Magnetic Fusion Application:

Center For
Extended Magnetohydrodynamic Modeling

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for the CEMM consortium

APDEC Kickoff Meeting

November 8, 2001

Lawrence Berkley Laboratory

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Outline:

1. **CEMM Objectives**

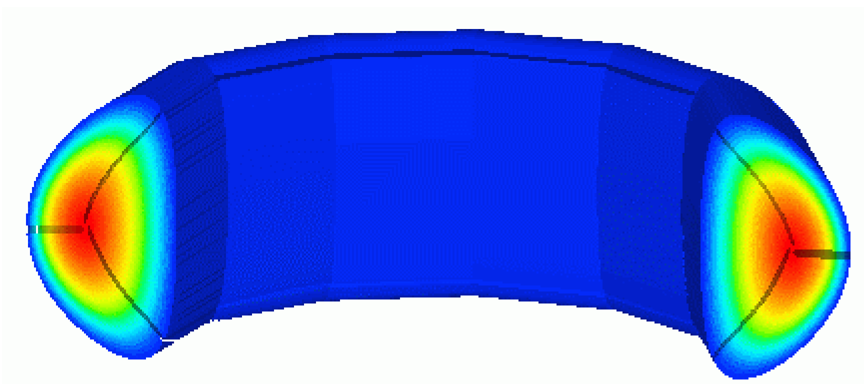
2. Typical Applications

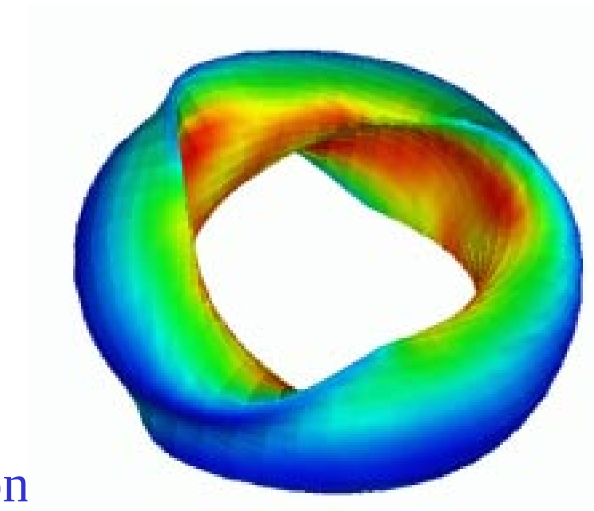
3. Equations

4. Role of APDEC Effort in CEMM

CEMM Objectives:

"…to <u>develop</u> and <u>deploy</u> predictive computational models for the study of low frequency, long wavelength fluid-like dynamics in the diverse geometries of modern magnetic fusion devices."





- Improved physics models and better resolution
- Large scale instabilities –not turbulence.
- Toroidal devices…tokamak, stellarator, FRC, RFP,…

NIMROD and **M3D** codes form basis: build on these assets

The computational challenges:

- temporal stiffness, or multiplicity of time scales,

$$\frac{\partial \vec{U}}{\partial t} + \overleftrightarrow{A} \bullet \frac{\partial \vec{U}}{\partial \vec{x}} = \ldots \quad \frac{\lambda_A^{\max}}{\lambda_A^{\min}} >> 1 \;\; (\sim 100), \qquad S = \frac{\tau_{RESISTIVE\ DIFFUSION}}{\tau_{ALFVEN\ WAVE\ TRANSIT}} >> 1 \quad (\sim 10^8)$$

- large differences in spatial scales lengths
  - internal reconnection layers develop with steep gradients
  - typical reconnection length scale $\frac{\delta}{L} \sim S^{-1/2}$
- anisotropy introduced by the strong magnetic field

