Status of Energetic-particle Sawtooth Calculations

Tom Jenkins — Tech-X Corporation

in collaboration with

Scott Kruger — Tech-X Corporation

Eric Held — Utah State University

CEMM meeting
March 23, 2014
San Diego, California

Housekeeping items

- This is not a talk on coupling NIMROD and GENRAY to do ECCD-induced NTM stabilization (you've all heard me talk about that a few times already…)

- NTM work is being halted; CEMM funds redirected to focus on giant sawtooth CEMM milestones (Kim, Schnack)

- Current funding levels allow ~45% level of effort toward CEMM; a recently funded SBIR Phase II grant will cover the rest – see our Monday poster (D. N. Smithe)

Useful (invaluable!) documentation

The Internal Kink Mode and Giant Sawteeth in Tokamaks

Dalton D. Schnack

March 5, 2012

1 Introduction

These rough notes are a result of my attempts to gain a rudimentary understanding of sawtooth oscillations in general, and giant sawteeth in particular. I have always learned best by writing down what I read: in going from printed page through brain to hand, perhaps some of it sticks. Simultaneousy, I decided to learn a little LaTeX. The result is what you have here.

The initial motivation was to do simulations of the giant sawtooth crash with the NIMROD code. I quickly found that I was groping in the dark (still am), and in need of some background. Why am I so ignorant? Why wasn't I taught this stuff in graduate school? Actually, it turns out that much of the work on the topic was done *after* I left grad school and was involved in other endeavors. Ignorance is bliss, and, yes, I should have paid more attention at those endless APS meetings. In any case, I need some self-education, and these notes are a summary of most of the important papers regarding the topic of the internal kink mode in tokamaks, and its stabilization by energetic particles. I make no pretense at mastery or expertise or completeness. The fundamental MHD is difficult, I know very little about kinetic theory, and I have found the mathematics to be a stretch. As the mouse said: "Mine is a long and sad tail."

Here is a quick summary of what I've learned so far:

Sawteeth are the experimental manifestation of the $m = 1$, $n = 1$ internal kink mode in a torus. The sawtooth crash occurs when $q(0) < 1$. The ideal internal kink mode has completely different properties in a torus than in a periodic cylinder; it becomes pressure, rather than current, driven. Resistivity is required to account for the experimentally measured amplitude and growth rate of the sawtooth crash. In modern tokamaks, and in ITER, extended MHD or kinetic theory must be used to describe the singular layer; nonetheless the linear theory has been worked out.

1

- 70 pages of explanations, references, derivations, head-scratching, NIMROD runs, and bemused observations about analytic theorists…
- Better documentation than one usually gets when taking over a project.
- A good reminder – document the important stuff you're doing.

Sawtooth basics

Normal sawtooth mode

-Plasma has q(0) > 1, peaked current density on axis

-Ohmic heating introduced (e.g. 80 keV neutral beam)

-Plasma near axis preferentially heated (higher J) ➔ decreased core resistivity (~ $T^{-3/2}$) ➔ further current peaking, decreased q(0)

-(1,1) internal kink instability triggered when q(0) < 1, which rearranges magnetic flux and flattens temperature profile

