Tokamak Rotation caused by Disruptions and other MHD Effects

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Introduction

It was observed that disruptions were accompanied by toroidal rotation [Gerhardt 2012,Granetz 1996, Gerasimov 2010].

There is a concern that this rotation may occur during ITER disruptions, causing a resonance between rotating toroidal perturbations and the resonant frequencies of the vacuum vessel.

In an MHD model, disruptions can produce rotation.

Rotation is MHD driven zonal flow.

Both toroidal and poloidal rotation are produced.

Toroidal rotation period in disruption is comparable to duration of wall force, a few linear growth times.

Toroidal rotation is sheared, peak value can be $10\times$ larger than average value.

ELMs can produce rotation.

MHD activity may produce intrinsic toroidal rotation [Rice 2007].

RMPs may also drive rotation, so there might be a momentum source without disruptions or ELMs.

Rotation driven by RMP might stabilize RWMs, possibly also VDEs.

Theory

Conservation of toroidal angular momentum:

$$\frac{\partial}{\partial t}L_\phi = \oint (RB_\phi B_n - \rho R v_\phi v_n) R dl d\phi \tag{1}$$

where the total toroidal angular momentum is

$$L_\phi = \int \rho R^2 v_\phi dR dZ d\phi \tag{2}$$

and the integral in (1) is over the boundary. Using the M3D magnetic field representation,

$$\mathbf{B} = \nabla\psi \times \nabla\phi + \frac{1}{R}\nabla_\perp F + G\nabla\phi \tag{3}$$

in (1) yields

$$\frac{\partial}{\partial t}L_\phi = \oint G\frac{\partial\psi}{\partial l} dl d\phi \tag{4}$$

where $\partial F/\partial n = 0$ at the boundary. We have assumed that $v_\phi = 0$ at the boundary, but not $v_n = 0$ at the boundary, although we have done so in simulations with M3D.

If $G = G(\psi)$, then toroidal angular momentum L_ϕ is conserved. This is the case in an equilibrium satisfying the Grad - Shafranov equation. If the plasma is not in equilibrium, such as during a disruption or ELM, then net flow can be generated.

The magnetic fluxes ψ and G can be split into equilibrium and toroidally varying parts, $\psi = \psi_0 + \psi_1$, $G = G_0 + G_1$. For simplicity we assume circular equilibrium cross sections, $dl = rd\theta$. The perturbed magnetic fluxes ψ_1 and G_1 approximately satisfy [Strauss 1977]

$$\psi_1 = \mathbf{B}_0 \cdot \nabla\xi \tag{5}$$

$$G_1 = -\nabla G_0 \times \nabla\xi \cdot \widehat{\phi} \tag{6}$$

where ξ is the perturbation displacement, given by

$$\xi = \sum_m \xi_{mn} \sin(m\theta - n\phi) \tag{7}$$

Now

$$\dot{L}_\phi = -\frac{G_0' B}{rR} \oint \frac{\partial\xi}{\partial\theta}\frac{\partial}{\partial\theta}\left(\frac{1}{q}\frac{\partial\xi}{\partial\theta} + \frac{\partial\xi}{\partial\phi}\right) d\theta d\phi \tag{8}$$

where we approximated $B/R =$ constant. The integral (8) vanishes except for a term

$$\dot{L}_\phi = \frac{G_0' B}{2rR} \oint \left(\frac{\partial\xi}{\partial\theta}\right)^2 \frac{1}{q^2}\frac{\partial q}{\partial\theta} d\theta d\phi. \tag{9}$$

The integral (9) does not vanish if we assume the plasma is displaced by a VDE with $(m, n) = (1, 0)$,

$$q = q(r - \xi_{10}\sin\theta), \xi = \xi(r - \xi_{10}\sin\theta, \theta, \phi), \psi(r - \xi_{10}\sin\theta, \theta, \phi) \tag{10}$$

Then $\partial q/\partial\theta = -\xi_{10}\cos\theta q'$. Assuming a rigid wall and $\gamma\tau_{wall} \gg 1$, then $\xi = 0, \psi \approx 0$ at the boundary, and $\psi = -\xi_{10}\sin\theta\psi'$, $\xi = -\xi_{10}\sin\theta\xi'$.

We must have at least two modes $(m, n), (m+1, n)$ contributing to ξ, which beat together to give a $\cos\theta$ term. It is useful to express (9) using (5) in terms of $B_\theta = -\psi'$,

