STUDIES OF NONLINEAR RESISTIVE AND EXTENDED MHD IN ADVANCED TOKAMAKS USING THE NIMROD CODE

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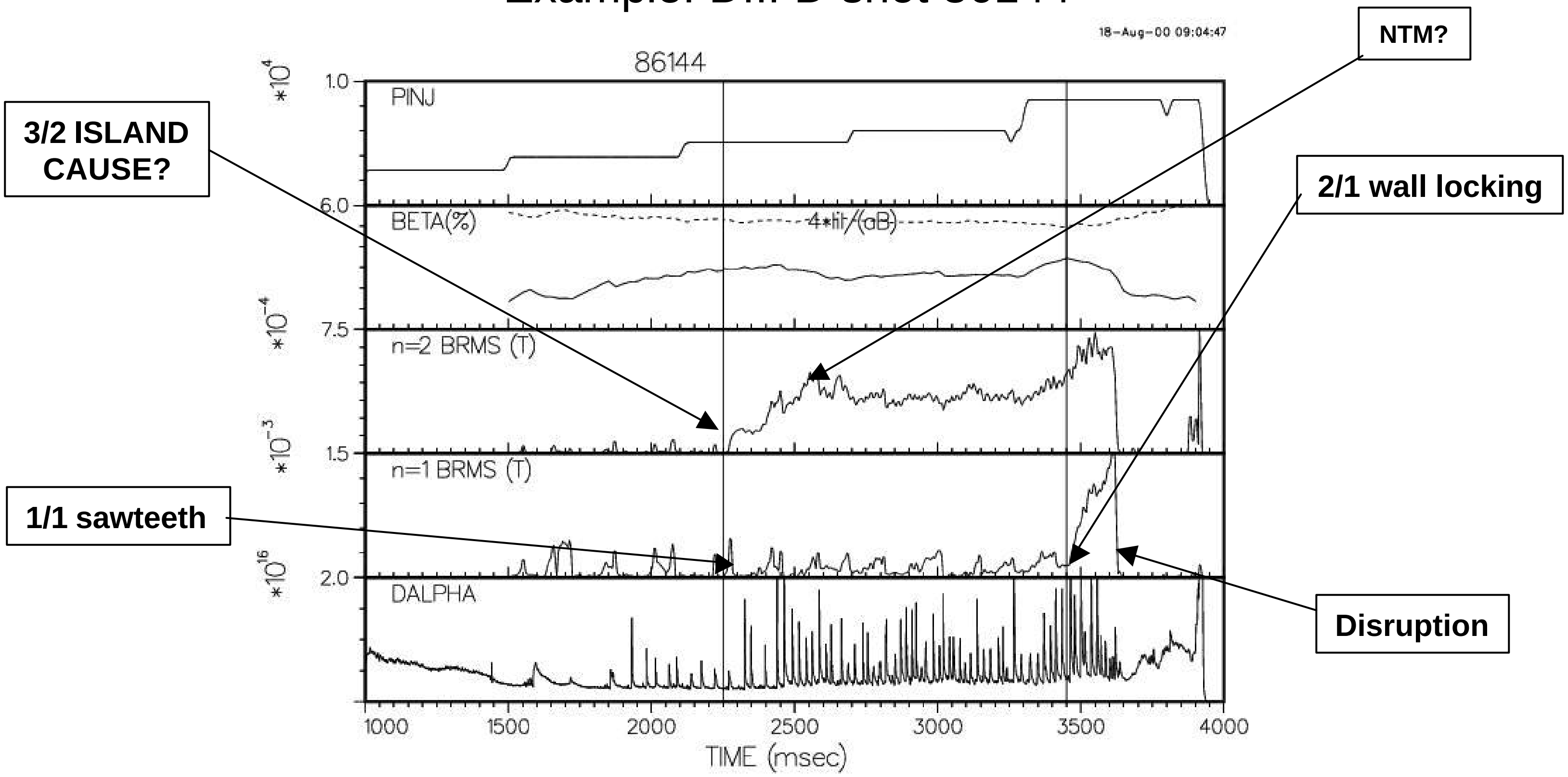
MODERN TOKAMAKS ARE RICH IN MHD ACTIVITY

Example: DIII-D shot 86144



- What is special about 2250 msec?
- What is the nature of the initial 3/2 island?
- Can this behavior be understood?
- Can this behavior be predicted?

MODELING REQUIREMENTS

- **Slow evolution, finite amplitude magnetic fluctuations**

  Nonlinear, multidimensional, electromagnetic <u>fluid</u> model required

- **Plasma shaping**

  Realistic geometry required

- **High temperature**

  Realistic *S* required

- **Low collisionality**

  Extensions to resistive MHD required

- **Strong magnetic field**

  Highly anisotropic transport required

- **Resistive wall**

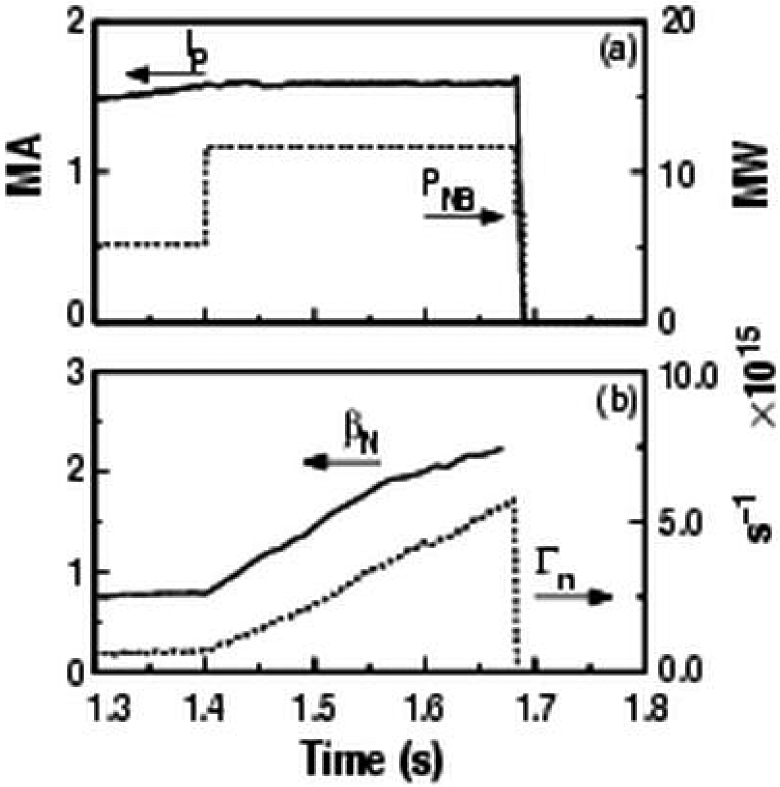
  Non-ideal boundary conditions required

