# Update on closure efforts at USU

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Sherwood CEMM Mtg., NYU, 2015

# Outline.

- Progress on analytical closures (J. Y. Ji):

  -accepted POP paper, "Electron Parallel Closures for Arbitrary Collisionality" provides fitted kernel functions for evaluating integral electron closures, and

  -submitted POP paper, "Electron Heat Transport in a Stochastic Magnetic Field" uses the electron parallel heat flow closure to estimate radial heat transport in the presence of magnetic field line fluctuations.

- Progress on numerical closures:

  -accepted POP paper, "Verification of Continuum Drift Kinetic Equation Solvers in NIMROD" benchmarks NIMROD's electron and ion DKE predictions against established neoclassical transport codes, and

  -progress on continuum hot particle benchmarks and implementation of equilibria that include hot particle current in Grad-Shafranov equation.

# Integral (nonlocal) parallel closures for arbitrary collisionality

J.-Y. Ji and E. D. Held, Phys. Plasmas 21, 122116 (2014)

- Closures for density, temperature, and flow velocity equations

$$h_\|(\ell) = -\frac{1}{2}Tv_T\int d\ell' K_{hh}\frac{n}{T}\frac{dT}{d\ell'} + Tv_T\int d\ell' K_{hR}Zn\frac{V_{\mathrm{ei}\|}}{v_T} + Tv_T\int d\ell' K_{h\pi}(\frac{3}{4}n\tau_{\mathrm{ee}}W_\|)$$

$$R_\|(\ell) = -\frac{mn}{\tau_{\mathrm{ei}}}V_{\mathrm{ei}\|} + \frac{mv_T}{\tau_{\mathrm{ei}}}\int d\ell'[-K_{Rh}\frac{n}{2T}\frac{dT}{dz'} + K_{RR}Zn\frac{V_{\mathrm{ei}\|}}{v_T} + K_{R\pi}(\frac{3}{4}n\tau_{\mathrm{ee}}W_\|)]$$

$$\pi_\|(\ell) = T\int d\ell' K_{\pi h}\frac{n}{T}\frac{dT}{d\ell'} - 2T\int d\ell' K_{\pi R}Zn\frac{V_{\mathrm{ei}\|}}{v_T} - T\int d\ell' K_{\pi\pi}(\frac{3}{4}n\tau_{\mathrm{ee}}W_\|)$$

where $\int d\ell' K_{AB}g_B \equiv \int d\ell' K_{AB}(\eta-\eta')g_B(\ell')$, $\eta=\eta(\ell)=\int^\ell d\ell/\lambda_{\mathrm{mfp}}$, $\eta'=\eta(\ell')$

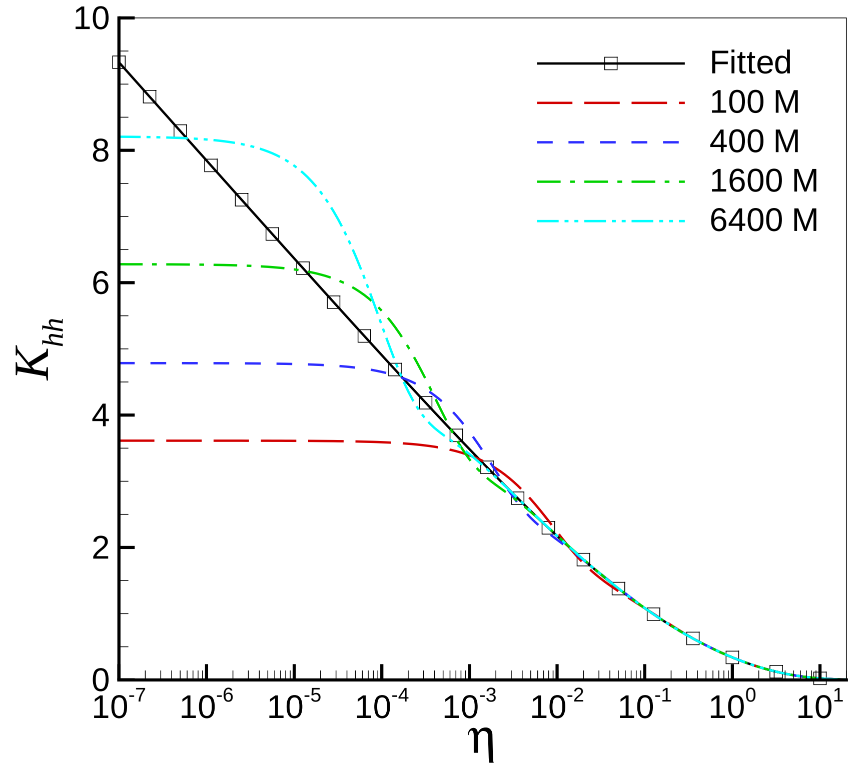
$\ell$ arclength, and $v_T$ thermal speed

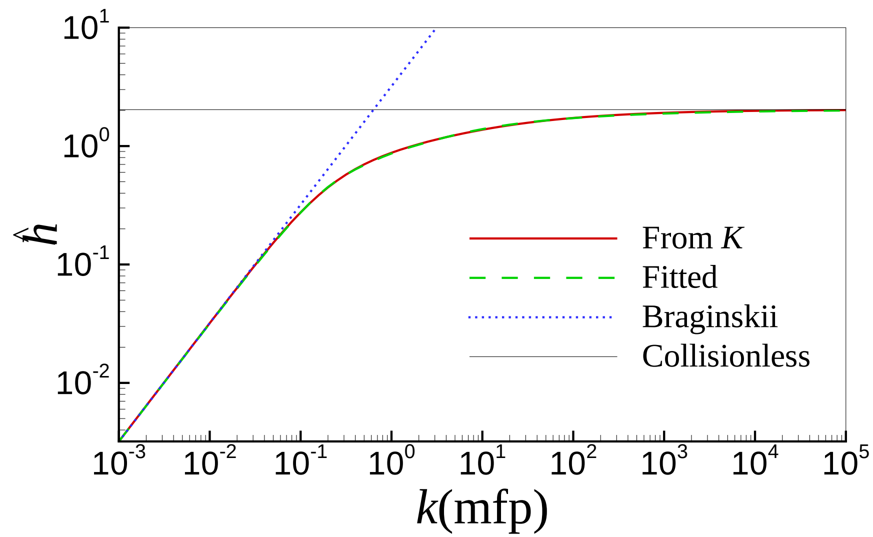
- Kernel functions fitted to the 6400 moment solution and the collisionless solution

$$K_{AB}(\eta) = -[d + a\exp(-b\eta^c)]\ln[1-\alpha\exp(-\beta\eta^\gamma)]$$

| | $a$ | $b$ | $c$ | $d$ | $\alpha$ | $\beta$ | $\gamma$ |
|---|---|---|---|---|---|---|---|
| $K_{hh}$ | -5.32 | 0.170 | 0.646 | 6.87 | 1 | 2.02 | 0.417 |
| $K_{hR}$ | 6.37 | 5.12 | 0.160 | 0.100 | 1 | 1 | 0.583 |
| $K_{h\pi}$ | -0.229 | 2.26 | 0.594 | 0.363 | 0.775 | 1.49 | 0.478 |
| $K_{RR}$ | 245 | 8.06 | 0.147 | 0.432 | 1 | 3.40 | 0.347 |
| $K_{R\pi}$ | -0.226 | 3.21 | 0.678 | 0.696 | 1 | 3.40 | 0.347 |
| $K_{\pi\pi}$ | 0.724 | 0.932 | 0.654 | 0.195 | 1 | 1.60 | 0.491 |

# Parallel heat flow $h_\parallel$ for a sinusoidal temperature fluctuation




- $h_\parallel(\ell) = -\dfrac{1}{2} n v_T \displaystyle\int d\ell' K_{hh}(\eta - \eta') \dfrac{dT(\ell')}{d\ell'}$

$$K_{hh}(\eta) = (-6.87 + 5.32 e^{-0.17\eta^{0.646}}) \times \ln(1 - e^{-2.02\eta^{0.417}})$$

- For $T = T_0 + T_\delta \sin(\frac{2\pi\ell}{\lambda} + \varphi_0),\ T_0 \gg T_\delta$

