Modeling Three Dimensional Equilibria and ELM Suppression in DIII-D

by
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Outline

- **Modeling 3D equilibria**
  - Validation of linear M3D-C1 calculations
  - Nonlinear M3D-C1 calculations
  - Code comparisons

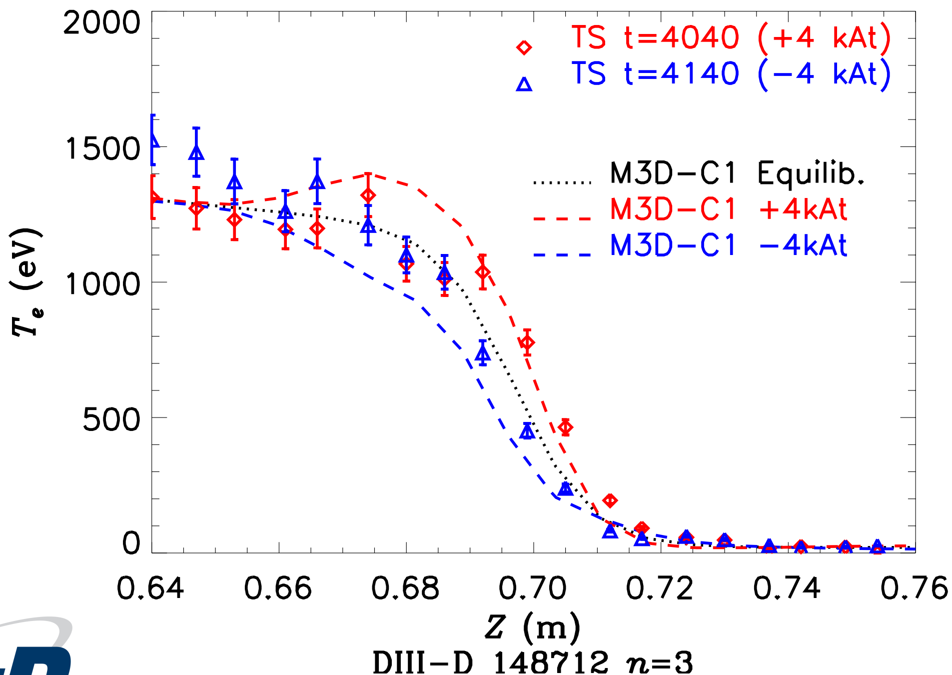
- **Developing and validating ELM suppression criteria**

- **Obtaining a predictive model for ELM suppression**

Modeling 3D Equilibria

Linear Calculations of Plasma Response Agree With Experimental Measurements of Profile Displacements

- **Changing the phase of applied 3D fields "displaces" the temperature and density profiles up to ~4 cm on DIII-D**
- **Linear M3D-C1 calculations show helical distortions that agree well with observed displacements**
  - Two-fluid effects and rotation affect agreement



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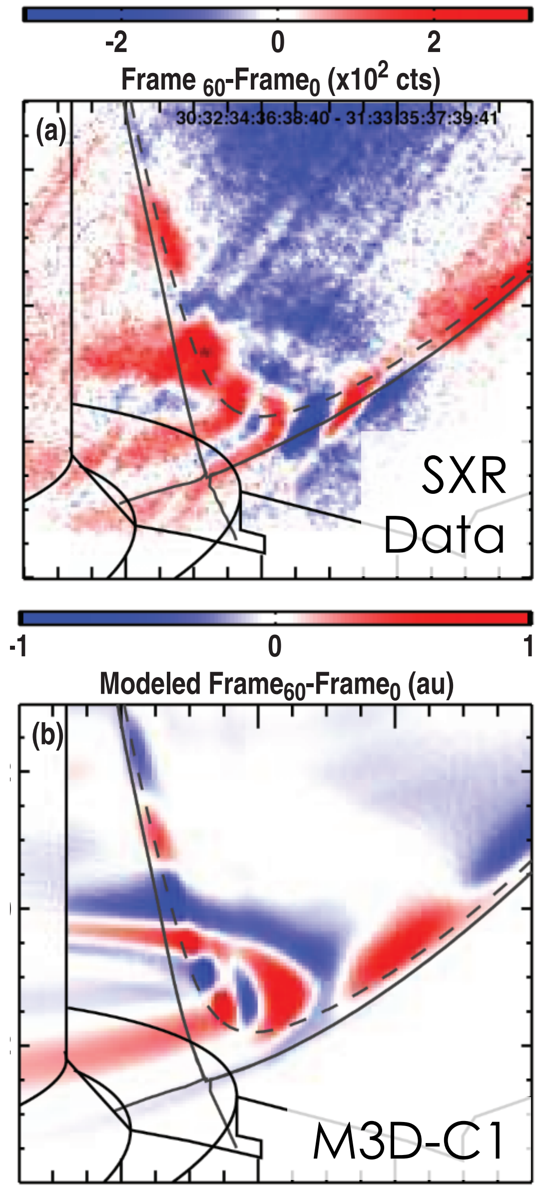

