30 not converged)
  - Const over edge, but Ω→0 smoothly (tanh) starting outside separatrix but well inside wall (●)
- Rotation is stabilizing
- Rotational shear effects weak, unless shear is very strong



MHD, no rotation ●
Rotation, exptl profile □
Rot, const over edge, zero at separatrix ♦
Rot, const over edge, smoothly to zero outside ●



Midplane profiles, expanded



DIII-D 126006 ELM

$\eta = 3 \times 10^{-8}$, $\mu = 6\times 10^{-6}$

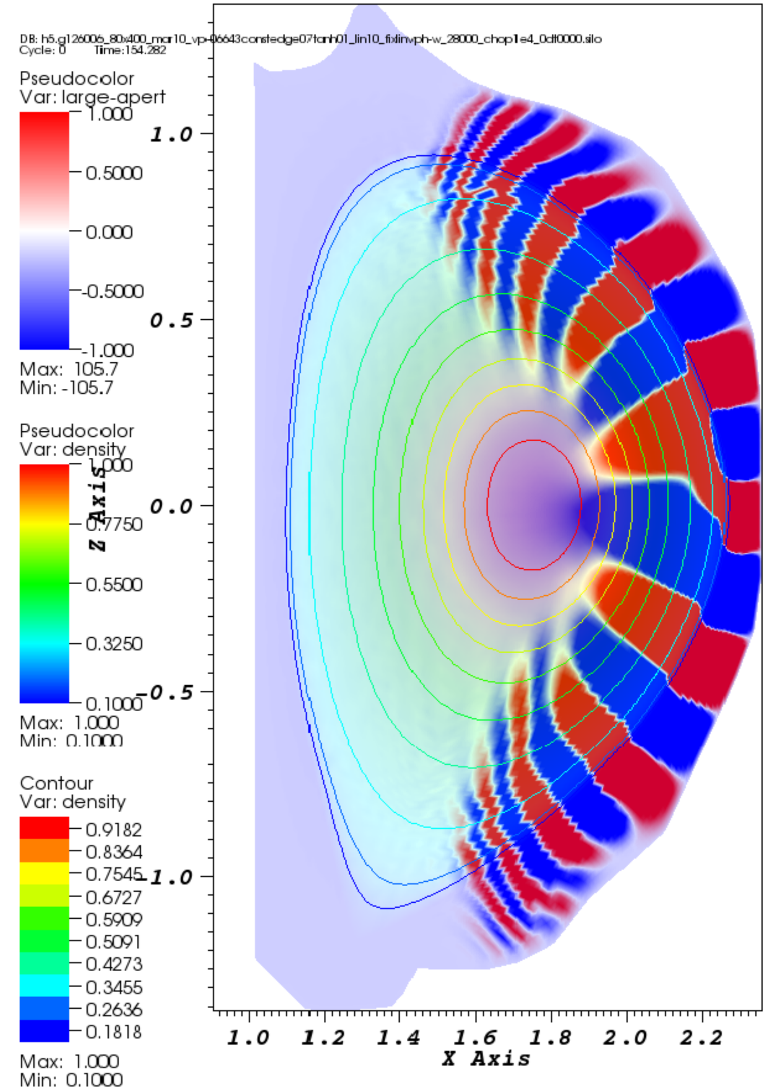
Linearization excludes the formation of a magnetic tangle

- Linearized magnetic perturbation can only be large where plasma perturbation is large, i.e., $\tilde{\mathbf{J}}$ and $\tilde{\psi}$ must match $\tilde{p}$:

$$\tilde{\mathbf{J}} \times \mathbf{B}_o + \mathbf{J}_o \times \tilde{\mathbf{B}} = \nabla \tilde{p} + \rho_o \left(\frac{\partial \tilde{\mathbf{v}}}{\partial t} + (\mathbf{v}_{\phi o} \cdot \nabla) \tilde{\mathbf{v}} \right) + \ldots$$

  - Test: Calculate linear perturbation in standard manner*, then multiply it by very large factor and plot $\rightarrow$