NIMROD APPLIED TO DIII-D DISCHARGES

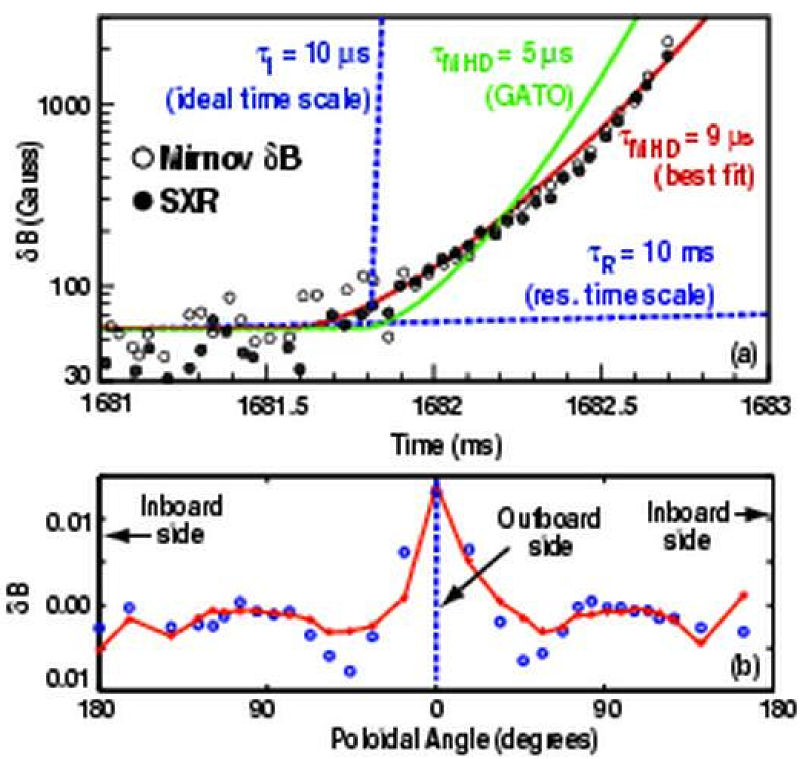
- **Gain understanding of dynamics of modern tokamak**
- **Validate code by benchmarking with experimental data**
- ***Shot 87009***
  - **Highly shaped plasma**
  - **Disruption when heated through b limit**
    - **Why is growth faster than simple exponential?**
    - **What causes disruption?**
  - **Test of nonlinear resistive MHD**
- ***Shot 86144***
  - **ITER-like discharge**
  - **Sawteeth**
    - **Nonlinear generation of secondary islands**
    - **Destabilization of NTM?**
  - **Tests both resistive MHD and closure models for Extended MHD**

DIII-D SHOT #87009

- High-b disruption when heated slowly through critical b_N



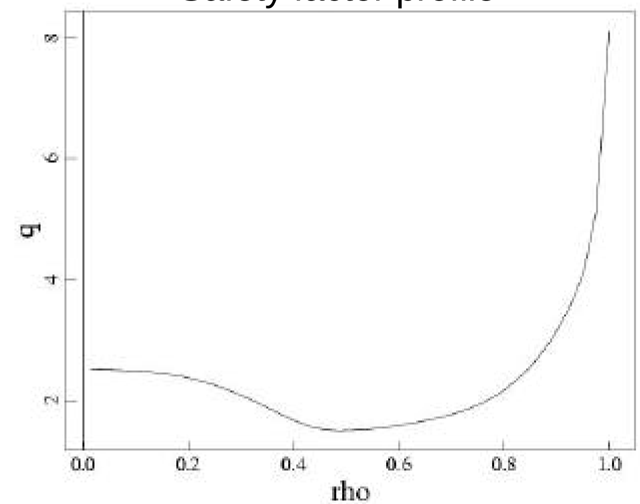
- Growth is faster than simple exponential



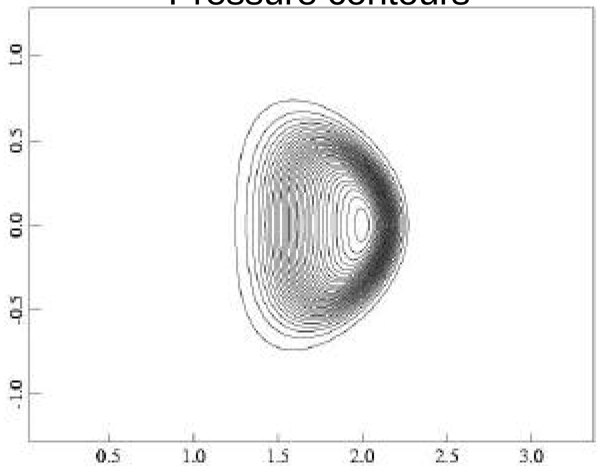
EQUILIBRIUM AT *t* = 1681.7 msec

- **Equilibrium reconstruction from experimental data**
- **Negative central shear**
- **Gridding based on equilibrium flux surfaces**
  - **Packed at rational surfaces**
  - **Bi-cubic finite elements**