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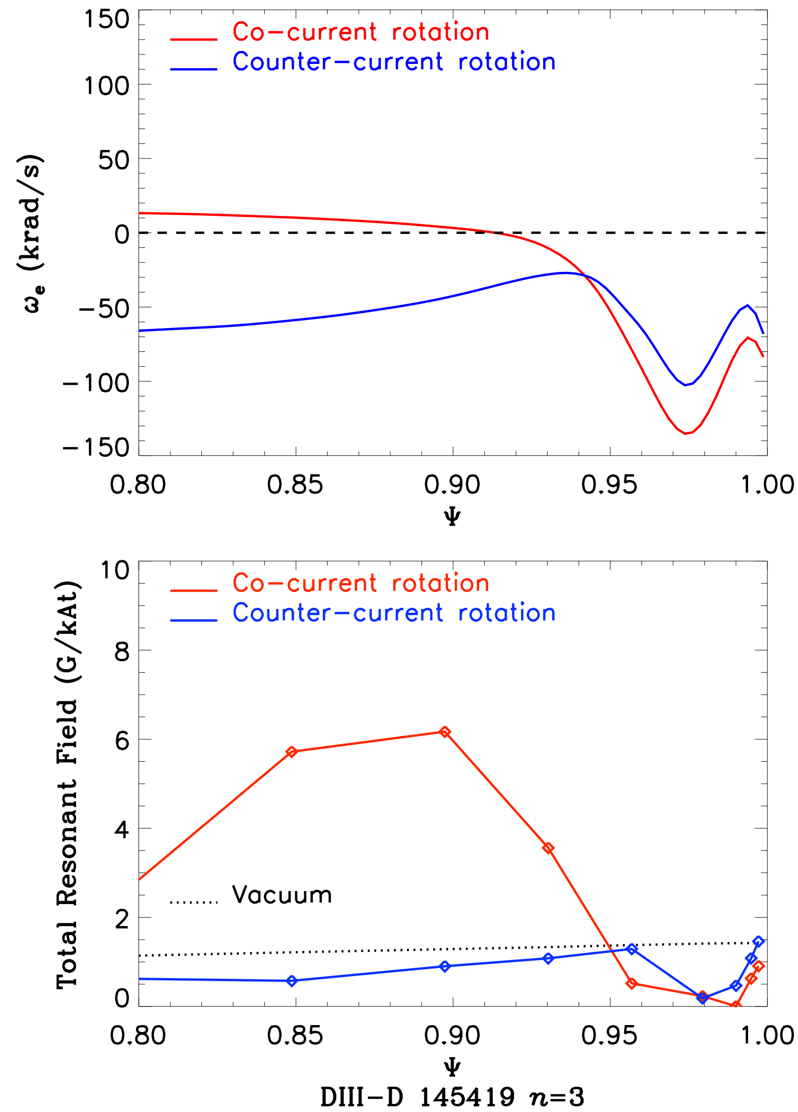
Plasma Response Calculations Predict Island Formation at Pedestal Top When Rotation is Co-Current

- **Perpendicular electron rotation screens islands ($\omega_e = \omega_{ExB} + \omega_{*e}$)**
- **Diamagnetic term → edge screening, reduced pedestal stochasticity**
- **In co-current rotating plasmas, $\omega_e = 0$ near top of pedestal**
  - Islands can penetrate and be amplified here
- **In counter-current rotation plasmas, ω_e never crosses zero**
- **Is this a mechanism for suppressing ELMs by limiting the pedestal?**
  - Consistent with lack of counter-current ELM suppression



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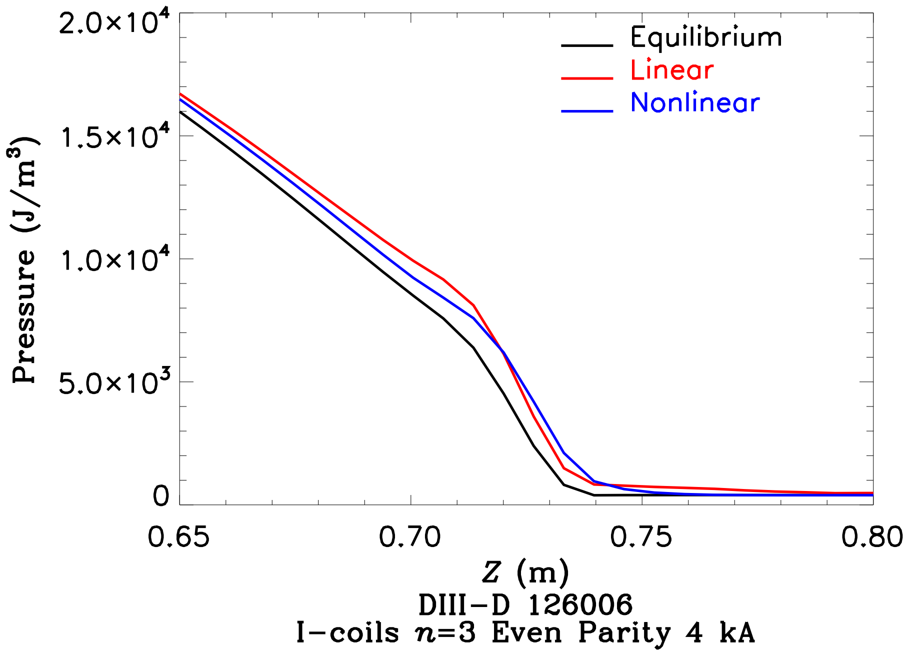
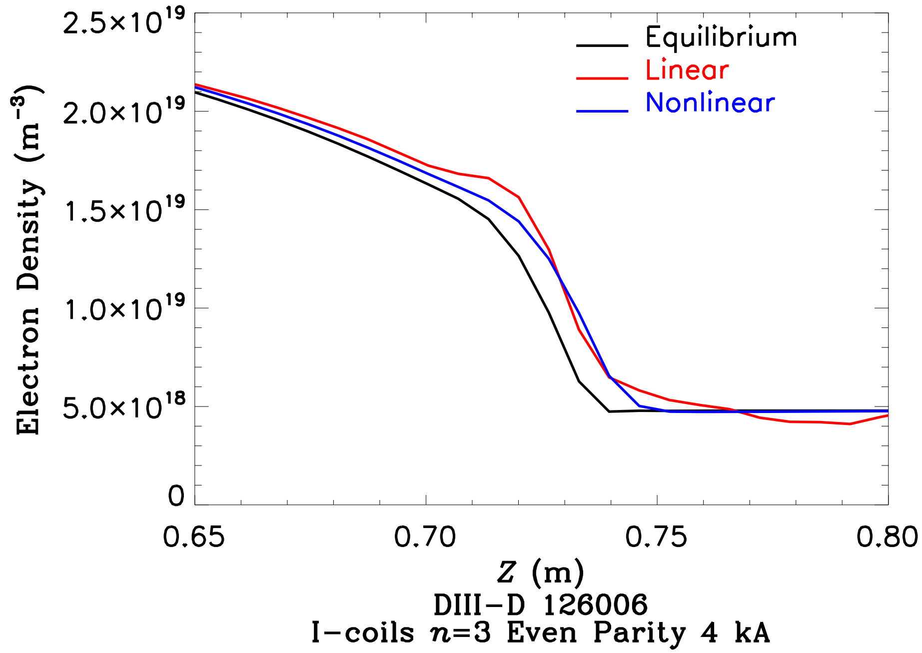
Linear and Nonlinear Calculations of Displacement are in Reasonable Agreement for 126006 at 4kA n=3

