Disruption Asymmetry Scalings

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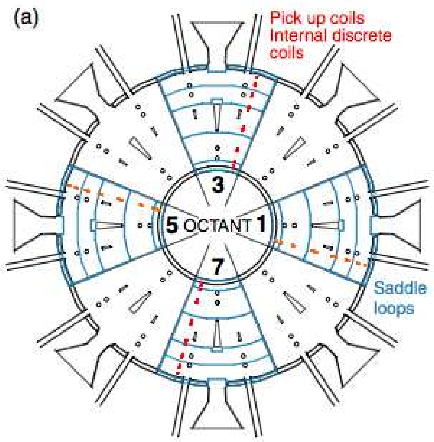
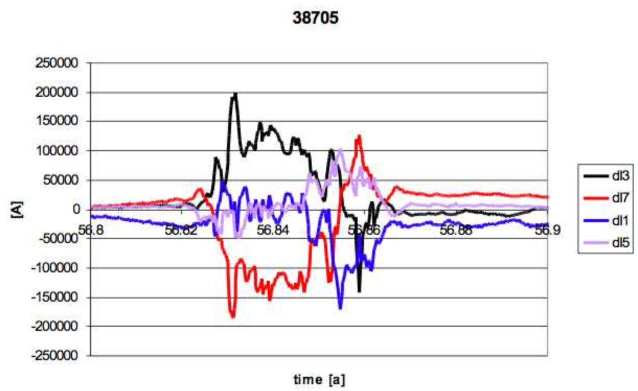
Outline

- Disruptions with VDE displacement ξ
  - relation of perturbed toroidal current $\tilde{I}_\phi$ and vertical current moment $\tilde{M}I_Z$
  - numerical simulations
- Sideways wall force and current asymmetry relation
  - sideways force $F_x \propto \tilde{M}_{IZ}$
  - comparison of $\tilde{M}_{IZ}$, δB, and ξ
- Rotation
  - fit of numerical data and analytic scaling with ξ,δB and β_N
  - comparison of force rotation and net toroidal velocity
- JET
  - Is JET a good predictor for ITER?
  - modes in multiple disruptions

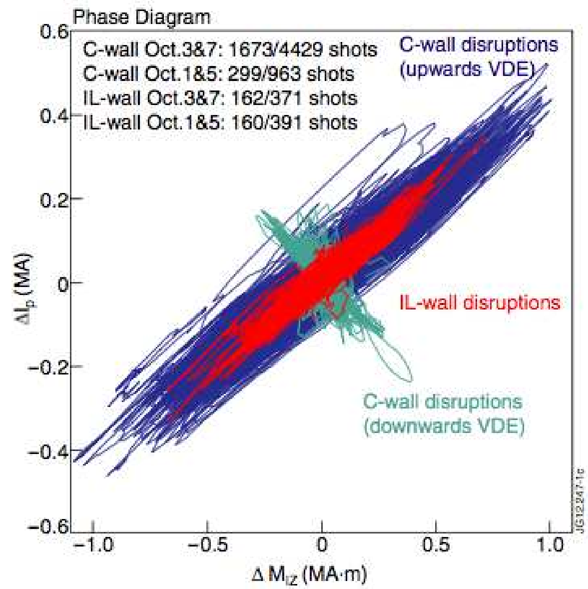
Toroidal variation of toroidal current in JET



(a)



(b)



(c)

(a) layout of JET I_ϕ measurements. (b) Current I_ϕ measured in quadrants of JET, showing $n = 1$ toroidal variation. (c) Toroidal current variation vs. the vertical moment of the current variations in asymmetric vertical displacement events (AVDE) [Gerasimov *et al.* N.F. 2014].

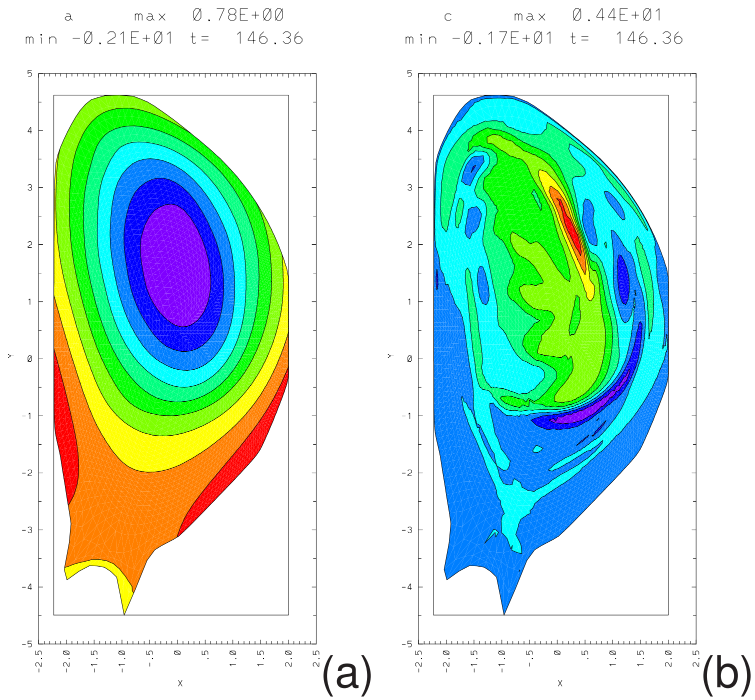
It was shown analytically and computationally [Strauss *et al.*, Phys. Plasmas **21**, 102509 (2014)] that the slope in (c) is Phys. Plasmas **21**, 102509 (2014)].