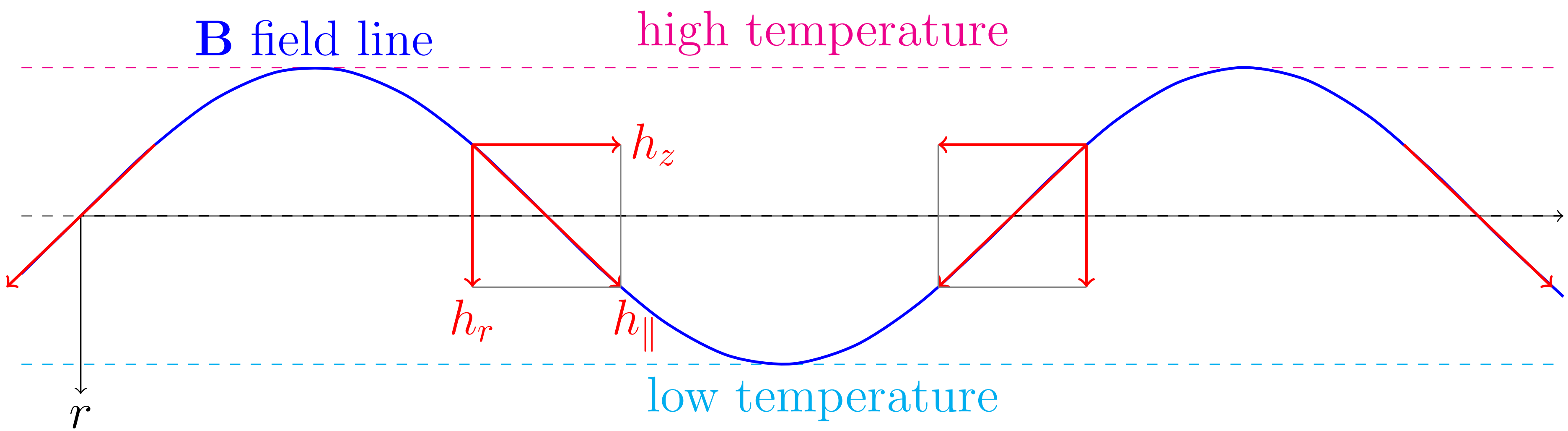
$$\begin{aligned} h_\parallel(\ell) &= -\frac{1}{2}\hat{h}(k) n_0 v_T T_\delta \cos(\frac{2\pi\ell}{\lambda} + \varphi_0) \\ &= -\frac{1}{4\pi}\hat{h}(k) n_0 v_T \lambda \partial_\parallel T \\ &= -\frac{\hat{h}(k)}{k}\frac{n_0 T \tau_{\text{ee}}}{m}\partial_\parallel T \end{aligned}$$

$$k = 2\pi \frac{\lambda_{\text{mfp}}}{\lambda}$$

$$\hat{h}(k) = \left(2.03 - \frac{5.67k^{1.28}}{1 + 4.22k^{1.59}}\right)\tanh(1.577k)$$

$$\rightarrow \begin{cases} \text{collisionless } (k \rightarrow \infty) \\ \quad 18/5\sqrt{\pi} \approx 2.03 \\ \text{collisional limit } (k \rightarrow 0) \\ \quad 4\pi\chi_\parallel / v_T\lambda \approx 3.20k \end{cases}$$

# Radial heat flow due to a sinusoidal magnetic field line




- Magnetic field fluctuation introduces temperature fluctuation

$$r = r_0 + r_\delta \sin\frac{2\pi}{\lambda}z \rightarrow T = T_0 + T_\delta \sin\frac{2\pi}{\lambda}z \text{ where } T_\delta = \frac{dT_0}{dr}r_\delta$$

- Arclength along the field line

$$\ell(z) = \int_0^z dz\sqrt{1+\left(\frac{dr}{dz}\right)^2} = \frac{\lambda}{2\pi}\sqrt{1+a^2}E\left(\frac{2\pi z}{\lambda}, \frac{a^2}{1+a^2}\right)$$

- Heat flow with the arclength approximation $\ell \approx \alpha z$ with $\alpha = \ell(\lambda)/\lambda$

$$h_r(z) = h_\parallel \frac{dr}{d\ell} = -\frac{1}{2}\hat{h}(\frac{k}{\alpha})n_0 v_T T_\delta a\gamma = -\frac{1}{4\pi}\hat{h}(\frac{k}{\alpha})n_0 v_T \gamma\lambda\left(\frac{B_\delta}{B_0}\right)^2\frac{dT_0}{dr}$$

where $a = \dfrac{2\pi r_\delta}{\lambda} = \dfrac{B_\delta}{B_0}$, $\gamma = \dfrac{\cos^2\varphi}{\sqrt{1+a^2\cos^2\varphi}}$, $\varphi = \dfrac{2\pi}{\lambda}z$

# Radial heat flow for small fluctuation

- Average over temporal fluctuations effectively results in spatial average
$$\langle h_r \rangle = \frac{1}{2\pi}\int_0^{2\pi} h_r d\varphi = -\frac{1}{4\pi}\hat{h}(\frac{k}{\alpha})n_0 v_T \langle\gamma\rangle \lambda \left(\frac{B_\delta}{B_0}\right)^2 \frac{dT_0}{dr}$$
where $0.38 \lesssim \langle\gamma\rangle \leq 1/2$ for $1 \gtrsim a \geq 0$
- For small fluctuation $a \ll 1$, $\alpha \approx 1$ (for $a = 0.1$, $\alpha \approx 1.003$, cf. RMP $a \sim 10^{-4}$)
$$\langle h_r \rangle = -\frac{1}{8\pi} n_0 v_T \hat{h}(k)\lambda \left(\frac{B_\delta}{B_0}\right)^2 \frac{dT_0}{dr}, \qquad \chi_{\mathrm{eff}} = \frac{1}{8\pi} v_T \hat{h}(k)\lambda \left(\frac{B_\delta}{B_0}\right)^2$$
- Collisional limit $\lim\limits_{k\to 0} \hat{h}(k) \approx 3.20k$
$$\chi_{\mathrm{eff}} = 1.60 \frac{T_0 \tau_{\mathrm{ee}}}{m_{\mathrm{e}}} \left(\frac{B_\delta}{B_0}\right)^2$$
- Collisionless limit $\lim\limits_{k\to\infty} \hat{h}(k) \approx 2.03 \approx 2$
$$\chi_{\mathrm{eff}} = \frac{1}{4\pi} v_T \lambda \left(\frac{B_\delta}{B_0}\right)^2$$
Rechester-Rosenbluth $\chi_{\mathrm{eff}}^{\mathrm{RR}} = \pi R v_T \left(\frac{B_\delta}{B_0}\right)^2$ ($2\pi R$ is the period of the system)
$$\frac{\chi_{\mathrm{eff}}}{\chi_{\mathrm{eff}}^{\mathrm{RR}}} = \frac{1}{4\pi^2}\frac{\lambda}{R} = \frac{1}{2\pi n} \text{ for } \lambda = \frac{2\pi R}{n}$$

# Numerical solutions to drift kinetic equations provide closures.

- NIMROD can solve for $F_e$, $F_i$ and $F_{hot}$ in 2D velocity space, $s = v/v_0$ and $\xi = v_{||}/v$:

$$\frac{\partial F}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \left[\nabla F - \frac{1-\xi^2}{2\xi}\nabla \ln B \frac{\partial F}{\partial \xi} - \frac{s}{2}\nabla \ln T_0 \frac{\partial F}{\partial s}\right] - C(F) +$$
$$\frac{1-\xi^2}{2\xi}\left[-\xi^2 \frac{\mathbf{b}}{B}\cdot\frac{\partial \mathbf{B}}{\partial t} + \frac{q}{s^2 T_0}(\mathbf{v}_{||} + \mathbf{v}_D)\cdot \mathbf{E} + \xi^2 \frac{\mathbf{E}\times\mathbf{B}}{B^2}\cdot \nabla \ln B\right]\frac{\partial F}{\partial \xi} +$$
$$\frac{s}{2}\left[-(1-\xi^2)\frac{\mathbf{b}}{B}\cdot\frac{\partial \mathbf{B}}{\partial t} + \frac{q}{s^2 T_0}(\mathbf{v}_{||} + \mathbf{v}_D)\cdot \mathbf{E} + (1+\xi^2)\frac{\mathbf{E}\times\mathbf{B}}{B^2}\cdot \nabla \ln B\right]\frac{\partial F}{\partial s} = 0,$$

where

$$\mathbf{v}_D = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{T_0 s^2}{qB^2}\left[\left(1+\xi^2\right)\mathbf{b}\times\nabla B + 2\xi^2\mu_0 \mathbf{J}_\perp + (1-\xi^2)\mu_0 \mathbf{J}_{||}\right] + \frac{m v_0 s \xi}{qB^2}\mathbf{b}\times\frac{\partial \mathbf{B}}{\partial t}$$

# $\delta F$ PIC approach for hot particle kink benchmark.

- Fu's approach (*Phys. Plasmas* **13**, 052517 (2006)) with M3D used:

  $F_0(\langle\psi\rangle, s) = P_0 \exp(\langle\psi\rangle/\psi_n)/(s^{3/2} + s_c^{3/2})$, where

  $\langle\psi\rangle = P_\zeta/e - \frac{m}{e}\langle v_{||} R \frac{B_\phi}{B}\rangle \approx P_\zeta/e$ for trapped particles and

  $\langle\psi\rangle = P_\zeta/e - \frac{m}{e}\langle v_{||} R \frac{B_\phi}{B}\rangle \approx P_\zeta/e - vR_0 sign(\frac{v_{||}}{v})\sqrt{1 - v_\perp^2 B_0/(Bv_{||}^2)}$ for passing particles.