$$B_\theta = \sum_{mn} B_{\theta mn}\sin(m\theta - n\phi) \tag{11}$$

with

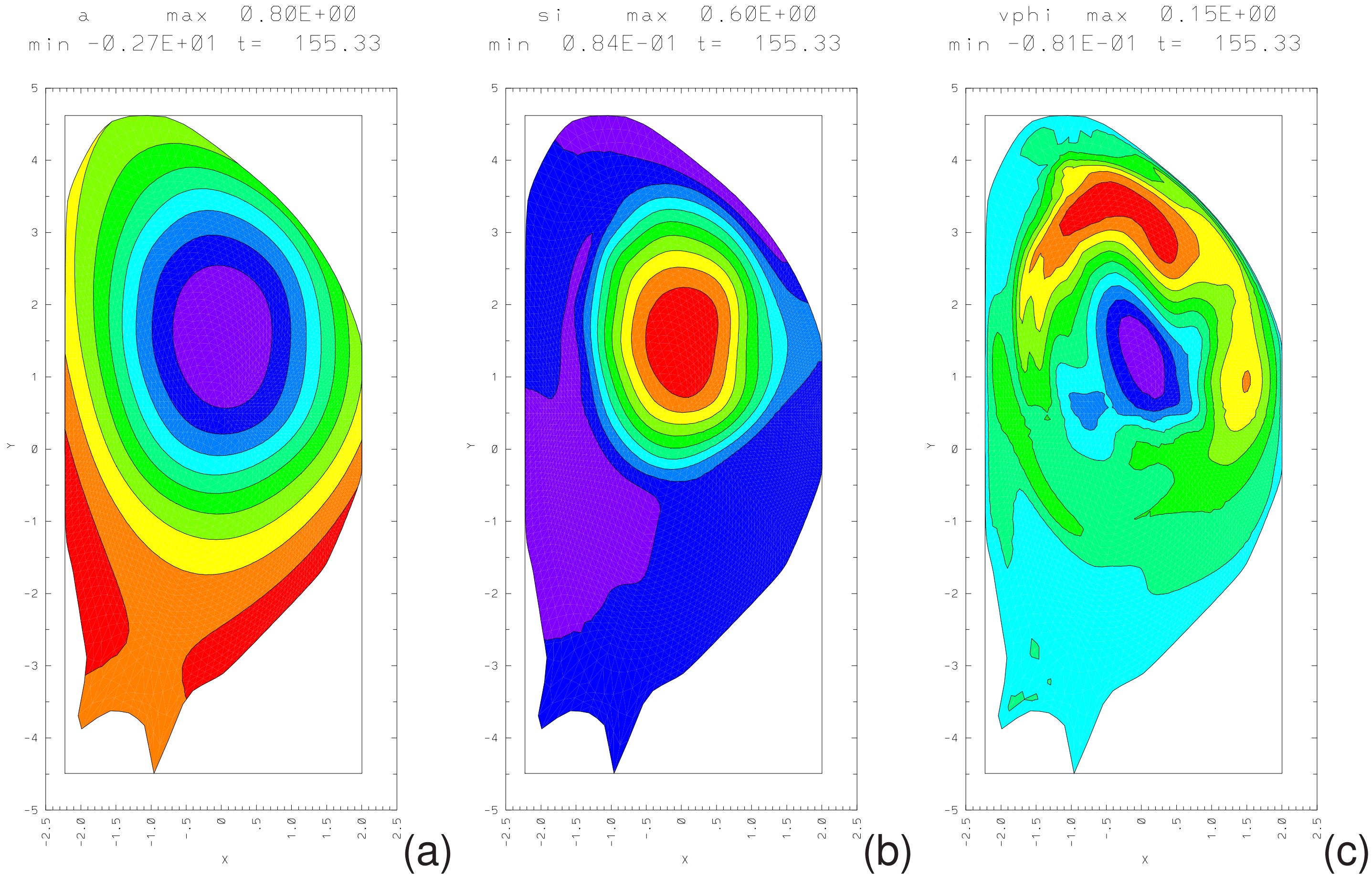
$$B_{\theta mn} = -\frac{B}{qR}(m - nq)\xi'_{mn} \tag{12}$$

which gives

$$\frac{dL_\phi}{dt} = -\pi^2 rq' G'_0 \xi_{10}^3 \frac{R}{B}\sum_{mn}\frac{m(m+1)B_{\theta mn}B_{\theta(m+1)n}}{(m-nq)(m+1-nq)} \tag{13}$$