$$\tilde{I} \propto \xi \tilde{M}_{IZ}$$

where ξ is the vertical displacement.

M3D simulations of current asymmetry and vertical current moment

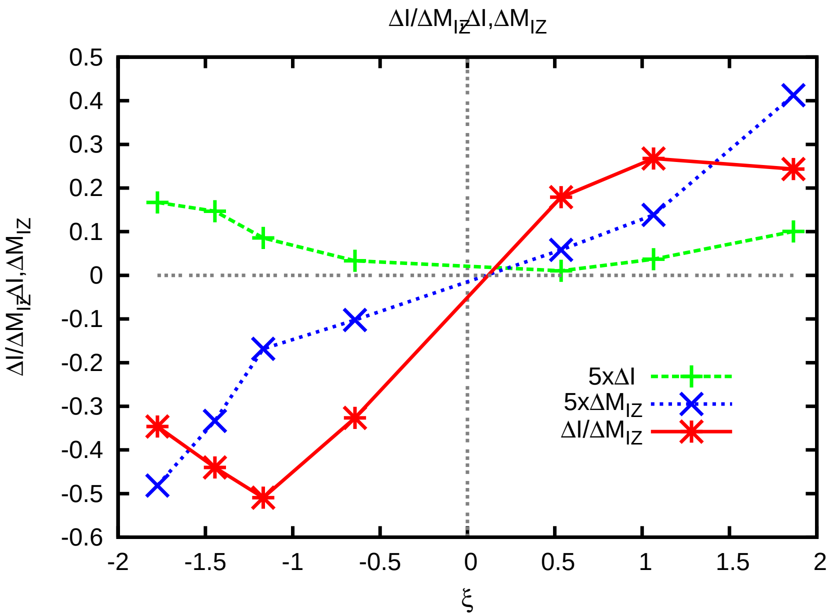
ITER FEAT15MA equilibrium was modified by setting toroidal current and pressure to zero outside the $q = 2$ surface, keeping the total toroidal current constant (MGI model) [Izzo *et al.* 2008]. Plasma was evolved with M3D in 2D to an initial VDE displacement, then evolved in 3D.
$\tau_{wall} = 10^4 \tau_A$. Velocity boundary condition $v_n = 0$.

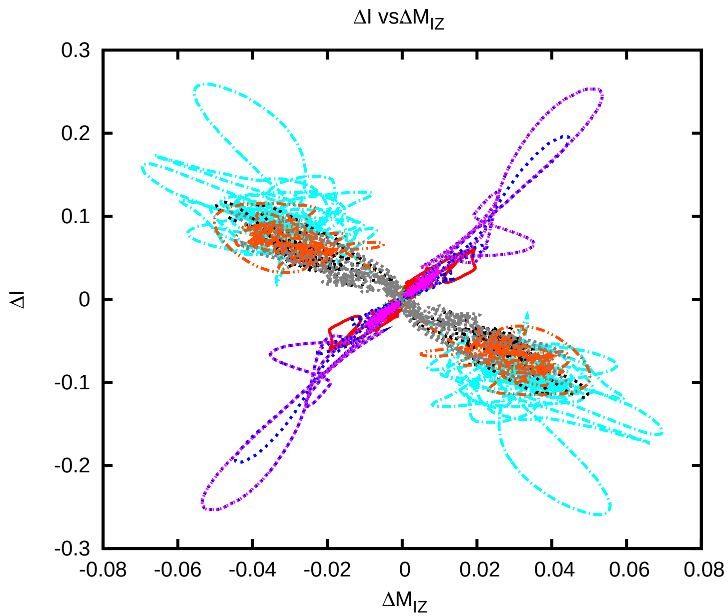


An additional set of states was made by setting current and pressure equal to zero outside the $q = 1.5$ surface. These states were unstable to downward VDEs. M3D simulations were done with $S = 10^6$, wall penetration time $\tau_{wall} = 10^4 \tau_A$. Velocity boundary condition $v_n = 0$. **Plasma is turbulent, not an equilibrium with surface current.**

Upward VDE: (a) ψ (b) J_ϕ with $\xi = 0.72a$, time $t = 146\tau_A$, toroidal angle $\phi = 0$.

