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KINETIC-MHD LINEAR SYSTEM WITH FAST EQUILIBRIUM ROTATION*

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MOTIVATION

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- A K-MHD SYSTEM THAT IS INTRINSICALLY QUASINEUTRAL AND CONSISTENT WITH MOMENTUM AND ENERGY CONSERVATION IS ADVOCATED HERE

- IN SUCH K-MHD SYSTEM, THE LINEARIZED DRIFT-KINETIC EQUATION ABOUT AN AXISYMMETRIC EQUILIBRIUM WITH FAST TOROIDAL FLOW HAS THE SAME DIMENSIONALITY AND IS SIMILAR TO THE AXISYMMETRIC, TIME-DEPENDENT DRIFT-KINETIC EQUATION IMPLEMENTED IN THE DK4D CODE [Lyons et al. 2015]

ZERO-LARMOR-RADIUS, COLLISIONLESS K-MHD MODEL

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- SINGLE ION SPECIES OF UNIT CHARGE
- ZERO-LARMOR-RADIUS-LIMIT AND ZERO COLLISION FREQUENCIES
- NO MASS RATIO APPROXIMATIONS
- MEAN FLOW VELOCITY OF THE ORDER OF THE SOUND SPEED
- KINETIC PRESSURES COMPARABLE TO MAGNETIC PRESSURE

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THEN, $\mathbf{u}_i \to \mathbf{u}_e \to \mathbf{u}$ (COMMON, SINGLE-FLUID MEAN VELOCITY)

BESIDES $n_i = n_e = n$ (FLUID QUASINEUTRALITY)

ZERO-LARMOR-RADIUS MAGNETOFLUID SYSTEM

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) \ , \qquad \mathbf{j} = \nabla \times \mathbf{B}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0 \ , \qquad \rho = (m_i + m_e) n$$

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right] - \mathbf{j} \times \mathbf{B} + \sum_{s=i,e} \nabla \cdot \mathbf{P}_s^{CGL} = 0$$

$$\mathbf{P}_s^{CGL} = p_{s\|}\mathbf{bb} + p_{s\perp}(\mathbf{I} - \mathbf{bb})$$

ZERO-LARMOR-RADIUS DRIFT-KINETIC EQUATION AND FLUID CLOSURE

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$$f_s^{(0)}(\mathbf{w}, \mathbf{x}, t) \;=\; \bar{f}_s(w_\|, w_\perp, \mathbf{x}, t)$$

where

$$\mathbf{w} \;=\; \mathbf{v} \;-\; \mathbf{u}(\mathbf{x}, t) \;=\; w_\| \;\mathbf{b}(\mathbf{x}, t) \;+\; w_\perp \;[\cos\alpha \;\mathbf{e}_1(\mathbf{x}, t) + \sin\alpha \;\mathbf{e}_2(\mathbf{x}, t)]$$

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$$\frac{\partial \bar{f}_s}{\partial t} \;+\; \Big(\mathbf{u} + w_\parallel \mathbf{b}\Big) \cdot \frac{\partial \bar{f}_s}{\partial \mathbf{x}} \;+\; \left[\frac{\mathbf{b} \cdot \Big(\nabla \cdot \mathsf{P}_s^{CGL}\Big)}{m_s n} - w_\parallel(\mathbf{bb}) : (\nabla \mathbf{u}) - \frac{w_\perp^2}{2}\mathbf{b} \cdot \nabla \ln B\right] \frac{\partial \bar{f}_s}{\partial w_\parallel} \;+$$

$$+\; \frac{w_\perp}{2}\; \Big[(\mathbf{bb} - \mathsf{I}) : (\nabla \mathbf{u}) + w_\parallel \mathbf{b} \cdot \nabla \ln B\Big]\; \frac{\partial \bar{f}_s}{\partial w_\perp} \;=\; 0$$

