Tokamak Pellet Fueling Simulations using 3D Adaptive Mesh Refinement

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Collaborators

- P. Colella and Applied Numerical Algorithms Group (LBNL)
- S. C. Jardin (PPPL)
- P. Parks (GA)
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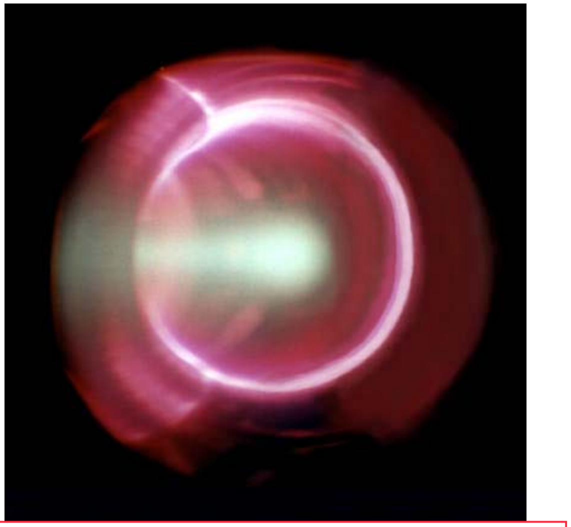
Outline

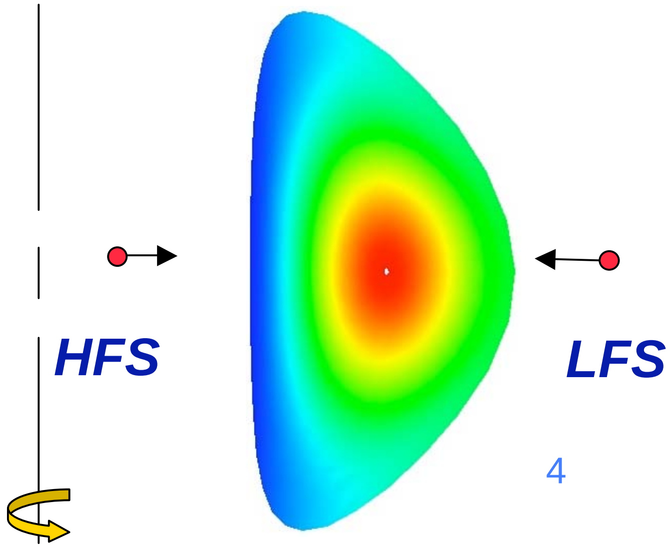
- Introduction and motivation
- Description of physical phenomenon
  - *Spatial and temporal scales*
- Equations and models
- Adaptive mesh refinement (AMR) for shaped plasma in flux-tube coordinates
- Results
  - *HFS vs. LFS Pellet injection*
- Future directions and conclusion

Pellet Injection: Objective and Motivation

- Motivation
  - *Injection of frozen hydrogen pellets is a viable method of fueling a tokamak*
  - *Presently there is no satisfactory simulation or comprehensive predictive model for pellet injection (esp. for ITER)*
- Objectives
  - *Develop a comprehensive simulation capability for pellet injection into tokamaks*
  - *Identify the mechanisms for mass distribution during pellet injection in tokamaks*
  - *Quantify the differences between “inside launch” (HFS) and “outside launch” (LFS)*



Pellet injection in TFTR



Physical Processes: Description

- Non-local electron transport along field lines rapidly heats the pellet cloud ($\underline{\tau_e}$).
  - *Frozen pellet encounters hot plasma and ablates rapidly*
  - *Neutral gas surrounding the solid pellet is ionized*
  - *Ionized, but cool plasma, continues to get heated by electrons*
  - *A high β "plasmoid" is created*
- Ionized plasmoid expands
  - *Fast magnetosonic time scale* $\underline{\tau_f}$.
- Pellet mass moves across flux surfaces $\underline{\tau_a}$.
  - *So-called "anomalous" transport across flux surfaces is accompanied by reconnection*
- Pellet mass expands along field lines $\underline{\tau_c}$.
  - *Pellet mass distribution continues along field lines until pressure equilibration*
- Pellet lifetime τ_p

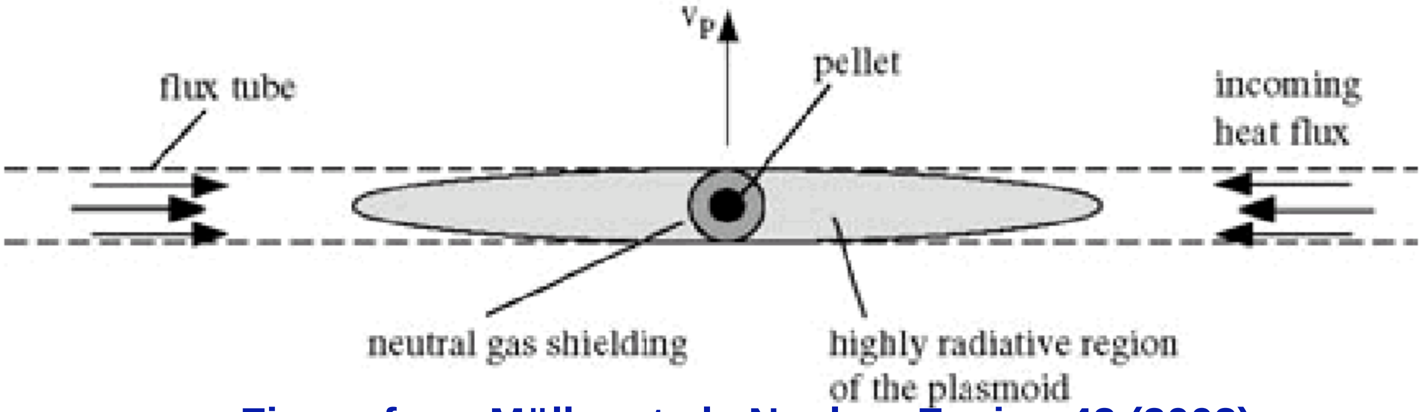


Figure from Müller et al., Nuclear Fusion 42 (2002)