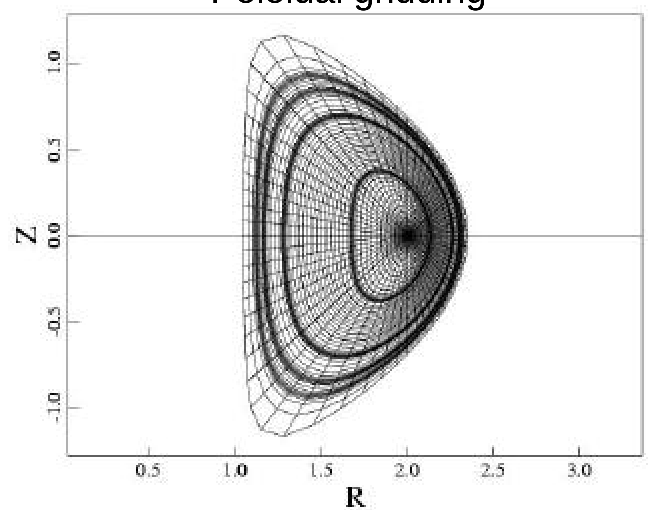
Safety factor profile



Pressure contours



Poloidal gridding



THEORY OF SUPER-EXPONENTIAL GROWTH

- **In experiment mode grows faster than exponential**
- **Theory of ideal growth in response to slow heating (*Callen, Hegna, Rice, Strait, and Turnbull, Phys. Plasmas 6, 2963 (1999)*):**

Heat slowly through critical b: $b = b_c(1+g_h t)$

Ideal MHD: $w^2 = -\hat{g}^2_{MHD}(b/b_c - 1)$ ➔ $g(t) = \hat{g}_{MHD}\sqrt{g_h t}$

Perturbation growth:

$$\frac{dx}{dt} = g(t)x \quad ➔ \quad x = x_0 \exp[(t/t)^{3/2}], \qquad t = (3/2)^{2/3}\hat{g}_{MHD}^{-2/3} g_h^{-1/3}$$

- **Good agreement with experimental data**

NONLINEAR SIMULATION WITH NIMROD

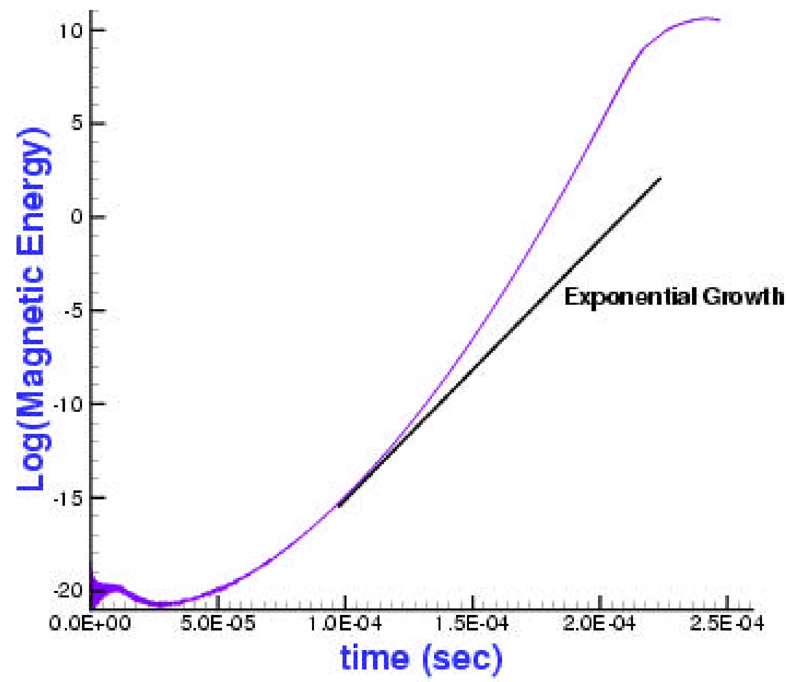
- **Initial condition: equilibrium below ideal marginal b_N**
- **Use resistive MHD**
- **Impose heating source proportional to equilibrium pressure profile**

$$\frac{\P P}{\P t} = \quad ..... \quad + g_H P_{eq}$$

$$\Rightarrow \qquad b_N = b_{Nc}(1 + g_H t)$$

- **Follow nonlinear evolution through heating, destabilization, and saturation**

Log of magnetic energy in *n* = 1 mode vs. time
$S = 10^6$ Pr = 200 $g_H = 10^3$ sec^{-1}



SCALING WITH HEATING RATE

- **NIMROD simulations also display super-exponential growth**
- **Simulation results with different heating rates are well fit by $x \sim \exp[(t-t_0)/t]^{3/2}$**
- **Time constant scales as**