-Cycle repeats

DIII-D shot #96043

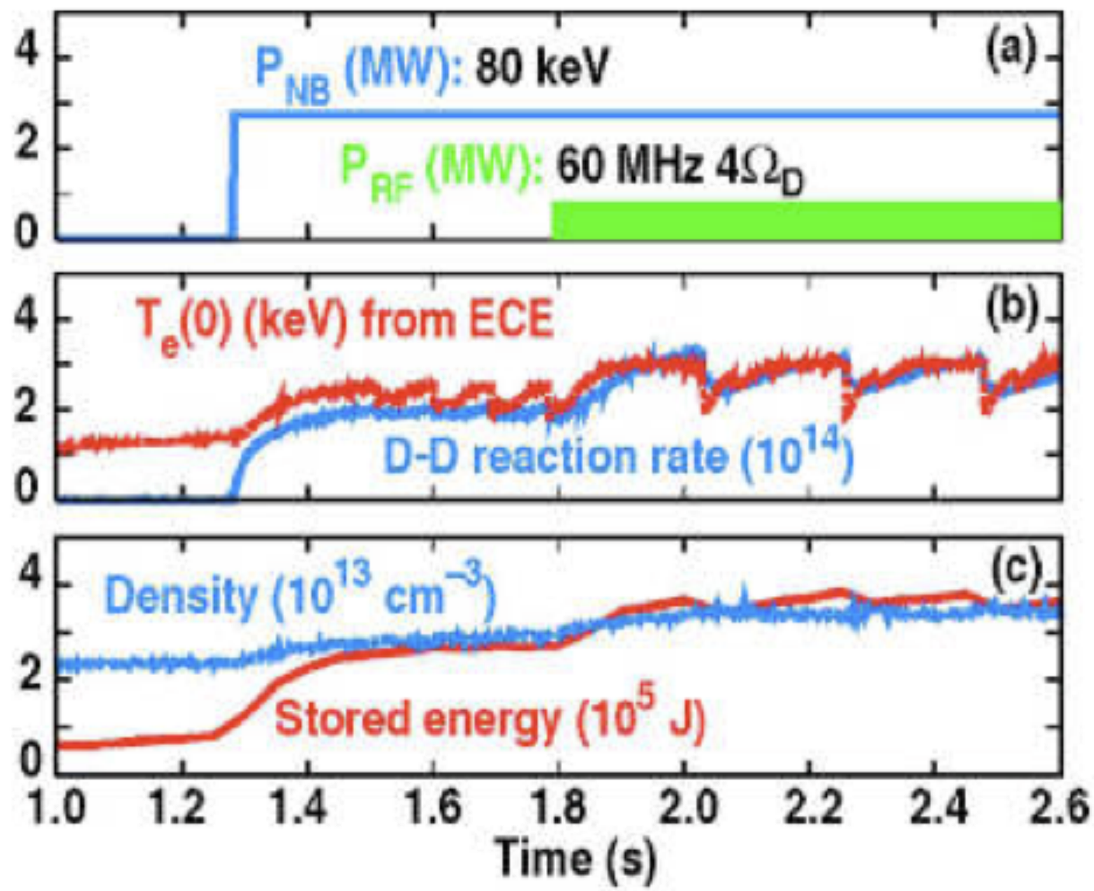


Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).

Giant sawtooth basics

Giant sawtooth mode

-Energetic particle population (e.g. induced by RF heating, or fusion reactions) alters stability of internal kink mode

-Higher temperatures and stored energies achievable even with q(0) < 1

-Terminates like a normal sawtooth crash, but with larger amplitude

-Potential trigger for ELMs, NTMs, large heat transfer to vessel wall

"slow leak" description
"soft β limit"