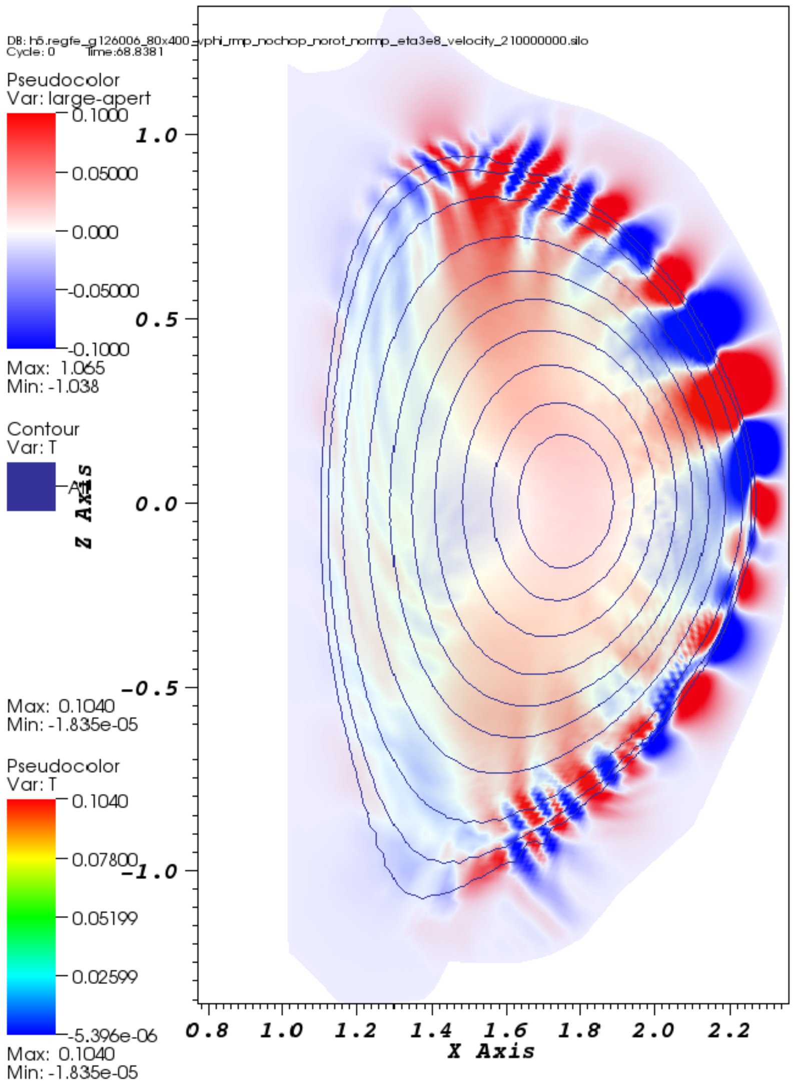
  - X-point regions and inboard side have $|\tilde{\psi}|/|\tilde{\psi}_{max}|$ less than 10-8. Poloidal extent is approximately the same for all variables.
  - $\tilde{\psi}$ plotted on density, n=10 mode, rotation (●)
  - *Linear calculation solves almost fully NL equations. After each Δt time step, resets n=0 part and filters to given harmonic n, also controls perturbation magnitude.