Scales and Resolution Requirements

- Time Scales $\tau_e < \tau_f < \tau_a < \tau_c < \tau_p$
- Spatial scales: Pellet radius r_p << Device size L ~$O(10^{-3})$
- Presence of magnetic reconnection further complicates things
  - *Thickness of resistive layer scales with* ~ $\eta^{1/2}$
  - *Time scale for reconnection is* ~ $\eta^{-1/2}$
- Pellet cloud density ~ $O(10^4)$ times ambient plasma density
- Electron heat flux is non-local
- Large pressure and density gradients in the vicinity of cloud
- Pellet lifetime ~ $O(10^{-3})$ s →long time integrations

Resolution estimates

Tokamak	Major Radius	N	N_{steps}	Spacetime Points
CDXU (Small)	0.3	2×10^7	2×10^5	4×10^{12}
DIIID (Medium)	1.75	3.3×10^9	7×10^6	2.3×10^{17}
ITER (Large)	6.2	1.5×10^{11}	9×10^7	1.4×10^{19}

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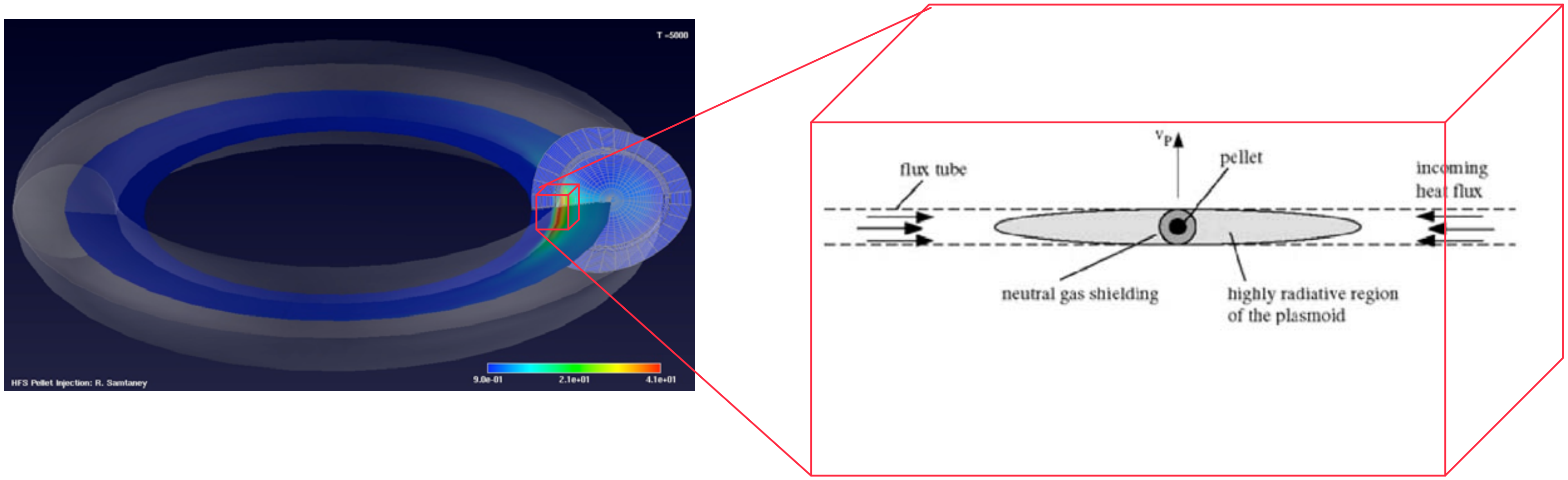
Related Work - Local vs. Global Simulations

- Earliest ablation model by Parks (Phys. Fluids 1978)
- Detailed multi-phase calculations in 2D of pellet ablation (MacAulay, PhD thesis, Princeton Univ 1993, Nuclear Fusion 1994)
- Detailed 2D Simulations of pellet ablation by Ishizaki, Parks et al. (Phys. Plasmas 2004)
  - *Included atomic processes – ablation, dissociation, ionization, pellet fluidization and distortion; semi-analytical model for electron heat flux from background plasma*
- In above studies, the domain of investigation was restricted to only a few cm around the pellet
  - *Also, in these studies the magnetic field was static*

- 3D Simulations by Strauss and Park (Phys. Plasmas, 1998)
  - *Solve an initial value problem. Initial condition consisted of a density "blob" to mimic a fully ablated pellet cloud which, compared with device scales, was relatively large due to resolution restrictions*
  - *No motion of pellet modeled*
- 3D Adaptive Mesh Simulation of pellet injection by Samtaney et al. (Comput. Phys. Comm, 2004)

Current Work

- Combine global MHD simulations in a tokamak geometry with detailed local physics including ablation, ionization and electron heating in the neighborhood of the pellet



- AMR techniques to mitigate the complexity of the multiple scales in the problem

Equations and Models

- Single fluid resistive MHD equations in conservation form

$$\frac{\partial U}{\partial t} + \underbrace{\frac{1}{R}\frac{\partial RF}{\partial R} + \frac{\partial H}{\partial z} + \frac{1}{R}\frac{\partial G}{\partial \phi}}_{\textbf{Hyperbolic terms}} = S + \underbrace{\frac{1}{R}\frac{\partial RF_D}{\partial R} + \frac{\partial H_D}{\partial z} + \frac{1}{R}\frac{\partial G_D}{\partial \phi}}_{\textbf{Diffusive terms}} + S_D + \boxed{S_{pellet}}$$

Density: Ablation
Energy :Electron heat flux

- Additional constraint ∇· **B** =0

- Mass source is given using the ablation model by Parks and Turnbull (Phy. Plasmas 1978) and Kuteev (Nuclear Fusion 1995)

$$\frac{dN}{dt} = -4\pi r_p^2 \frac{dr_p}{dt} 2n_m = 1.12\times10^{16} n_e^{0.333} T_e^{1.64} r_p^{1.33} M_i^{-0.333}$$

  - *Above equation uses cgs units*
- Abalation occurs on the pellet surface $S_n = \dot{N}\delta(x - x_p)$
  - *Regularized as a truncated Gaussian of width 10 r_p*
  - *Pellet shape is spherical for all t*
  - *Pellet trajectory is specified as either HFS or LFS*
  - *Monte Carlo integration to determine average source in each finite volume*