$$\vec{q}_{||} \gg \vec{q}_{\perp}$$

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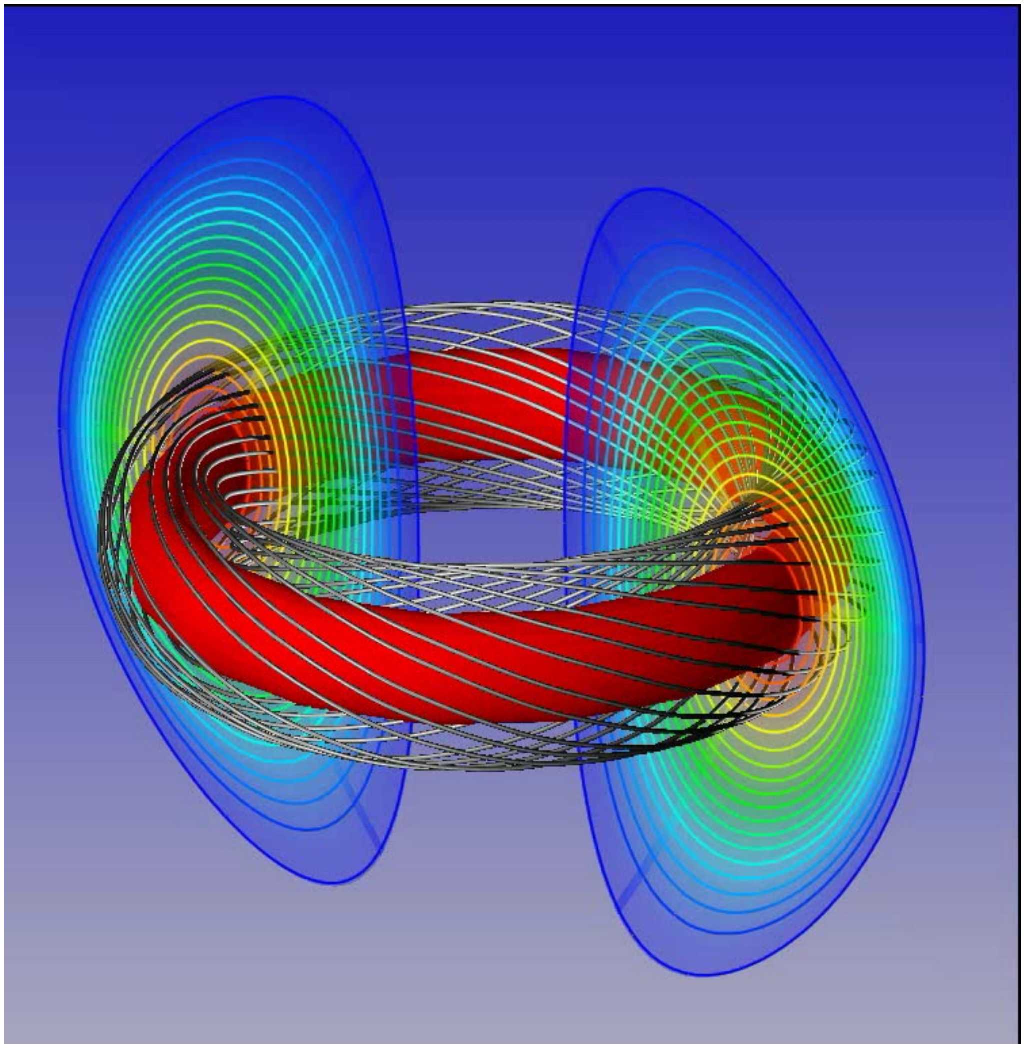
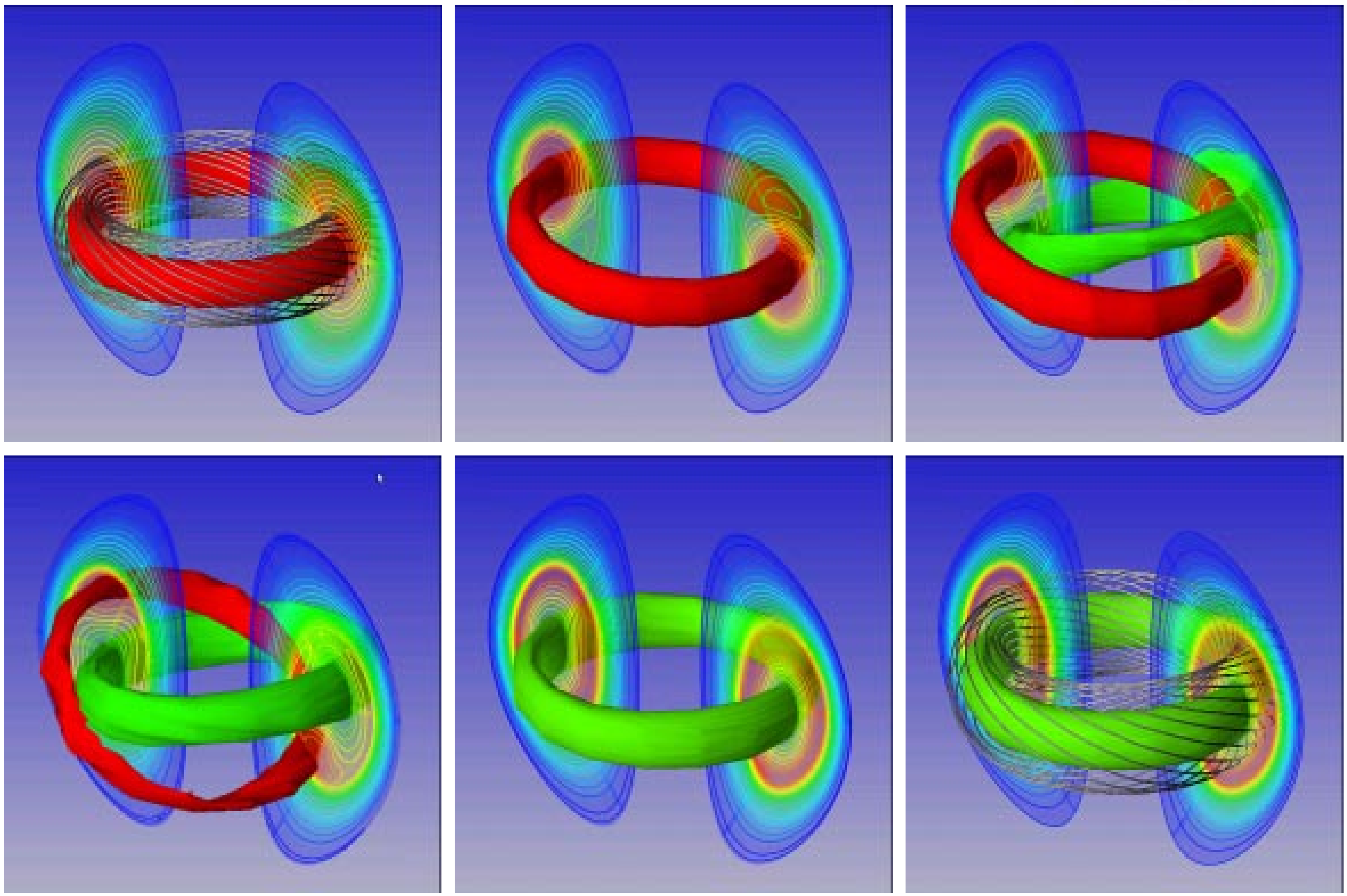
m=1 mode (sawtooth) in tokamak is high priority objective



- caused by tendency of plasma current to peak in center and become unstable
- involves reconnection layer, 2-fluid, hot-particles
- better predictive model of m=1 mode is needed for next step tokamak burning plasma
- benign self-regulating event or plasma termination?

Shown are constant pressure surfaces and some magnetic field lines

Hot inner region interchanges with colder outer region via magnetic reconnection

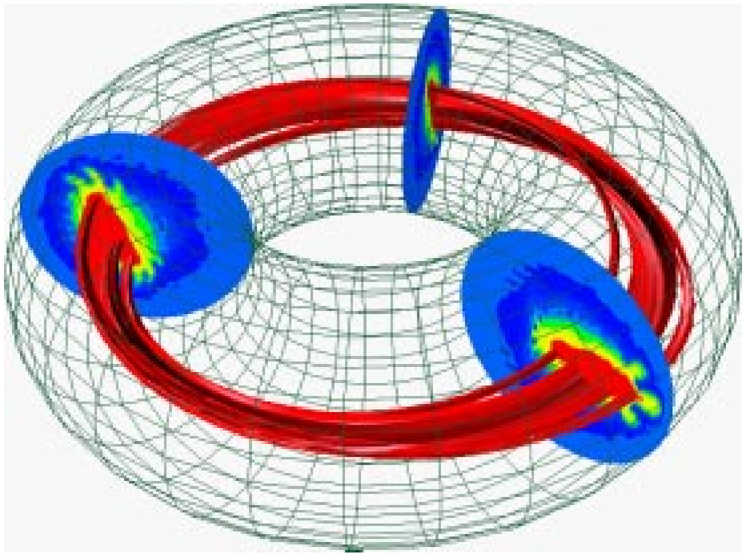


m=1 mode can also destablize short wavelength modes and lead to plasma termination