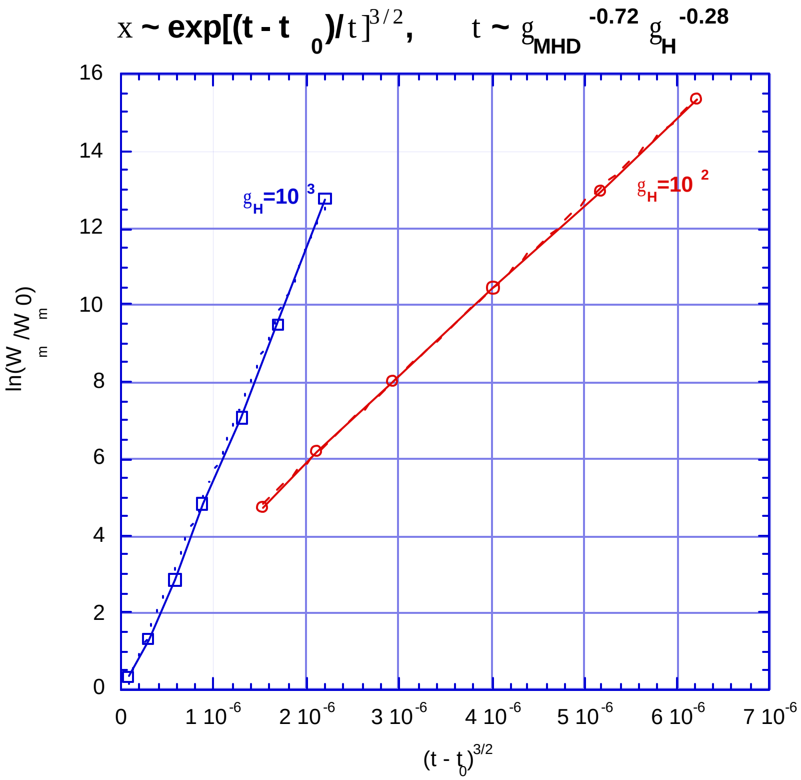
$$t \sim g_{MHD}^{-0.72} g_H^{-0.28}$$

- **Compare with theory:**

$$t = (3/2)^{2/3} \hat{g}_{MHD}^{-2/3} g_h^{-1/3}$$

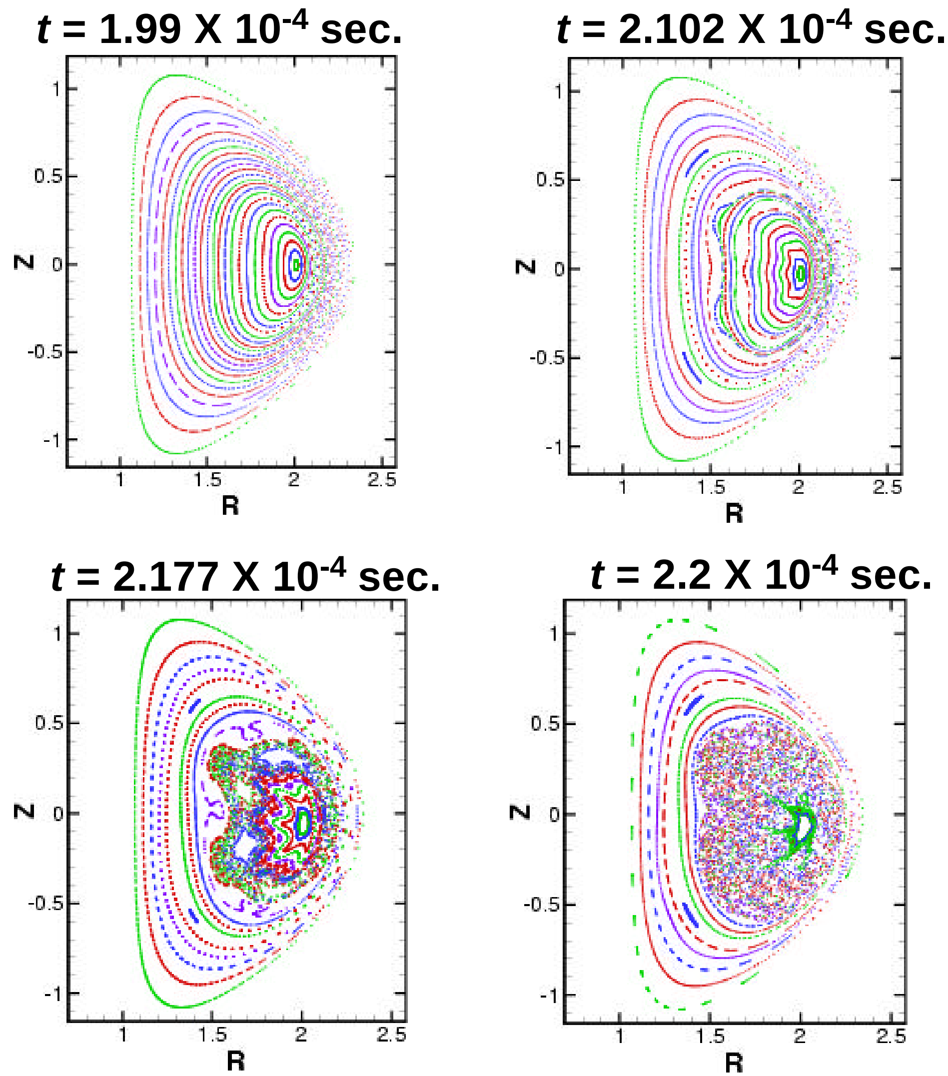
- **Discrepancy possibly due to non-ideal effects**