Electron Heat Flux Model

- Semi-analytical model by Parks et al. (Phys. Plasmas 2000)
  - *Assumes Maxwellian electrons and neglects pitch angle scattering*

$$\boxed{-\nabla \cdot q_e = \frac{q_\infty n}{\tau_\infty}\left[g(u_+) + g(u_-)\right]}$$

Where $g(u) = u^{\frac{1}{2}} K_1(u^{\frac{1}{2}})/4$ *,* $u_\pm = \frac{\tau_\pm}{\tau_\infty}$ *and* $\tau_\pm = \pm \int_{\mp\infty}^{\boldsymbol{x}} n(s)ds$

- Solve for opacities as a “steady-state” solution to an advection-reaction equation
  - *Solve by using an upwind method*
    - *Advection velocity is* ***b***

$$\frac{d\tau}{ds} = n(\boldsymbol{x}) \qquad \hat{b} \cdot \nabla\tau = n(\boldsymbol{x})$$

$$\boxed{\frac{d\tau}{d\zeta} + \hat{\boldsymbol{b}} \cdot \nabla\tau = n(\boldsymbol{x})}$$

- Ansatz for energy conservation
  - *Sink term on flux surface outside cloud*

$$\boxed{-\nabla \cdot q_e = \frac{1}{V_\psi - V_{cloud,\psi}} \int_{cloud,\psi} \nabla . q_e}$$

Curvilinear coordinates for shaped plasma

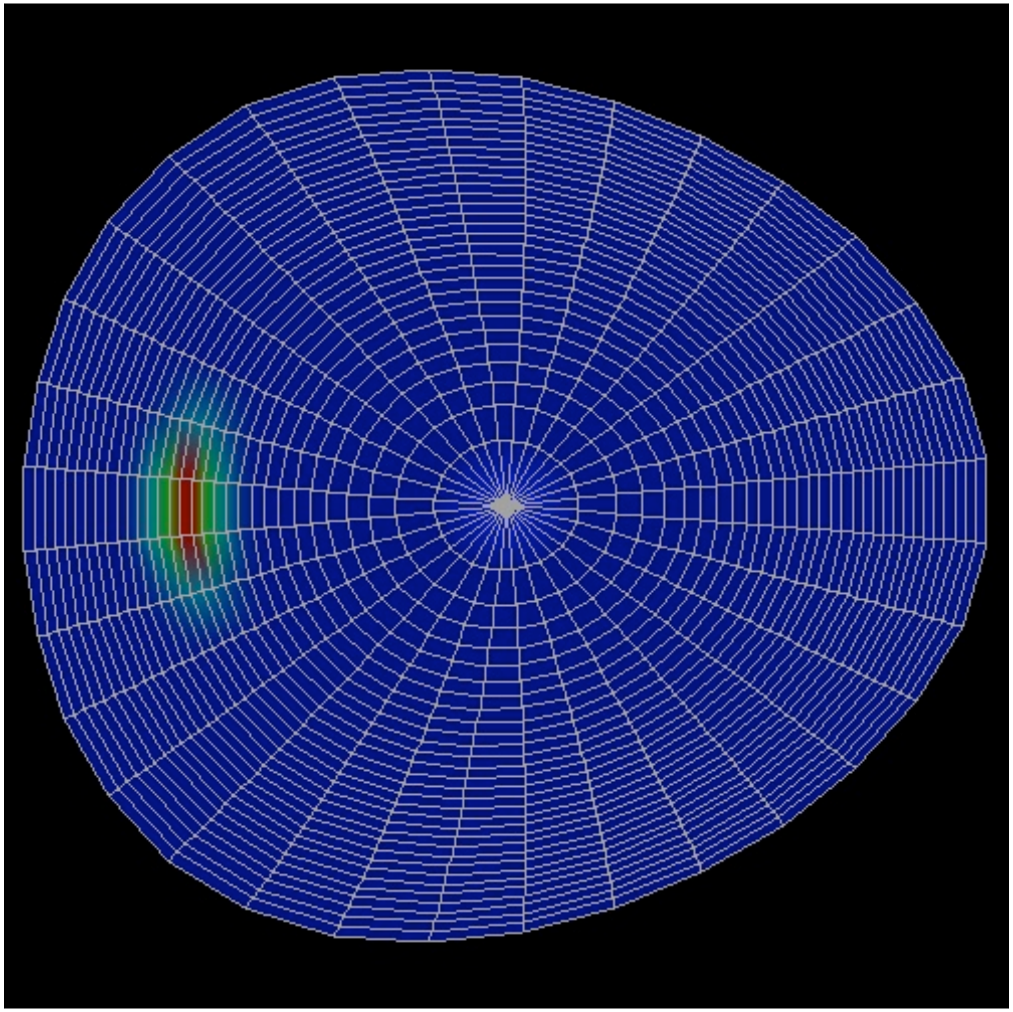
- Adopt a flux-tube coordinate system (flux surfaces ψ are determined from a separate equilibrium calculation)
  - *$R \equiv R(\xi, \eta)$, and $Z \equiv Z(\xi, \eta)$*
  - *$\xi \equiv \xi(R,Z)$, and $\eta \equiv \eta(R,Z)$*
  - Flux surfaces: $\psi = \psi_0\, \xi$
  - *ϕ coordinate is retained as before*
- Equations in transformed coordinates

$$\frac{\partial UJ}{\partial t} + \frac{1}{R}\frac{\partial R\tilde{F}}{\partial \xi} + \frac{1}{R}\frac{\partial R\tilde{H}}{\partial \eta} + \frac{1}{R}\frac{\partial \tilde{G}}{\partial \phi} = \tilde{S}$$