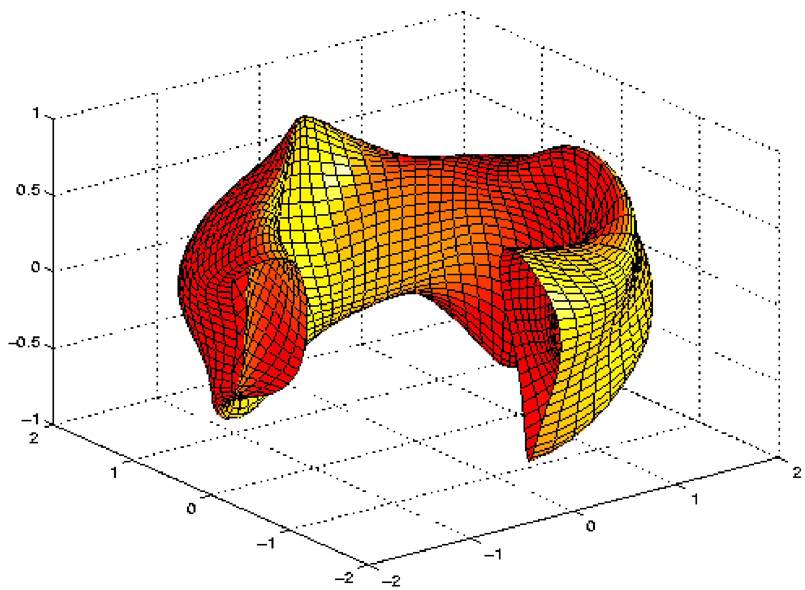
• If plasma pressure is already high and near stability limit, m=1 helical distortion can make it locally unstable to pressure-driven-modes

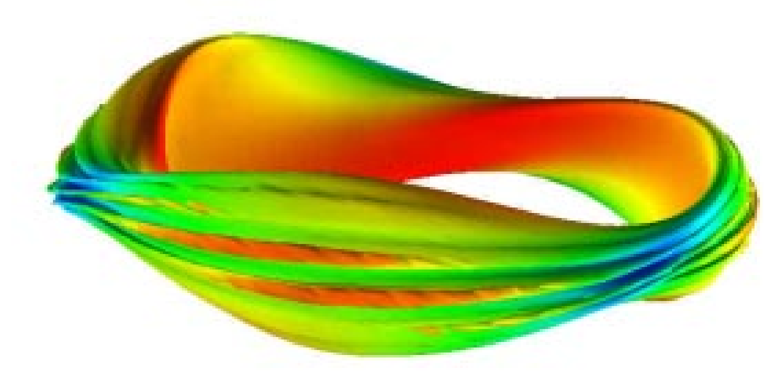
• These modes steepen nonlinearly in a ribbon like structure driving field line stochasticity and leading to plasma termination.

• The plasma termination event in the record making 10 MW fusion power DT TFTR discharge has been explained by this mechanism



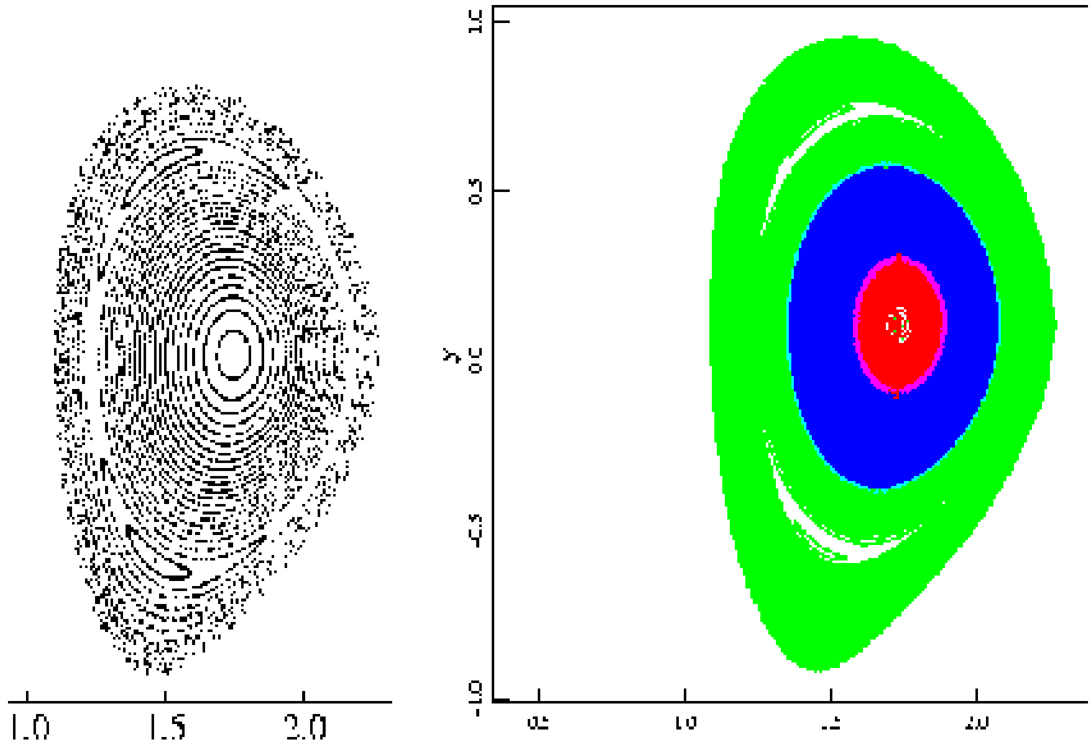
Quasi-Axisymmetric Stellarator NCSX now being designed





- **Stellarator has “twisted” outer surface formed by 3D coil set…does not need to carry net plasma current like tokamak**
- **No sawtooth modes…but instabilities can be excited when the pressure locally exceeds stability limit**
- **Instabilities cause high pressure areas to further steepen nonlinearly …consequence ?**