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$$+\; \frac{w_\perp}{2} \;\left[(\mathbf{b}\mathbf{b} - \mathsf{I}) : (\nabla \mathbf{u}) + w_\parallel \mathbf{b} \cdot \nabla \ln B\right] \;\frac{\partial \bar{f}_s}{\partial w_\perp} \;=\; 0$$

and

$$p_{s\parallel} \;=\; m_s \int d^3\mathbf{w} \;w_\parallel^2 \;\bar{f}_s \;=\; 2\pi m_s \int_0^\infty dw_\perp \;w_\perp \int_{-\infty}^\infty dw_\parallel \;w_\parallel^2 \;\bar{f}_s$$

$$p_{s\perp} = \;\frac{m_s}{2} \int d^3\mathbf{w} \;w_\perp^2 \;\bar{f}_s \;=\; \pi m_s \int_0^\infty dw_\perp \;w_\perp \int_{-\infty}^\infty dw_\parallel \;w_\perp^2 \;\bar{f}_s$$

- THIS NON-LINEAR, FAST-DYNAMICS, K-MHD SYSTEM IS OF THE "FULL-f" KIND, WITH DISTRIBUTION FUNCTIONS THAT CAN BE ARBITRARILY DIFFERENT FROM MAXWELLIANS (HOWEVER, IT WILL BE LINEARIZED ABOUT A MAXWELLIAN EQUILIBRIUM)

- **THIS NON-LINEAR, FAST-DYNAMICS, K-MHD SYSTEM IS OF THE "FULL-f" KIND, WITH DISTRIBUTION FUNCTIONS THAT CAN BE ARBITRARILY DIFFERENT FROM MAXWELLIANS (HOWEVER, IT WILL BE LINEARIZED ABOUT A MAXWELLIAN EQUILIBRIUM)**

- **DEFINING** $n_s^{kin} \equiv \int d^3\mathbf{w}\ \bar{f}_s$ **,** $p_s \equiv (m_s/3) \int d^3\mathbf{w}\ w^2\ \bar{f}_s$ **[i.e.** $p_s = (p_{s\|} + 2p_{s\perp})/3$ **] AND** $q_{s\|} \equiv (m_s/2) \int d^3\mathbf{w}\ w^2\ w_\|\ \bar{f}_s$ **, THE** 1 **,** $w_\|$ **AND** w^2 **MOMENTS OF THE DRIFT-KINETIC EQUATION YIELD**

$$\frac{\partial n_s^{kin}}{\partial t} + \nabla \cdot (n_s^{kin}\mathbf{u}) = 0 \qquad \Rightarrow \qquad n_i^{kin} = n_e^{kin} = n$$

$$\int d^3\mathbf{w}\ w_\|\ \bar{f}_s = 0$$

$$\frac{3}{2}\left[\frac{\partial p_s}{\partial t} + \nabla \cdot (p_s\mathbf{u})\right] + \mathbf{P}_s^{CGL} : (\nabla\mathbf{u}) + \nabla \cdot (q_{s\|}\mathbf{b}) = 0$$

LINEAR ANALYSIS

AXISYMMETRIC, MAXWELLIAN EQUILIBRIUM WITH TOROIDAL FLOW

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$$\mathbf{B}_0 \;=\; \nabla\psi \times \nabla\zeta \;+\; I(\psi)\;\nabla\zeta \;, \qquad\qquad \mathbf{j}_0 \;=\; \frac{dI}{d\psi}\,\nabla\psi \times \nabla\zeta \;-\; \Delta^*\psi\;\nabla\zeta$$

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$$\mathbf{u}_0 \ = \ \Omega(\psi)\ R^2\ \nabla\zeta\ , \qquad (\mathbf{u}_0 \cdot \nabla)\mathbf{u}_0 \ = \ -\Omega^2\ R\ \nabla R\ , \qquad \nabla \cdot \mathbf{u}_0 \ = \ (\mathbf{b}_0\mathbf{b}_0) : (\nabla \mathbf{u}_0) \ = 0$$

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$$\bar{f}_{s0} \;=\; f_{Ms0} \;=\; \left(\frac{m_s}{2\pi}\right)^{3/2}\;\frac{n_0}{T_{s0}^{3/2}}\;\exp\left(-\frac{m_s w^2}{2T_{s0}}\right)$$

$$T_{s0} \;=\; T_{s0}(\psi)\;, \qquad n_0 \;=\; n_0(\psi, R) \;=\; N(\psi)\;\exp\left\{\frac{(m_i + m_e)R^2\Omega^2(\psi)}{2\;[T_{i0}(\psi) + T_{e0}(\psi)]}\right\}\;, \qquad \rho_0 \;=\; (m_i + m_e)n_0$$