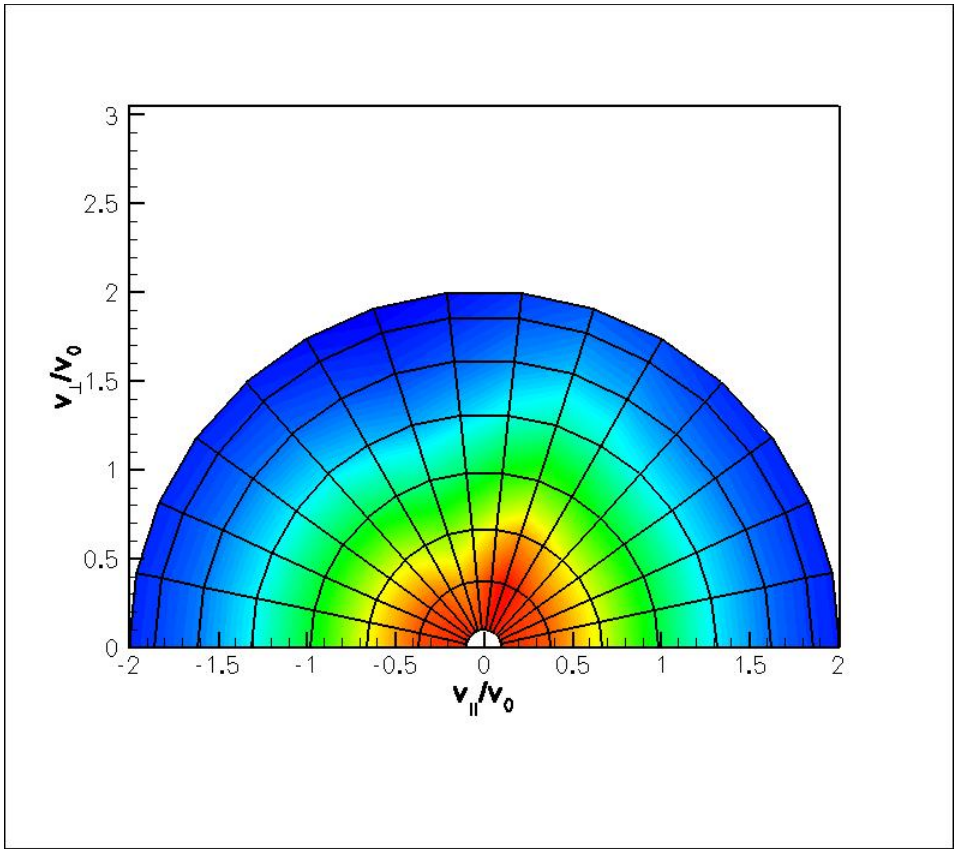
- Perpendicular, $P_\perp$, and parallel $P_{||}$, pressure moments appear as anisotropic hot particle pressure tensor in NIMROD:

  $\delta P_\perp = \int d\mathbf{v} \mu B \frac{\delta F}{F_0(P_\zeta)} F_0(\psi)$

  $\delta P_{||} = \int d\mathbf{v} v_{||}^2 \frac{\delta F}{F_0(P_\zeta)} F_0(\psi)$.

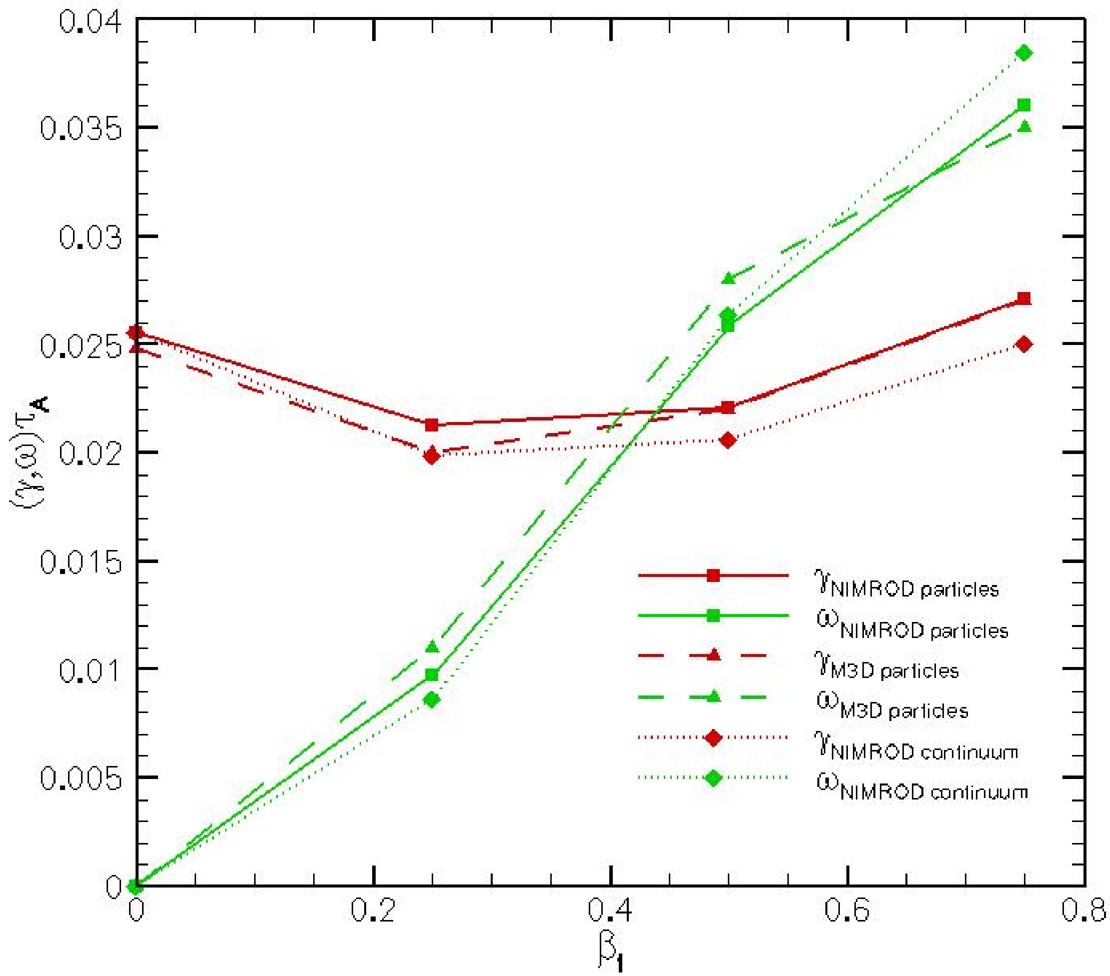
# 2D velocity grid.

- 1D finite elements used in pitch angle: $\frac{v_{||}}{v} = \sum_i (\frac{v_{||}}{v})_i \phi_i(x)$.
- Quadrature points in $s = v/v_0$ associated with nodes of nonclassical polynomials.