DIII-D shot #96043

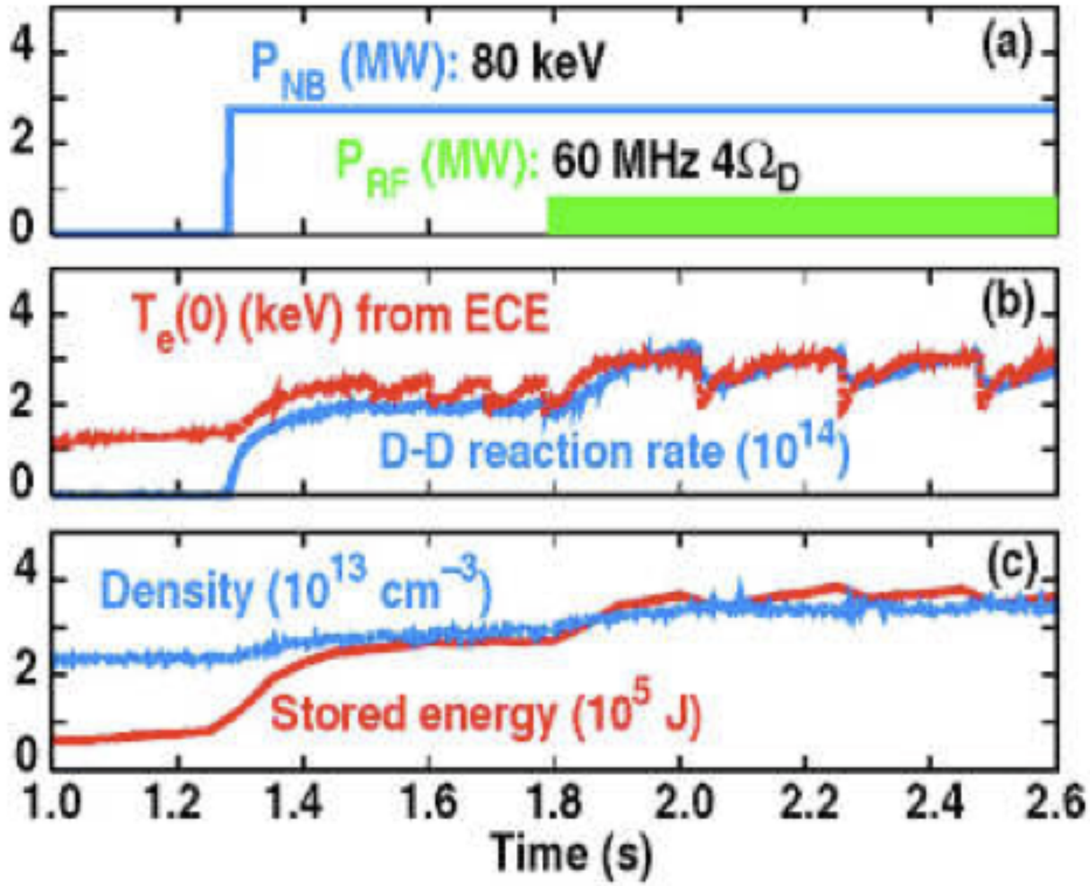
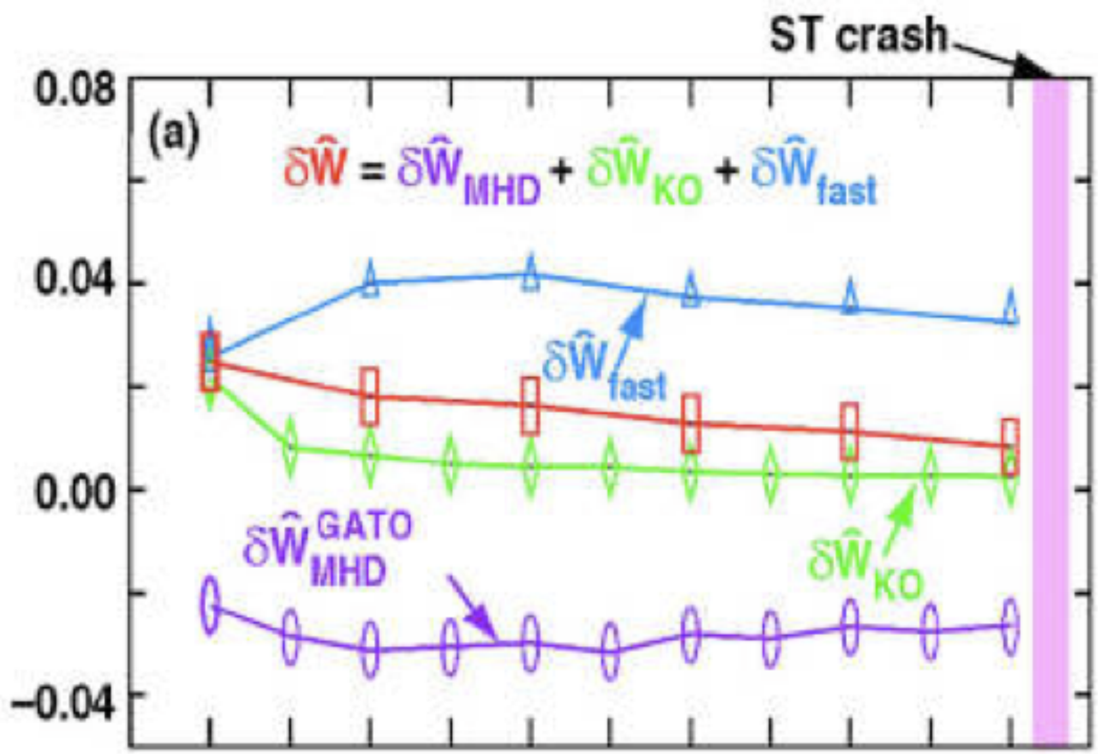


Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).

Sawtooth stability



Hot-particle effects

Bulk pressure tensor effects

Ideal MHD effects

Total stability parameter

- Does ideal MHD + hot-particle kinetics explain everything?

- Role of two-fluid effects?

Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).

Hot-particle sawtooth stabilization in NIMROD: computational approach

Momentum equation has an extra term:

$$\rho \frac{\partial \vec{V}}{\partial t} + \rho(\vec{V} \cdot \nabla)\vec{V} = \vec{J} \times \vec{B} - \nabla \cdot \overleftrightarrow{P} - \nabla \cdot \overleftrightarrow{P}_{hot}$$

Continuum kinetic:

$\overleftrightarrow{P}_{hot}$ represented by moments of Eric's solution to drift-kinetic equation

Kinetic PIC:

$\overleftrightarrow{P}_{hot}$ represented by moments of evolving PIC distribution

Current form of hot-particle pressure tensor contribution: slowing-down distribution

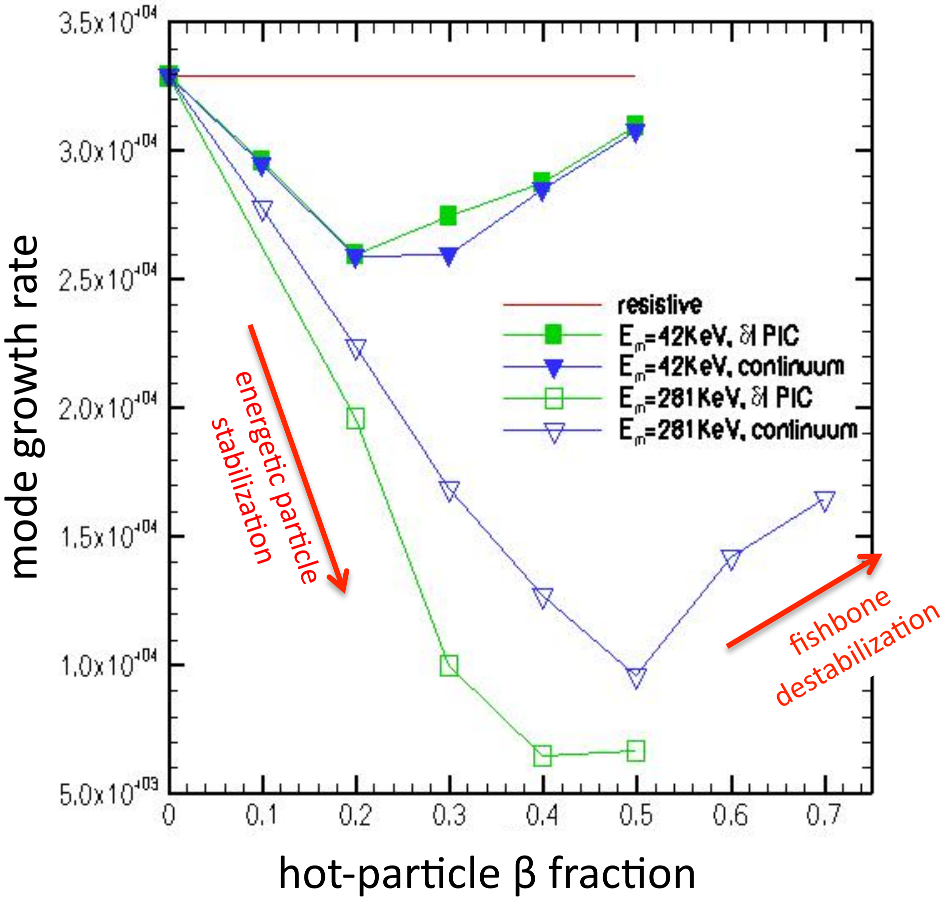
$$f_\alpha = \frac{P_0 \exp(P_\zeta / \psi_n)}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}$$