$$\tilde{F} = J(\xi_R F + \xi_z H) = z_\eta F - R_\eta H,$$

$$\tilde{H} = J(\eta_R F + \eta_z H) = -z_\xi F + R_\xi H,$$

$$\tilde{G} = JG, \ \tilde{S} = JS.$$

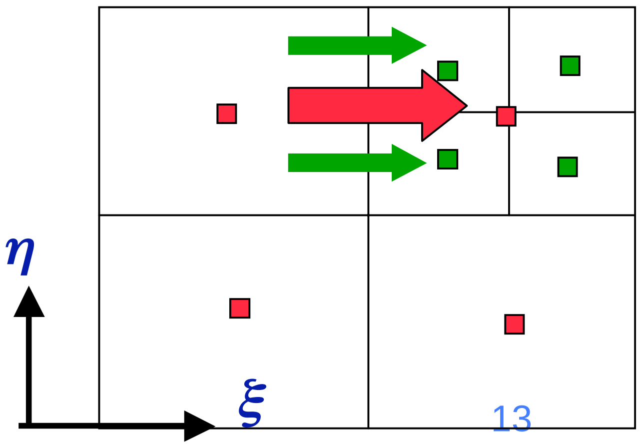


Numerical method

- Finite volume approach
- Explicit second order or third order TVD Runge-Kutta time stepping
- The hyperbolic fluxes are evaluated using upwinding methods
  - *seven-wave Riemann solver*
  - *Harten-Lee-vanLeer (HLL) Method (SIAM Review 1983)*
- Diffusive fluxes computed using standard second order central differences
- The solenoidal condition on B is imposed using the *Central Difference* version of Constrained Transport (Toth JCP 161, 2000)
  - $\nabla \cdot \mathbf{B} \neq 0$ *on coarse mesh cells adjacent to coarse-fine interfaces*
- **Initial Conditions:** Express $B=1/R(\phi \times \nabla \psi + g(\psi)\, \phi) \neq \mathrm{fnc}(\phi)$. Initial state is an MHD equilibrium obtained from a Grad-Shafranov solver.
- **Boundary Conditions**: Perfectly conducting for $\xi=\xi_o$, zero flux (due to zero area) at $\xi=\xi_i$, and periodic in η and ϕ

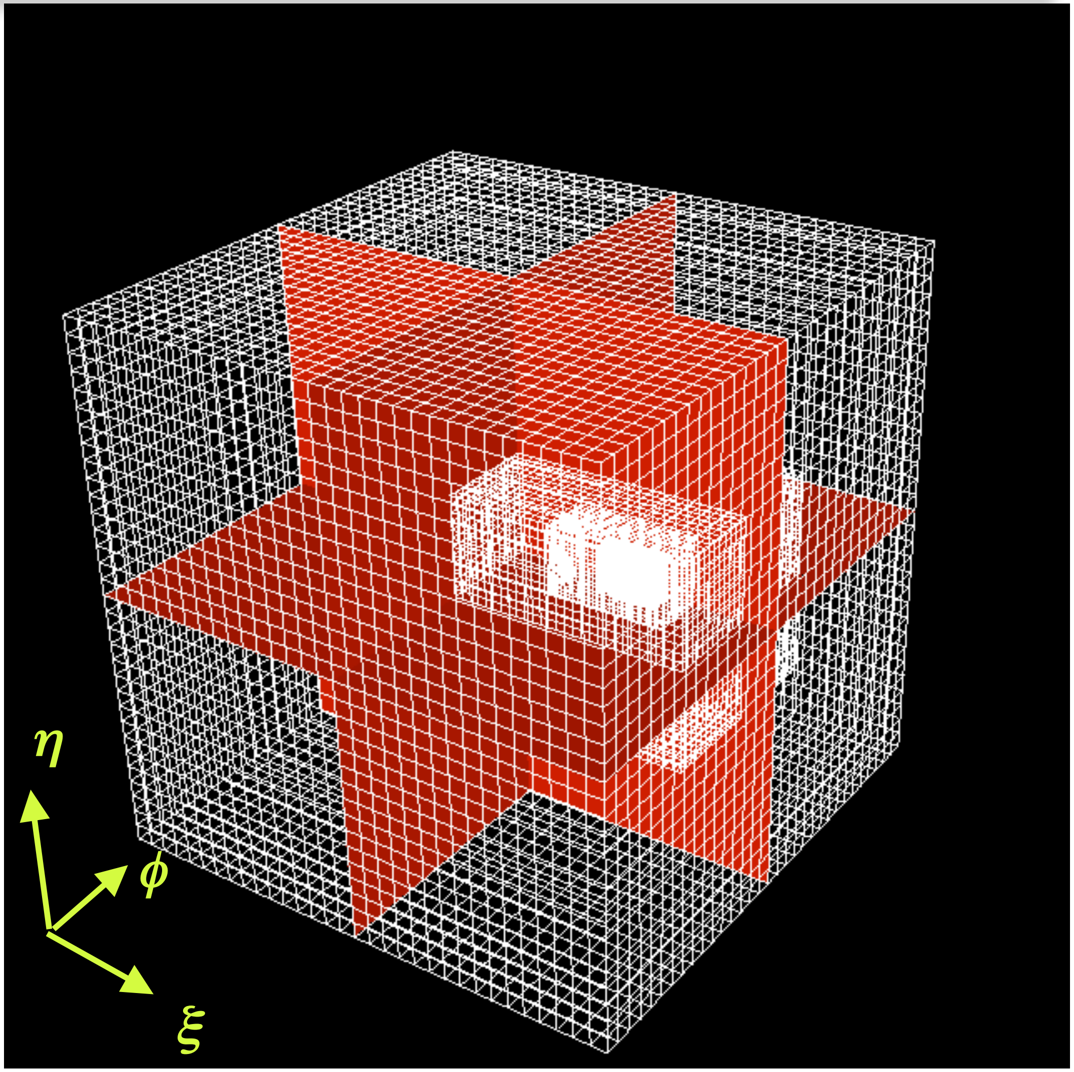
Adaptive Mesh Refinement with Chombo

- ***Chombo*** is a collection of C++ libraries for implementing block-structured adaptive mesh refinement (AMR) finite difference calculations (http://www.seesar.lbl.gov/ANAG/chombo)
  - *(Chombo is an AMR developer's toolkit)*
- Adaptivity in both space and time
- Mesh generation: necessary to ensure volume preservation and areas of faces upon refinement
- Flux-refluxing step at end of time step ensures conservation



Pellet Injection: AMR

- Meshes clustered around pellet
- Computational space mesh structure shown on right
- Mesh stats
  - *32^3 – base mesh with 5 levels, and refinement factor 2*
  - *Effective resolution: 1024^3*
  - *Total number of finite volume cells:113408*
  - *Finest mesh covers 0.015 % of the total volume*
  - *Time adaptivity:*
    $1\ (\Delta\ t)_{base} = 32\ (\Delta\ t)_{finest}$