- **Nonlinear, one-fluid calculations with Spitzer resistivity and realistic dissipation (~5 m^2/s) have been carried out**
- **Displacements are similar to linear results, in this case**

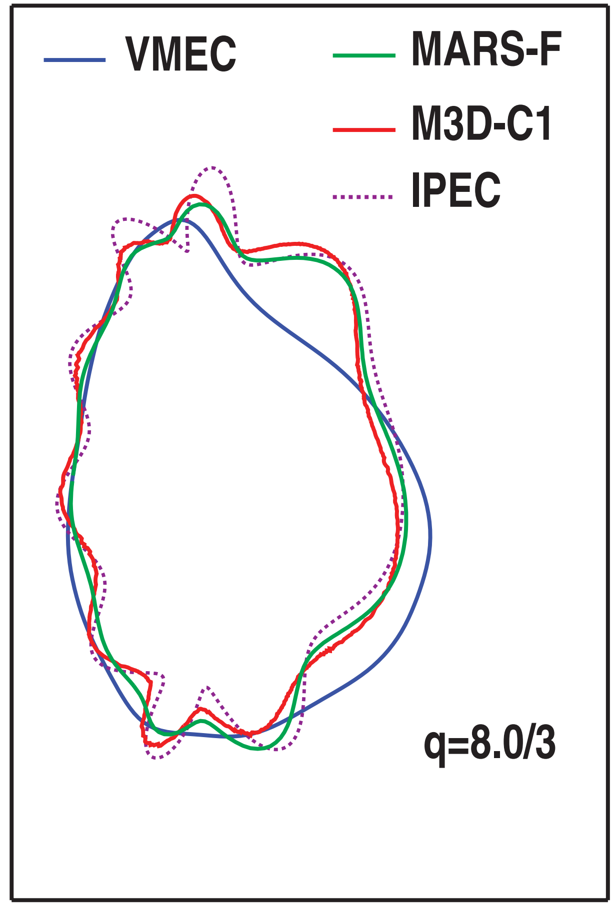




- **$|d\xi/dr|$ never exceeds ~0.3 in this case, so we expect the linear result to be valid**

2012 Theory Milestone Compared 3D Equilibrium Calculations From IPEC, MARS, M3D-C1, and VMEC

- **VMEC results were qualitatively different from results of linear codes (IPEC, MARS, M3D-C1)**
- **Nonlinear M3D-C1 calculations agreed well with linear M3D-C1 calculations (although linear validity was questionable at edge)**
- **VMEC was found to converge very slowly to analytic result in circular, large aspect-ratio test case as radial resolution was increased (S. Lazerson)**
- **My conclusion: VMEC struggles to resolve resonant currents in perturbed tokamaks**



Turnbull, *et al.* Submitted to *Phys. Plasmas*

Developing and Validating ELM Suppression Criteria

ELM Suppression Criteria Are Tested on DIII-D Discharges Taking Plasma Response Into Account

- **"Island Overlap Width" criterion was found to correlate with RMP ELM Suppression**
  - Considered only the "vacuum" fields
  - Predicted level of stochasticity is
  - *This metric was not intended to describe the actual physical situation, but rather to find a characteristic of the coil spectrum that correlates with ELM suppression*

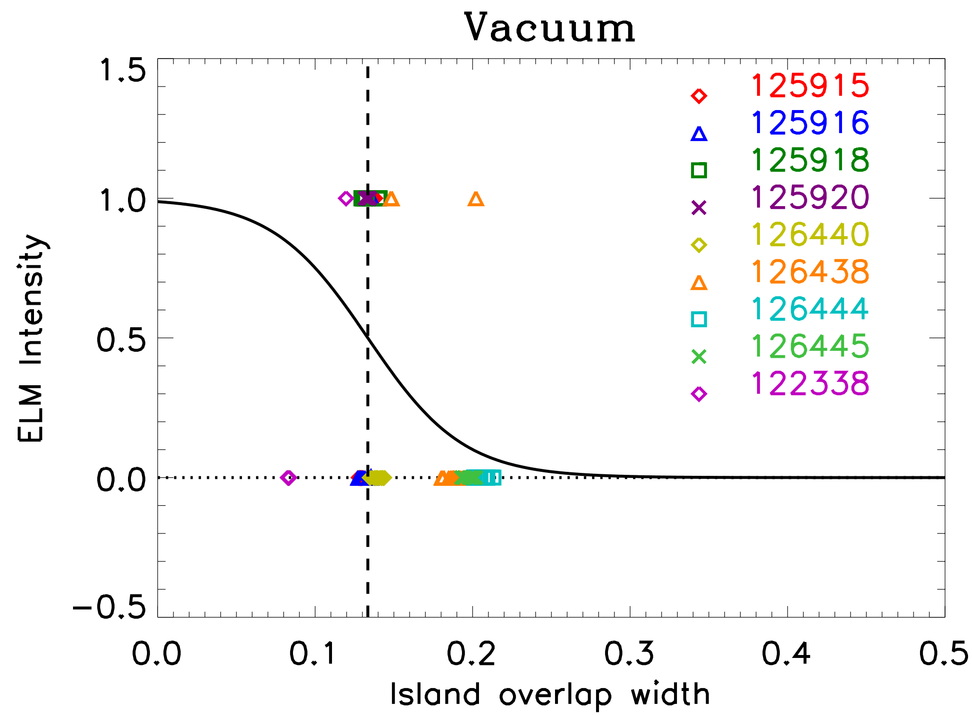
- **To develop a better ELM suppression criterion:**
  - Take into account the plasma response
  - Explore metrics related to our ELM suppression hypothesis

- **We can evaluate various criteria on a database of discharges with RMP applied (some are ELM suppressed, others aren't)**

Features of Plasma Response Are Correlated With ELM Suppression For a Set of DIII-D Discharges