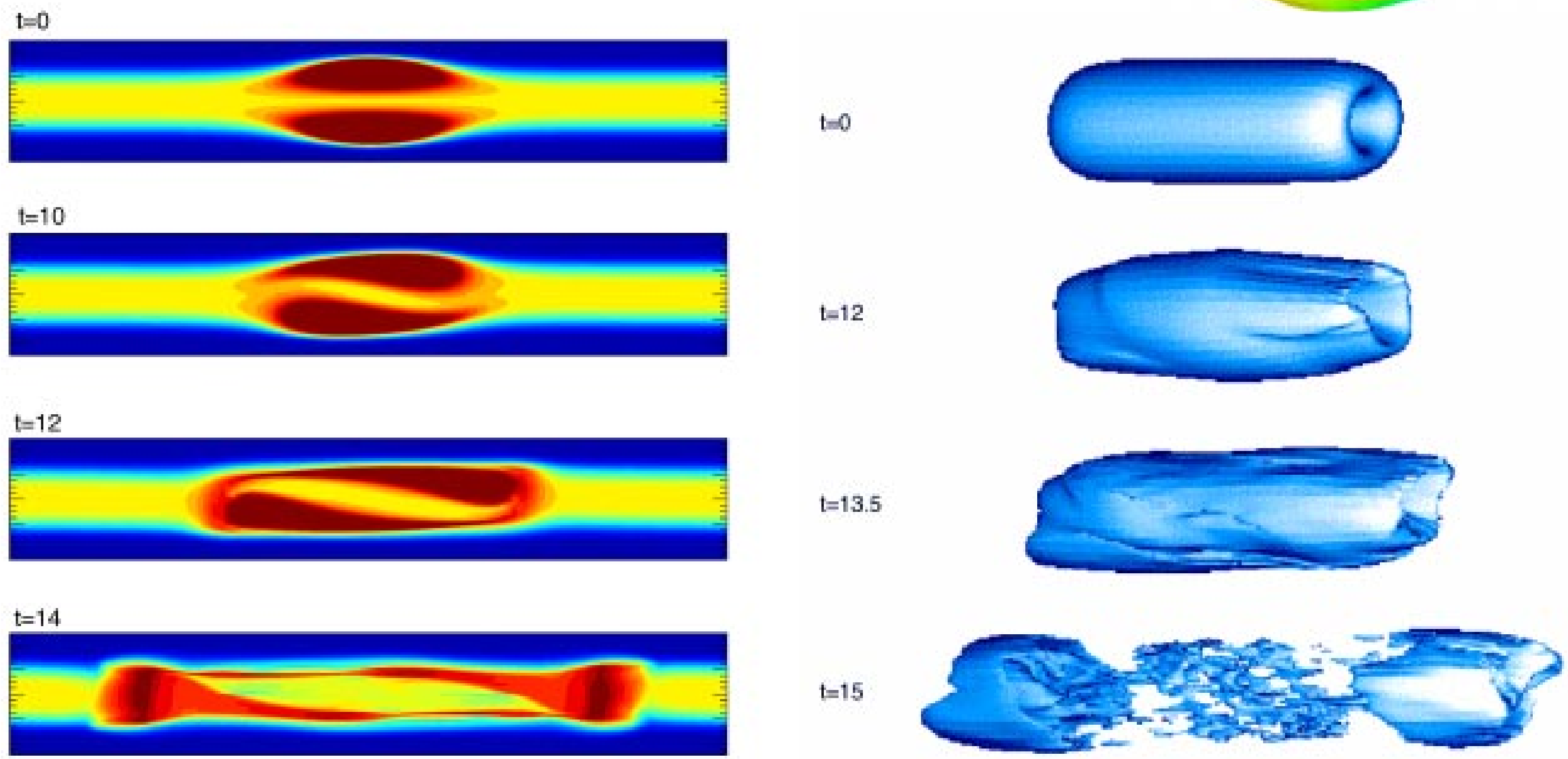
Spontaneous development of Magnetic Islands (tearing modes)



NIMROD

- "neo-classical tearing modes" driven by small differences in the plasma current-carrying capability inside the islands
- comparing results 3 different fluid closures with exp. data

Resistive MHD simulations of Field Reversed Configuration (FRC)

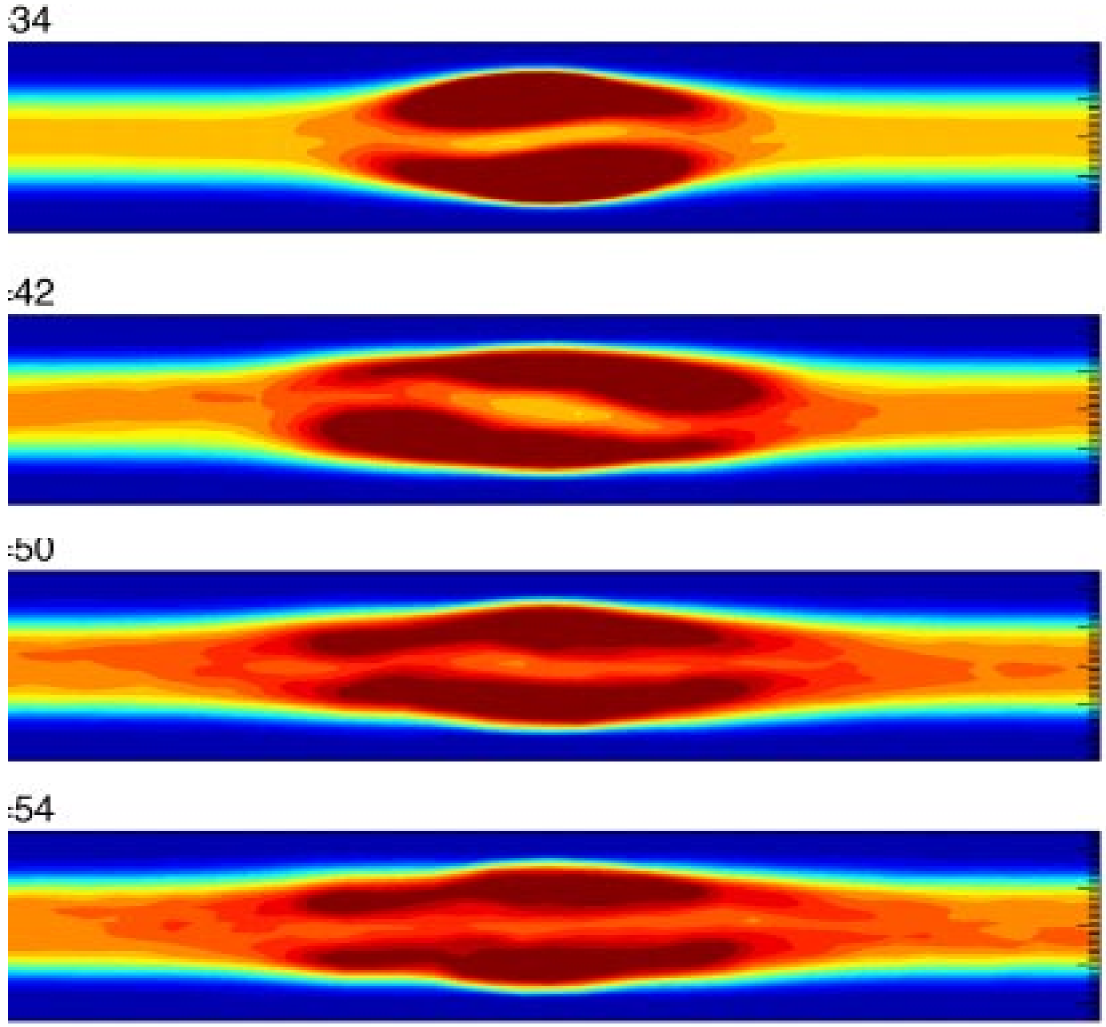


Belova

Field Reversed Configuration has no strong toroidal field.