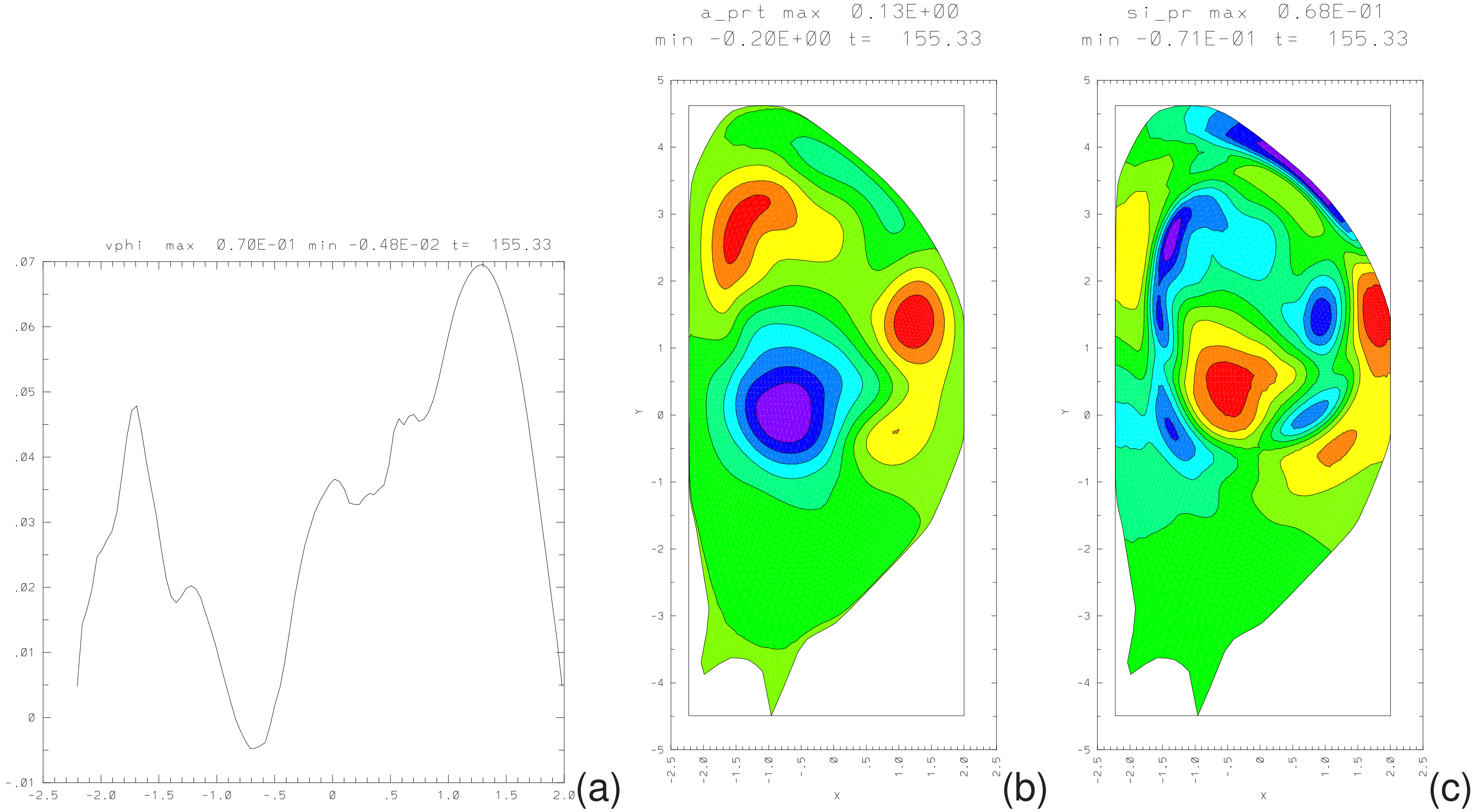
Magnetic field perturbations at the boundary, as well as a vertical asymmetry, can allow for a net plasma rotation.