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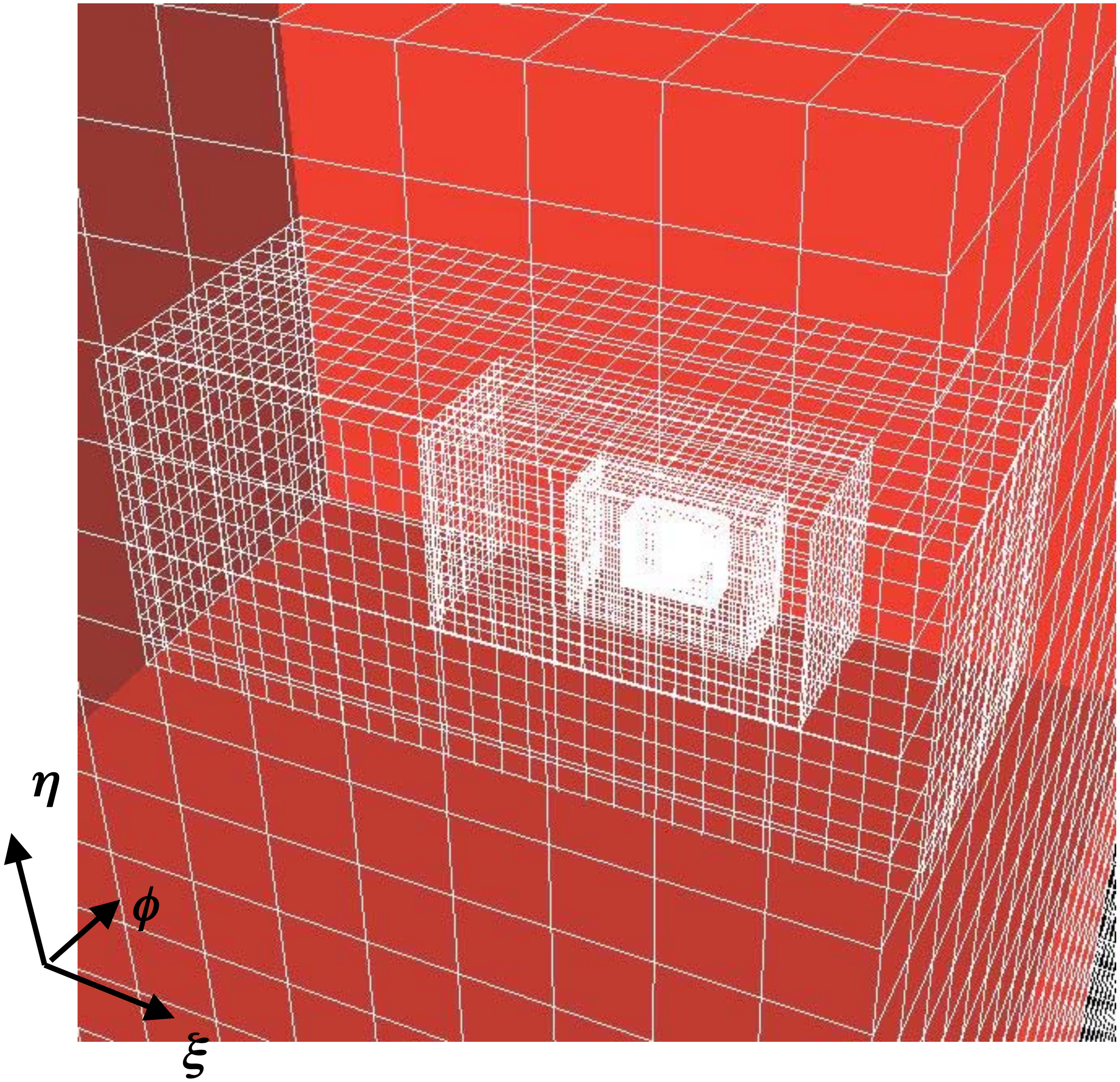