- **For each discharge/timeslice, an axisymmetric equilibrium is reconstructed with EFIT**
- **ELM Intensity at each timeslice**
  - 1 if plasma was ELMing
  - 0 if plasma was suppressed
- **Each criterion is evaluated by tanh fit to ELM intensity**
- **Threshold = center of tanh fit**
- **Accuracy =**

$$\frac{\text{\# of correctly classified timeslices}}{\text{\# of timeslices}}$$



Vacuum island overlap width
Threshold = 0.133 (Ψ_N)
Accuracy = 60%

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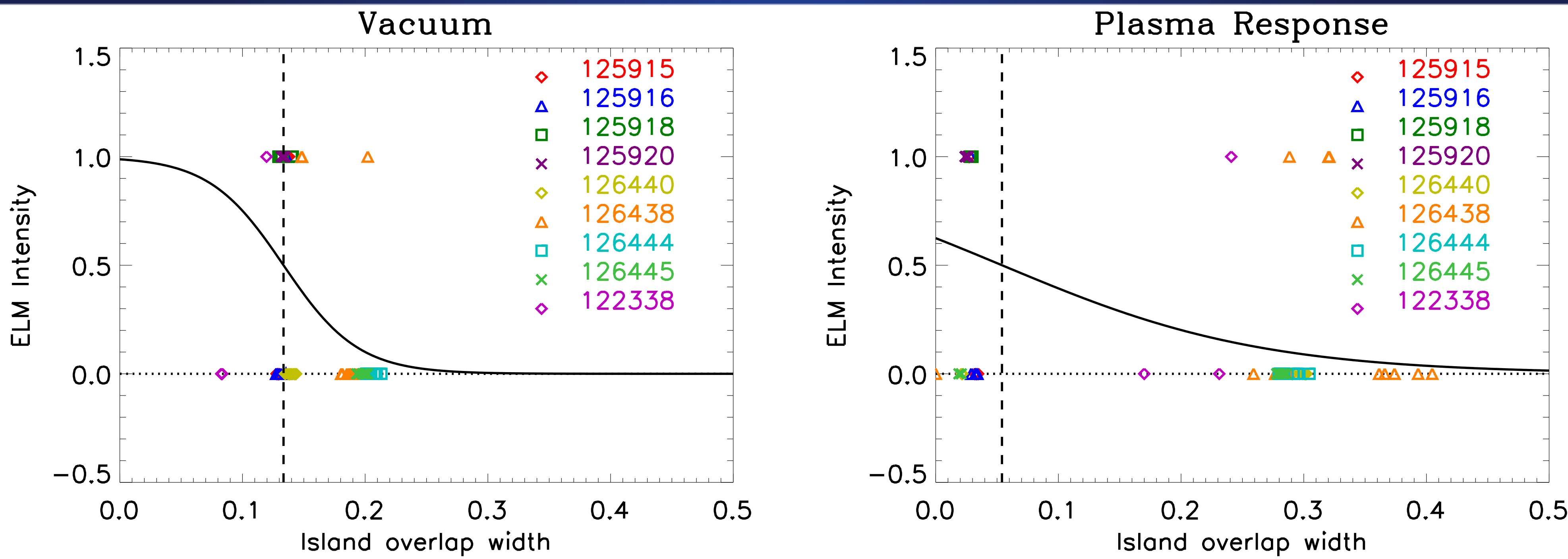
Island Overlap Criterion is Improved by Including Plasma Response



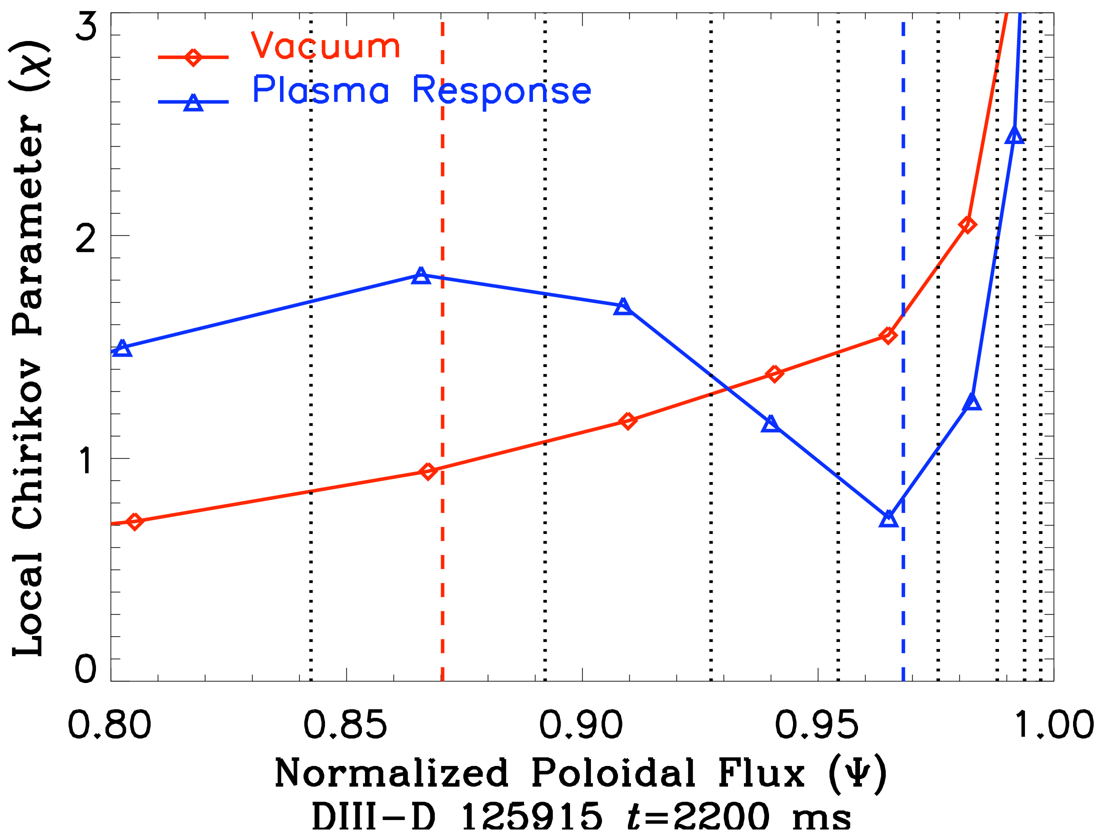
Vacuum island overlap width
Threshold = 0.133 (Ψ_N)
Accuracy = 60%