Simple "MHD" model of FRC shows they should be <u>unstable</u>

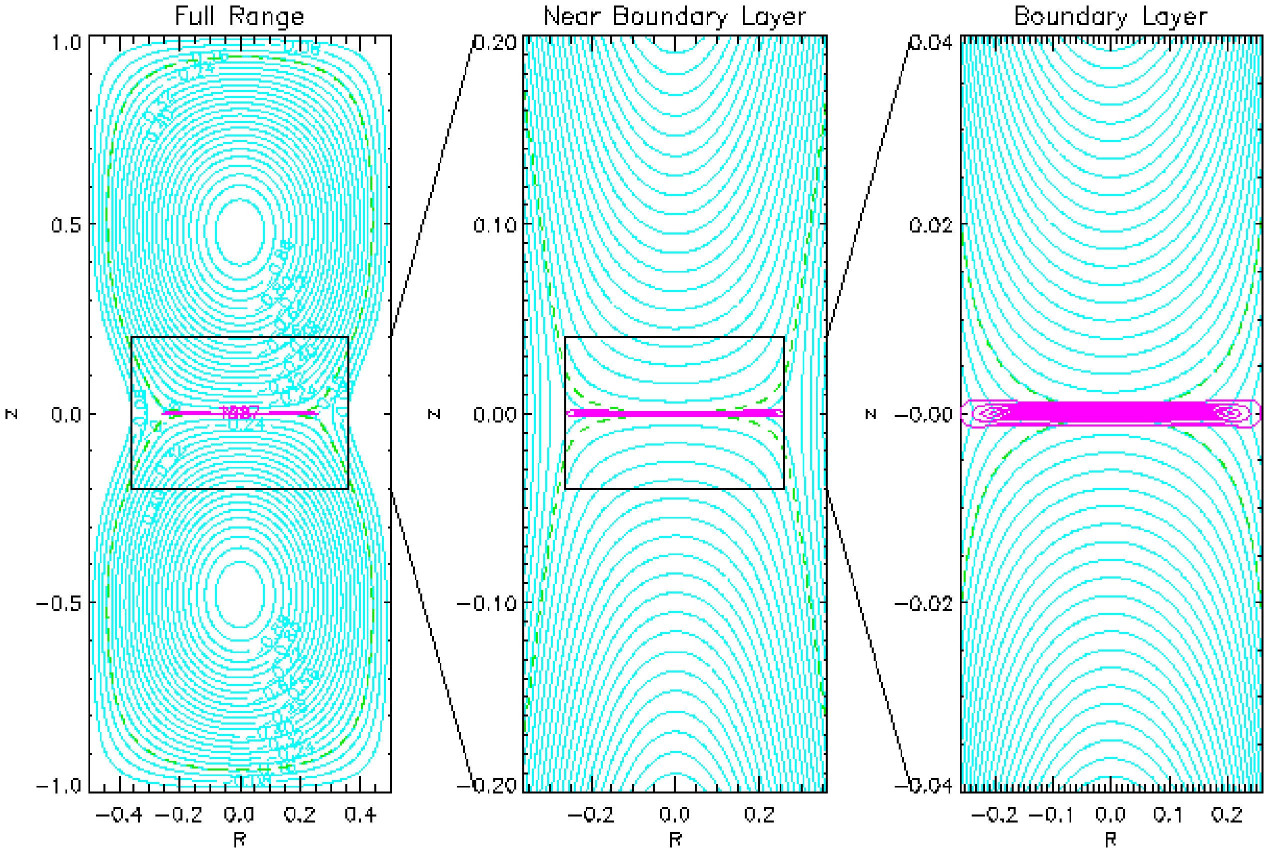
Fully kinetic-ion simulation shows much different behavior



- Simulation starts out with linear growing instability with reduced growth rate
- However, instability saturates nonlinearly!
- Shows remarkable agreement with experimental data
- Illustrates importance of correct plasma model

Model 2D problem: merging spheromaks with 2-fluid MHD equations, high-resolution

$\eta = 10^{-5}$



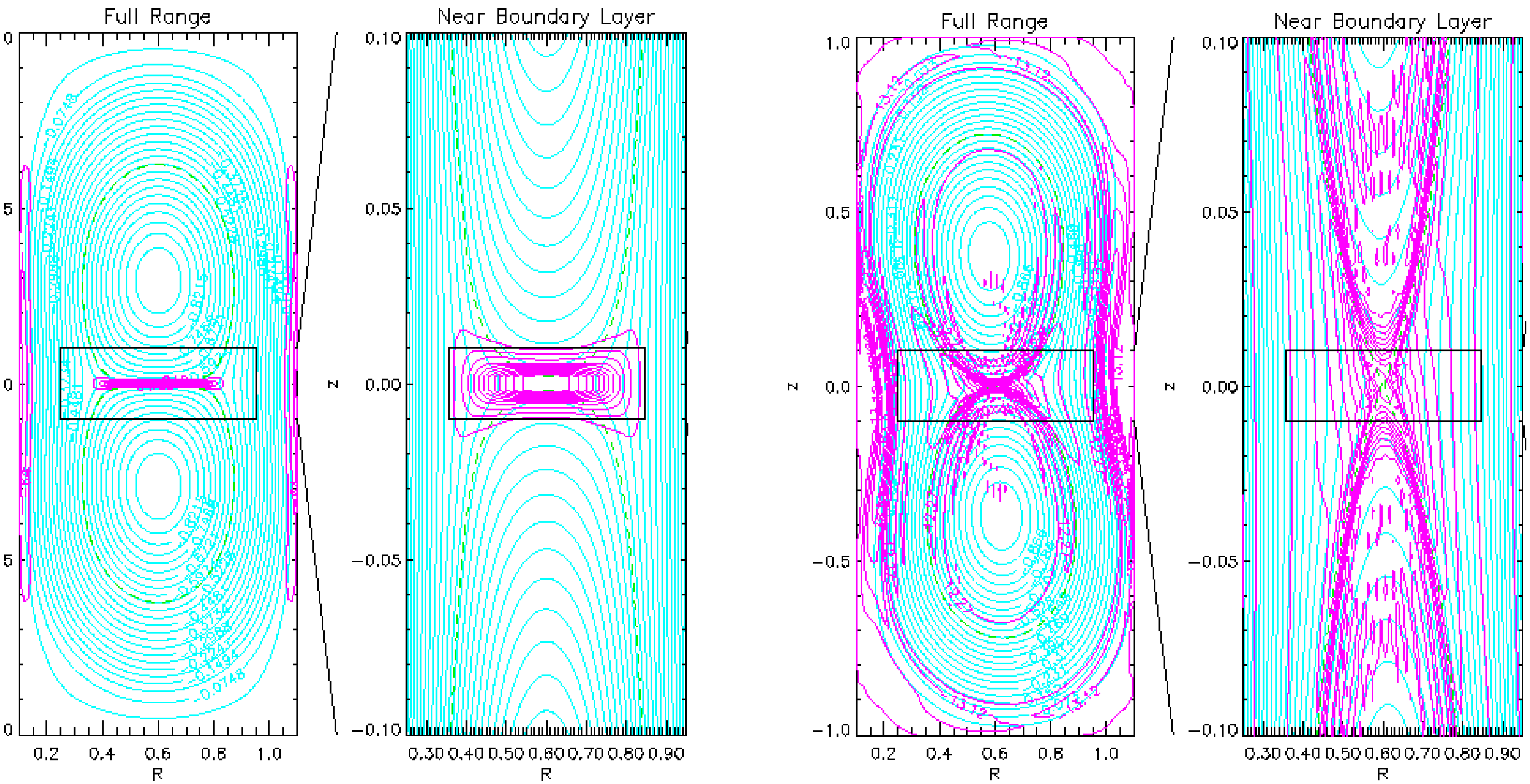
- Variable resolution grid allows resolution of disparate space scales.
- note: cyan: flux purple: current