Time averaged $\tilde{I}_\phi/\tilde{M}_{IZ}$ and time histories $\tilde{I}_\phi, \tilde{M}_{IZ}$





(a) Time averages of $\tilde{I}_\phi, \tilde{M}_{IZ}$. Showing $\tilde{I}_\phi/\tilde{M}_{IZ} \propto \xi$, for $|\xi| \lesssim 1$, when plasma current channel reaches the wall. (b) Time histories of $\tilde{I}_\phi, \tilde{M}_{IZ}$ for the cases in (a). **This is similar to JET data.**

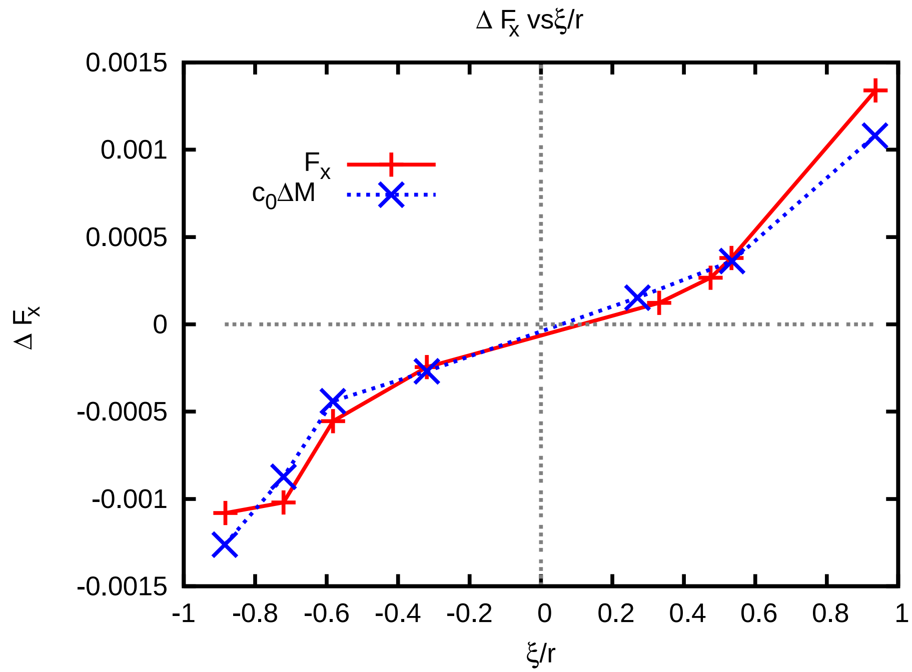
Noll relation of sideways force $F_x \propto \tilde{M}_{IZ}$

Measurements of these simulations were also made of asymmetric wall force $F_x(\xi)$, and net toroidal velocity $V_\phi(\xi)$. These were compared with theory.

A Noll like relation [Noll et al., 1996] for the sideways force F_x

$$F_x = \frac{\pi B}{q} \tilde{M}_{IZ} \tag{1}$$

is obtained from the data, with $q = 1.3$. The force F_x and $\tilde{M}_{IZ}$ are approximately linear in VDE displacement ξ. Note $F_x \approx 0$ for $\xi = 0$.