P_ζ	canonical toroidal momentum
ψ_n	normalized poloidal flux
ε_c	critical slowing-down energy

Relevant CEMM milestones

	Year 4	Year 5
Sawtooth	•Apply continuum closure models for energetic and thermal ions to the Giant Sawtooth problem (Tech-X).	•Continue linear modeling of sawtooth stabilization in DIII-D shot 96043 (Tech-X). •Demonstrate nonlinear evolution of sawtooth with continuum kinetic closures and extended MHD Ohm's law (Tech-X/USU).
Model development - continuum kinetic (with Eric Held)	•Improve parallel scaling of kinetic closures (USU).	•Demonstrate applicability by applying to a 3D coupled problem (USU/Tech-X)
Model development - kinetic PIC	•Begin new particle parallelization development for NIMROD (Tech-X).	•Complete, test, and apply the new particle parallelization in NIMROD (Tech-X).

Continuum and kinetic PIC benchmarks



E_m = peak energy of slowing-down distribution function

<u>Related issues</u>

- Energetic particle kink stabilization, fishbone destabilization as β fraction increased

- Stabilization (3rd adiabatic invariant – toroidal precession of energetic trapped particles modifies MHD) requires

  $\omega_{pd}/\gamma_R >> 1$

  (growth slow compared to precession) but for these cases,

  $\omega_{pd}/\gamma_R = 1.5$ (42 keV)

  $\omega_{pd}/\gamma_R = 10$ (281 keV)

Can we run at high enough energies and Lundquist numbers to achieve full stabilization? (particle population in phase space)

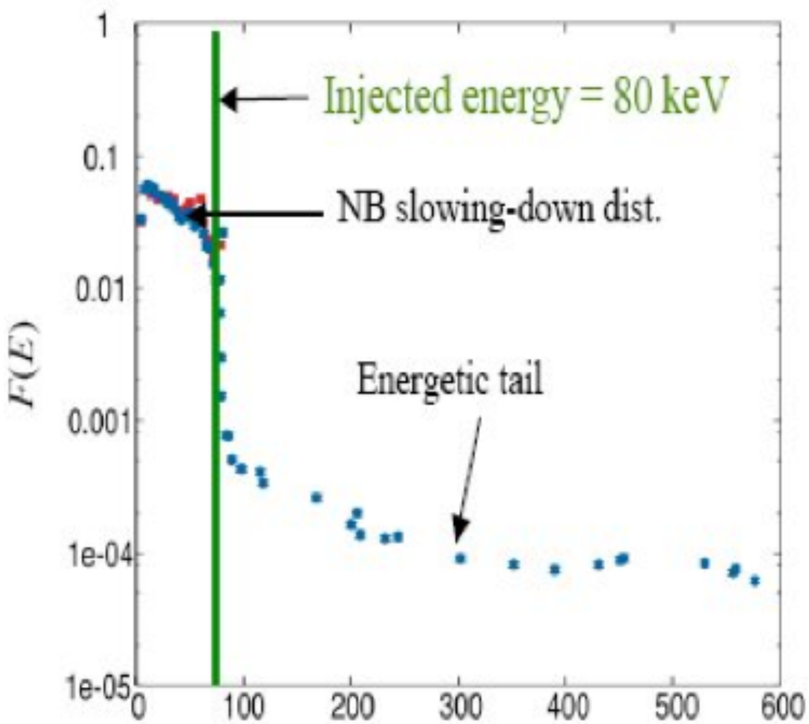
How much does the form of the hot-particle distribution function matter?