Breslau

More complete physics (two-fluid) can change he qualitative nature of the reconnection physics

$\chi = 0$ (resistive MHD)

$\chi = 0.2$ (2-fluid MHD)



reconnection rate with 2-fluid MHD ($\chi > 0$) can increase reconnection rate by order of magnitude..or more!

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Less complex model, valid for high-collisionality, strong fields, long times

More computationally demanding. Required to describe many important but subtle phenomena.

Single Fluid Resistive MHD	Two Fluid MHD (electrons and ions)	Two Fluid MHD plus energetic gyro-particles	Gyro-particle ions and fluid electrons	Full orbit particle ions and fluid electrons
External kink modes	**Neoclassical tearing mode (including rotation)** **m =1 mode**	**MHD modes destabilized by wave-particle resonance with energetic species**	**Kinetic stabilization of internal MHD modes by ions**	**Tilting and interchange modes in FRC**

Several variations of the Extended-MHD model exist.

Plasma Models: XMHD

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$\vec{E} + \vec{V} \times \vec{B} = \eta \vec{J} + \frac{1}{ne}\left[\vec{J} \times \vec{B} - \nabla \bullet \overleftrightarrow{P}_e \right]$$

$$\mu_0 \vec{J} = \nabla \times \vec{B}$$

$$\overleftrightarrow{P} = p\overleftrightarrow{I} + \overleftrightarrow{\Pi}$$

$$\rho\left(\frac{\partial \vec{V}}{\partial t} + \vec{V} \bullet \nabla \vec{V}\right) = \nabla \bullet \overleftrightarrow{P} + \vec{J} \times \vec{B} + \mu \nabla^2 \vec{V}$$

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \vec{V}) = S_M$$

$$\frac{3}{2}\frac{\partial p}{\partial t} + \nabla \bullet \left(\vec{q} + \frac{5}{2}\overleftrightarrow{P} \bullet \vec{V} \right) = \vec{J} \bullet \vec{E} + S_E$$

$$\frac{3}{2}\frac{\partial p_e}{\partial t} + \nabla \bullet \left(\vec{q}_e + \frac{5}{2}\overleftrightarrow{P}_e \bullet \vec{V}_e \right) = \vec{J} \bullet \vec{E} + S_E$$

Two-fluid XMHD: define closure relations for $\mathbf{\Pi_i}$,$\mathbf{\Pi_e}$, $\mathbf{q_i}$, $\mathbf{q_e}$

Hybrid particle/fluid XMHD: model ions with kinetic equations, electrons either fluid or by drift-kinetic equation

Simplest 2-fluid Closure for ions and electrons

$$\rho\left(\frac{\partial \vec{V}_i}{\partial t} + \left((\vec{V}_i - \vec{V}_*)\bullet\nabla\right)\vec{V}_i\right) + \nabla P = \vec{J}\times\vec{B}$$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla\times\vec{E}$$

$$\vec{E} + \vec{V}_i\times\vec{B} = \eta\vec{J} + \frac{1}{ne}\left(\underline{\vec{J}\times\vec{B}} - \nabla p_e\right)$$

"Hall Term" in Ohm's Law brings in essential new physics in 2-fluid equations

$$\vec{J} = \nabla\times\vec{B}$$

$$\vec{V}_* \equiv \vec{B}\times\nabla p_i / enB^2$$

$$P = p_e + p_i$$

$$\nabla \times \vec{B} = \nabla P = \vec{V} = 0$$

2-fluid zero-pressure dispersion relation:

$$\left[\frac{\omega^2}{V_A^2} - (k_x^2 + k_z^2)\right]\left[\frac{\omega^2}{V_A^2} - k_z^2\right] - \frac{\omega^2}{V_A^2}\left(\frac{V_A^2}{\Omega^2}\right) k_z^2 (k_x^2 + k_z^2) = 0$$

$$\vec{B}_0 = (0, 0, B),$$
$$\vec{k} = (k_x, 0, k_z)$$

the Hall modified fast wave (+) and shear Alfven wave (-) are given by:

$$\omega^2 / V_A^2 = \frac{1}{2}\left[k_x^2 + 2k_z^2 + \frac{V_A^2}{\Omega^2} k_z^2 \left(k_x^2 + k_z^2\right)\right]$$
$$\pm \frac{1}{2}\left[k_x^4 + 2\frac{V_A^2}{\Omega^2}\left(k_x^2 + 2k_z^2\right) k_z^2 \left(k_x^2 + k_z^2\right) + \frac{V_A^4}{\Omega^4} k_z^4 \left(k_x^2 + k_z^2\right)^2\right]^{1/2}$$