VDE - kink disruption

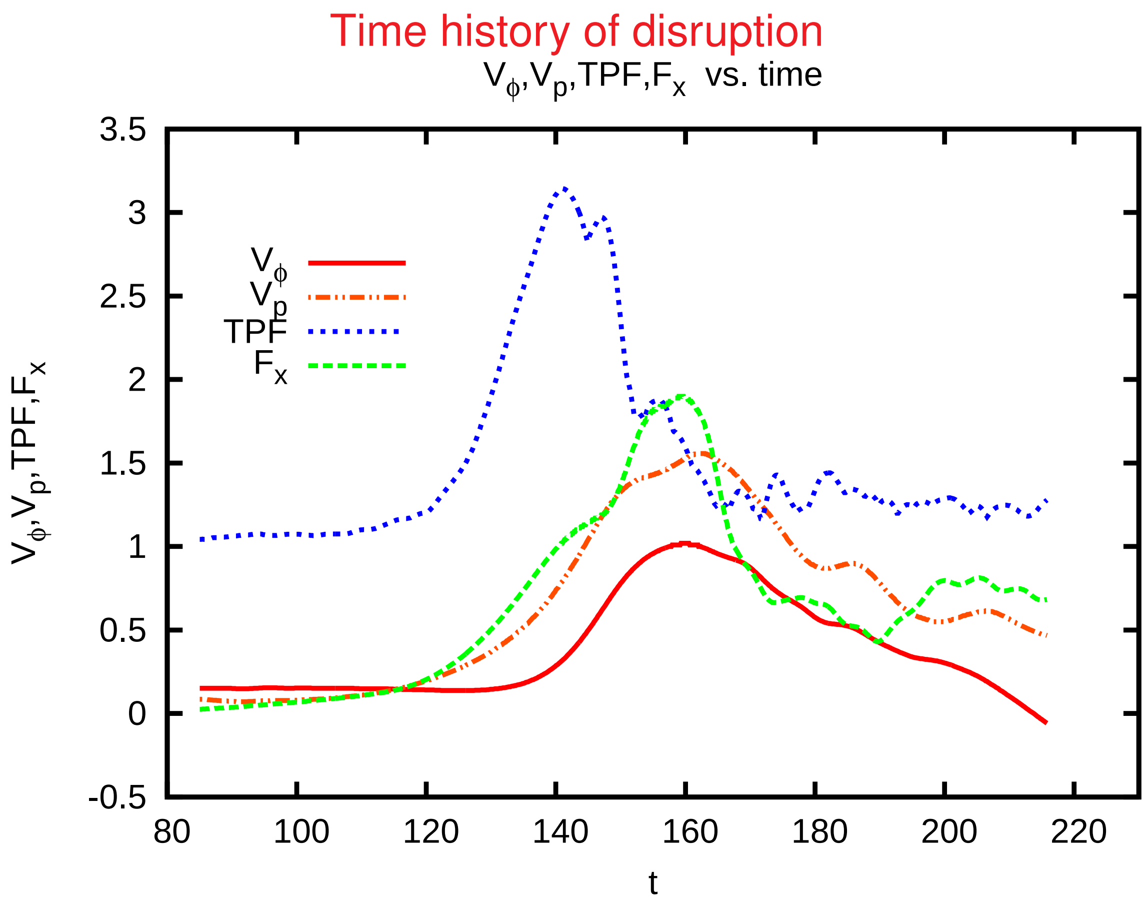


(a) The poloidal magnetic flux ψ during a disruption at toroidal angle $\phi = 0$, time $t = 155\tau_A$. Parameters: $S = 10^5, \tau_w = 10^3\tau_A$. (b) Toroidal magnetic flux G at the same toroidal angle and the same time. The contours of G and ψ are different, indicating that toroidal angular momentum can be generated. (c) Toroidal velocity v_ϕ at the same toroidal angle and time. The flow is sheared; it is zero on axis and small near the wall.

VDE - kink disruption



(a) slice plot of $v_\phi(R,0,0)$ at time $t = 155\tau_A$ showing sheared structure. (b) Toroidally varying part of ψ at the same time. (c) Perturbed toroidal magnetic flux G at the same toroidal angle and the same time. The contours of G and ψ are different, indicating that toroidal angular momentum can be generated.



Time history of the simulation shown in previous figures. Shown are V_ϕ, V_p, TPF, and F_x. $V_\phi =< \rho v_\phi > / < \rho >$, where the bracket is the volume average, and $V_p =< \sqrt{v_\perp^2} >$ are multiplied by 100. F_x, the sideways wall force, is multiplied by 10^4. The rotation drive is largest when TPF is largest, and the peak rotation coincides with the peak F_x.

Toroidal Rotation in Disruptions: Summary

$V_\phi = 0.01 v_A$, duration of V_ϕ is about $1/\gamma = 100\tau_A$.

The plasma makes one rotation during the time when the sideways force F_x is substantial.

The rotation is sheared; spatial maximum of $v_\phi \approx 5\times < v_\phi >$, hence 3D structures could rotate several times during the disruption, consistent with experimental observations on JET.

The time history of V_ϕ, V_p is insensitive to τ_{wall}, when $\tau_{wall} \gg \tau_A$.

Note $V_\phi \approx V_p$. In disruptions, the poloidal and toroidal velocities are comparable.

The peak density drive is associated with largest TPF. The peak toroidal velocity coincides with the peak wall force.

The toroidal velocity is damped on a longer time scale, by viscosity, mode locking to resistive wall [Hender 1989], or other effects [Boozer, 2010].

Poloidal Rotation

The vorticity is

$$\dot{w} = -(\nabla \times \frac{\partial \rho \mathbf{v}}{\partial t}) \cdot \nabla \phi = \nabla \cdot (\frac{B_\phi}{R}\mathbf{J} - \frac{J_\phi}{R}\mathbf{B}) \tag{14}$$

Then

$$\dot{W} = \int \dot{w} R dR dZ d\phi = \oint (B_\phi J_n - J_\phi B_n) dl d\phi \tag{15}$$

We find that only the first term on the right is nonzero, with only the term

$$J_n = \frac{1}{R}\frac{\partial^2 \psi}{\partial n \partial \phi} + \ldots \tag{16}$$

contributing. Expressing G in terms of ξ as in (5),(6), setting $\psi' = -B_\theta$ and expanding $\xi = -\xi_{10} \sin\theta \xi'$ as before