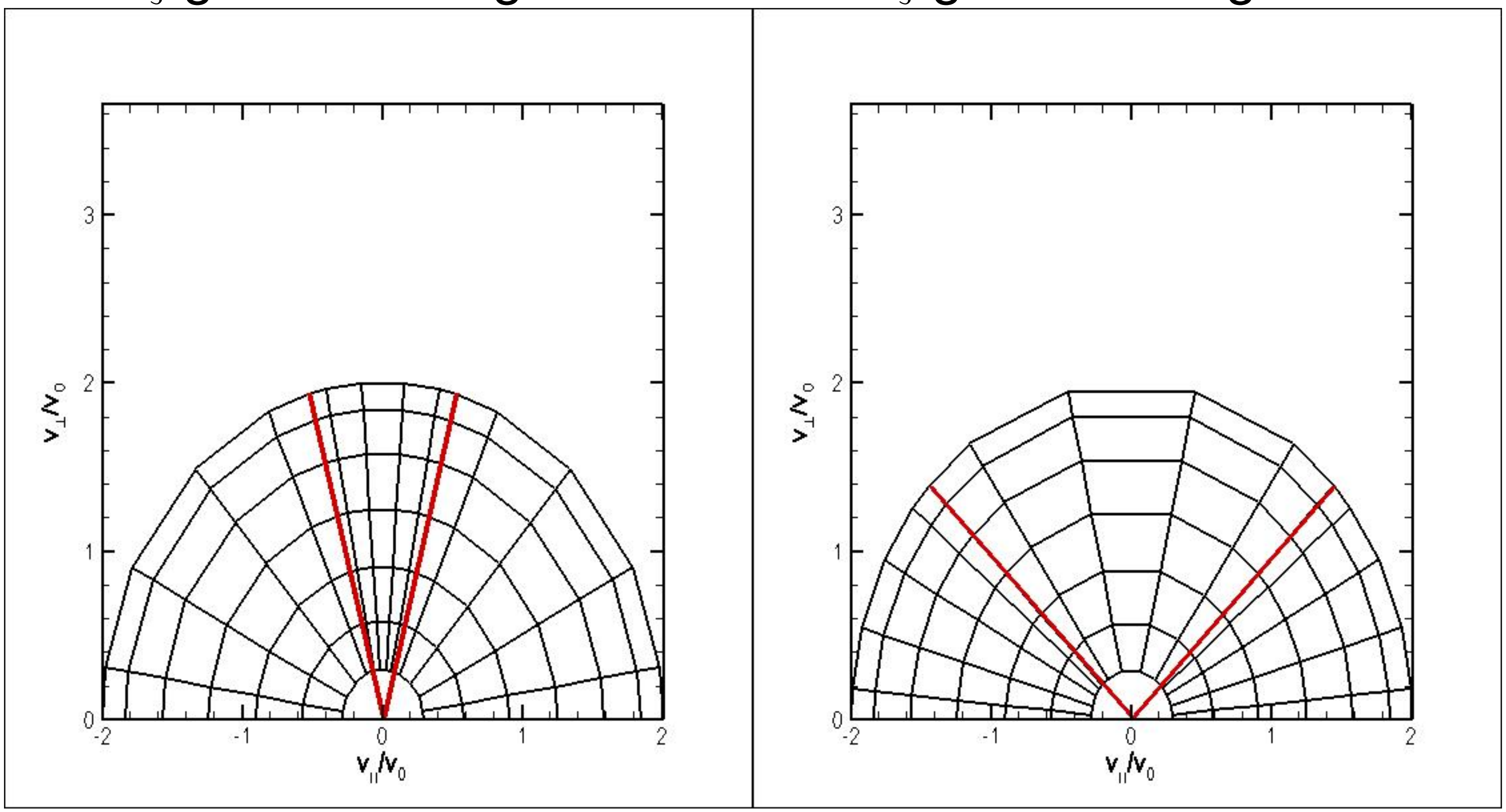
# Continuum results agree with $\delta F$ PIC.

- Increasing hot particle pressure first stabilizes and then destabilizes mode.
- Growth rates and real frequencies similar.



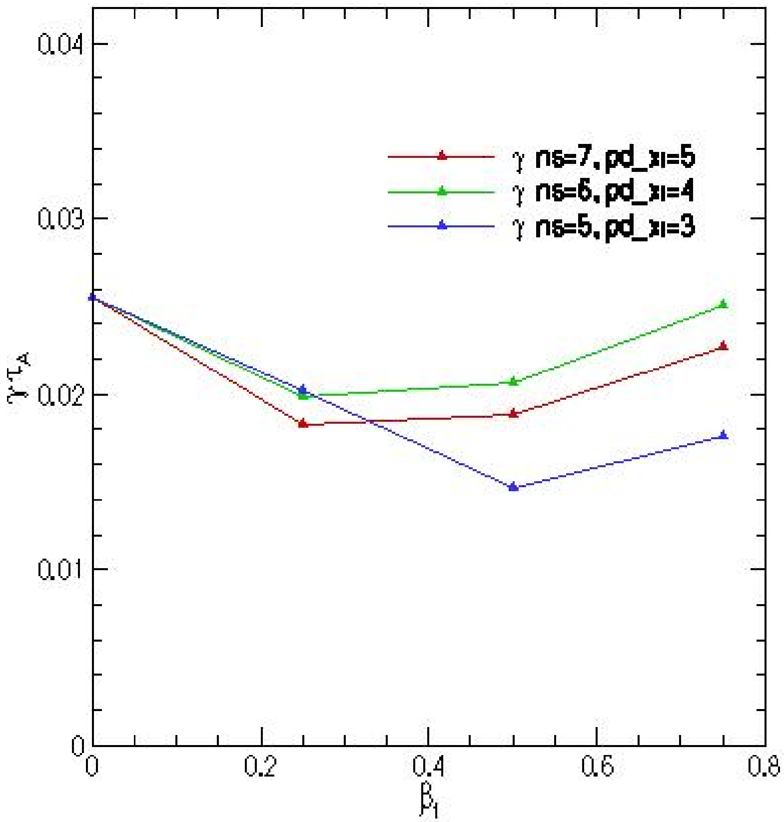
# Vertex nodes in $\xi$ grid at $\pm\xi_t(\psi)$ help convergence.