Heuristic Noll relation of sideways force $F_x \propto M_{IZ}$

Noll relation [Noll et al., 1996] for the sideways force F_x in terms of M_{IZ}

$$I_Z = I_\phi \xi \cos\phi$$

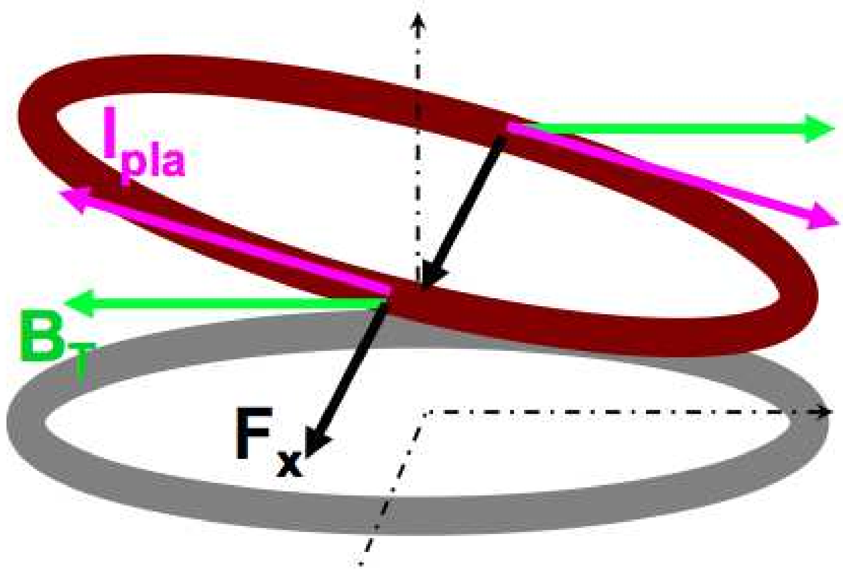
$$F_R = I_Z B_\phi$$

$$F_x = \oint F_R \cos\phi d\phi$$

$$M_{IZ} = \xi I_\phi$$

$$F_x = \pi B M_{IZ}$$

M_{IZ} is more easily measured than F_x, used in analysis of JET data [Gerasimov *et al.* 2014]. Note $\pi/2$ shift in ϕ between maximum F_x and maximum vertical displacement ξ.



Theory of Noll relation $F_x \propto \tilde{M}_{IZ}$

The total wall force is given by

$$\mathbf{F} = \frac{\delta_{wall}}{2\pi L} \int dl d\phi \mathbf{J}^{wall} \times \mathbf{B}^{wall} \tag{2}$$

and

$$\mathbf{J}^{wall} \times \mathbf{B}^{wall} \approx \hat{\mathbf{r}} \tilde{J}_{\phi}^{wall} B_{\theta}^{wall}.$$

where δ_{wall} is the wall thickness, $\tau_{wall} = \delta_{wall} r/(m\eta_{wall})$. For long wall penetration time, $\gamma\tau_{wall} >> 1$, the vacuum field does not contribute to the perturbed current,

$$\tilde{J}_{\phi}^{wall} = \frac{\tilde{B}_{\theta}}{\delta_{wall}}$$

The net horizontal force F_x is

$$F_x = \frac{B_{\theta}^{wall}}{4\pi^2} \oint \tilde{B}_{\theta} \cos\theta \cos\phi d\theta d\phi \tag{3}$$

Expressing $\tilde{B}_{\theta}$ in terms of M_{IZ}, gives (1).

$$\tilde{M}_{IZ} = \int J_{\phi} \sin\theta r^2 dr d\theta = r^2 \oint \tilde{B}_{\theta} \sin\theta d\theta \tag{4}$$

Relation of $\tilde{B}_\theta$ to $\tilde{M}_{IZ}$

In a circular cross section with large aspect ratio, let

$\tilde{B}_\theta = B_{11}\sin(\theta+\phi) + B_{21}\cos(2\theta+\phi)$.

Displacing

$$B_{21}(r - \xi\sin\theta) \approx B_{21} - B'_{21}\xi\sin\theta$$

by a VDE, then

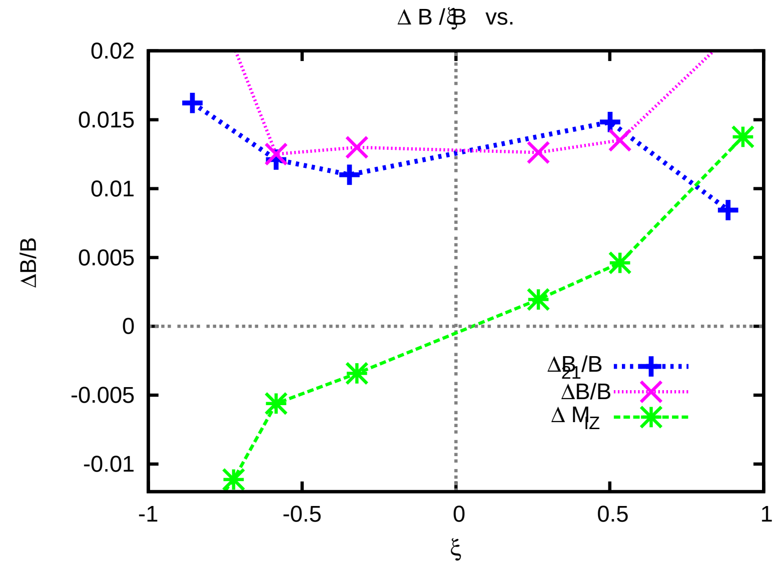
$$\tilde{M}_{IZ} = \pi a^2 (2B_{11} - \xi B'_{21}) \qquad (5)$$

