
Update on giant sawtooth, importance of GS resolves, and improvements to PIC closures

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Outline

- Brief report on testing the accuracy of NIMROD equilibria with resolves of the Grad-Shafranov equation.
- Brief report on continuum kinetics applied to giant sawtooth problem.
- Improvements to NIMROD's delta-f PIC algorithm.
- Plans for continuum kinetics

Computation of vanishing contribution to ion distribution function.

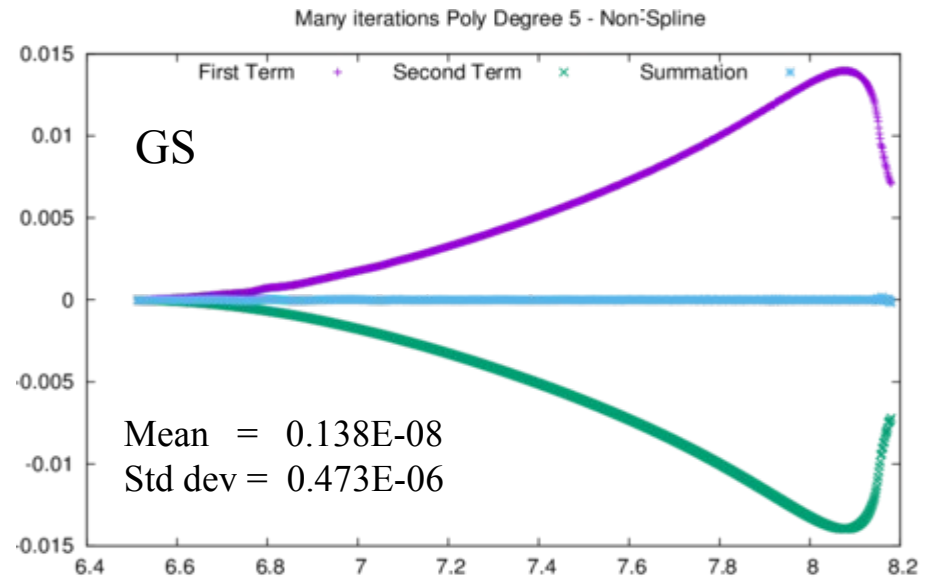
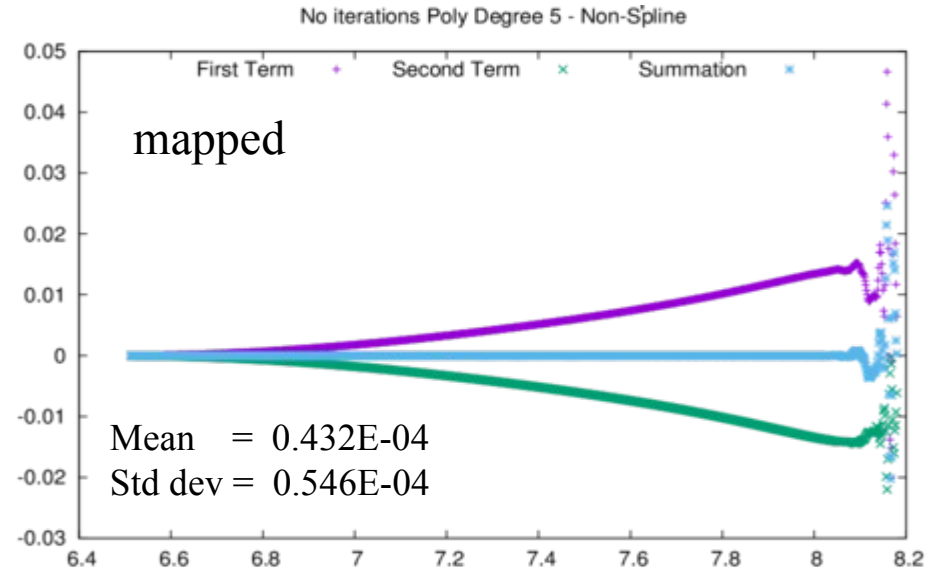
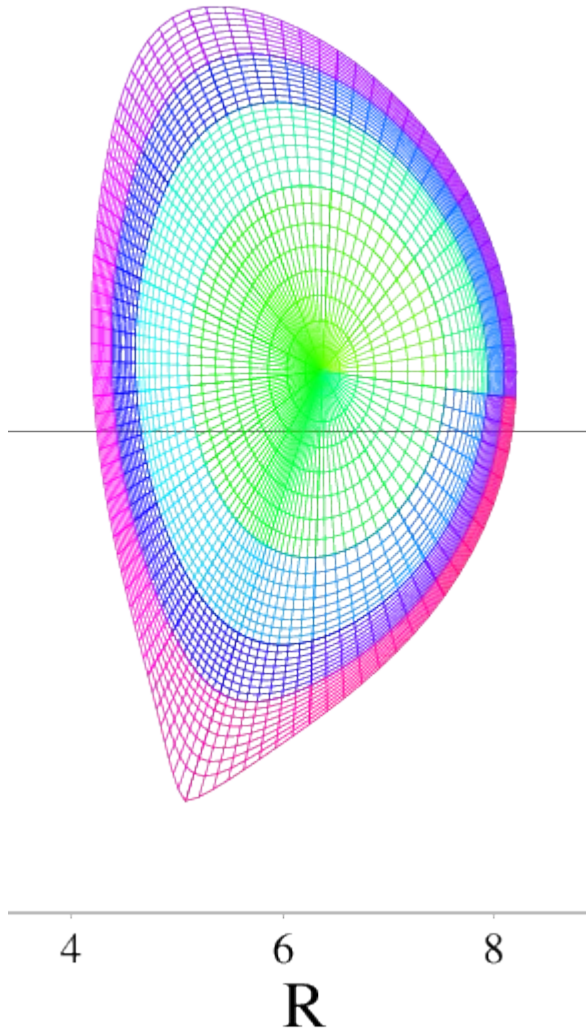
- Test accuracy of NIMROD resolves of Grad-Shafranov equilibrium on vanishing neoclassical term predicted by Ramos for $\nu \sim \rho/L \ll 1$ (Phys. Plasmas 22, 070702 (2015)) :