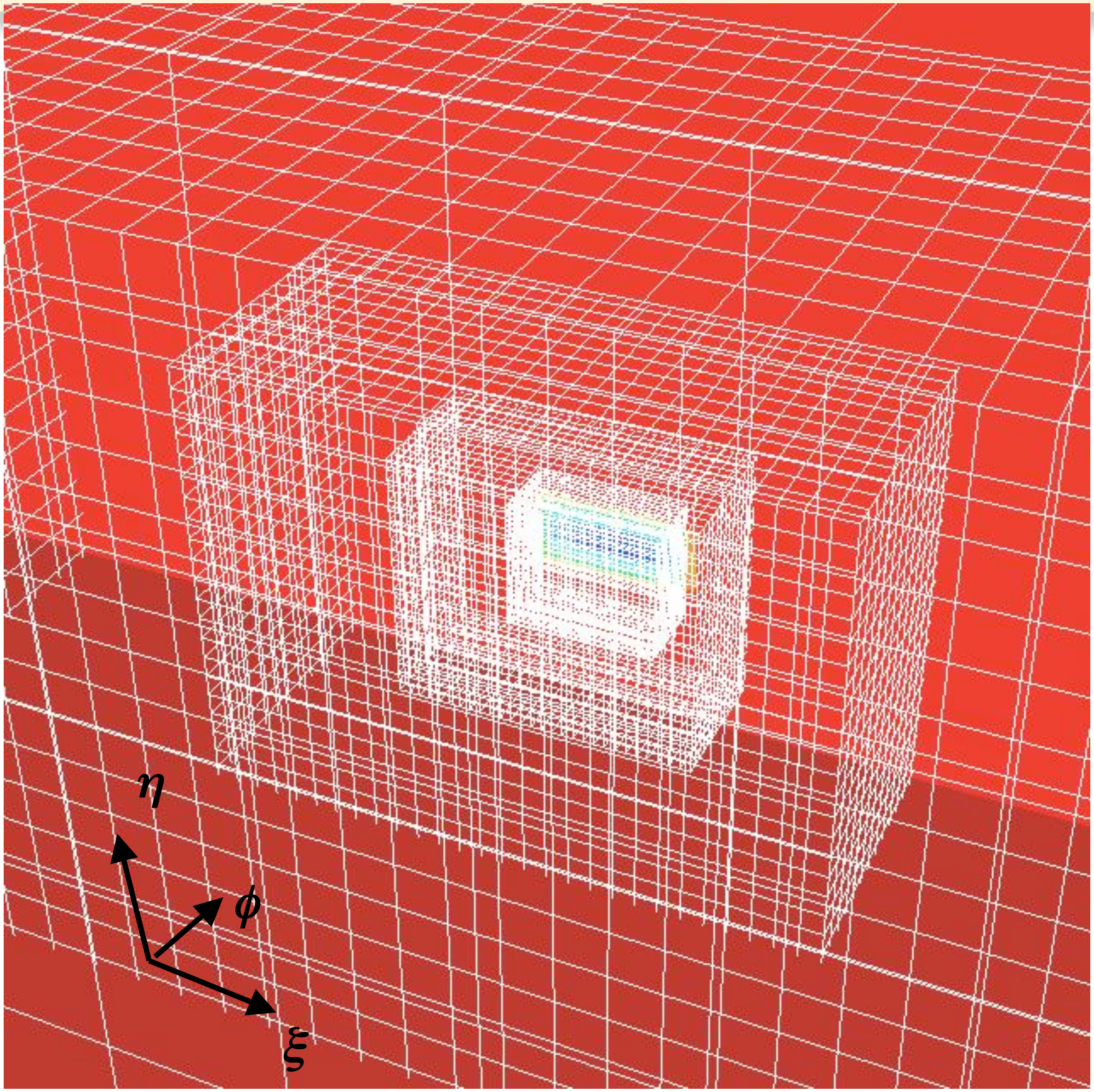
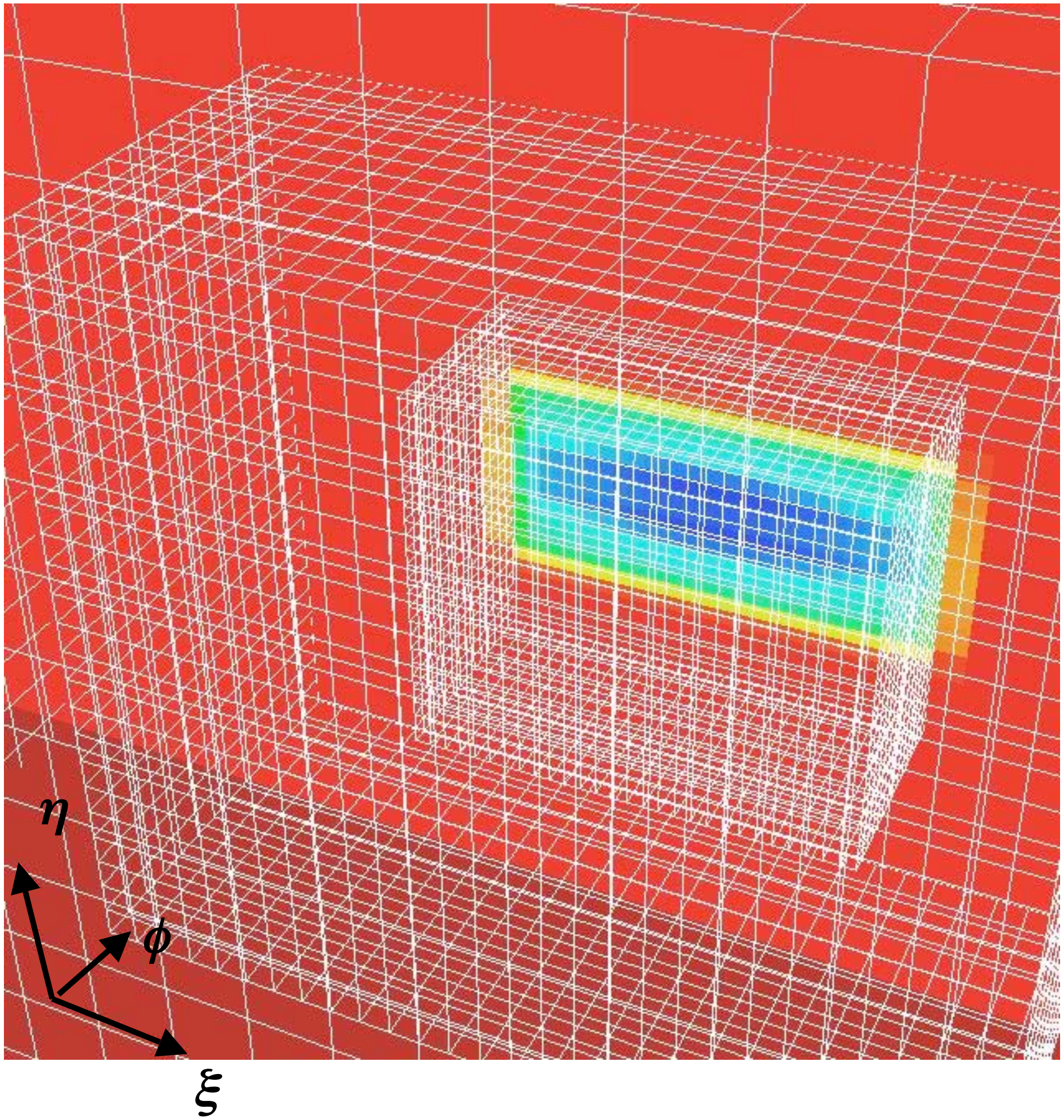
Pellet Injection: Zoom into Pellet Region