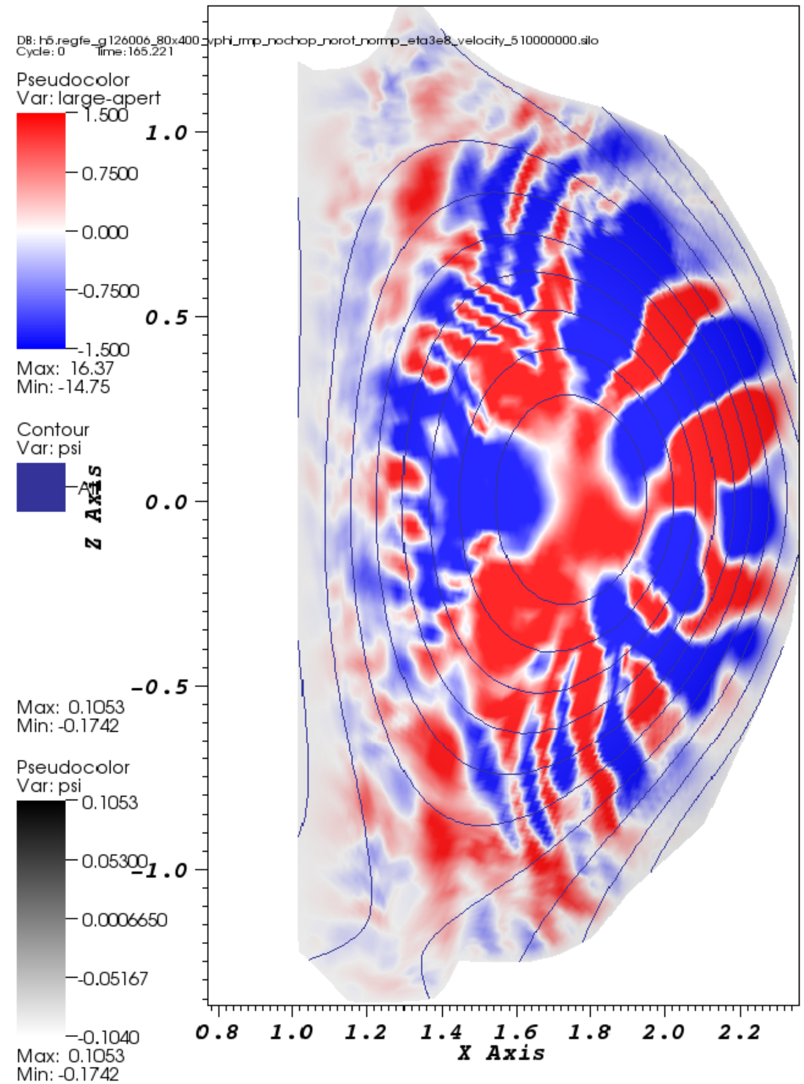
$\tilde{\psi}$ on density, n=10

DIII-D 126006

- Nonlinear ELM forms a tangle early (inboard and near-X fraction of $\tilde{\psi}$ is 1/20-1/200 of maximum instead of less than 10^{-8}; fraction declines later, but absolute magnitude increases



Just before crash, t=68.3



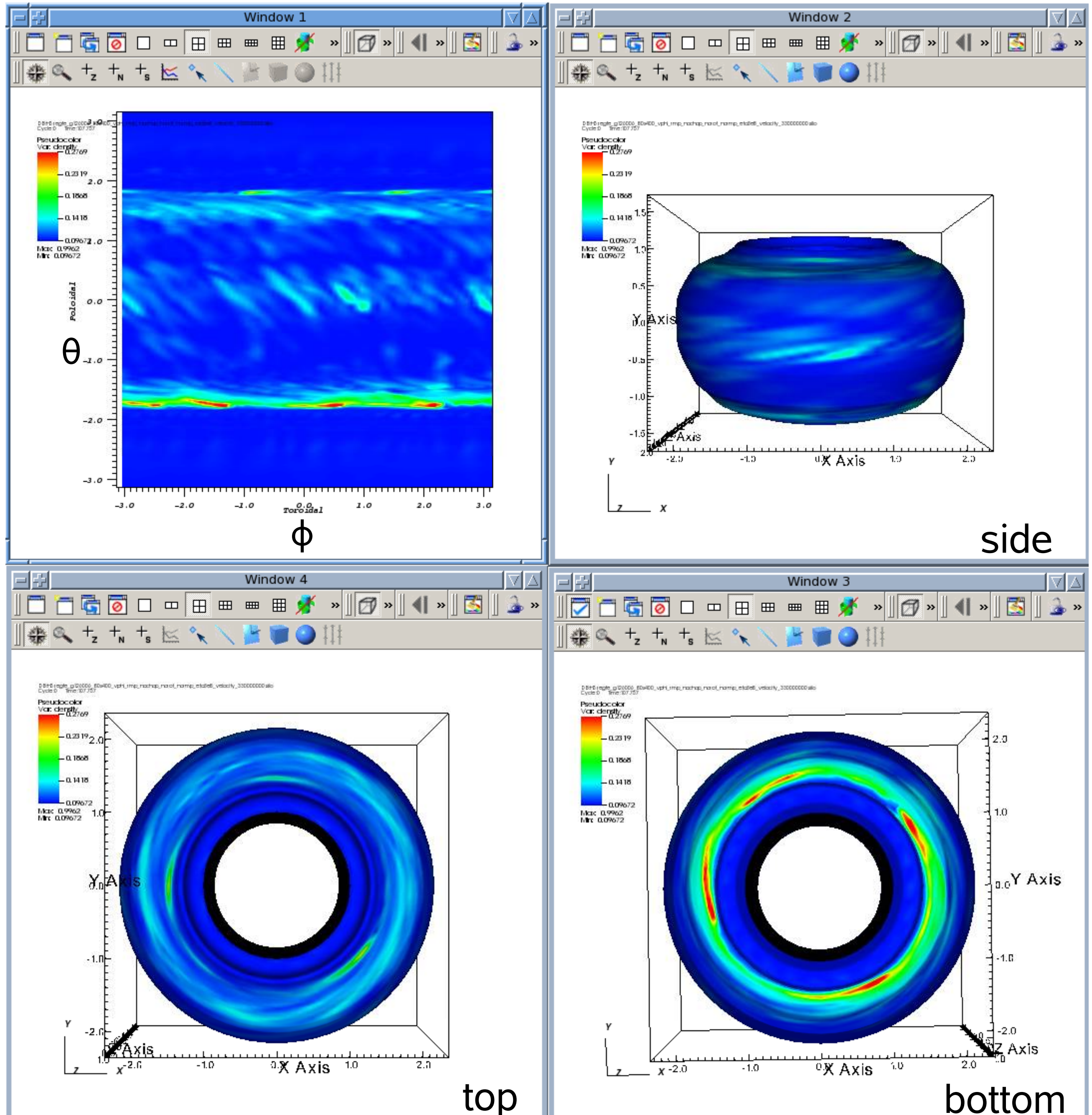
After initial outburst, t=165
max $\tilde{\psi}$ x16 vs earlier time