$$\Gamma(\psi) = 2 \frac{d}{d\psi} \int dl B \cdot \nabla R^2 + \int \frac{dl}{B} \left[2b \cdot \nabla (\nabla \psi \cdot \nabla \ln R^2) + \nabla \psi \cdot \nabla (b \cdot \nabla \ln B) \right]$$

- Ramos writes, “Even though the result was masked by numerical noise, it was sufficiently suggestive of $\Gamma(\psi)$ being zero to motivate the search for the analytic proof that this geometrical function vanishes identically.”
- Examine improvement to equilibria provided by GS resolves by testing the extent to which the two terms above cancel..

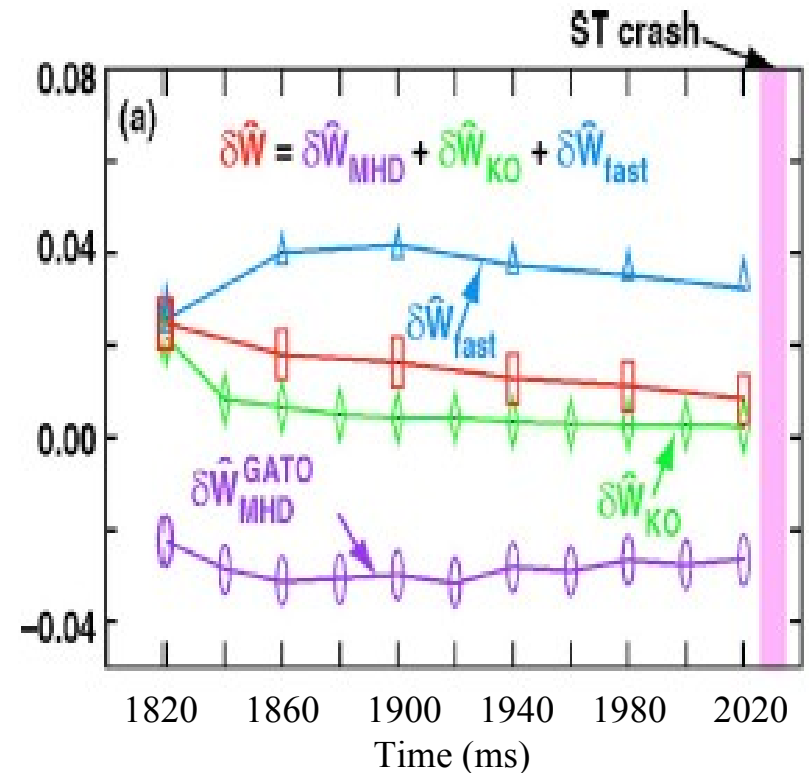
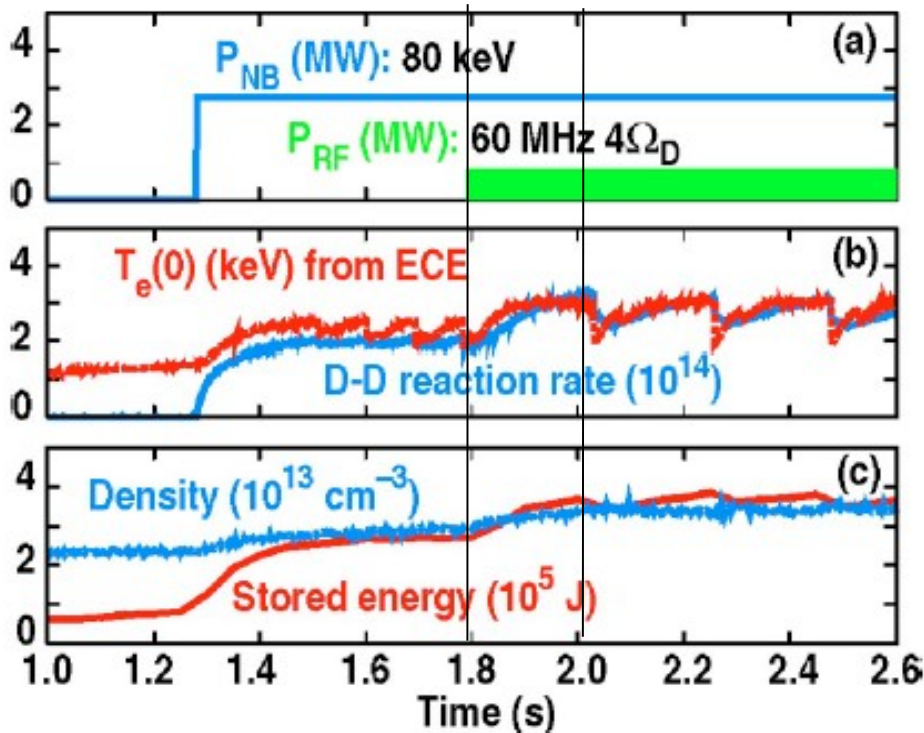
Results show importance of GS resolves.