$\xi$ grid near magnetic axis $\quad$ $\xi$ grid near edge

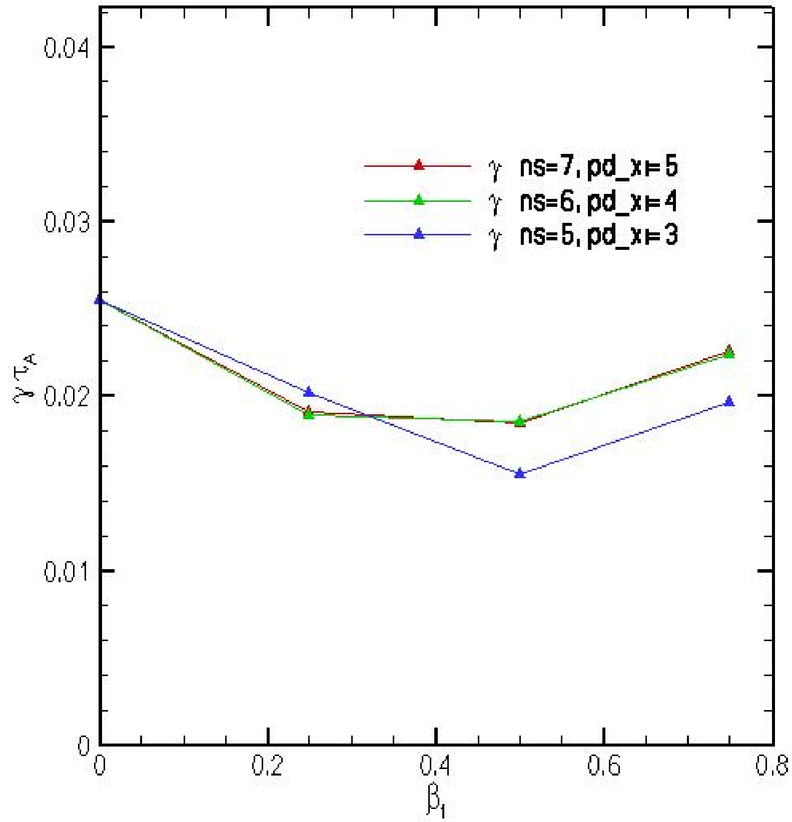


# Growth rate convergence

vertex nodes uniform in $cos^{-1}(\xi)$

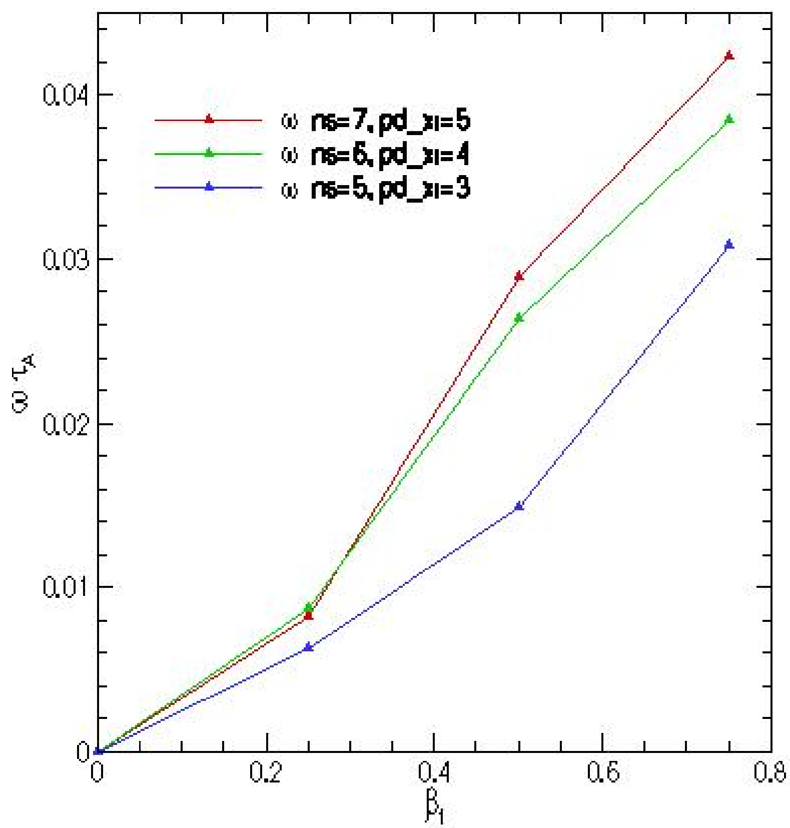



vertex nodes at $\pm\xi_t(\psi)$

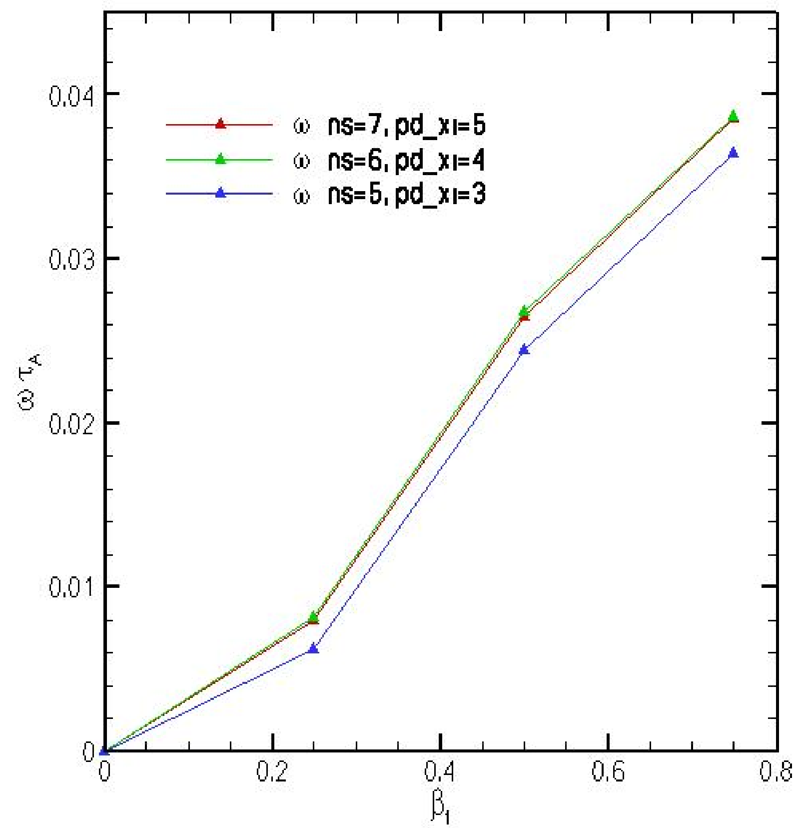


# Real frequency convergence.

vertex nodes uniform in $cos^{-1}(\xi)$



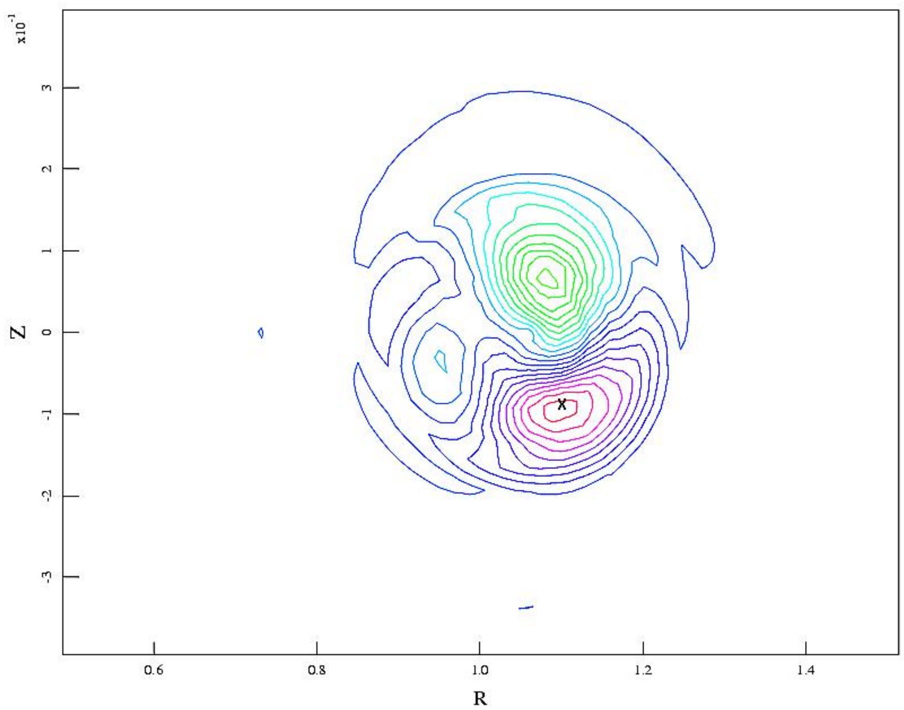

vertex nodes at $\pm\xi_t(\psi)$



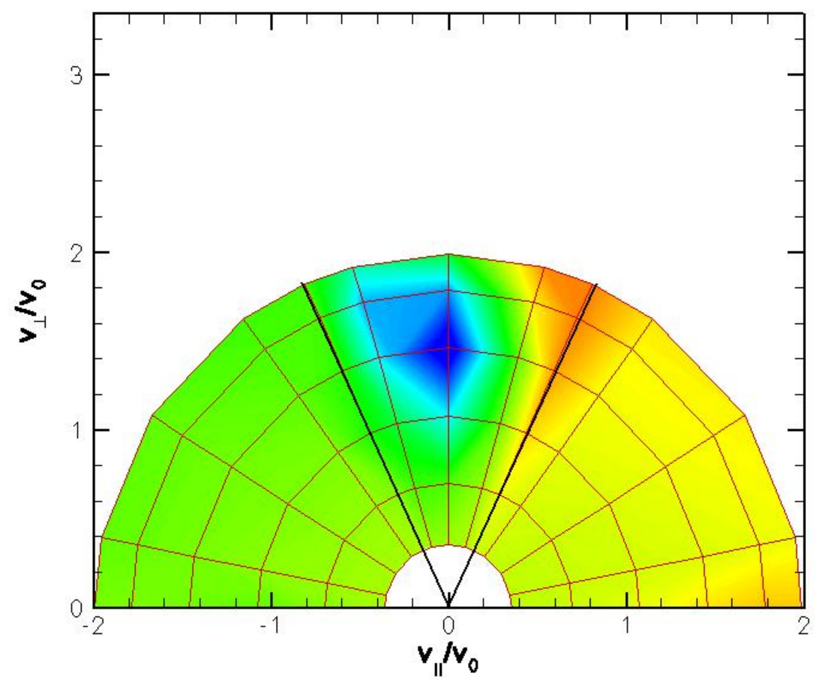
# Trapped particles dominate anisotropic pressure response.