Let $V = \pi a^2$ and

$$\delta B = (a/\xi V)\tilde{M}_{IZ} \approx (1/2)|B_{21}|.$$

Magnetic perturbations were also calculated directly from

$$\delta B_{21} = \frac{1}{abL}\oint (R - R_0) Z \tilde{B}_l dl$$

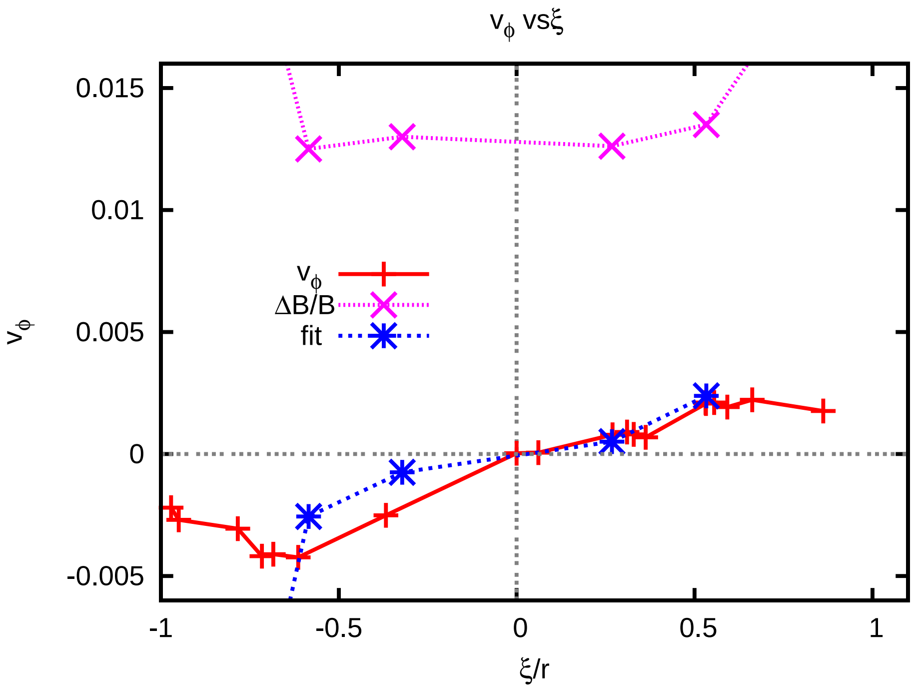


Simulations show that $\tilde{M}_{IZ} \approx 0$ for $\xi = 0$, which implies $B_{11} \approx 0$, because it is an internal mode. The plot compares δB, δB_{21}, and $\tilde{M}_{IZ}$ which agree for ξ/a not too large.

For nonzero ξ, B_{11} can be nonzero. δB will be used in the following.

Dependence of toroidal velocity on vertical displacement

In [Strauss *et al.* 2014] the toroidal rotation caused by disruptions was calculated. This was motivated by a concern that if the asymmetric wall force F_x varies with a frequency ~ 100 Hz, there could be a resonance with ITER structures.