Log of magnetic energy vs. $(t - t_0)^{3/2}$ for 2 different heating rates

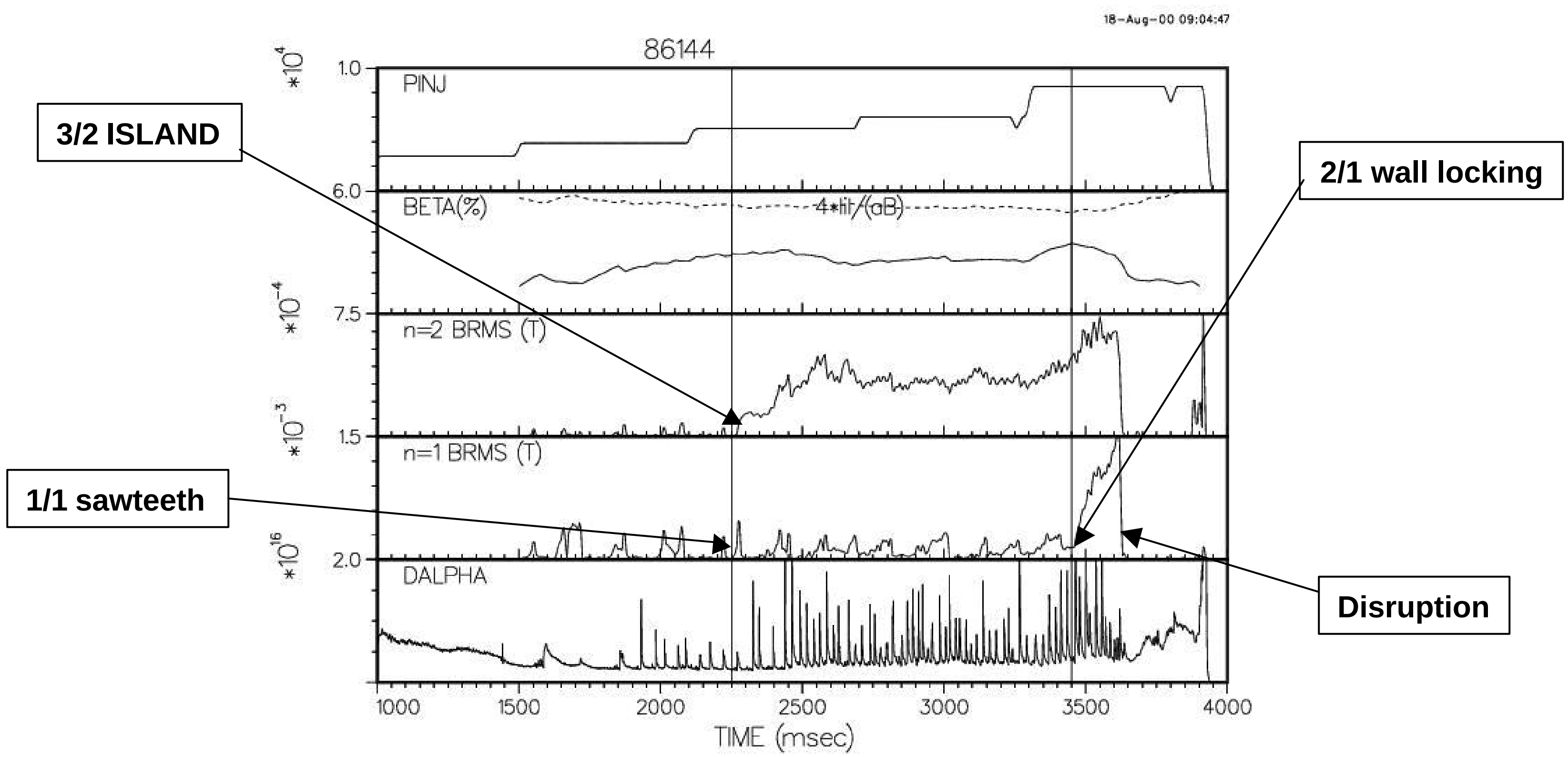


EVOLUTION OF MAGNETIC FIELD LINES

- **Simulation with small but finite resistivity**
- **Ideal mode yields stochastic field lines in late nonlinear stage**
- **Implications for degraded confinement**
- **Disruption?**



DIII-D SHOT #86144



- What is special about 2250 msec?
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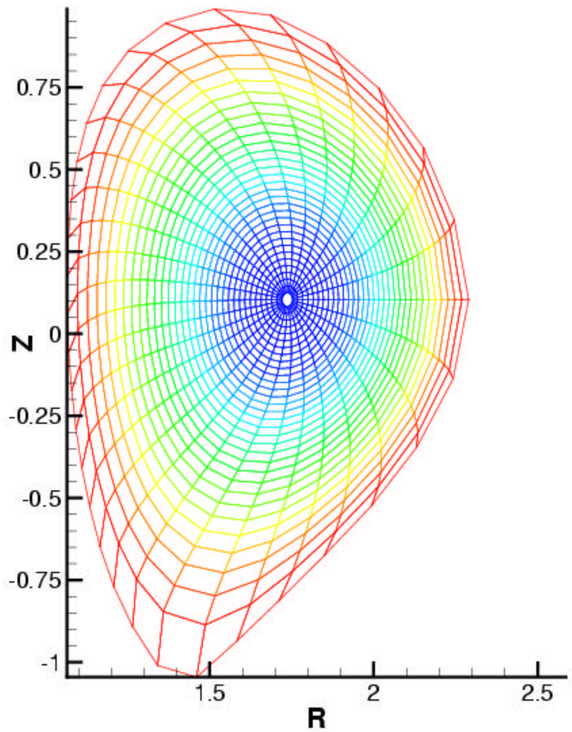
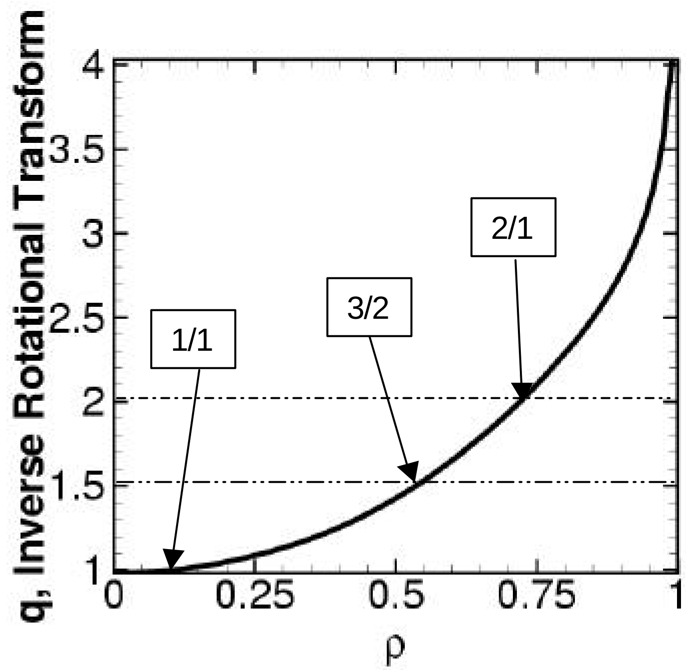
EQUILIBRIUM AT *t* = 2250 msec