- ITER equilibrium.



Background on giant sawtooth problem

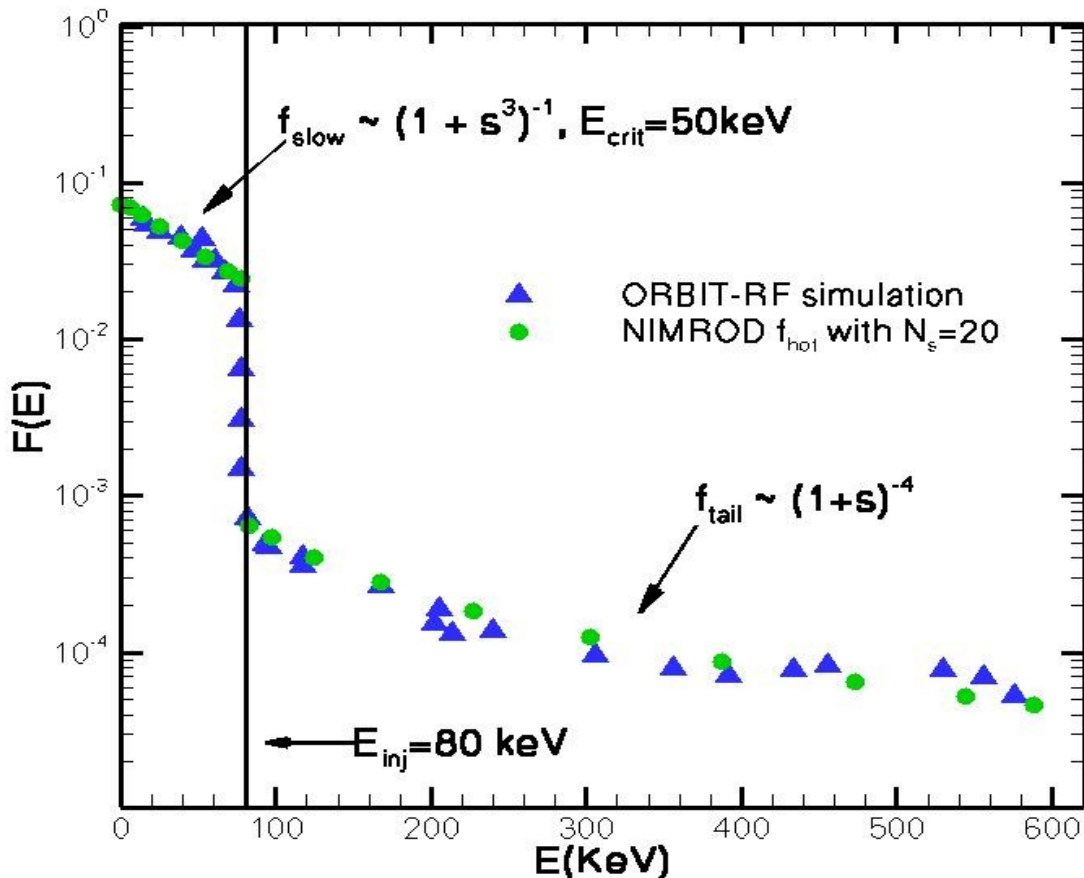
- RF-driven tail, slowing-down and thermal ions provide stabilizing effect on small sawteeth in DIII-D discharge #96043.
- Goal: use NIMROD to improve our understanding of this effect and the giant sawtooth cycle.