Plasma island overlap width
Threshold = 0.059 (Ψ_N)
Accuracy = 72.6%

- **Plasma response reduces island overlap width threshold**
- **This criterion yields many false negatives (ELM suppressed despite not meeting threshold)**

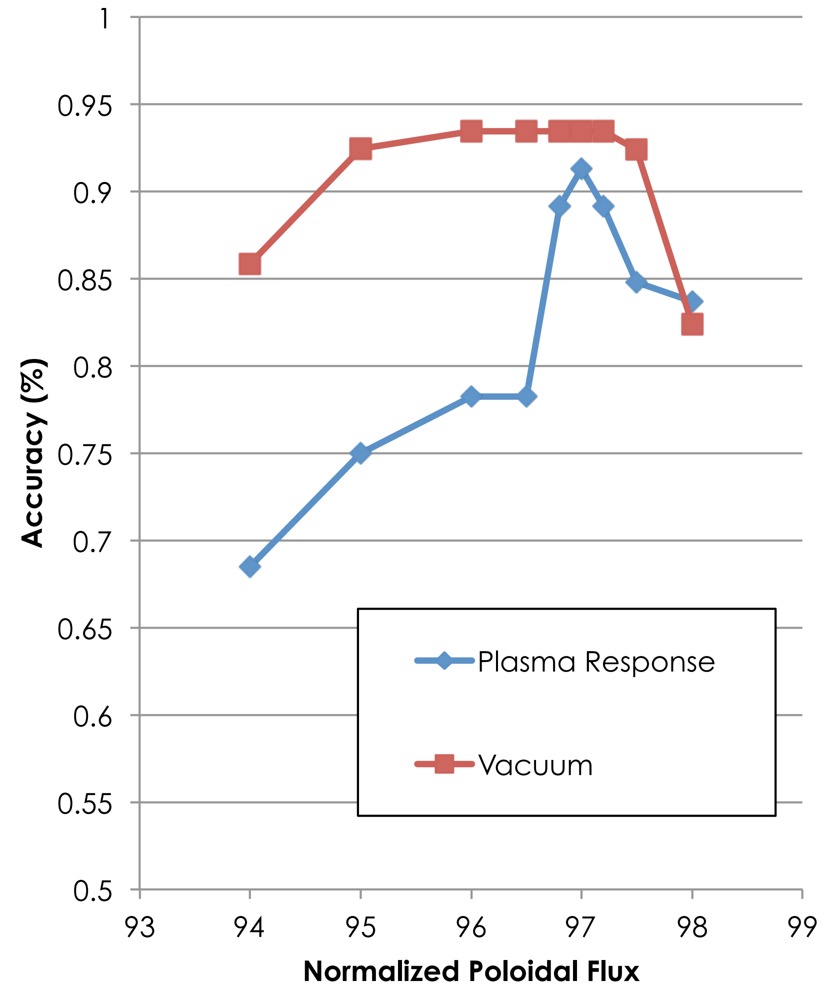
Local Chirikov Parameter $\chi(\Psi)$ Characterizes Local Stochasticity

- **Island Overlap Width requires stochasticity across entire pedestal**
- **"Local Chirikov Parameter" evaluates stochasticity at particular location**
- **Chirikov parameter (symbols) is evaluated for each pair of adjacent rational surfaces (dotted lines)**
- **$\chi(\Psi)$ (solid lines) is defined by linear interpolation of Chirikov parameters**