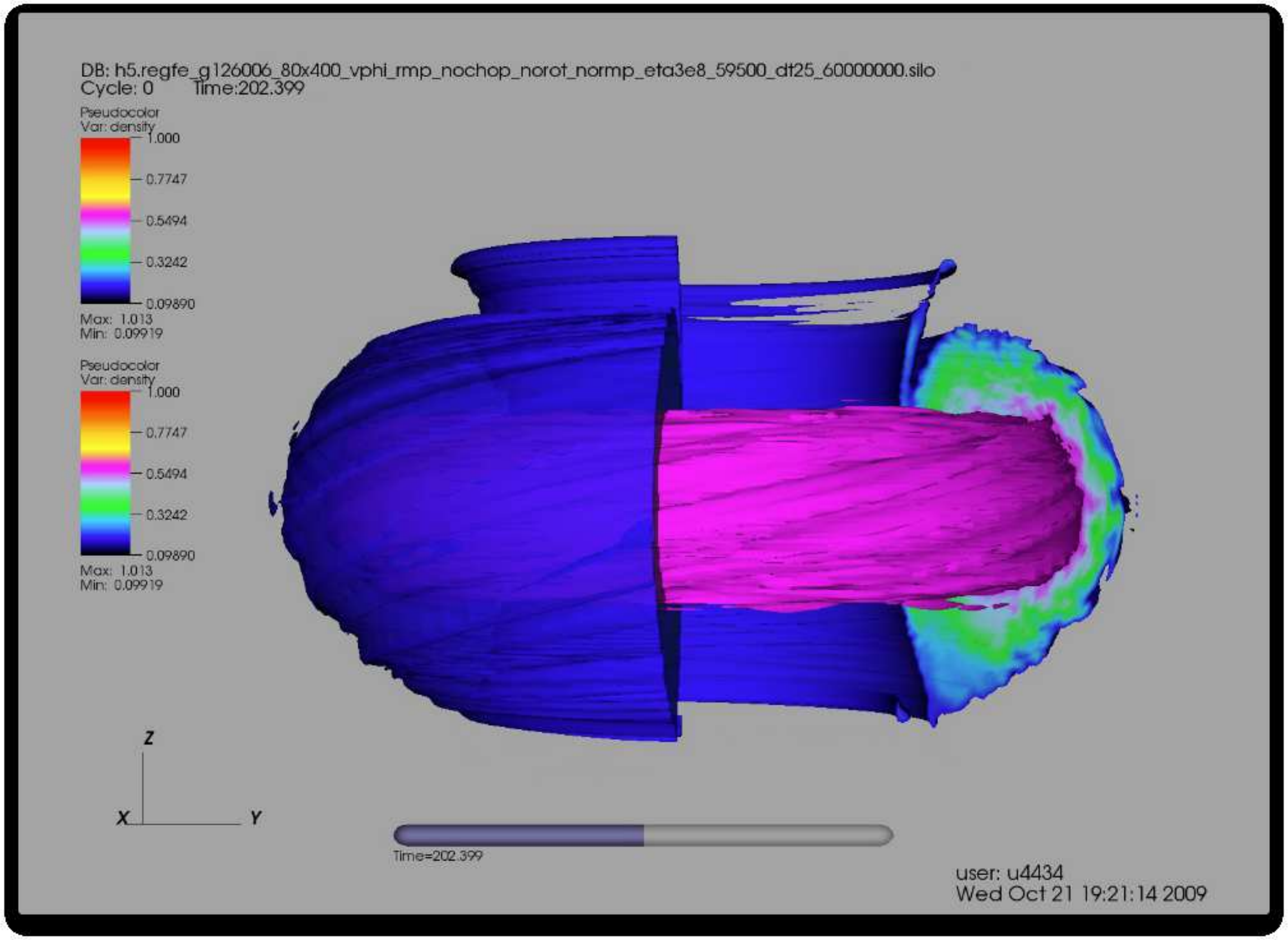
$$\dot{W} = \frac{G_0' \xi_{10}}{R^2} \oint \frac{\partial}{\partial \theta}(\sin\theta \xi') \frac{\partial B_\theta}{\partial \phi} d\theta d\phi \tag{17}$$

Expanding ξ in Fourier series and expressing ξ'_{mn} in terms of $B_{\theta mn}$ as in (12), we find that

$$\dot{W} = -\frac{\pi^2 q G_0' \xi_{10}}{BR} \sum_{mn} \frac{(2m+1-nq) n B_{\theta mn} B_{\theta(m+1)n}}{(m-nq)(m+1-nq)} \tag{18}$$

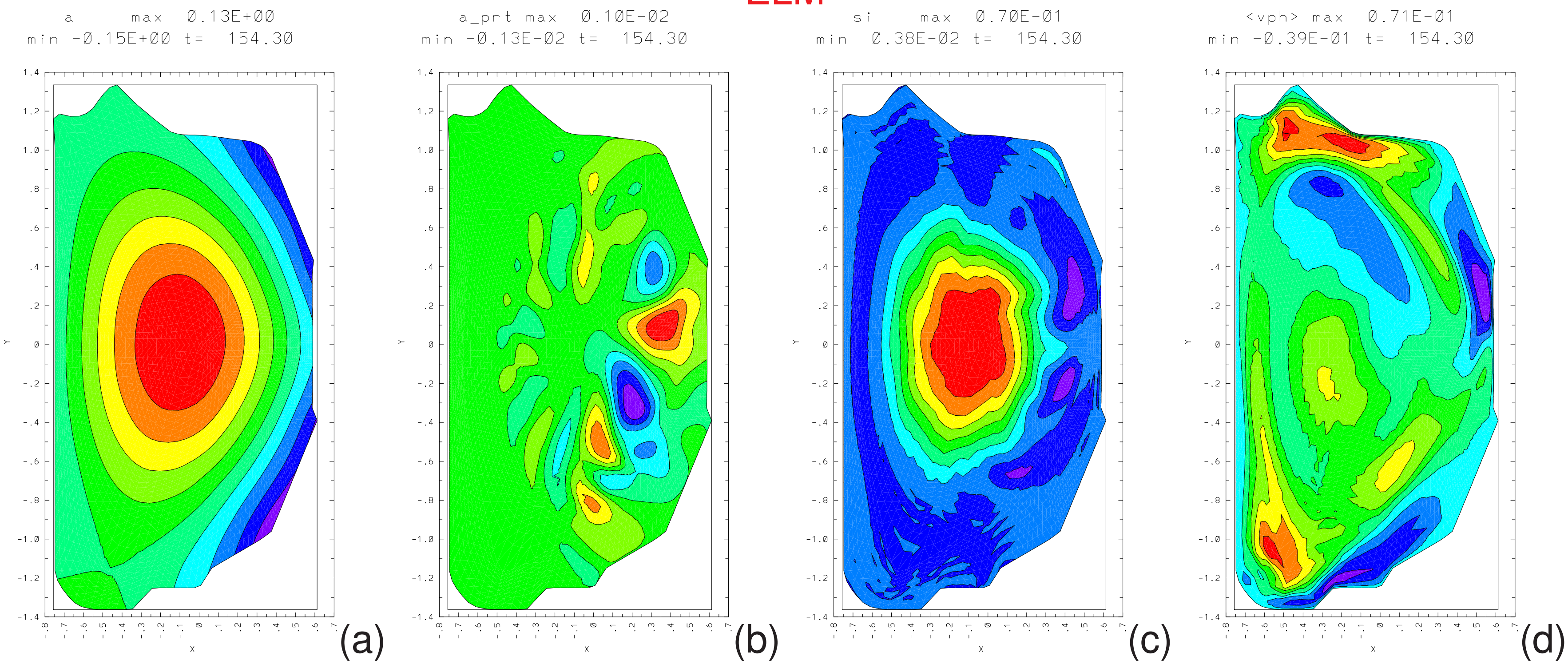
which is similar in form to (13).