(Choi et al. POP, 2007).

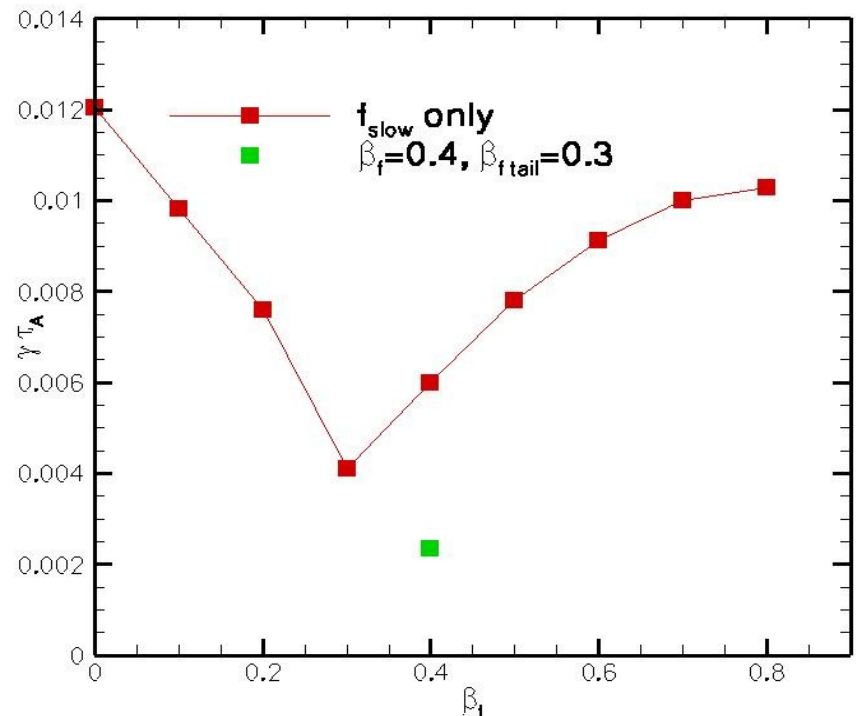
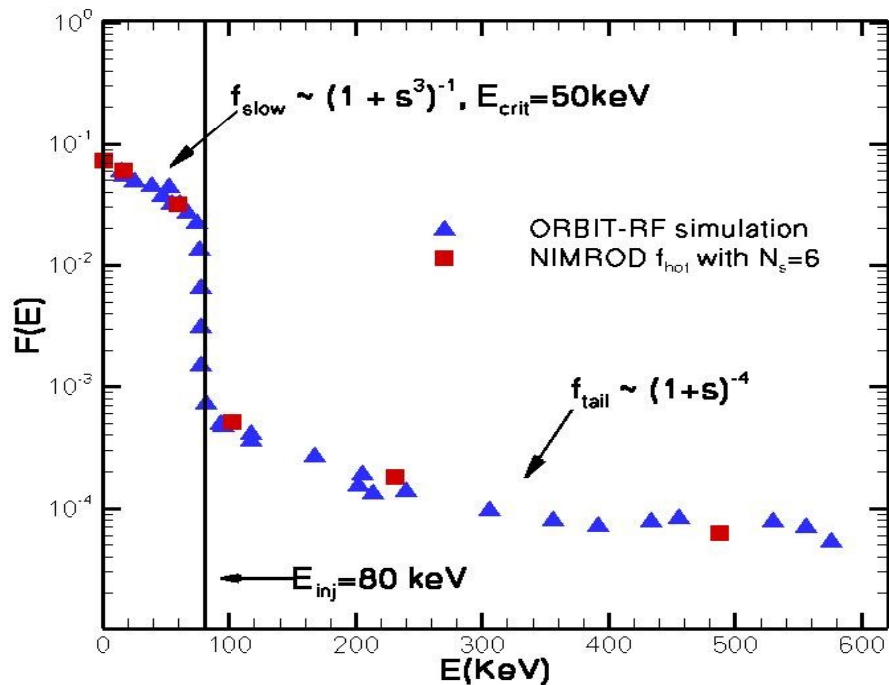
NIMROD's continuum kinetics applied to giant sawtooth.