Comments

- Absence of the magnetic tangle in linear simulation is unlikely to result from numerical reasons (e.g., not enough resolution near X-point), but cannot be completely ruled.
- Linear theory does not predict tangle; major restrictions agree with those in simulation, so conclude 'No linearized tangle'.
  - Tangle requires propagation along B; too slow to grow as $e^{\gamma t}$
- Magnetic tangle should be stabilizing nonlinearly, since requires additional work to drive a field perturbation away from main plasma instability
  - Seen in an indirect test of nonlinear evolution (Sugiyama, PoP 2010)
- Some other results suggest that the density evolution may have strong stabilizing effects on linear edge perturbations (not part of standard MHD linear model)
  - Here, no $\partial n/\partial t$ in linear case, since very steep edge density gradient in ELM case needs to be better resolved for linear convergence.

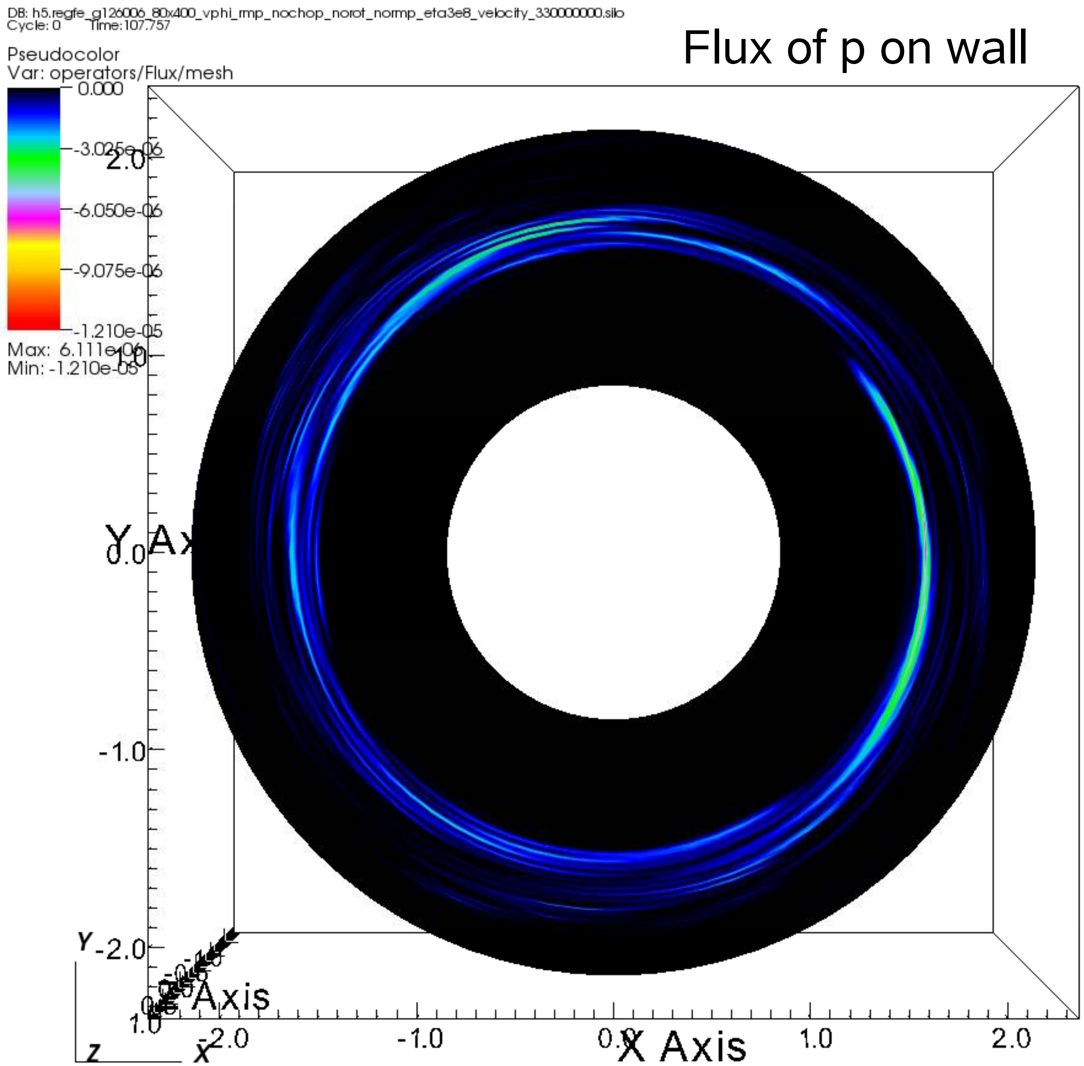
ELM wall loads are strongly asymmetric

- New wall diagnostic in VisIt for M3D (LBL Vis group, H. Childs)
- Strong asymmetry in divertor (when density first hits lower outer divertor) Concentrated points during crash!
- Helical stripes on top and bottom divertors follow field lines, overall
- Locations and magnitudes change on fast MHD time scale