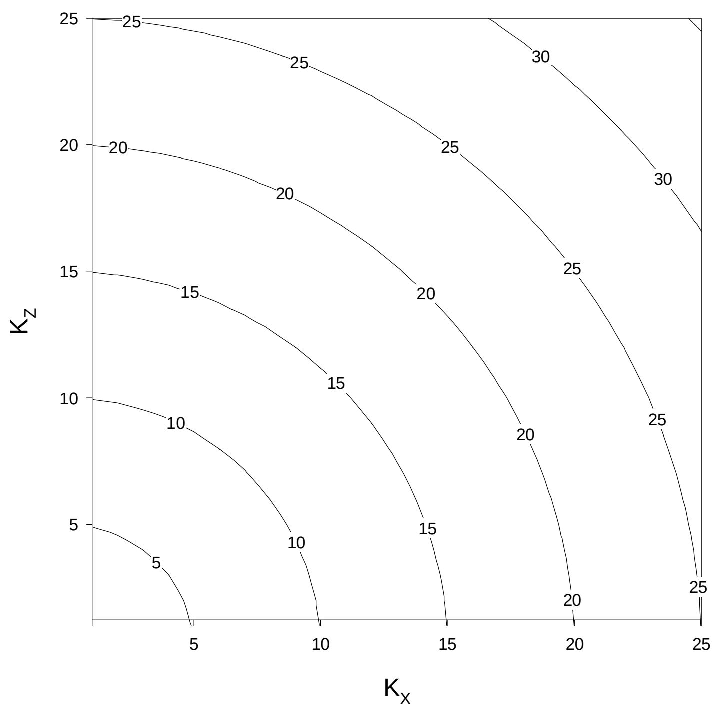
large k limit:

$$k^2 \gg \left(\frac{V_A^2}{\Omega^2}\right)^{-1}$$

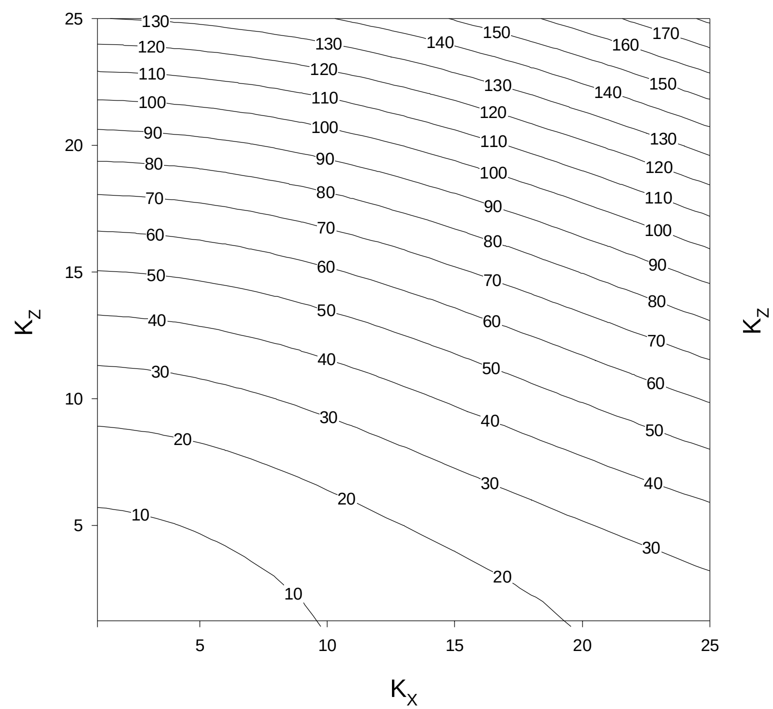
$$\frac{\omega^2}{V_A^2} \sim \left(1 + \frac{V_A^2}{\Omega^2} k_z^2\right)\left(k_x^2 + k_z^2\right) + \ldots$$ Fast wave

$$\frac{\omega^2}{V_A^2} \sim \left(\frac{V_A^2}{\Omega^2}\right)^{-1} - \left(\frac{V_A^2}{\Omega^2}\right)^{-2} \frac{\left(k_x^2 + 2k_z^2\right)}{k_z^2 \left(k_x^2 + k_z^2\right)} + \ldots$$ Shear Alfven

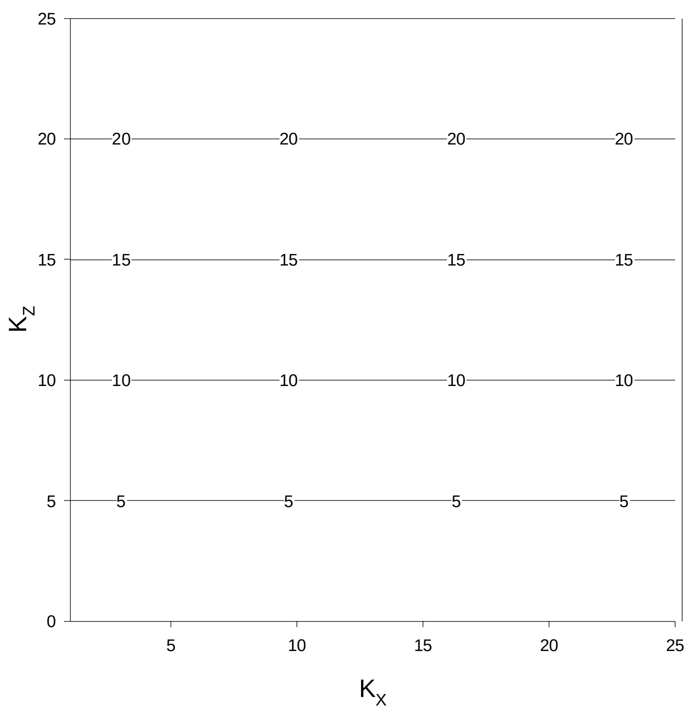
ω^2/V_A^2 for Fast Wave with $(V_A^2/\Omega^2=0)$



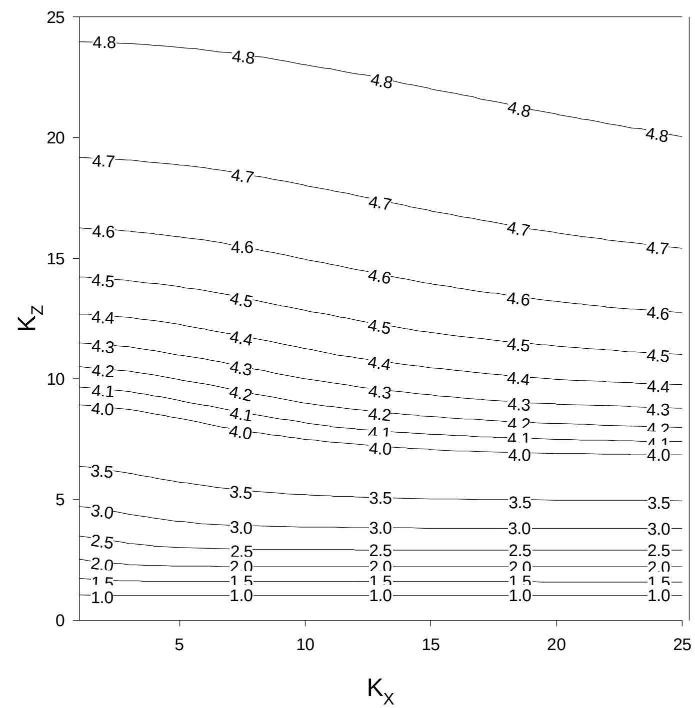
ω^2/V_A^2 for Hall modified Fast Wave with $(V_A^2/\Omega^2=0.04)$