- Match energy dependence of slowing-down and RF tail ions.
- Use as lowest-order solution to energetic ion drift kinetic equation.
- Growth rates at $t=1900\text{ms}$ insensitive to pitch-angle dependence.



Growth rate results.

- Low resolution cases (3 speed grid points) show stabilizing/destabilizing effect for slowing-down-only distribution in a hot particle β scan (red).
- RF tail provides further stabilization (green).



Is PIC compatible with NIMROD philosophy?

- PIC: Solution of the Boltzmann equation
 - Eulerian in real space, Lagrangian in velocity space
 - Fundamentally explicit
 - Has value in few ops/time step
 - Convergence in number of particles is slow, error $\sim N^{-1/2}$
 - Solutions are noisy
- NIMROD:
 - Heavily implicit
 - Many ops/time step
 - High-order finite elements
 - Convergence is fast when solutions are smooth
- Why use PIC in NIMROD?
 - Good at capturing fluid-particle resonances with small amount of velocity space resolution.

Previous efforts: Use different grids

- G-Y Fu, Parker, etc. (M3D)
 - Use uniform cartesian grid for particles
 - Interpolate fluid quantities from cartesian grid to non-uniform fluid grid
- Kim, Parker, Sovinec (NIMROD)
 - Use linear finite elements for particles in poloidal plane. Fourier modes for toroidal.
 - Bilinear FE grid used on top of NIMROD high-order grid
 - Never had Fourier parallelization working
 - Benchmarked with G-Y Fu's code (and NIMROD's continuum kinetics) for linear kink mode
- New development:
 - Use NIMROD's HO-FE for particle push (advance particles) and weight advance.
 - Retain linear elements for deposition
 - Cannot easily put noisy data within NIMROD cell with $pd > 1$
 - Mass matrix inversion projects this onto NIMROD's solution space.

Implementation details

- Implementation uses `map_mod` routine originally developed for `nimfl` and integral closure
 - Use single rblock to advance particles in global domain.
 - Toroidal parallelization comes along for free.
 - Scalability limited, but sufficient for planned usage: Double check continuum, tracer particles, etc.
- Problem: High-order FE evals are expensive
- Solutions:
 - Take all quantities to be evaluated at point and place into single vector
 - Cache certain data at field eval step in push to allow usage at the deposition step to optimize over current implementation
 - Certain other optimizations made possible by cleaning up coding

Implementation details

- Single vector includes B,J,E:

```
rb(ibl)%partvar%fs(1:3,::,:) = rb(ibl)%be%fs,  
rb(ibl)%partvar%fs(4:6,::,:) = rb(ibl)%ja%fs,  
rb(ibl)%partvar%fs(7:9,::,:) = rb(ibl)%eef%fs
```

- Evaluation of fields:

equilibrium: CALL get_field(xyp,rb_cel(1)%partvar_eq,outfld,bigr,1._r8)

perturbed: CALL get_field(xyp,,rb_cel(1)%partvar, outfld)

Does lagr_quad_eval that has nice type vectorization: only 1 lagr_1D for each direction instead of 3 for each direction:

```
CALL lagr_1D(pd,x-ix,alx,dalx,dmode)
```

```
CALL lagr_1D(pd,y-iy,aly,daly,dmode)
```

```
laq%f = (laq%fs(:,ix ,iy ,:)*alx(0) + laq%fs(:,ix+1,iy ,:)*alx(1))*aly(0)  
+ (laq%fs(:,ix ,iy+1,:)*alx(0) + laq%fs(:,ix+1,iy+1,:)*alx(1))*aly(1)
```

Plans for continuum kinetics

- Continuum energetic ions
 - quantitative verification with NIMROD's delta-f PIC (higher-order fields in push and weight advance may help)
 - explore anisotropic pressure versus current coupling
 - finish RSAE benchmark and giant sawtooth
- Continuum electrons
 - finish implementation of Ramos form for electron DKE (requires rewrite of acceleration term)
 - anisotropic conduction test problems using kinetic parallel heat flow closure in island geometry
 - applications with 3D fields: NTMs, RMPs
 - implement relativistic electron DKE
- Continuum ions
 - neoclassical toroidal viscosity studies
 - implement Ramos form for ion DKE (considerably messier than electron version)