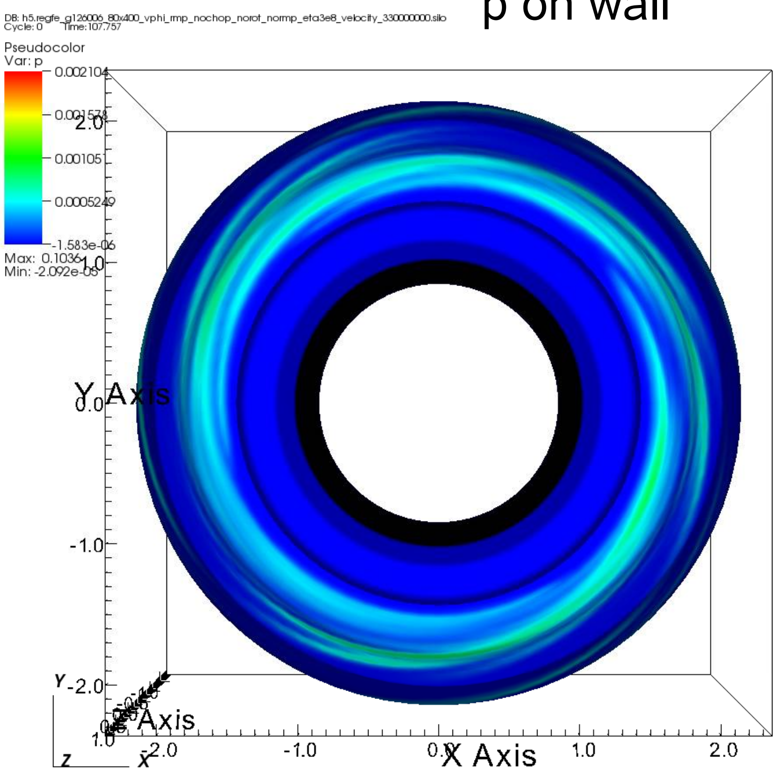


Density
DIII 126006 ELM_33000

Flux of pressure ($p \cdot v_n$) shows multiple striations in divertor



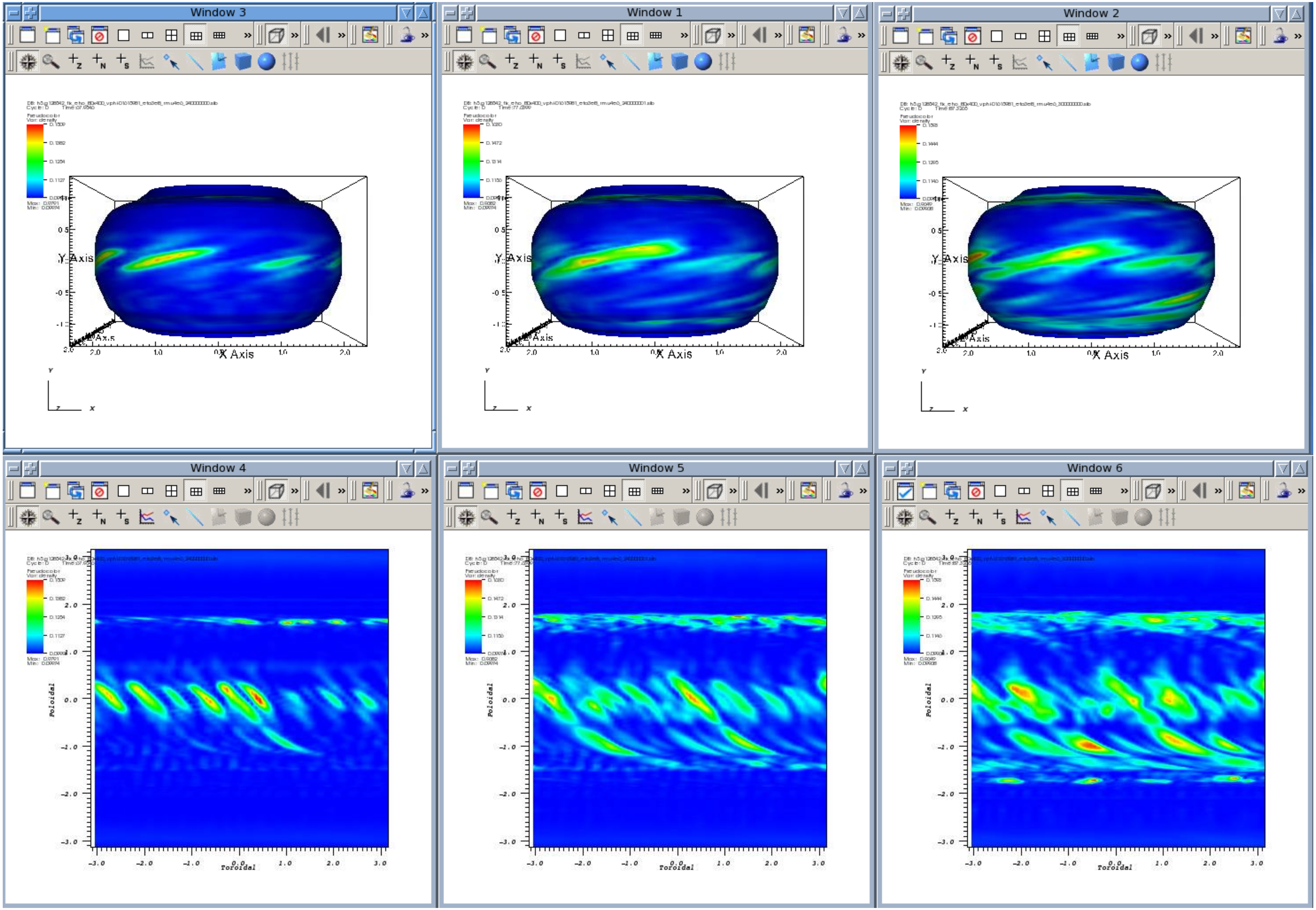
- Raw variable p (or n) is broader, smoother