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$$\nabla \cdot (n_0\mathbf{u}_0) \;=0\;, \qquad\qquad \nabla \times (\mathbf{u}_0 \times \mathbf{B}_0) \;=0$$

AXISYMMETRIC, MAXWELLIAN EQUILIBRIUM WITH TOROIDAL FLOW

$$\mathbf{B}_0 \;=\; \nabla\psi \times \nabla\zeta \;+\; I(\psi)\;\nabla\zeta\;, \qquad\qquad \mathbf{j}_0 \;=\; \frac{dI}{d\psi}\;\nabla\psi \times \nabla\zeta \;-\; \Delta^*\psi\;\nabla\zeta$$

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$$\nabla \cdot (n_0\mathbf{u}_0)\;\; = 0\;, \qquad\qquad \nabla \times (\mathbf{u}_0 \times \mathbf{B}_0)\;\; = 0$$

$$\mathbf{j}_0 \times \mathbf{B}_0 \;=\; \rho_0\;(\mathbf{u}_0 \cdot \nabla)\mathbf{u}_0 \;+\; \nabla[n_0(T_{i0} + T_{e0})] \quad \Rightarrow \quad -\frac{1}{R^2}\left(I\frac{dI}{d\psi} + \Delta^*\psi\right) \;=\; \frac{\partial[n_0(T_{i0} + T_{e0})]}{\partial\psi}\;\Big|_R$$

LINEARIZED MAGNETOFLUID SYSTEM

$$\frac{\partial \mathbf{B}_1}{\partial t} = \nabla \times (\mathbf{u}_1 \times \mathbf{B}_0 + \mathbf{u}_0 \times \mathbf{B}_1) \ , \qquad \mathbf{j}_1 = \nabla \times \mathbf{B}_1$$

$$\frac{\partial n_1}{\partial t} = - \nabla \cdot (n_0 \mathbf{u}_1 + n_1 \mathbf{u}_0) \ , \qquad \rho_1 = (m_i + m_e) n_1$$

$$\rho_0 \left[\frac{\partial \mathbf{u}_1}{\partial t} + (\mathbf{u}_1 \cdot \nabla)\mathbf{u}_0 + (\mathbf{u}_0 \cdot \nabla)\mathbf{u}_1 \right] + \rho_1 \ (\mathbf{u}_0 \cdot \nabla)\mathbf{u}_0 =$$

$$= \mathbf{j}_0 \times \mathbf{B}_1 + \mathbf{j}_1 \times \mathbf{B}_0 - \sum_{s=i,e} \left\{ \nabla p_{s\perp 1} + \nabla \cdot \left[(p_{s\parallel 1} - p_{s\perp 1}) \mathbf{b}_0 \mathbf{b}_0 \right] \right\}$$

$$p_{s\parallel 1} = m_s \int d^3\mathbf{w} \ w_\parallel^2 \ \bar{f}_{s1} \ , \qquad p_{s\perp 1} = \frac{m_s}{2} \int d^3\mathbf{w} \ w_\perp^2 \ \bar{f}_{s1}$$

LINEARIZED DRIFT-KINETIC EQUATION

$$\frac{\partial \bar{f}_{s1}}{\partial t} \;+\; \left(\mathbf{u}_0 + w_\parallel\ \mathbf{b}_0\right) \cdot \frac{\partial \bar{f}_{s1}}{\partial \mathbf{x}} \;+\; \frac{T_{s0}}{m_s} \left(\mathbf{b}_0 \cdot \nabla \ln n_0\right) \frac{\partial \bar{f}_{s1}}{\partial w_\parallel} \;+\; \frac{w_\perp}{2} \left(\mathbf{b}_0 \cdot \nabla \ln B_0\right) \left(w_\parallel \frac{\partial \bar{f}_{s1}}{\partial w_\perp} - w_\perp \frac{\partial \bar{f}_{s1}}{\partial w_\parallel}\right) \;=$$

$$= \Big\{ - \left[\mathbf{u}_1 + \frac{n_1}{n_0}\ w_\parallel\ \mathbf{b}_0\right] \cdot \nabla \ln n_0 \;+\; \left[\left(\frac{3}{2} - \frac{m_s w^2}{2T_{s0}}\right)\mathbf{u}_1 + \left(\frac{5}{2} - \frac{m_s w^2}{2T_{s0}}\right)\ \frac{w_\parallel}{B_0}\ \mathbf{B}_1\right] \cdot \nabla \ln T_{s0} \;+$$