ELM example

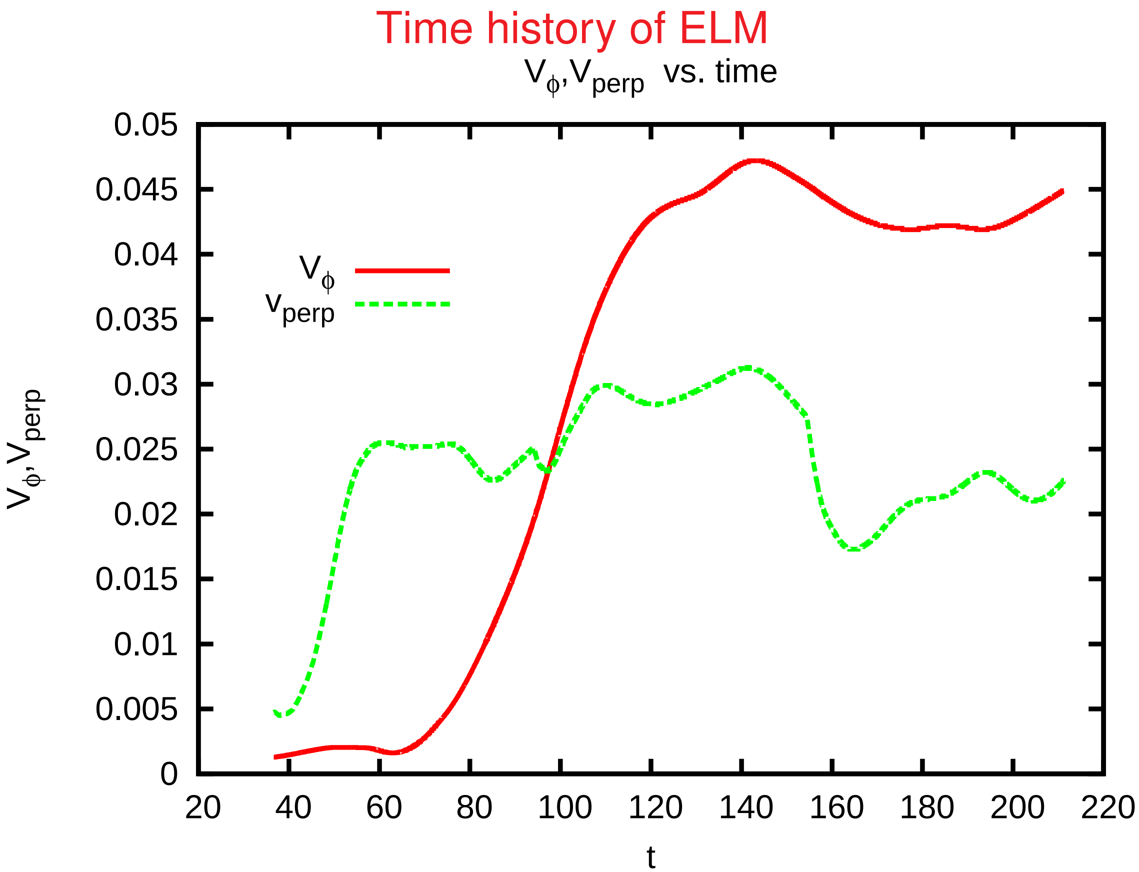


This is from a movie of a DIII-D ELM simulation in $/project/projectdirs/mp288/sugiyama/movie$ $\_pasden - psi\_126006\_movie1a\_3drot\_n018\_n058.mpeg$. Rotation is obvious in the movie, and looks like both poloidal and toroidal rotation. Two isosurfaces of density are plotted.

ELM



ELM simulation of DIII-D 126006, $S = 10^5$, $\tau_{wall} = 100\tau_A$. (a) The poloidal magnetic flux ψ at toroidal angle $\phi = 0$, time $t = 154\tau_A$. (b) perturbation of ψ. (c) toroidal magnetic flux G (d) Toroidally averaged toroidal velocity.



Time history of the simulation shown in previous figure. Shown are V_ϕ and $V_\perp$. The maximum value of $V_\phi = 0.05 v_A$. The results are insensitive to τ_{wall}/τ_A.

Intrinsic toroidal rotation

Scaling law of rotational Alfvén Mach number $M_\phi \propto \beta_N$ has been obtained where $M_\phi = v_\phi / v_A$. " ... scalings of intrinsic rotation with normalized gyro - radius or collisionality show no correlation. Whether this suggests the predominant role of MHD phenomena such as ballooning transport over turbulent processes in driving the rotation remains an open question." [Rice 2007] This was a comparative study of intrinsic toroidal rotation in H mode plasmas, in several experiments.