- Need time integrals to match experimental observations

DIII-D 126006 30000

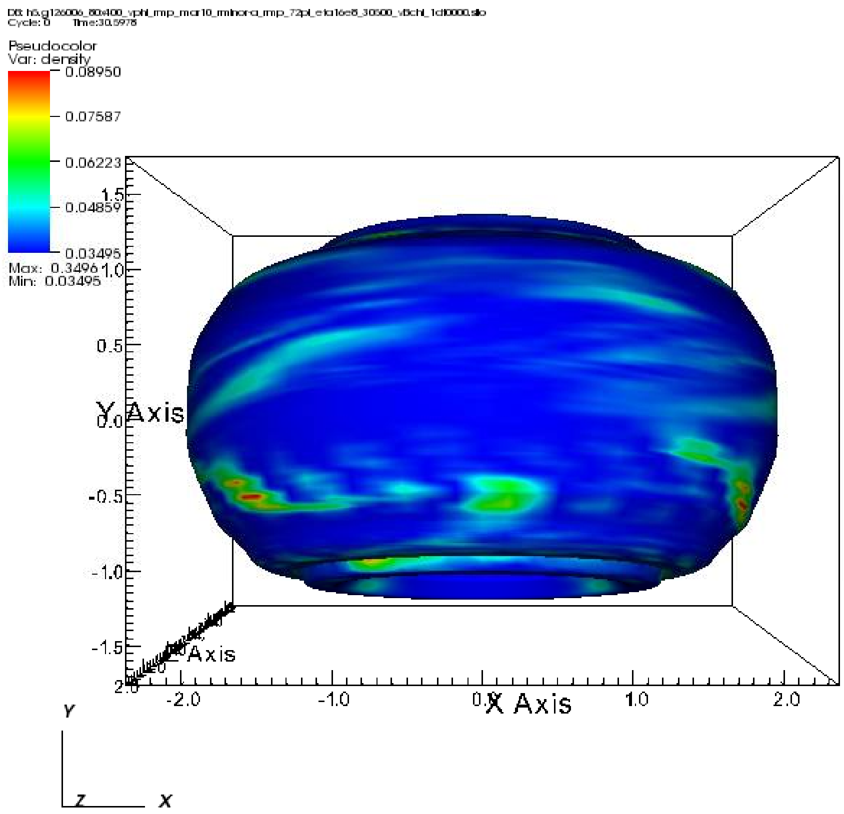
DIII-D 128542 EHO instability: Density on wall. (Experiment has dominant n=2 mode.)