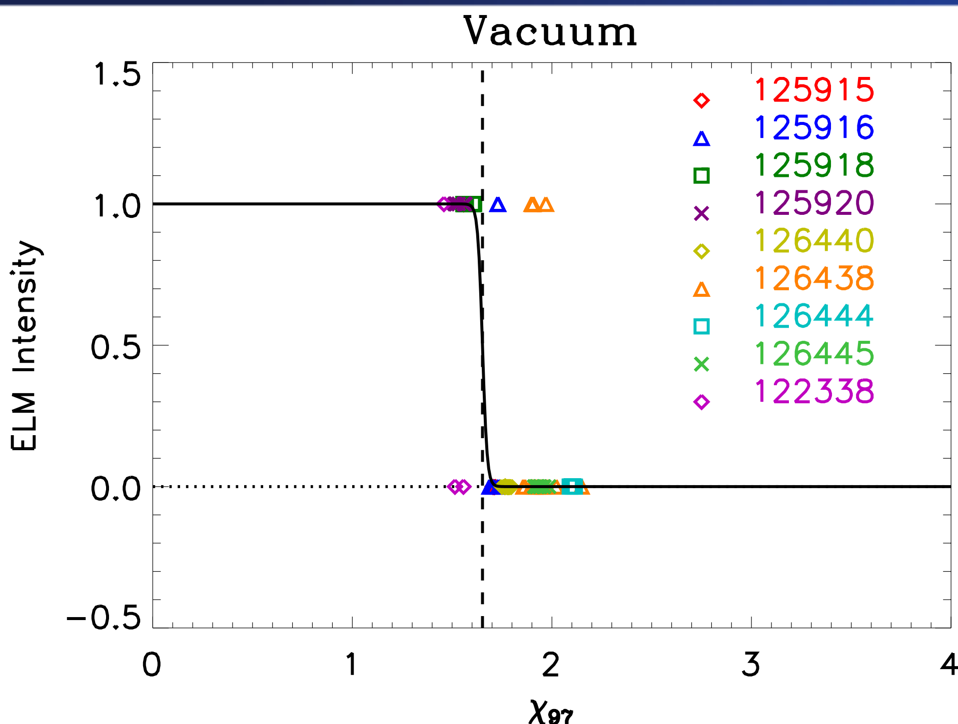


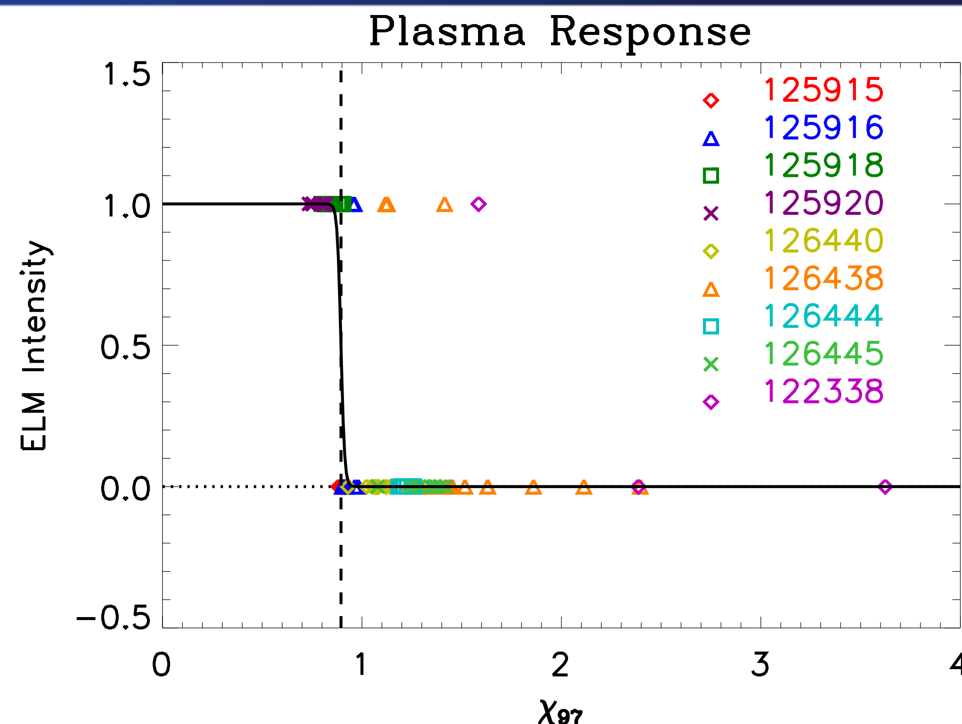
χ_{97}: Correlation of $\chi(\Psi)$ with ELM Suppression is Best at Ψ=0.97

- **$\chi(\Psi)$ with plasma response has sharp increase in accuracy near Ψ=0.97**
- **$\chi(\Psi)$ without plasma response is actually more accurate than with plasma response**
  - Vacuum calculation is more robust
  - Little variation with Ψ
- **$\chi(\Psi=0.97)$ is much more accurate than island overlap width**



ELM Suppression More Strongly Correlates With χ_{97} Than With Island Overlap Width