- Anisotropic hot particle pressure shifted to outboard side of torus.

contours of $P_{||} - P_{\perp}$




$F_{hot}$



# Consistent equilibria for hot particle simulations needed.

- Whenever $F_{hot}$ is function of pitch-angle ($v_{||}/v$), pressure tensor for hot particles is anisotropic.
- One should re-solve for equilibrium force balance with the effects of an anisotropic hot particle pressure.
- Can accomplish this by incorporating hot particle (beam) current into Grad-Shafranov equation in NIMROD's nimeq (Howell and Sovinec) / fgnimeq (J. King) functionality.

# Equations for hot particle equilibria.

- Formulation in Belova *et al.*, *Phys. Plasmas* **10** 3240 (2003):

  $$0 = -\nabla p_p + (\mathbf{J} - \mathbf{J}_b) \times \mathbf{B}$$

  where $p_p$ is the thermal plasma pressure and $\mathbf{J}_b$ is beam current.

- Modified right side of Grad-Shafranov equation looks like

  $$\mathrm{gs\_rhs} = -\mu_0 R^2 p' - FF' - \mu_0 FG' + \mu_0 R J_{\phi,b}$$

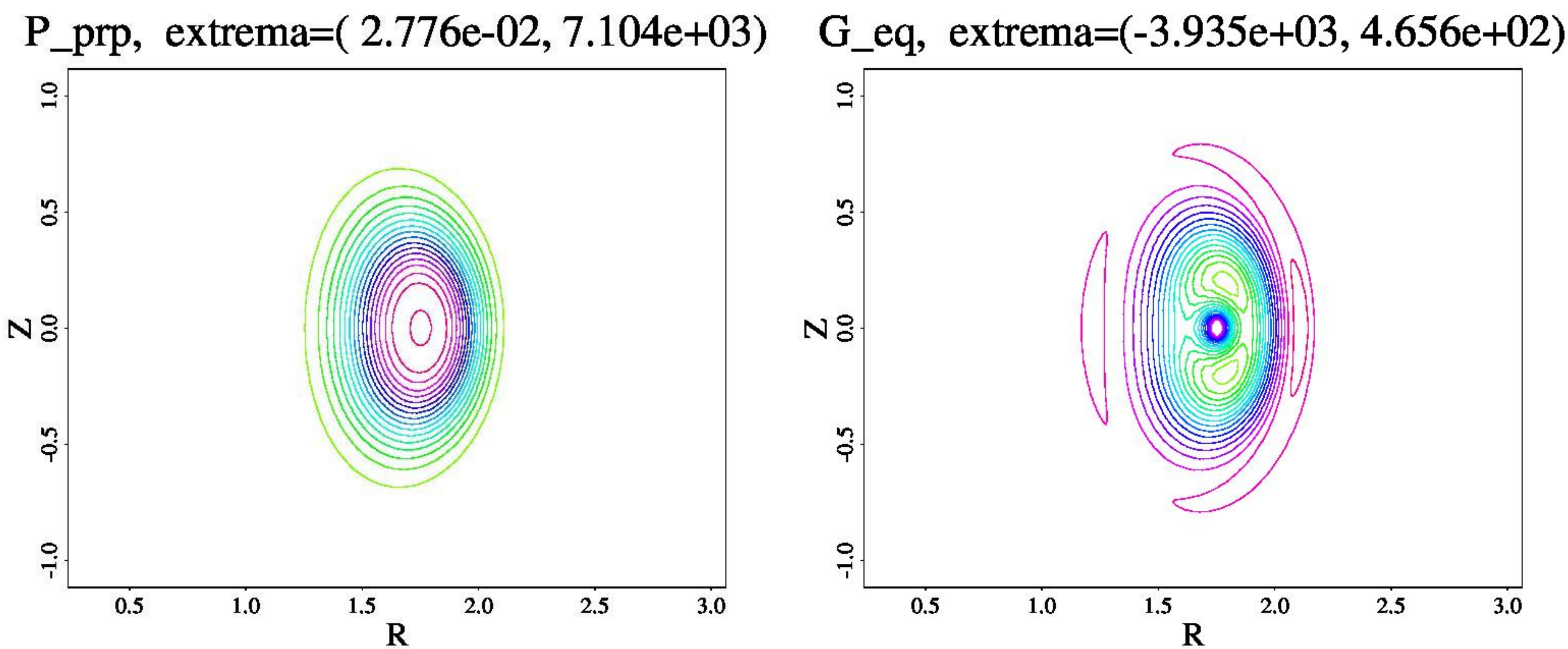
  with the poloidal stream function, $G$, determined from

  $$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial G}{\partial R}\right) + \frac{\partial^2 G}{\partial^2 Z} = -R^2 \nabla\phi \cdot \nabla \times \mathbf{J}_{pol,b}$$

  In the drift kinetic approximation, the beam current is

  $$\mathbf{J}_b = NV_{||}\mathbf{b} + (P_{||} - P_{\perp})\nabla \times \mathbf{b} + \mathbf{b} \times \nabla P_{\perp}$$

# Implementation is complete, testing continues.



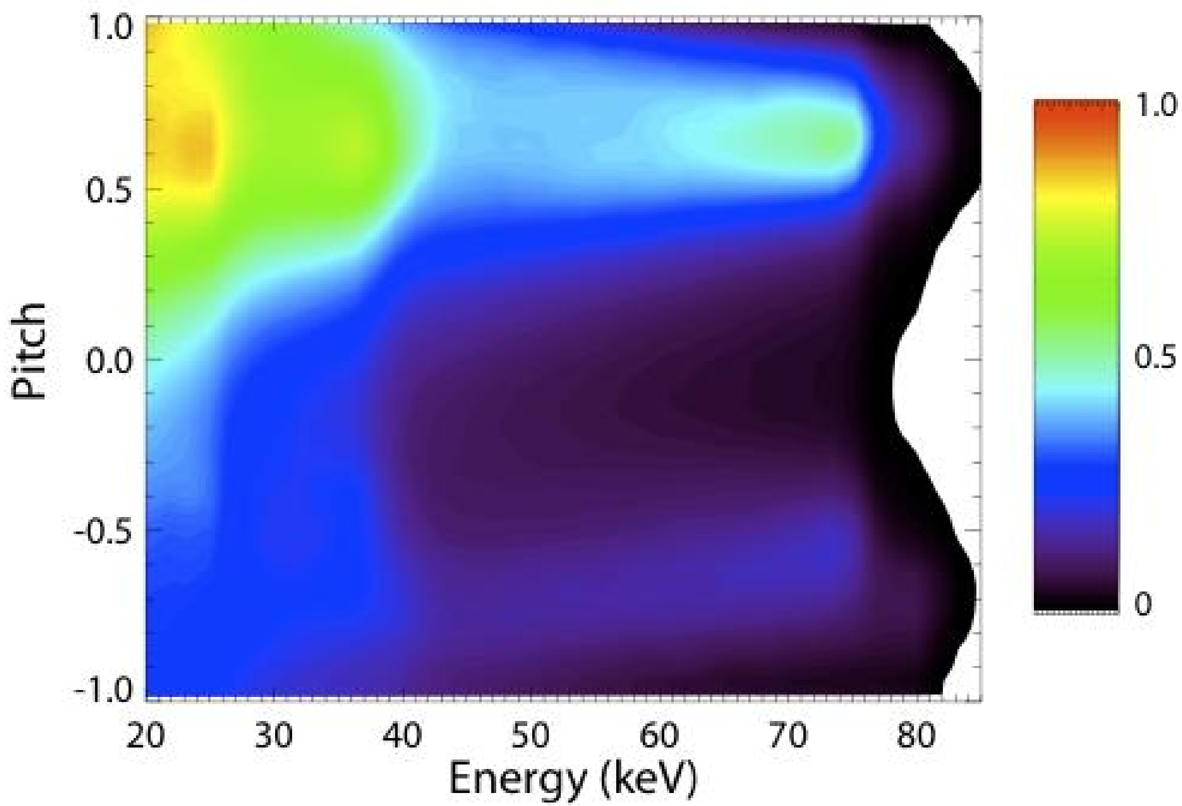
# Verify hot particles in NIMROD on RSAE case

- From "Verification and validation of linear gyrokinetic simulation of Alfvén eigenmodes in the DIII-D tokamak," Spong, *et al.*, Phys. Plasmas 19, 082511 (2012)