$$+\ \frac{w_\parallel}{n T_{s0}}\ \left[\mathbf{b}_0 \cdot \nabla p_{s\parallel 1} - \left(p_{s\parallel 1} - p_{s\perp 1}\right) \mathbf{b}_0 \cdot \nabla \ln B_0\right] \;-\; \frac{m_s}{2T_{s0}}\ \left[w_\perp^2 \nabla \cdot \mathbf{u}_1 + \left(2w_\parallel^2 - w_\perp^2\right) \left(\mathbf{b}_0 \mathbf{b}_0\right) : \left(\nabla \mathbf{u}_1\right)\right] \Big\}\ f_{Ms0}$$

LINEARIZED DRIFT-KINETIC EQUATION

$$
\frac{\partial \bar{f}_{s1}}{\partial t} + (\mathbf{u}_0 + w_\parallel \, \mathbf{b}_0) \cdot \frac{\partial \bar{f}_{s1}}{\partial \mathbf{x}} + \frac{T_{s0}}{m_s} (\mathbf{b}_0 \cdot \nabla \ln n_0) \frac{\partial \bar{f}_{s1}}{\partial w_\parallel} + \frac{w_\perp}{2} (\mathbf{b}_0 \cdot \nabla \ln B_0) \left(w_\parallel \frac{\partial \bar{f}_{s1}}{\partial w_\perp} - w_\perp \frac{\partial \bar{f}_{s1}}{\partial w_\parallel} \right) =
$$

$$
= \left\{ - \left[\mathbf{u}_1 + \frac{n_1}{n_0} \, w_\parallel \, \mathbf{b}_0 \right] \cdot \nabla \ln n_0 + \left[\left(\frac{3}{2} - \frac{m_s w^2}{2 T_{s0}} \right) \mathbf{u}_1 + \left(\frac{5}{2} - \frac{m_s w^2}{2 T_{s0}} \right) \frac{w_\parallel}{B_0} \, \mathbf{B}_1 \right] \cdot \nabla \ln T_{s0} + \right.
$$

$$
\left. + \frac{w_\parallel}{n T_{s0}} \left[\mathbf{b}_0 \cdot \nabla p_{s\parallel 1} - \left(p_{s\parallel 1} - p_{s\perp 1} \right) \mathbf{b}_0 \cdot \nabla \ln B_0 \right] - \frac{m_s}{2 T_{s0}} \left[w_\perp^2 \nabla \cdot \mathbf{u}_1 + \left(2 w_\parallel^2 - w_\perp^2 \right) (\mathbf{b}_0 \mathbf{b}_0) : (\nabla \mathbf{u}_1) \right] \right\} f_{Ms0}
$$

FOR A TOROIDAL FOURIER MODE, $\partial / \partial \zeta = i n$ **:**

$$
\frac{\partial \bar{f}_{s1}}{\partial t} + w_\parallel \, (\mathbf{b}_0 \cdot \nabla \theta) \frac{\partial \bar{f}_{s1}}{\partial \theta} + \frac{w_\perp}{2} (\mathbf{b}_0 \cdot \nabla \ln B_0) \left(w_\parallel \frac{\partial \bar{f}_{s1}}{\partial w_\perp} - w_\perp \frac{\partial \bar{f}_{s1}}{\partial w_\parallel} \right) +
$$

$$
+ i n \left(\Omega + \frac{w_\parallel I}{B_0 R^2} \right) \bar{f}_{s1} + \frac{T_{s0}}{m_s} (\mathbf{b}_0 \cdot \nabla \ln n_0) \frac{\partial \bar{f}_{s1}}{\partial w_\parallel} =
$$

$$
= \left\{ - \left[\mathbf{u}_1 + \frac{n_1}{n_0} \, w_\parallel \, \mathbf{b}_0 \right] \cdot \nabla \ln n_0 + \left[\left(\frac{3}{2} - \frac{m_s w^2}{2 T_{s0}} \right) \mathbf{u}_1 + \left(\frac{5}{2} - \frac{m_s w^2}{2 T_{s0}} \right) \frac{w_\parallel}{B_0} \, \mathbf{B}_1 \right] \cdot \nabla \ln T_{s0} + \right.
$$

$$
\left. + \frac{w_\parallel}{n T_{s0}} \left[\mathbf{b}_0 \cdot \nabla p_{s\parallel 1} - \left(p_{s\parallel 1} - p_{s\perp 1} \right) \mathbf{b}_0 \cdot \nabla \ln B_0 \right] - \frac{m_s}{2 T_{s0}} \left[w_\perp^2 \nabla \cdot \mathbf{u}_1 + \left(2 w_\parallel^2 - w_\perp^2 \right) (\mathbf{b}_0 \mathbf{b}_0) : (\nabla \mathbf{u}_1) \right] \right\} f_{Ms0}
$$