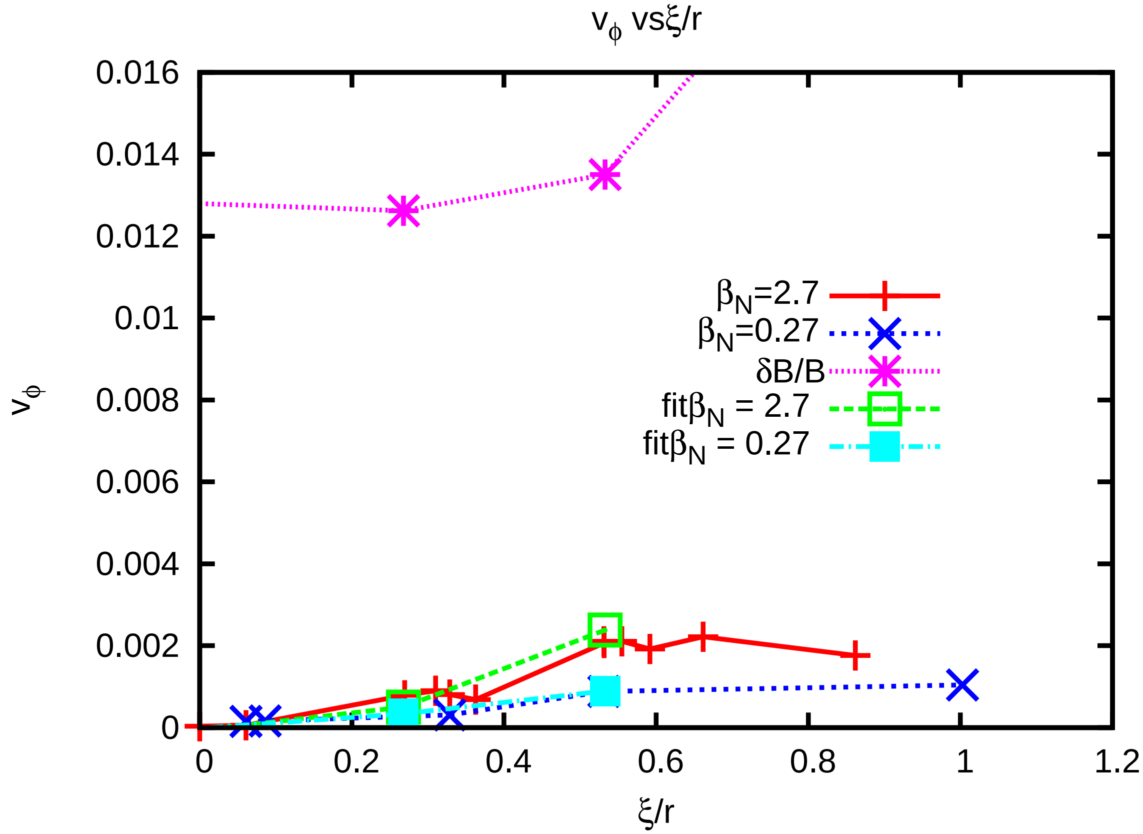


The previous set of states was used to calculate V_ϕ, the maximum in time of the volume average of the toroidal velocity. V_ϕ as a function of vertical displacement. Also shown is the magnetic perturbation δB. Simulational points and V_ϕ fit to (6) are shown. The fit has $A_1 = 7.5$, $A_2 = 3$ in the function

$$\frac{V_\phi}{v_A} = A_1 \frac{\xi}{r}\left[1 + A_2 \beta_N \left(\frac{\xi}{r}\right)^2\right]\left(\frac{\delta B}{B}\right)^2. \tag{6}$$

Scaling of toroidal velocity with ξ and β_N

Two sets of cases were compared to get the scaling with β_N.



These cases compare equilibria with $\beta_N = 2.7$ and $\beta_N = 0.27$. In the low β_N case, $A_2 \approx 0$. The fit is very good for $\xi/r < 0.6$. There is only low β_N data for $\xi > 0$. The $\beta_N = 2.7$ and δB data is the same as on the previous slide, for $\xi > 0$.

Theory: Conservation of toroidal angular momentum

$$\frac{\partial}{\partial t} L_\phi = \oint B_\phi B_n R^2 dl d\phi \tag{7}$$

assuming $v_n = 0$ or $v_\phi = 0$ at the boundary, where the total toroidal angular momentum is

$$L_\phi = \int \rho R^2 v_\phi dR dZ d\phi \tag{8}$$

For simplicity assume circular equilibrium cross sections, $dl = r d\theta$. To obtain a tractable equation for G, assume radial force balance,

$$B_\phi^2 + B_\theta^2 + 2p \approx 0 \tag{9}$$

and large aspect ratio so that $R \approx R_0 =$ constant. Then $\dot{L}_\phi$ can be split into two parts, $\dot{L}_\phi = \dot{L}_{\phi B} + \dot{L}_{\phi p}$ where

$$\dot{L}_{\phi B} = -\frac{Rr}{2B_{\phi 0}} \oint B_r B_{\theta 1}^2 d\theta d\phi \tag{10}$$

$$\dot{L}_{\phi p} = -\frac{Rr}{B_{\phi 0}} \oint B_r p d\theta d\phi \tag{11}$$

The plasma and poloidal flux ψ are displaced by a VDE with $\xi \sin\theta$, $\psi_0 = \psi_0(r - \xi \sin\theta)$. Hence with $B_r = (1/r)\partial\psi/\partial\theta$, $B_\theta = -\partial\psi/\partial r$, $B_r = (\xi/r)\cos\theta B_\theta$. There must be at least two modes $(m, n), (m+1, n)$ contributing to $B_{\theta 1}$ which beat together to give a $\cos\theta$ term. Expanding $B_{\theta 1} = \sum_{mn} B_{\theta mn} \cos(m\theta - n\phi)$ then (10) becomes

$$\dot{L}_{\phi B} = \frac{\pi^2 \xi r R}{2q} \sum_{mn} B_{\theta mn} B_{\theta(m+1)n} \tag{12}$$

To compare with the scaling (6), let $\dot{v}_\phi = \gamma v_\phi$, in (7). Then (12) yields

$$\boldsymbol{A_1 = \frac{1}{4\gamma\tau_A q}} \tag{13}$$

and taking $q = 2.7$, $\gamma\tau_A = 0.014$ gives agreement with $A_1 = 7.5$ in (6). The calculation of (11), is given in [Strauss *et al.* 2014].