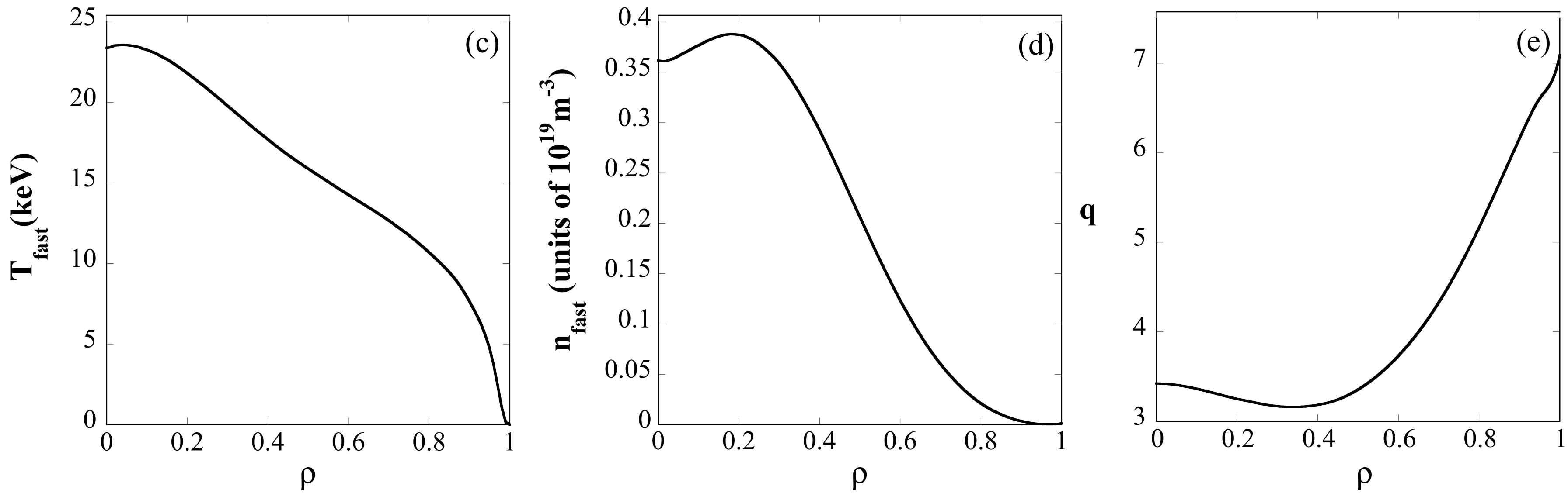
"Fully self-consistent simulation of energetic particle turbulence and transport in burning plasmas must incorporate three new physics elements: kinetic effects of thermal particles at the thermal ion gyro-radius (micro scale), nonlinear interactions of many meso scale (energetic particle gyro-radius) shear Alfvén waves induced by the kinetic effects at the micro scale, and meso-micro couplings of the micro-turbulence and shear Alfvén wave turbulence. The large dynamical ranges of spatial-temporal processes further require global simulation codes to be efficient in utilizing massively parallel computers. Therefore, the studies of energetic particle physics in the burning plasma regime require a new approach using gyrokinetic turbulence simulation.In this paper, we document progress in the verification and validation of the simulation of Alfvén eigenmodes using the advanced tokamak regime of the DIII-D experiment as a reference case."

# Beam Ions Drive RSAE's in DIII-D (#142111)

- RSAE's driven by 4.6 MW of deuterium neutrals injected at 75-81 keV.
- Verification uses Maxwellian fast particles although TRANSP-NUBEAM predicts anisotropic distribution.

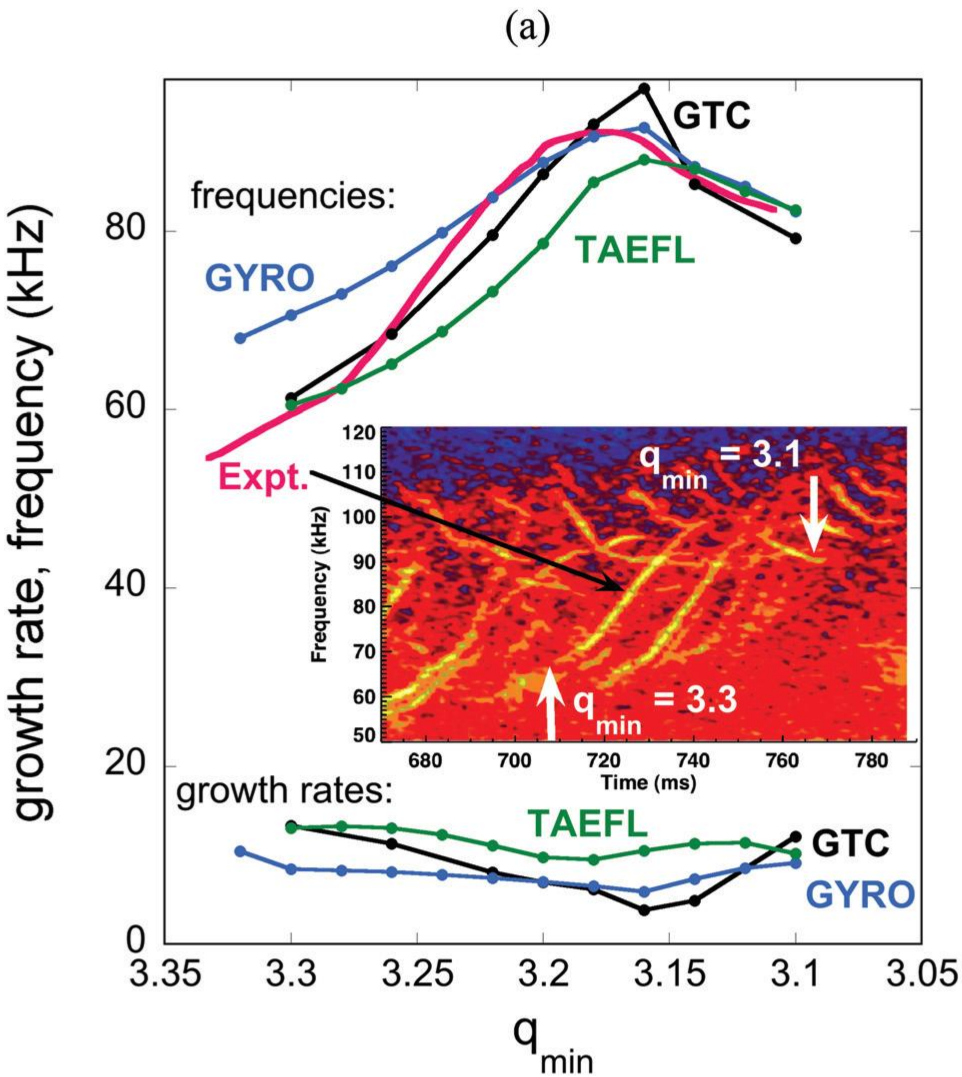


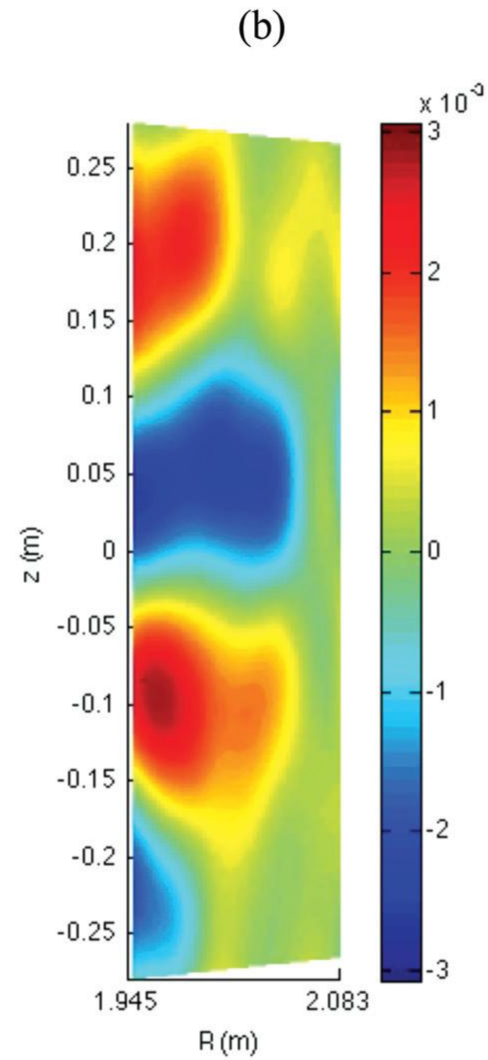
# Hot particle Maxwellian used in validation/verification exercise