USING AS PHASE-SPACE COORDINATES THE KINETIC ENERGY AND THE MAGNETIC MOMENT,

$$\varepsilon \;=\; \frac{m_s}{2}\,(w_\parallel^2 + w_\perp^2)\;, \qquad\qquad \mu \;=\; \frac{m_s}{2B_0}\,w_\perp^2\;,$$

$$\frac{\partial \bar{f}_{s1}}{\partial t} \;+\; in\left(\Omega + \frac{w_\parallel I}{B_0 R^2}\right)\bar{f}_{s1} \;+\; w_\parallel\left[(\mathbf{b}_0\cdot\nabla\theta)\,\frac{\partial \bar{f}_{s1}}{\partial\theta}\Big|_{\varepsilon,\mu} \;+\; T_{s0}\,(\mathbf{b}_0\cdot\nabla\ln n_0)\,\frac{\partial \bar{f}_{s1}}{\partial\varepsilon}\Big|_{\mathbf{x},\mu}\right] \;=$$

$$= \Big\{ -\left[\mathbf{u}_1 + \frac{n_1}{n_0}\,w_\parallel\;\mathbf{b}_0\right]\cdot\nabla\ln n_0 \;+\; \left[\left(\frac{3}{2} - \frac{\varepsilon}{T_{s0}}\right)\mathbf{u}_1 + \left(\frac{5}{2} - \frac{\varepsilon}{T_{s0}}\right)\,\frac{w_\parallel}{B_0}\;\mathbf{B}_1\right]\cdot\nabla\ln T_{s0} \;+$$

$$+\;\frac{w_\parallel}{nT_{s0}}\;\left[\mathbf{b}_0\cdot\nabla p_{s\parallel 1} - \left(p_{s\parallel 1} - p_{s\perp 1}\right)\mathbf{b}_0\cdot\nabla\ln B_0\right] \;-\; \frac{1}{T_{s0}}\;\left[\mu B_0\nabla\cdot\mathbf{u}_1 + (2\varepsilon - 3\mu B_0)\,(\mathbf{b}_0\mathbf{b}_0) : (\nabla\mathbf{u}_1)\right]\Big\}\; f_{Ms0}$$

where $\qquad w_\parallel \;=\; \pm[2(\varepsilon - \mu B_0)/m_s]^{1/2}$

SUMMARY

- A KINETIC-MHD MODEL IS PROPOSED TO ANALIZE THE LINEAR STABILITY OF AXISYMMETRIC EQUILIBRIA WITH FAST TOROIDAL FLOW

- THE MAGNETOFLUID PART OF THE SYSTEM COMPRISES THE LINEARIZED FORMS OF THE FARADAY-OHM LAW, THE CONTINUITY EQUATION AND THE MOMENTUM CONSERVATION EQUATION, THAT EVOLVE B_1, n_1 AND $\mathbf{u}_1$

- THE KINETIC PART YIELDS THE FLUID CLOSURES $p_{s\|1}$ AND $p_{s\perp 1}$, AS MOMENTS OF THE GYROPHASE-INDEPENDENT DISTRIBUTION FUNCTIONS $\bar{f}_{s1}$. THESE EVOLVE WITH LINEARIZED DRIFT-KINETIC EQUATIONS THAT ARE CONSISTENT WITH THE FLUID CONSERVATION LAWS. FOR A GIVEN TOROIDAL MODE, THE DKE'S DEPEND ON THREE PHASE-SPACE COORDINATES $(w_\|, w_\perp, \ell_p)$ AND TIME

- THE DKE PHASE-SPACE DIMENSIONALITY CAN BE REDUCED TO (ε, ℓ_p) AT CONSTANT μ, BY USING ENERGY AND MAGNETIC MOMENT COORDINATES