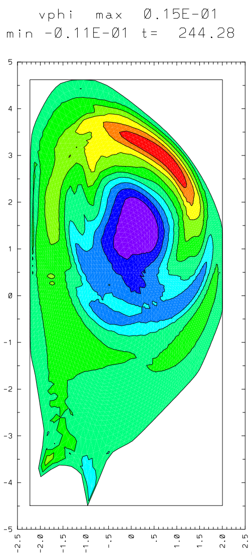
The ratio is $\dot{L}_{\phi p}/\dot{L}_{\phi B} = A_2 \beta_N (\xi/r)^2$, with

$$\boldsymbol{A_2 = \frac{q[1 + m(m+1)]}{2(m-q)(m+1-q)} (\ln\beta_N)'(\ln\delta B)' r^2} \tag{14}$$

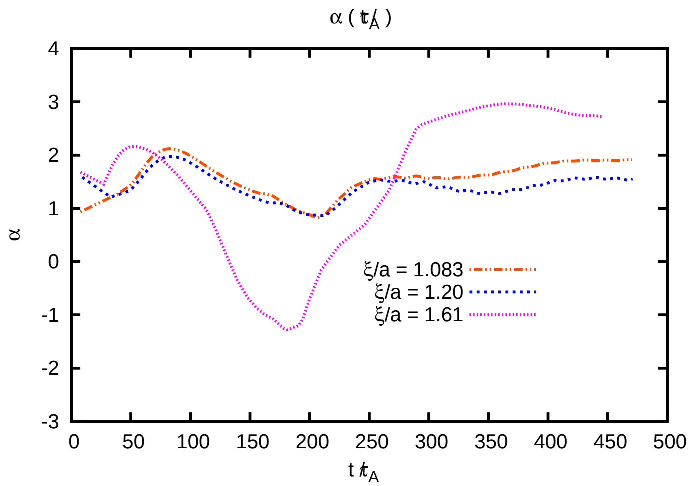
Taking $m = 1$, $\beta_N = 2.7$, $(\ln\delta B)'r = (\ln\beta_N)' = -1$, gives $A_2 = 3$ in agreement with (6).

Comparison of average V_ϕ and rotation of sideways force

The frequency of oscillation of the sideways force is higher than the frequency of rotation calculated from the volume averaged toroidal velocity, because of its zonal structure. The force angle is $\alpha = \tan^{-1}(F_y/F_x)$. The half period $\tau_\alpha/2$ from between minimum and maximum α was measured. The oscillation of the force is about $3-4$ times faster than calculated from the net rotation V_ϕ, as in [Strauss *et al.* 2014].



(a)



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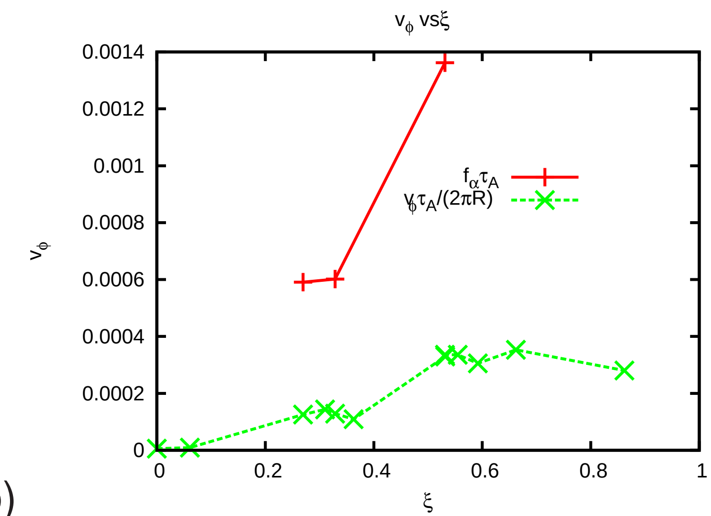


(c)

(a) v_ϕ in a disruption, showing zonal structure (b)Force angle α as a function of time, for several values of ξ. (c) Frequency calculated from α and from average toroidal velocity v_ϕ. **The force oscillation frequency is ~ 1kHz, not a problem for ITER.**

Are JET disruptions predictive for ITER?

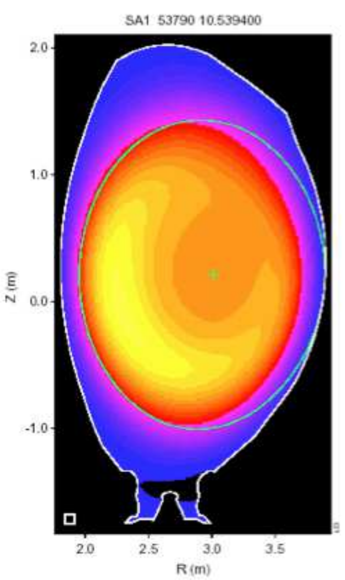
JET has short wall time: $\tau_{w-JET} = 3ms, \tau_{w-ITER} = 300ms.$

JET and ITER τ_A are comparable, $\tau_A \approx 1\mu s.$

For MHD instabilities, $\gamma\tau_w = \mathcal{O}(1)$ in JET, $\gamma\tau_w = \mathcal{O}(100)$ in ITER

Hence the scaled sideways force is much larger in JET.