Grid (Flux Surfaces)

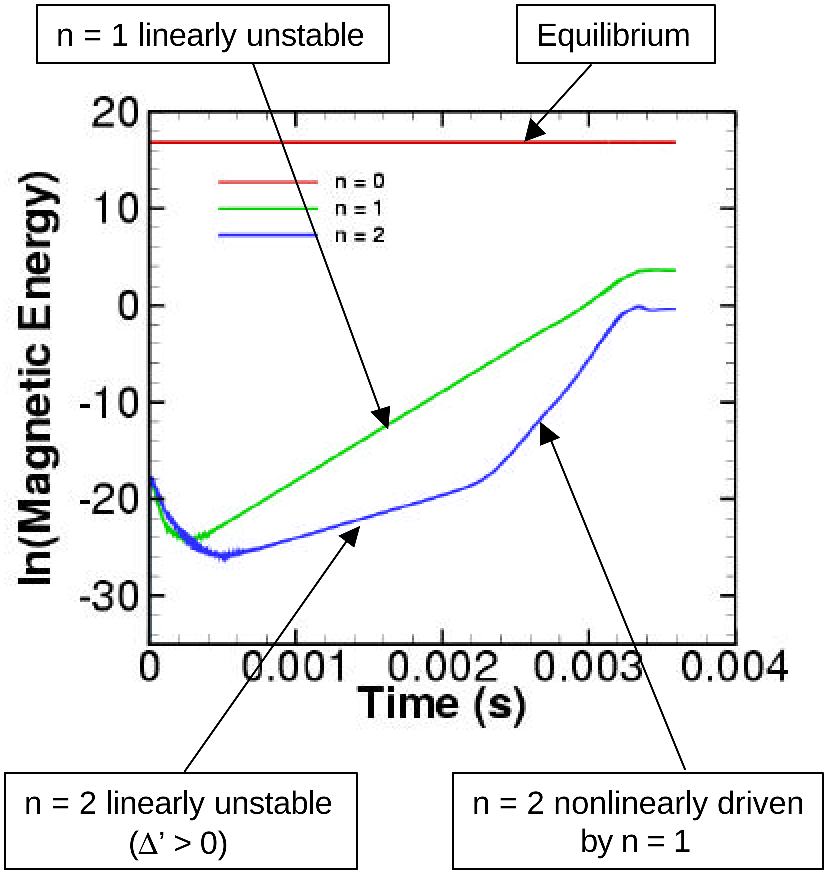


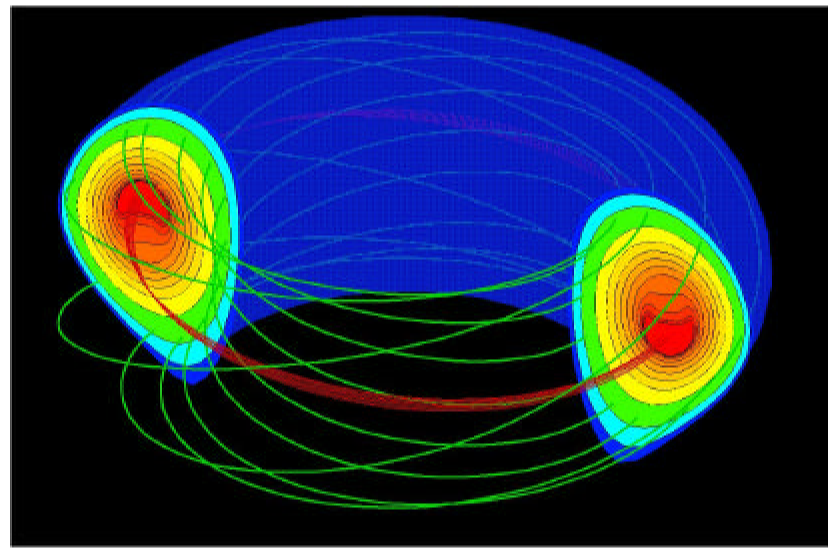
q - profile



- ITER-like discharge
- *q*(0) slightly below 1

DISCHARGE IS UNSTABLE TO RESISTIVE MHD



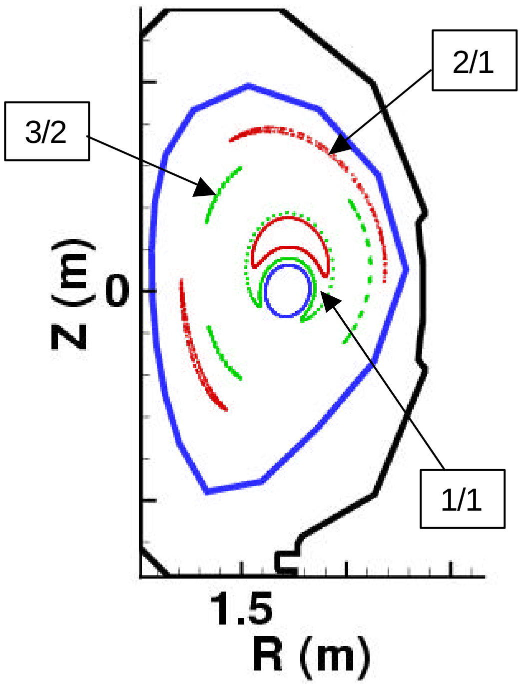


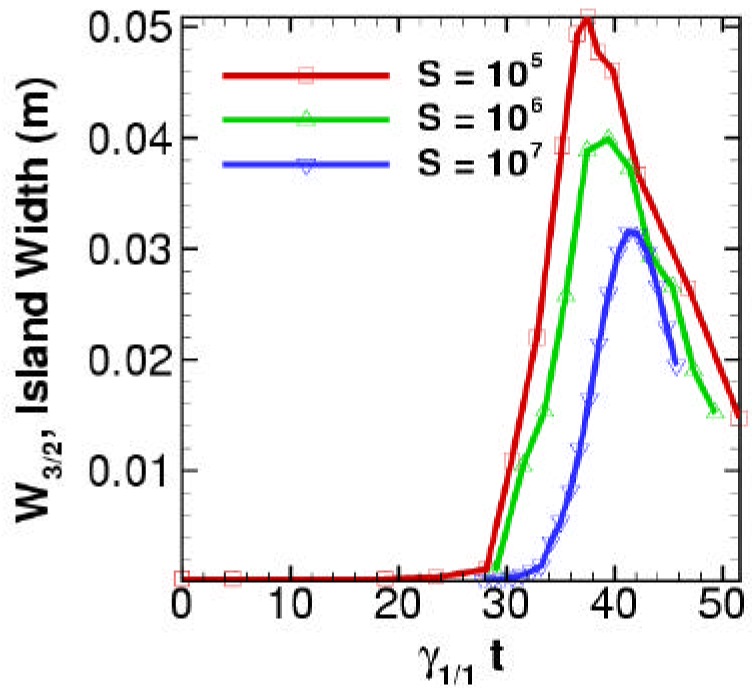
Pressure and field lines

$$S = 10^7 \quad \mathrm{Pr} = 10^3$$

$$g = 4.58 \times 10^3 \text{ / sec} \quad g_{exp} \sim 1.68 \times 10^4 \text{ / sec}$$

SECONDARY ISLANDS IN RESISTIVE MHD





- Secondary islands are small in resistive MHD
    - —W_{exp} ~ 0.06 - 0.1 m
- 3/2 island width decreases with increasing *S*
- Need extended MHD to match experiment?

NUMERICALLY TRACTABLE CLOSURES

- **Resistive MHD is insufficient to explain DIII-D shot 86144**
  - **3/2 magnetic island is too small**