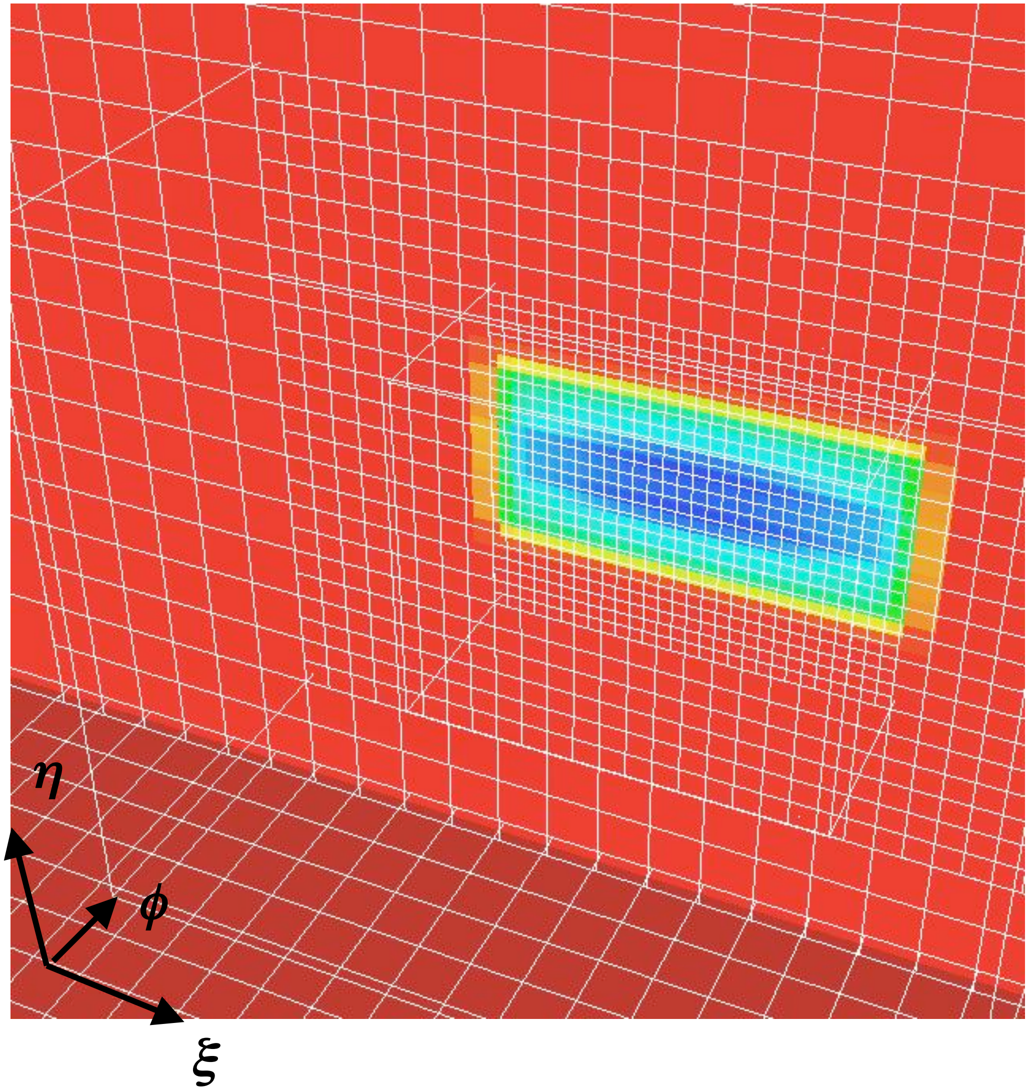
Pellet Injection: Zoom in



Pellet Injection: Pellet in Finest Mesh

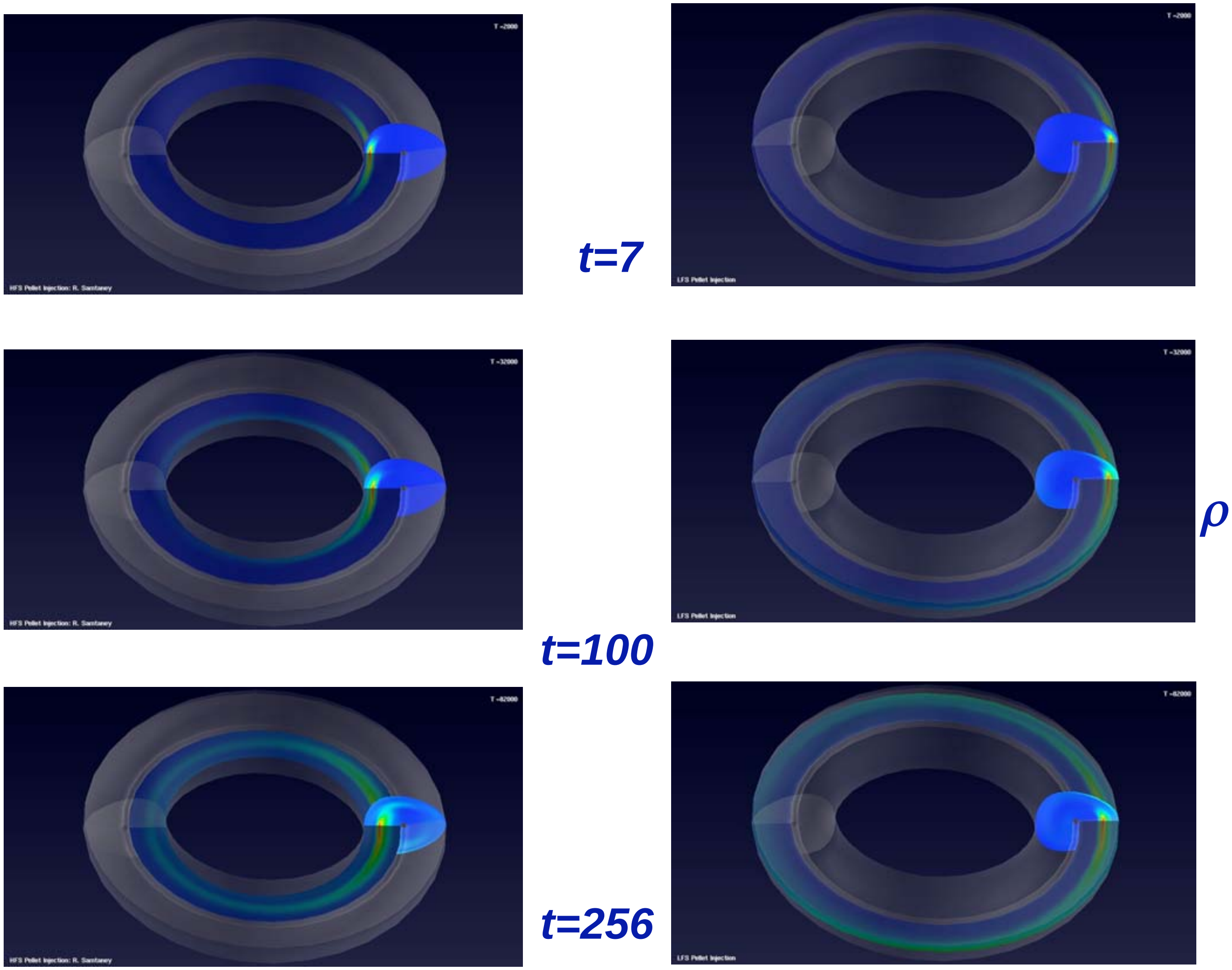


Pellet Injection: Pellet Cloud Density



Results - HFS vs. LFS

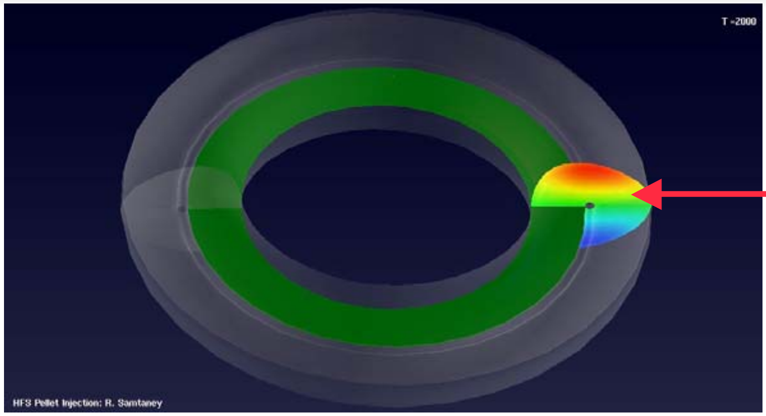
B_T = 0.375T