ω^2/V_A^2 for Alfven Wave with $(V_A^2/\Omega^2=0)$



ω^2/V_A^2 for Hall modified Alfven Wave with $(V_A^2/\Omega^2=,04)$



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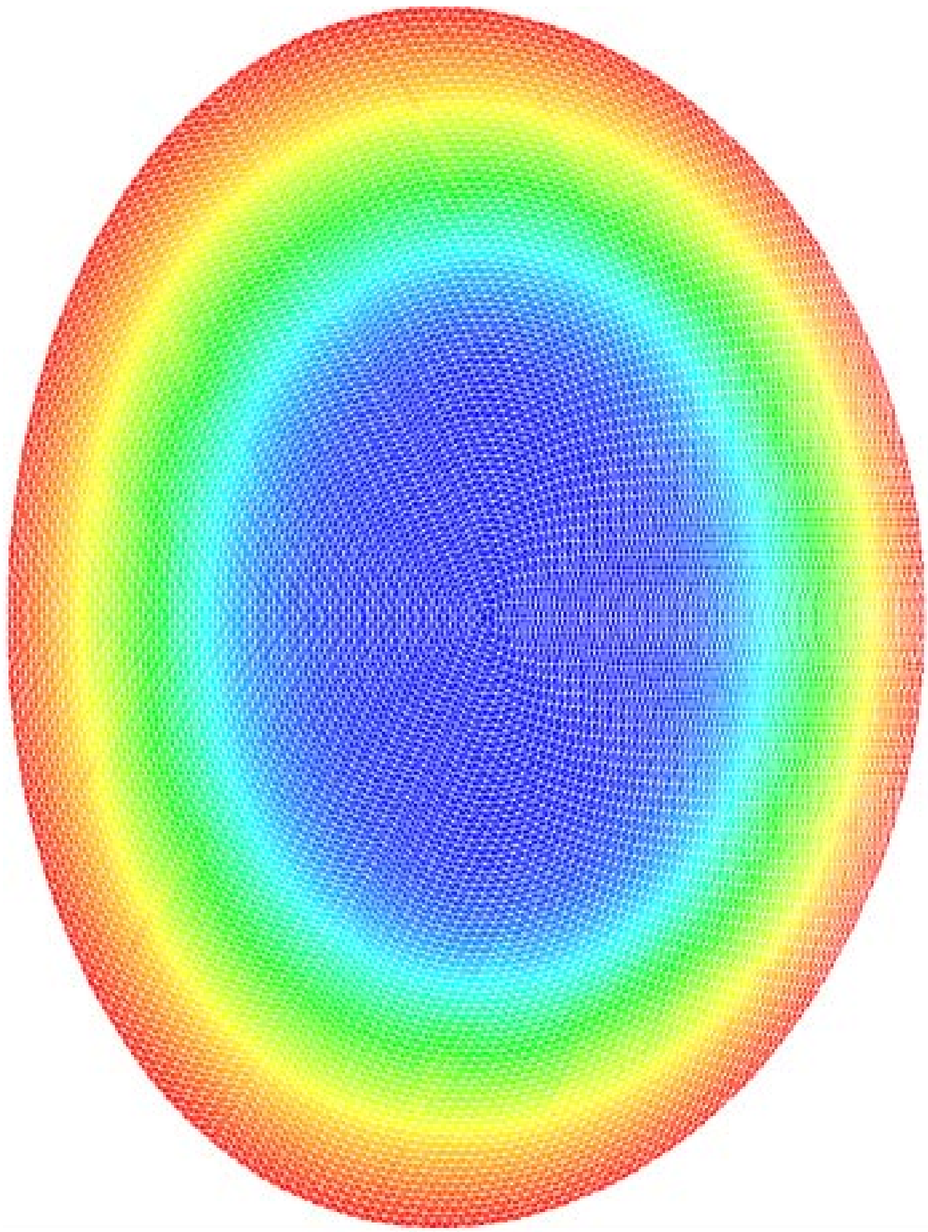
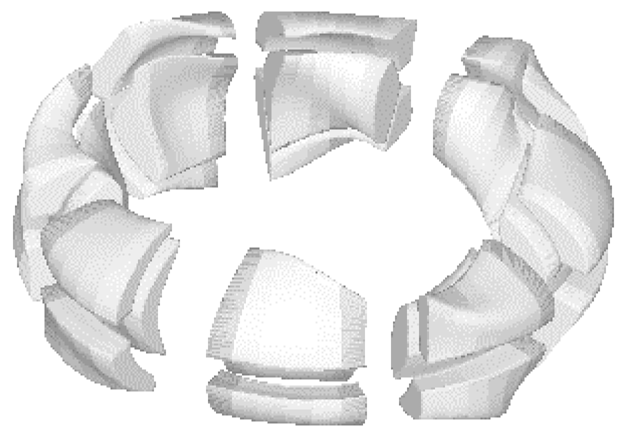


Typical M3D Mesh in Poloidal Plane

- Unstructured
- Not adaptive



Relation of APDEC Activity to Baseline

M3D

- quasi-implicit (Krylov)
- stream function/ potential
- triangular finite elements in poloidal plane
- domain decomposition in poloidal plane using MPI
- Finite difference in toroidal direction
- scales good on 256-512 processors on T3E & SP2
- resistive MHD, two-fluid (Hall term) & hybrid/particles
- uses PETSC framework

NIMROD:

- strongly implicit (Krylov)
- uses B and V
- triangular and quad finite elements in poloidal plane
- domain decompositon in poloidal plane using MPI
- pseudo-spectral (FFT) in toroidal direction
- scales good on 256-512 processors on T3E & SP2
- resistive MHD, two-fluid (Hall term) & hybrid/particles

APDEC Activity:

- adaptive mesh
- structured mesh with embedded boundary
- evaluate generalized upwind FD methods

Must eventually dea with

- partially implicit solve
- Hall term in Ohm's la
- Anisotropic heat conduction
- hybrid particle/fluid description
- must interface with existing code(s)

Summary:

1. **CEMM Objectives**

   \- improved physics models, better resolution

2. **Typical Applications**

   \- global instabilities with localized reconnection layers:

3. **Equations**

   \- “two-fluid” (Hall MHD) and Hybrid

4. **Role of APDEC Effort in CEMM**

   - Initial role is to demonstrate new techniques
   - Next phase is to incorporate in M3D and/or NIMROD