- **Parallel variation of *B* leads to trapped particle effects**
- **Particle trapping causes neo-classical effects**
  - **Poloidal flow damping**
  - **Enhancement of polarization current**
  - **Bootstrap current**
- **Simplified model captures most neo-classical effects**

  (T. A. Gianakon, S. E. Kruger, C. C. Hegna, Phys. Plasmas (to appear) (2002))

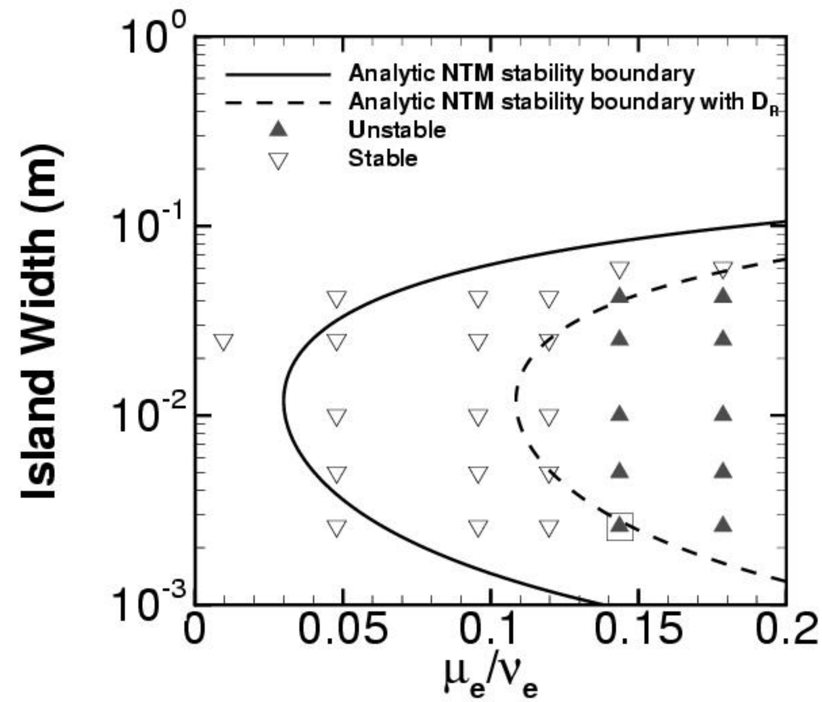
$$\nabla \cdot \Pi_a = m_a n_a m_a \langle B_0 \rangle^2 \frac{\mathbf{v}_a \cdot \nabla q}{(\mathbf{B}_0 \cdot \nabla q)^2} \nabla q$$

- **For electrons, ideal MHD equilibrium yields bootstrap current**

$$\nabla \cdot \Pi_e = -\frac{r_e m_e}{ne} \frac{\langle B \rangle^2}{B^2} \frac{\mathbf{B}_0 \times \nabla p \cdot \nabla q}{(\mathbf{B}_0 \cdot \nabla q)^2} \nabla q$$