PIC approach is considerably more expensive computationally – initial focus on continuum

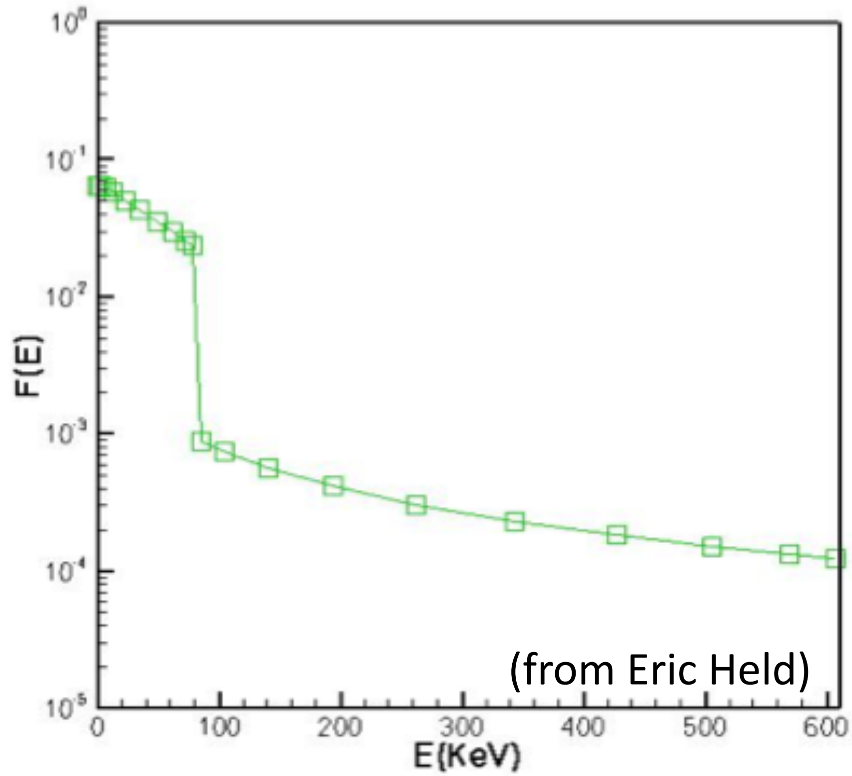
Continuum closures – hot-particle RF tail

- High-energy tail and/or thermal ions needed for stabilization (only partial stabilization achieved from slowing-down distribution)

- Continuum closure developments include high-energy tail model – further testing needed to assess efficacy



(from Eric Held)



(from Eric Held)

Plan of action going forward

- Dalton's runs essentially characterize the 2-fluid and MHD behavior of sawtooth modes
  (and also provide a solid foundation as I come up to speed on this project)
- From this data, can explore the underlying physics of linear sawtooth stability
- Near-term goal – publish these results in Physics of Plasmas
  - -Working with NERSC to arrange for ownership transfer of existing data; then,
    will need to sort out what is useful
  - -May need guidance on analytic work while sorting out the salient physics
- Longer-term goal – gaining physics insights with NIMROD
  - -get experience using continuum kinetic model capabilities
  - -assess code performance improvements necessary to address PIC model issues
  - -examine the effect of more general hot-particle distribution functions
- Milestone – DIII-D shot 96043 modeling