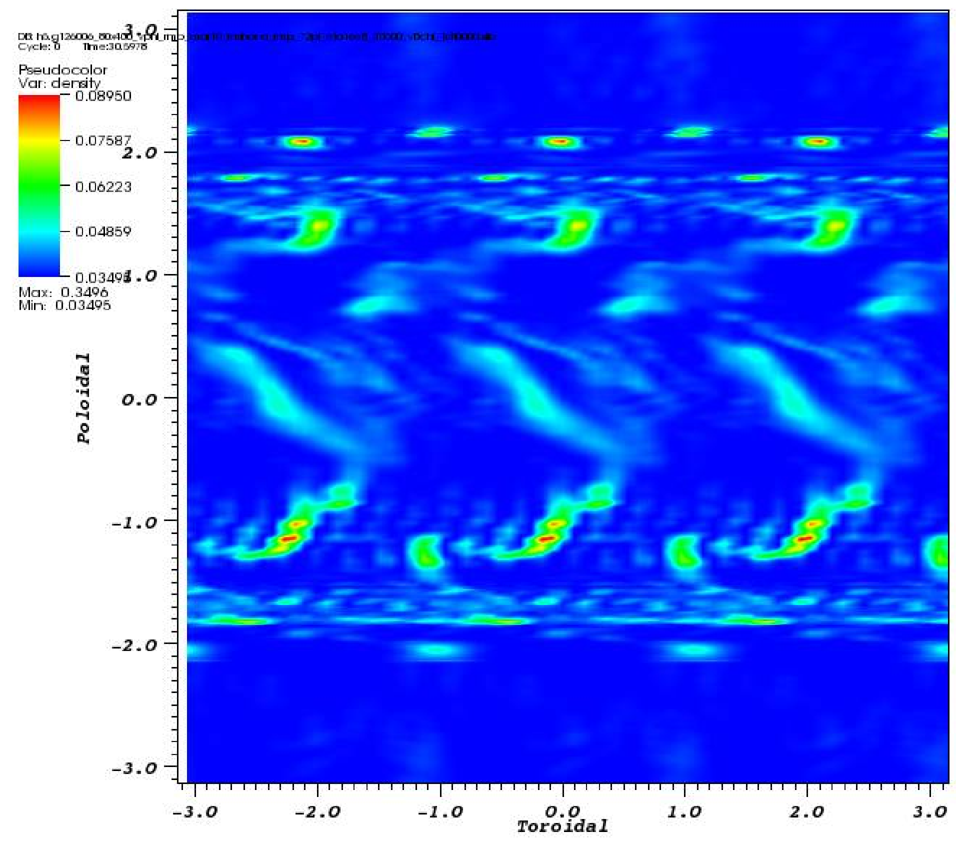


time →

DIII-D RMP fields from I-coil

- Full I-coil field with new algorithm for M3D by D. Orlov: B_ϕ and A_ϕ (magnetic vector potential, from J. Hansen (2002) algorithm)
- Analysis of spectrum shows n=3,9, ..., B_ϕ has many harmonics near coils
- Preliminary example: density pump-out to wall with n=3 (*old* RMP!)





Finite time Lyapunov exponents (FTLEs)

- How to characterize magnetic tangle and other superficially stochastic structures?
- Local definition of structures: Finite time Lyapunov exponents measure how fast local structures move apart or together.
  - Unlike regular Lyapunov exponents, which measure infinite-time growth or convergence
- New techniques to determine FTLEs for ordinary fluids recently developed and tested against experiments (mostly 2D, starting to go to 3D). Increasingly useful for real problems.
  - Haller (2001); Mathematical foundation (Shadden 2005)
- Older applications to plasmas targeted mainly homogeneous turbulence. Now, becoming practical to apply to instabilities with real structures.
- Bridge modern ideas of fluid turbulence/mixing and plasmas
- Study fundamental questions: Incompressible vs compressible MHD (also differences in GK and MHD magnetic evolution), num stability

FTLEs for plasmas

- Visualization techniques to compute FTLEs for fluid velocity field **v** are active area of research
  - Trace 'particle' paths in flow field, extract relative motion.
- Apply to plasma: not only **v**, but **B**
  - Some extensions can be developed.
  - Incompressible MHD: $\mathbf{v} \pm \mathbf{B}/\rho^{1/2}$, vorticity $w \pm J_\phi$, etc.
- Nonlinear MHD simulations are a good test bed
  - H. Krishnan, LBL Vis Group (post-doc) working on M3D data
- Finding FTLEs is related to certain types of feature extraction (identify hills,valleys, level contours), so FTLE techniques can help analyze dynamic plasma structures, independent of the Lyapunov meaning
- Techniques will improve as computation capability improves
  - Next generation computing: highly parallelizable