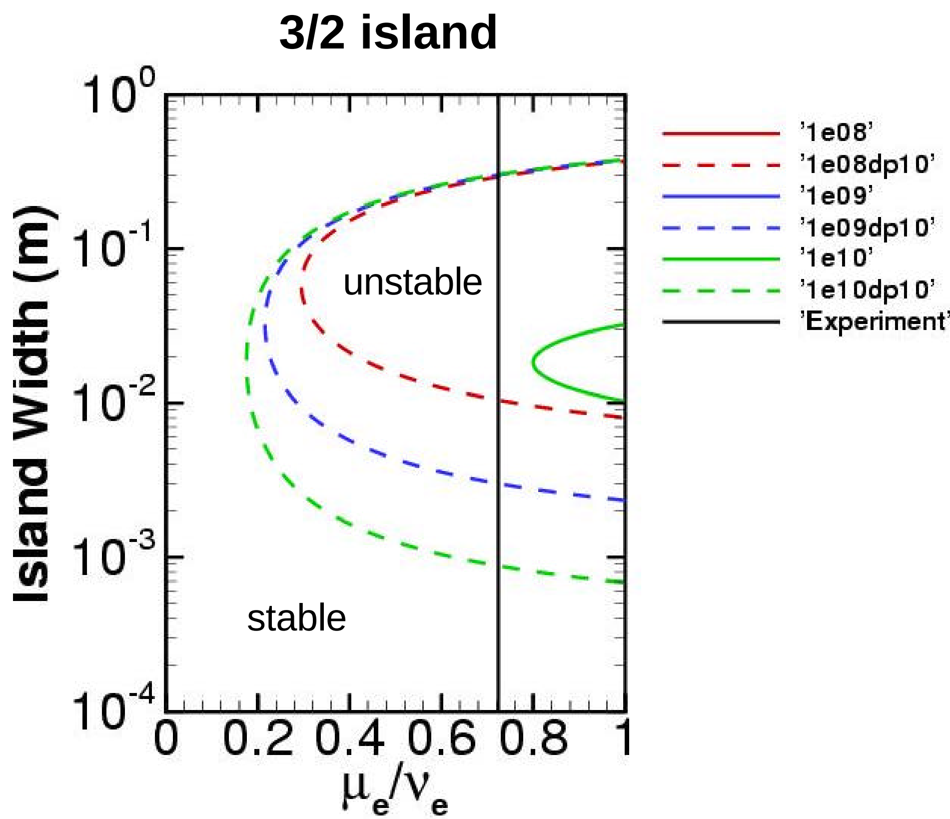
CLOSURES REPRODUCE NTM INSTABILITY

- TFTR-like equilibrium
- Comparison with modified Rutherford equation
- Initialize NIMROD with various seed island sizes
- Look for growth or damping
- Seek self-consistent seed and growth



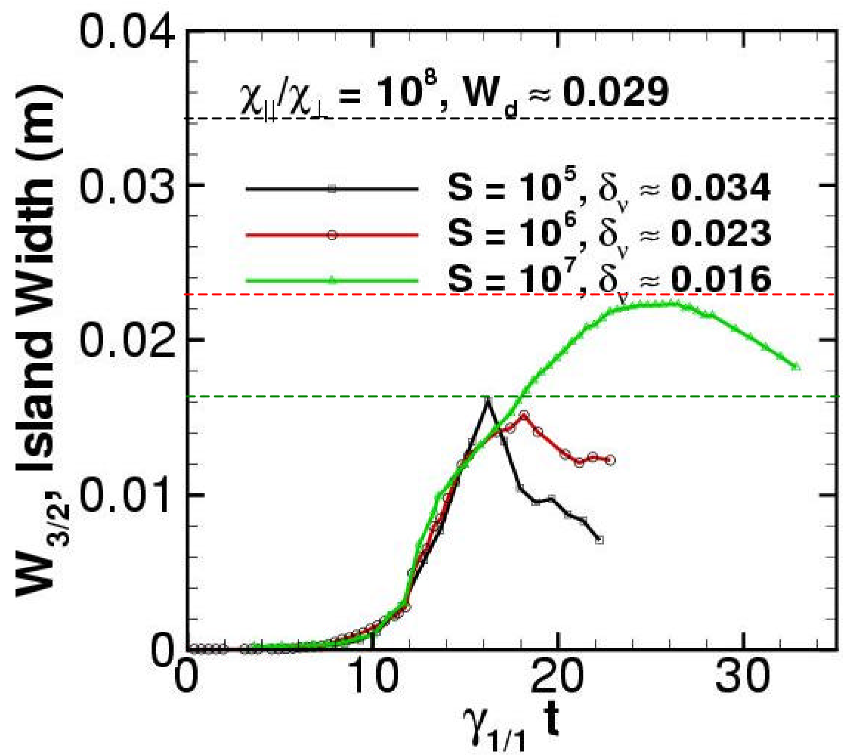
86144.2250 NTM STABILITY BOUNDARIES

- **Use modified Rutherford equation**
- **3 values of anisotropic heat flux**
- **2 values of D¢**
  - **Vacuum**
  - **Reduced by factor of 10**
- **Experimental island width ~ 0.06 - 0.1 m**



SELF-CONSISTENT NTM MAY REQUIRE HIGHER *S*

- **Nonlinear simulation with NIMROD code**
- **Look for 3/2 neoclassical mode driven by 1/1 sawtooth**
- **Use PFD (analytic) closure**
- **Threshold island width ~ 2-4 cm (uncertainty in D¢)**
- **$W_{3/2}$ ~ 6 - 10 cm in experiment**
- **Still need larger *S*, more anisotropy**



Cannot cheat on parameters!!!

Nonlinear NTM calculations are extremely challenging!

SUMMARY

- **Nonlinear modeling of experimental discharges is possible, but extremely challenging**
- ***DIII-D shot #87009***
  - **Heating through b limit**
  - **Super-exponential growth, in agreement with experiment and theory**
  - **Nonlinear state leads to stochastic fields**
  - **Calculations with anisotropic thermal transport underway**
- ***DIII-D shot #86144***
  - **Secondary islands driven by sawtooth crash**
  - **Source of driven island still unresolved (NTM? RMHD?)**
  - **Must go to large *S* (~10^7), large anisotropy (> 10^8) to get proper length scales**
    - **Calculations are underway**

Realistic calculations require maximum resources