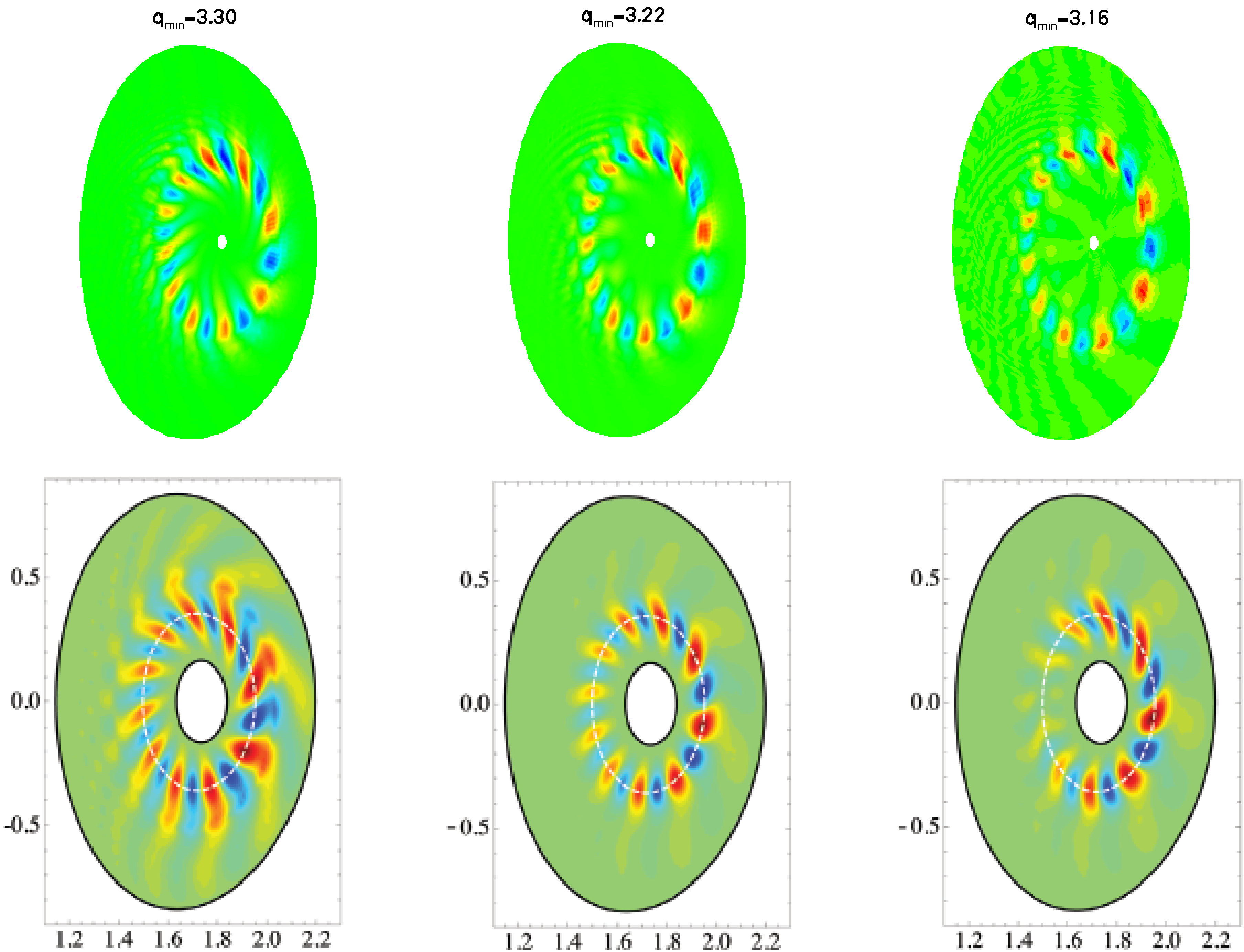
# Real frequency sweeps as $q_{min}$ evolves.

- Real frequency of RSAE/TAE changes as minimum in q decreases.



# NIMROD/GYRO eigenmodes compare favorably

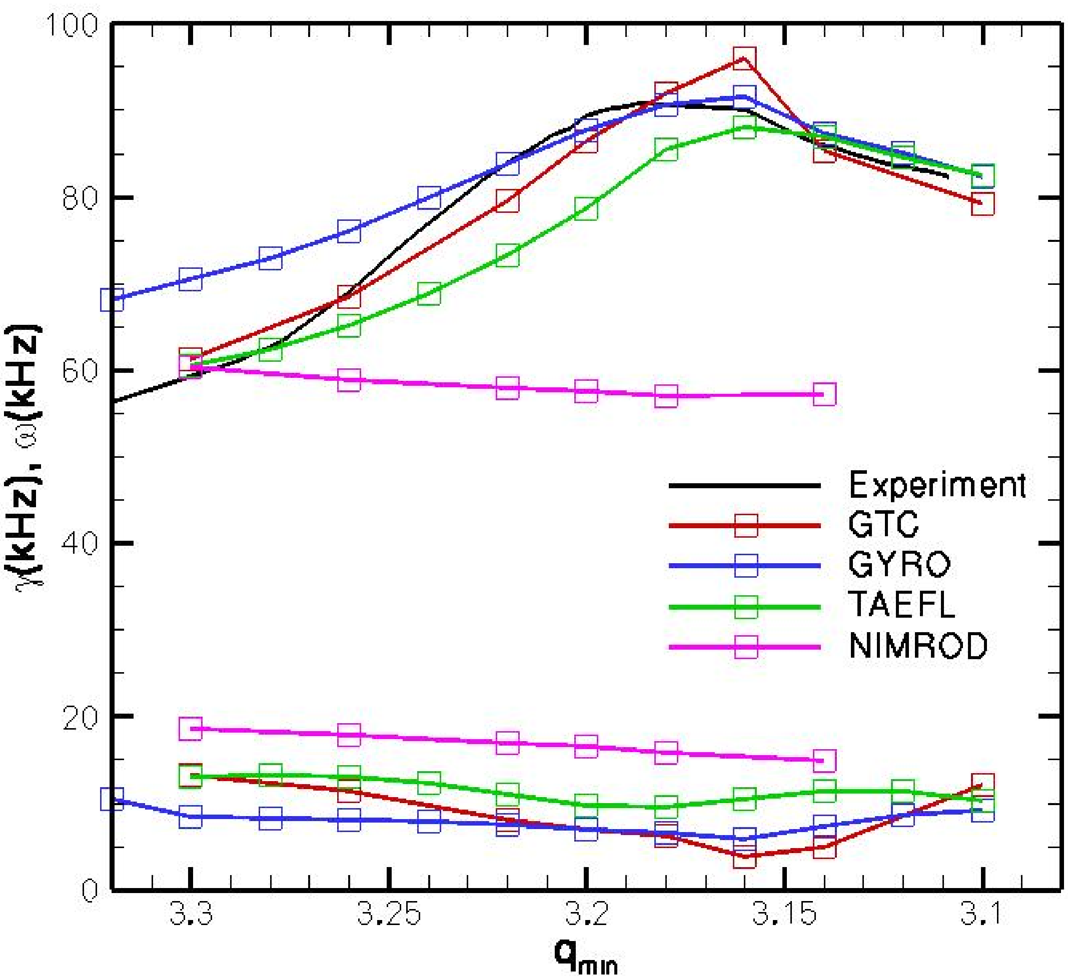


## At present, real frequency results relatively insensitive to $q_{min}$.

- Additional physics needed:

  -coupling to geodesic acoustic mode important: $C_s^2 = n_e k(T_e + 7T_i/4)/m_i$.

  -evolve $F_e$ and $F_i$ and close for heat flows and parallel stresses.



# Summary

- Analytic closures

  -accepted POP paper on fitted kernel functions for evaluating integral electron closures.

  -submitted POP paper on radial heat transport in the presence of magnetic field line fluctuations.

- Numerical closures:

  -accepted POP paper on neoclassical benchmark between NIMROD, NEO, and DK4D.

  -progress on implementing GS equilibria with beam current and reverse shear Alfvén eigenmode benchmark with TAEFL, GTC, GYRO and DIII-D experiment.