In a high β large aspect ratio approximation [Strauss 1977], $G_0 = -Rp/B$, so a β scaling emerges naturally. This tends to be a better approximation in an H mode pedestal, where there is a relatively large pressure gradient. The VDE could be replaced by vertical asymmetry, and the 3D perturbations could be ballooning modes which occur in ELMs. Writing (13) in terms of the normalized time $1/(\gamma\tau_A)$, and dividing both sides by $\rho\gamma\tau_A$ gives the scaling

$$M_\phi \approx \frac{1}{\gamma\tau_A}\frac{R}{r}\frac{\xi_{10}^3}{r^3}\frac{B_{\theta mn}^2}{B^2}\beta_N, \tag{19}$$

where $\beta_N = \epsilon p R/(B I_\phi)$.

Taking $\gamma\tau_A = 0.01$ as above, $B_\theta/B = 0.01$, $\xi_{10}/r = 0.5$ and $\beta_N = 3$ yields

$$M_\phi \approx 10^{-2} \tag{20}$$

consistent with the simulations above and with the ITER prediction of [Rice 2007].

RMP toroidal rotation

It is possible to get a nonzero RMP net velocity. Like the disruption and ELM cases, it seems to require two modes and a vertical asymmetry. The total toroidal momentum is

$$\dot{L}_\phi = \oint \lambda' \frac{\partial \lambda}{\partial \phi} r d\theta d\phi. \tag{21}$$

where the prime is a normal derivative, and in the vacuum,

$$\mathbf{B} = \nabla \overline{\psi} \times \nabla \phi + G_0 \nabla \phi + \nabla \lambda \tag{22}$$

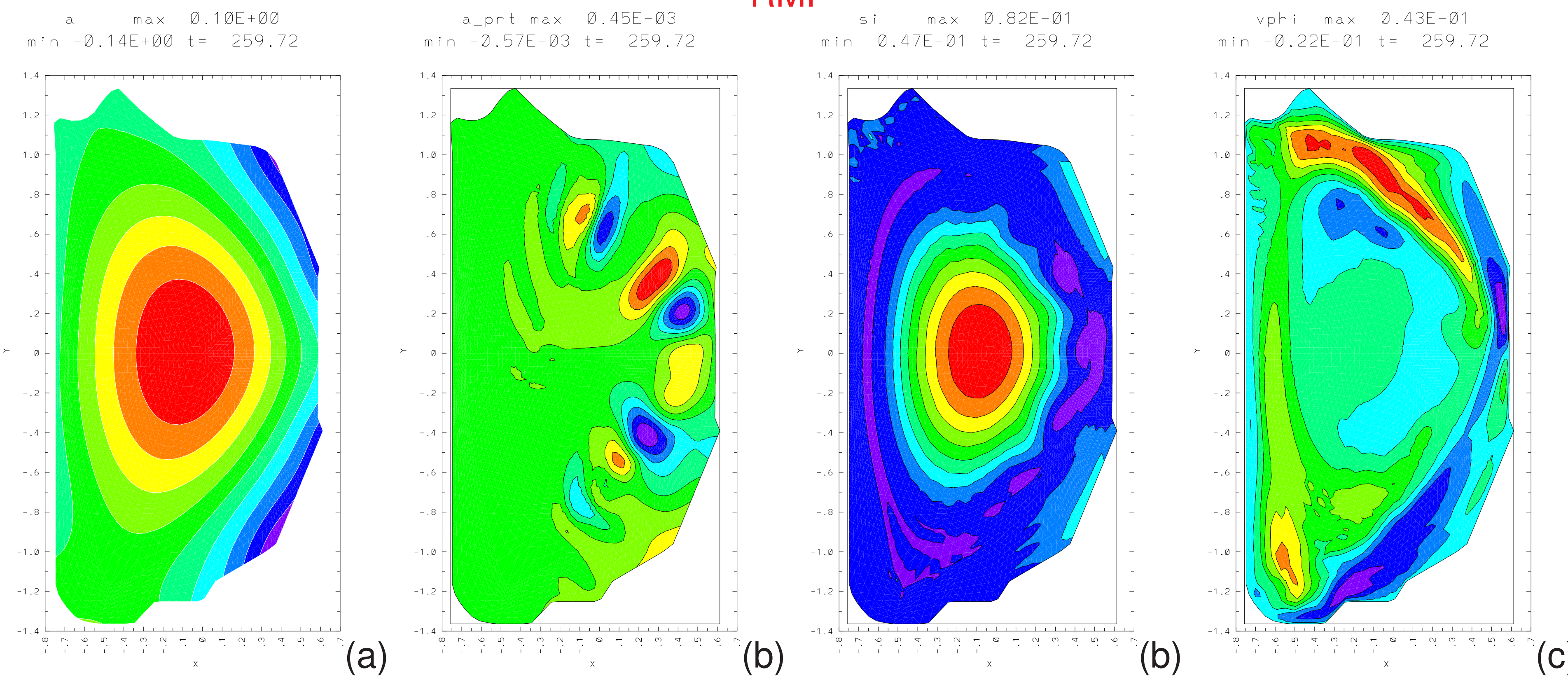
where $G_0 =$ constant, and $\overline{\psi}$ is independent of ϕ. Again assume a vertical asymmetry, $\lambda = \lambda(r - \xi_{10} \sin \theta, \theta, \phi)$. Then

$$\dot{L}_\phi = -\xi_{10} \oint \sin \theta (\lambda' \frac{\partial \lambda'}{\partial \phi} + \lambda'' \frac{\partial \lambda}{\partial \phi}) r d\theta d\phi \tag{23}$$