FTLE example: Fluid Jet

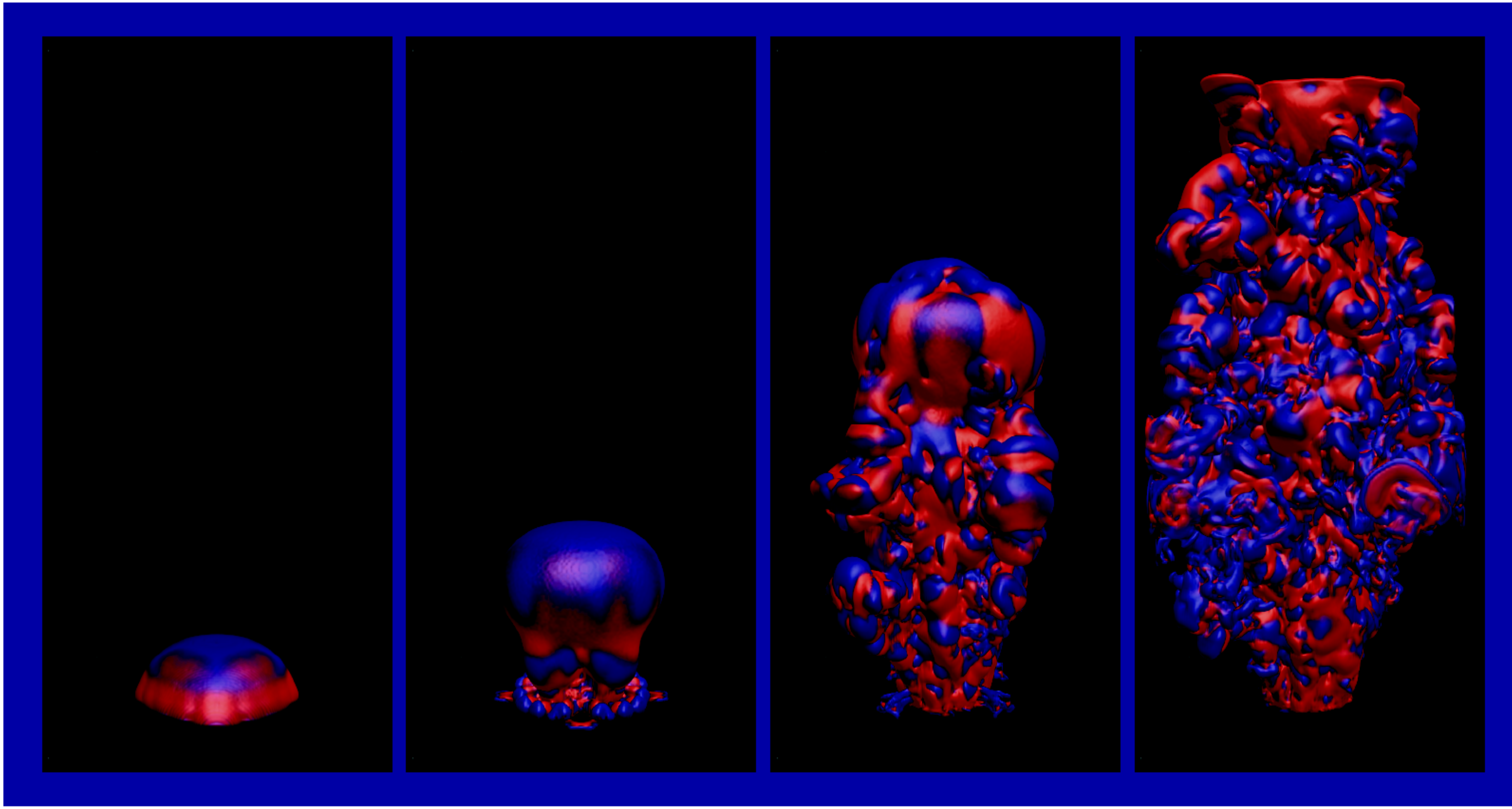


Figure 4. *This image shows direct volume rendering of the time-varying Finite-Time Lyapunov Exponent fields (red indicates the forward-time exponent, blue shows the backward-time exponent) for four time steps, illustrating the formation of turbulence in a high-speed jet of entering a domain of stationary fluid. Individual turbulent structures and structure size and distribution can be observed directly from the volume rendering. Data set: C. Garth (UC Davis).*

(VACETS SciDAC center (2007))

Summary

- Linear vs nonlinear edge instability – important differences
  - Magnetic tangle is nonlinear
- MHD plasma edge stability/instability ongoing
  - Edge (ELM, ELM-free oscillations)
  - Edge + interior mode coupling
  - Initial two-fluid shows two-fluid+rotation is important
- Developing theoretical and practical tools to study questions raised: visualization with help from LBL/NERSC Vis group
  - Wall load and wall-flux diagnostic (working)
  - Finite time Lyapunov exponents
    - MHD structures and evolution: identification, local stability
    - Develop extensions from fluids to plasmas
- Other areas not discussed here...