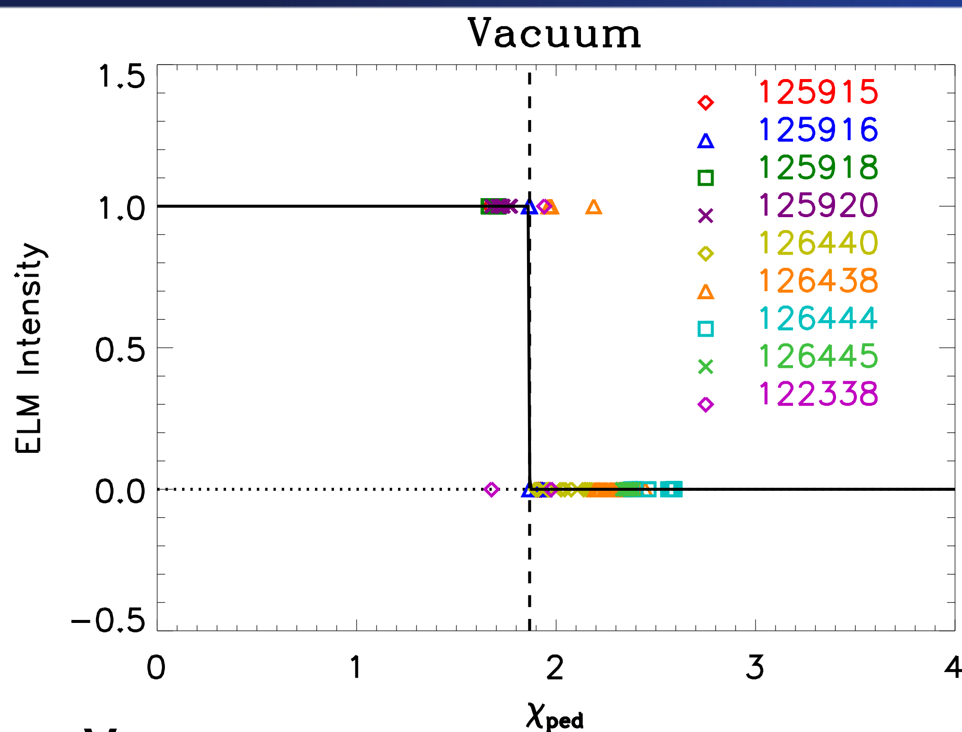


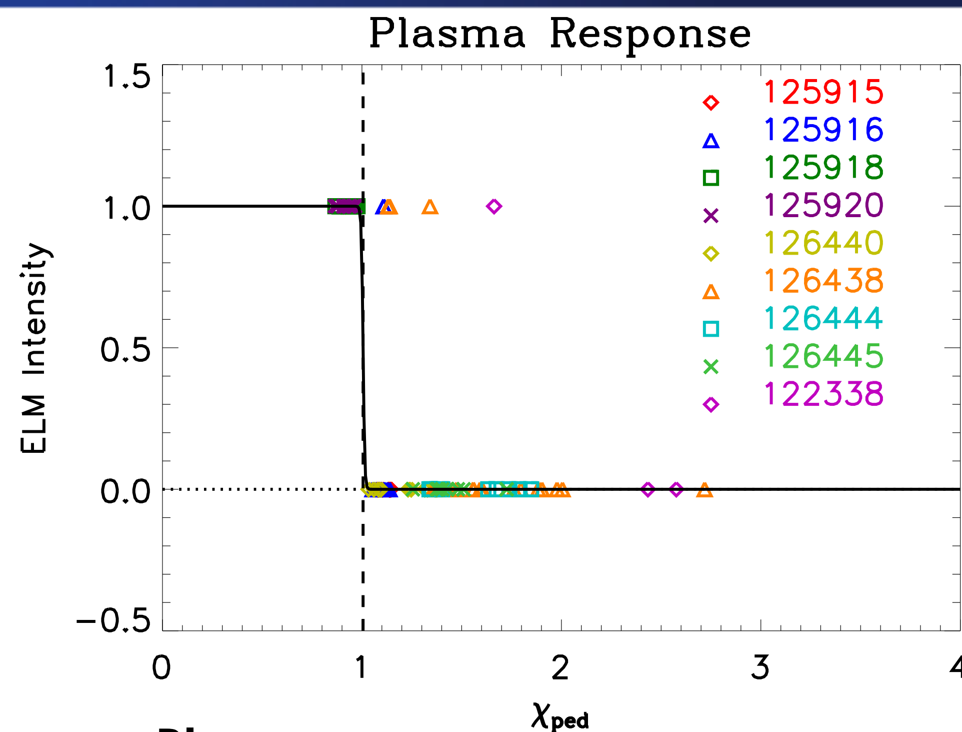


Vacuum χ_{97}
Threshold = 1.65
Accuracy = 93.5%

Plasma χ_{97}
Threshold = 0.895
Accuracy = 91.3%

χ_{ped}: Local Chirikov Parameter is Further Improved By Evaluating χ at Pedestal Top





Vacuum χ_{ped}
Threshold = 1.867
Accuracy = 91.6%

Plasma χ_{ped}
Threshold = 1.01
Accuracy = 92.6%

- **Choosing "Ψ=pedestal top" replaces arbitrary Ψ=0.97**
- **Accuracy of χ_{ped} with plasma response exceeds $\chi(\Psi)$ for any fixed Ψ**

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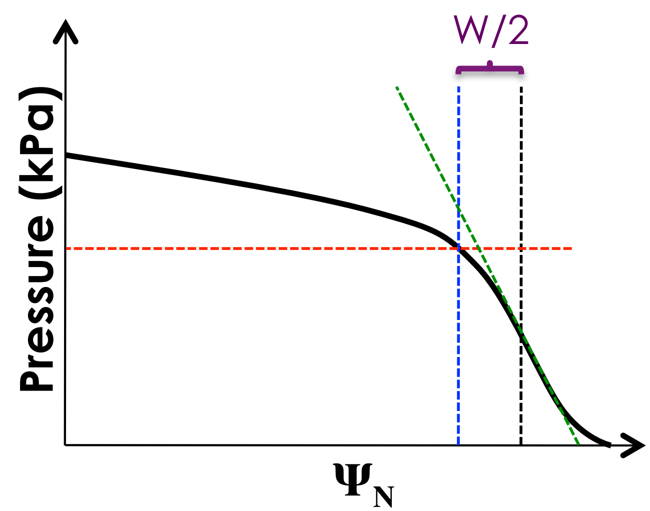
ELM Suppression Correlates Best With Position of Pedestal Center

	Threshold	Accuracy
Pedestal Gradient	$> 1{,}305$ kPa/Ψ_N	66.3%
Pedestal Height	> 38.4 kPa	68.4%
Pedestal Width	$> 0.0201\ \Psi_N$	77.9%
Pedestal Top	$< 0.966\ \Psi_N$	91.5%
Pedestal Center	$< 0.976\ \Psi_N$	95.8%



- **These are purely measures of the axisymmetric equilibrium**
- **Correlation with "Pedestal Center" is not a physically intuitive result**
  - Pedestal width, height, or gradient would seem more natural
- ***A predictive model of ELM suppression must be able to predict the response of the axisymmetric pedestal profiles to 3D fields***

Obtaining a Predictive Model of ELM Suppression

Ultimate Goal Is To Understand RMP ELM Suppression

- **Need to know:**
  - Will coils suppress ELMs?
  - What is the associated loss of confinement?
  - What are the resulting heat/particle fluxes to the walls?

- **This will require:**
  - ☑ Calculating 3D MHD response
  - ☐ Calculating 2D transport response
  - ☐ (Maybe) calculating 3D peeling-ballooning stability

- **Efforts to calculate transport in 3D fields are underway**
  - Gyrokinetics (GENE)
  - Fast ion transport (SPIRAL)
  - Ballooning mode stability (T. Bird)

- **Maybe 3D effects on turbulence can be integrated into EPED**

Summary

- **Good progress on calculating 3D MHD response**
  - Various codes in relatively good agreement (except VMEC)
  - Quantitative agreement with experimental results
  - Linear response may be sufficient for some aspects of plasma response
  - Single-fluid nonlinear calculations with realistic parameters are feasible

- **We have obtained criteria that correlate with ELM suppression better than island overlap width does**
  - Fact that best correlation is with pedestal position implies that understanding transport in 3D is necessary for predictive model of ELM suppression