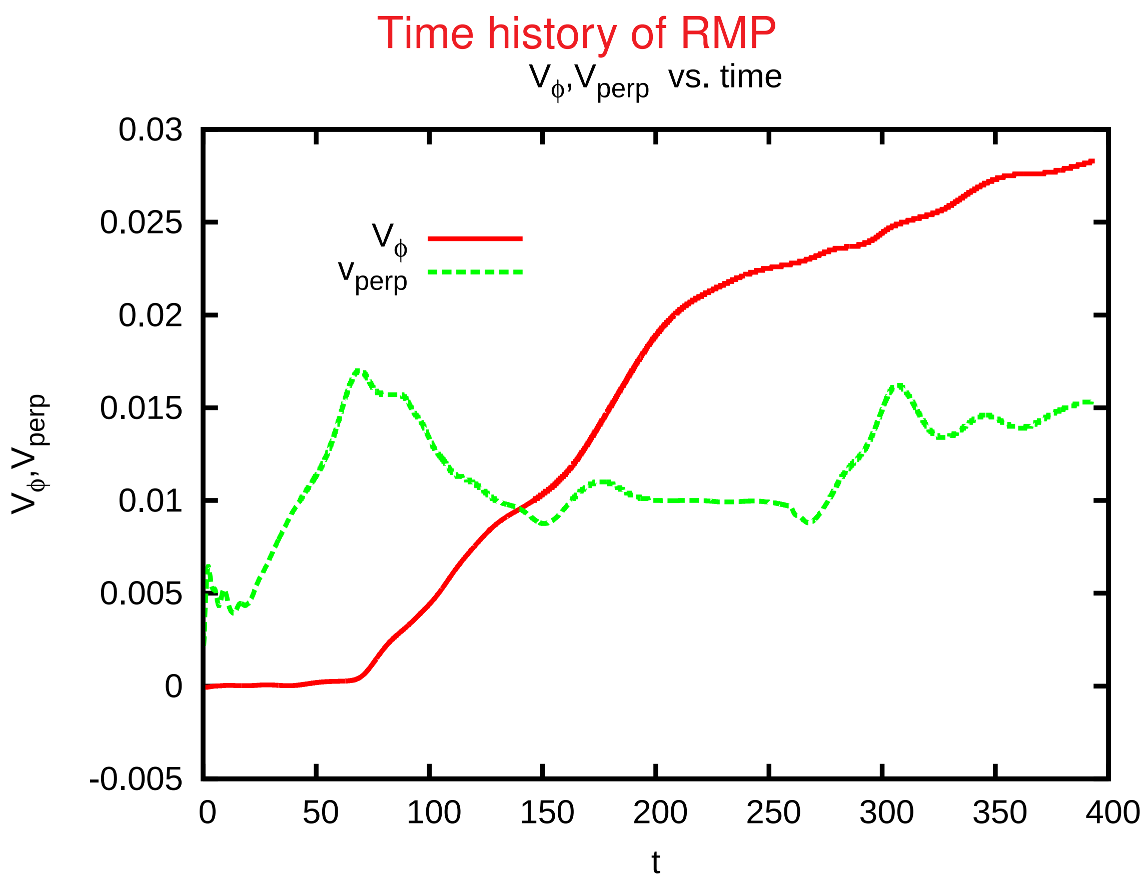
Let $\lambda = \sum_{mn} \lambda_{mn} \cos(m\theta - n\phi)$, and $\lambda'_{mn} \approx (m/r)\lambda_{mn}$. The result is

$$\dot{L}_\phi = \frac{\pi^2}{4r} \xi_{10} \sum_{mn} n(2m+1)^2 \lambda_{mn} \lambda_{(m+1)n} \tag{24}$$

RMP



Application of RMP to DIII-D 126006 [Strauss *et al.* 2009]. (a) poloidal magnetic flux ψ at toroidal angle $\phi = 0$, time $t = 259\tau_A$. (b) perturbed $n = 3$ poloidal flux (c) toroidal magnetic flux G at same time. (d) toroidal velocity. In this simulation there is no initial rotation. The RMP excites ELM - like perturbations, which in turn produce rotation. In previous RMP simulations [Strauss *et al.* 2009] rotation was included initially, and it screened the RMP from the plasma.



Time history of the simulation shown in previous figure. Shown are V_ϕ and $V_\perp$. The maximum value of $V_\phi = 0.025 v_A$, similar to disruption and ELM.

Conclusions

MHD can drive toroidal and poloidal rotation.

Rotation is MHD driven zonal flow.

Need nonzero B perturbations at the wall to get a net rotation.

Toroidal rotation period in disruption is comparable to duration of wall force, a few linear growth times. Rotation is damped by viscosity.

Toroidal rotation is sheared, peak value can be $10\times$ larger than average value.

MHD activity may produce intrinsic toroidal rotation [Rice 2007].

RMPs also drive rotation, perhaps there could be rotation without ELMs.

The rotation might be enough to stabilize RWMs in ITER.