JET toroidal rotation in disruptions $= 100Hz$, NSTX and C-Mod, $1kHz$.



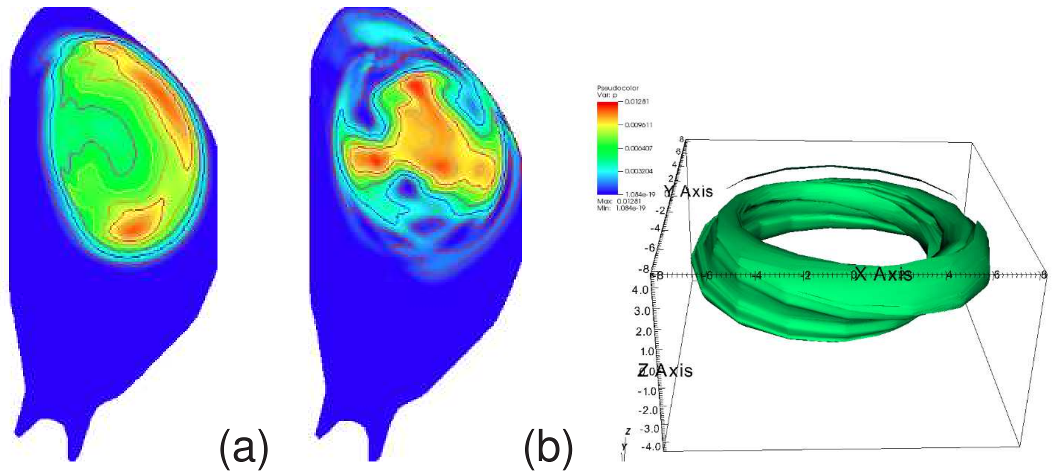
Radiation from a JET disruption, which looks like an $m, n = 1, 1$ island, suggesting $q \approx 1$. From Plyusnin *et al.* , IAEA 2004.

JET disruptions have $q_{LCFS} \approx 1 - 1.5$, while ”most” disruptions occur when $q_{LCFS} \approx 2$.

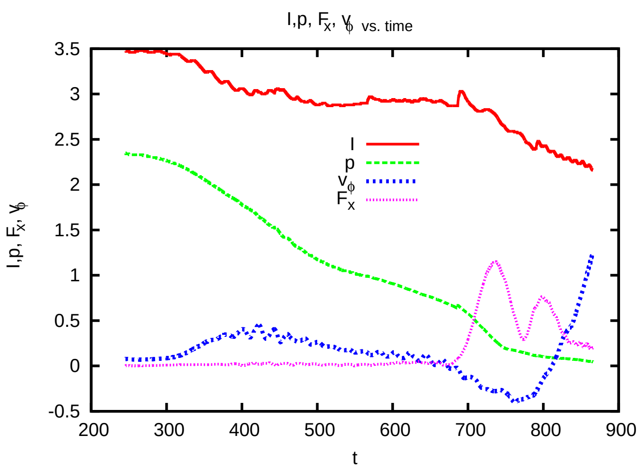
how is a state with $q_{LCFS} \approx 1 - 1.5$ produced?

Multiple TQ's when toroidal current is sustained

(a) pressure p at $t = 343\tau_A$ during 1st TQ. (b) pressure p at $t = 686\tau_A$ during 2nd TQ. (c) iso plot of p at $t = 686\tau_A$ showing $n = 2$ structure.



(a) (b) (c)



(d)

(d) time history of I_ϕ, p, F_x, V_ϕ.

The first disruption has large $(m, n) = (1, 1)$ perturbations, although $q_{LCFS} \approx 2$. If the current is sustained by loop voltage, there can be a second TQ. Apparently the second disruption is produced by an $(m, n) = (3, 2)$ mode. This suggests that magnetic flux is scraped off so that $q_{LCFS} \approx 1.5$. $(2, 1)$ and $(3, 2)$ modes could interact nonlinearly to produce a large $(1, 1)$ force perturbation.

Conclusions

- Relation of $\tilde{I}$ to $\tilde{M}_{IZ}$.
  - $\tilde{I} \propto \xi \tilde{M}_{IZ}$ where ξ is VDE displacement
- Noll relation $F_x \propto \tilde{M}_{IZ}$
  - F_x and $\tilde{M}_{IZ}$ approximately $\propto \xi$
  - estimate δB from $\tilde{M}_{IZ}$
- Scaling of V_ϕ with $\xi, \delta B, \beta_N$.
  - Force oscillations are not a problem for ITER
- JET disruptions seem to have $q_{LCFS} = 1$ or 1.5
  - might be explained by current sustainment, long enought for flux scrape off and multiple disruptions