Extra Slides

M3D-C1 Solves Two-Fluid Equations

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}) = 0$$

$$n_i m_i \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi$$

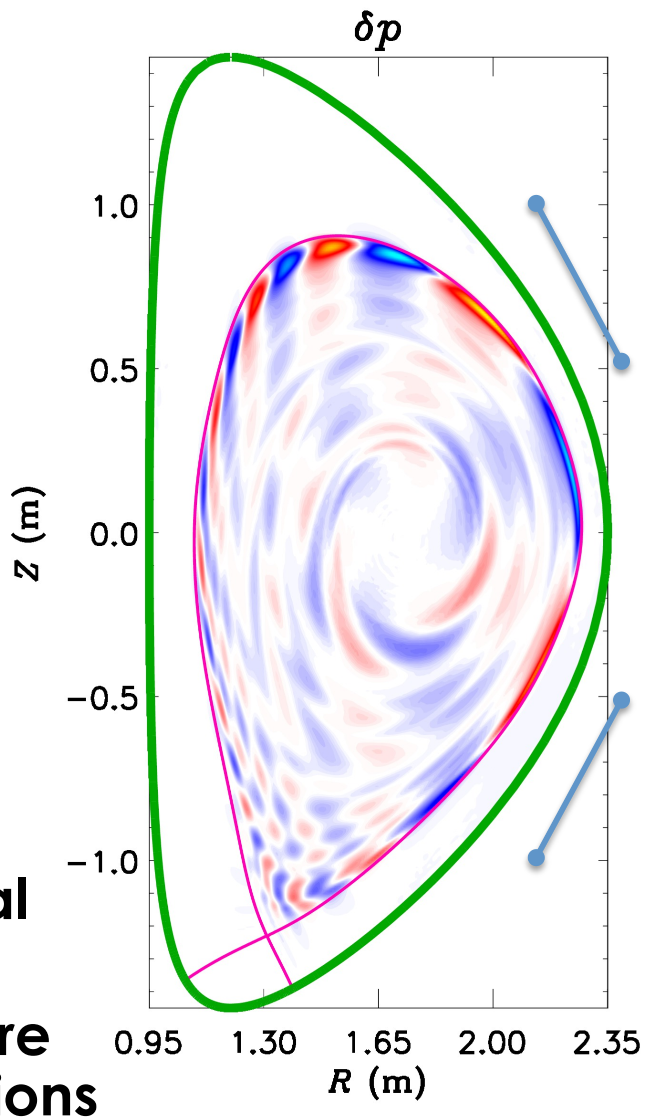
$$\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p = -\Gamma p \nabla \cdot \mathbf{u} - \frac{1}{n_e e} \mathbf{J} \cdot \left(\Gamma p_e \frac{\nabla n_e}{n_e} - \nabla p_e \right) + (\Gamma - 1)\left(\eta J^2 - \Pi : \nabla \mathbf{u} - \nabla \cdot \mathbf{q} \right)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[-\mathbf{u} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{n_e e} (\mathbf{J} \times \mathbf{B} - \nabla p_e) \right]$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

$$\Pi = -\mu \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right]$$

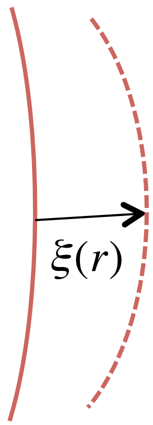
$$\mathbf{q} = -\kappa \nabla (T_e + T_i) - \kappa_\parallel \mathbf{b}\mathbf{b} \cdot \nabla T_e$$



- **Boundary conditions: normal B from external coils is held constant at boundary**
- **Here, linear, time-independent equations are solved directly, subject to boundary conditions**

The Validity of Linear Temperature Response Can Be Quantified

- **"Displacement" may be defined by movement of isotherms:**



$$T_0(r+\xi) + \delta T(r+\xi) = T_0(r)$$

$$\left[T_0(r) + \frac{dT_0}{dr}\xi\right] + \delta T(r) = T_0(r)$$

$$\xi = -\frac{\delta T}{dT_0/dr}$$

Criterion for validity of linear temperature response

- **Overlap of adjacent isotherms is implied unless** $\left|\frac{d\xi}{dr}\right| < 1$

- **In ideal MHD, violating this criterion implies that perturbed surfaces are no longer well described by ξ**

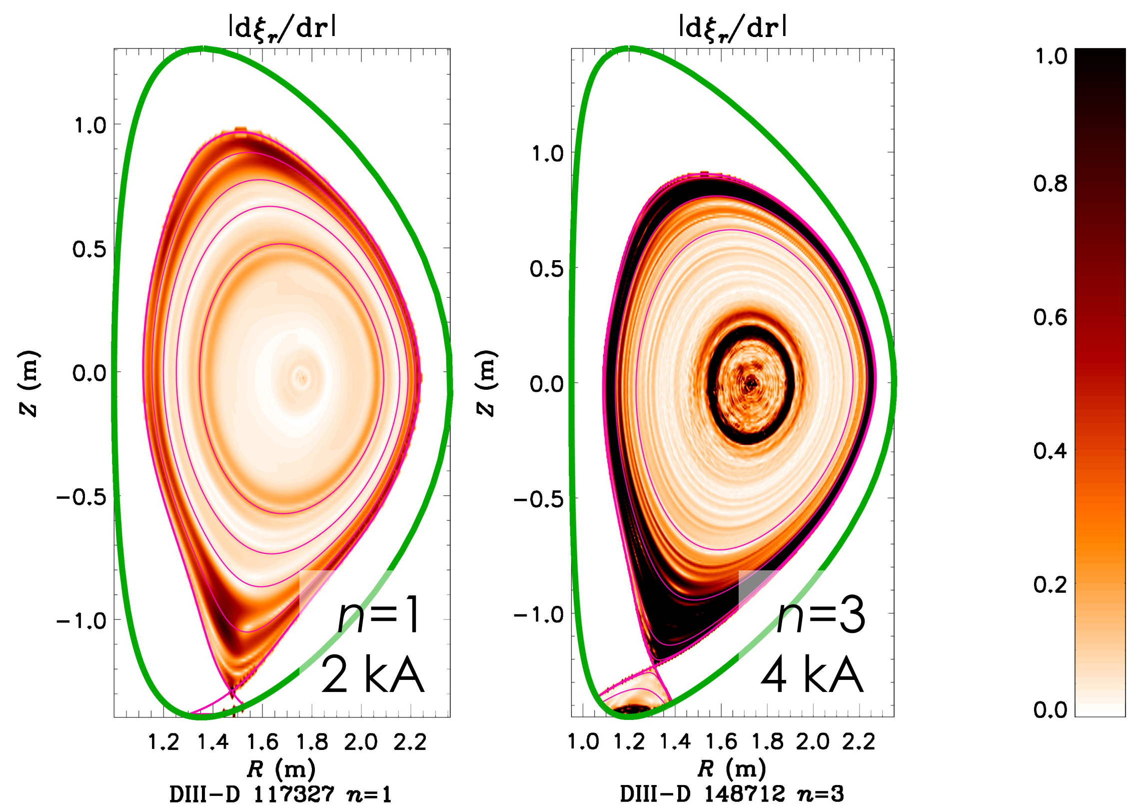
- **In M3D-C1, violating this criterion implies breakdown of linear approximation to $\mathbf{B}\cdot\nabla T = 0$**

Linear Validity of Temperature Response Is Often Violated in the Pedestal for Typical DIII-D Cases

- **For typical DIII-D parameters, linear validity is often exceeded in the pedestal region and near mode-rational surfaces**

- **Validity may also be exceeded near mode-rational surfaces**

- **Magnetic response is likely valid nearly everywhere since $\delta B/B$ is always small**
  - Exception may be resonant field components



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