n_0=1.5× 10^{19}/m^3
$T_{e\infty}$=1.3Kev
β=0.05
R_0=1m, a=0.3 m
Pellet: r_p=1mm,
v_p=1000m/s



Results - B-field Distortion

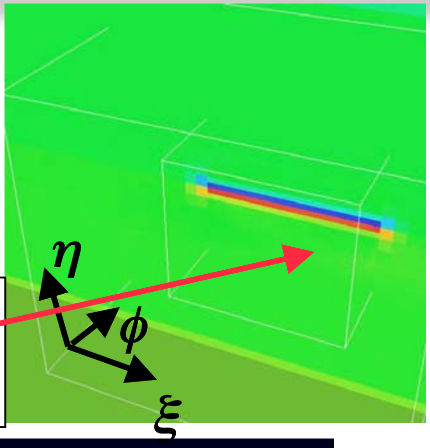
B_R

t=6.2

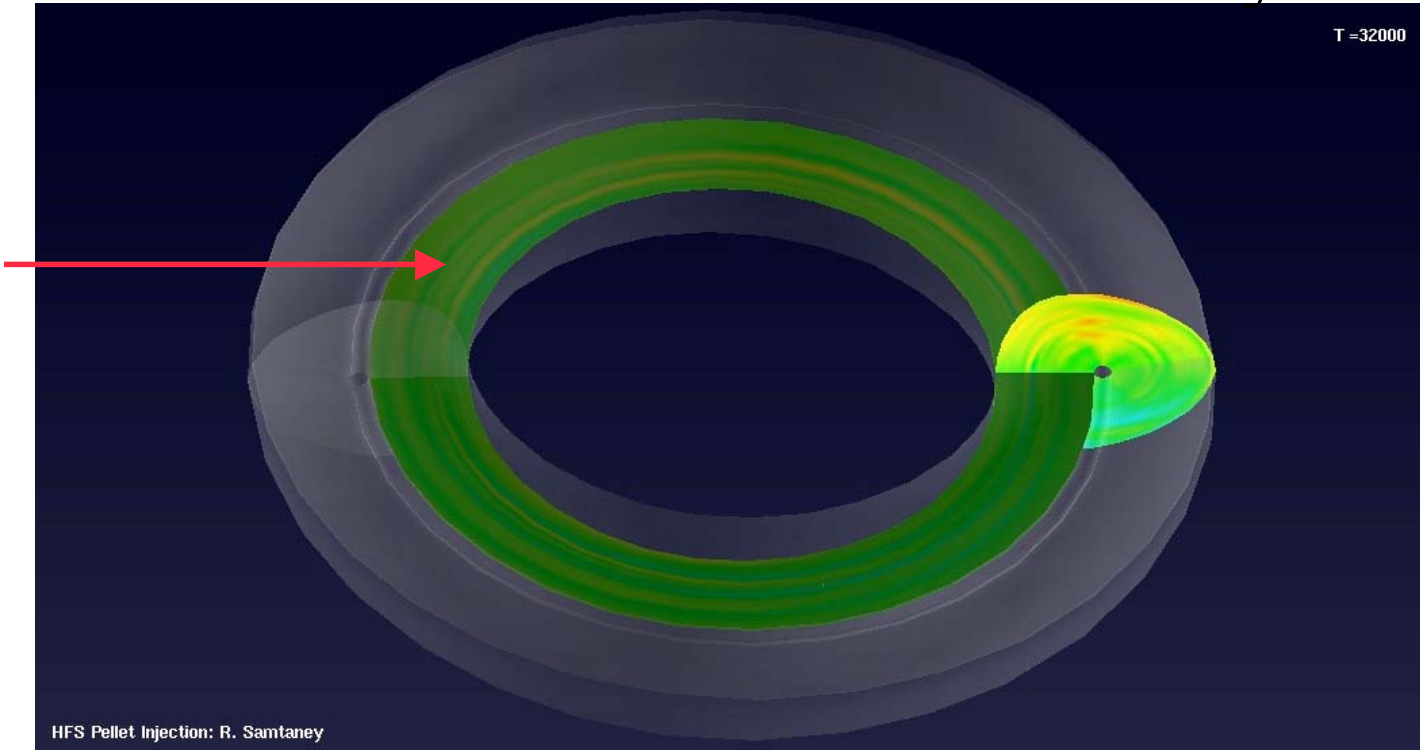


Initial Equilibrium

Zoom-in near pellet

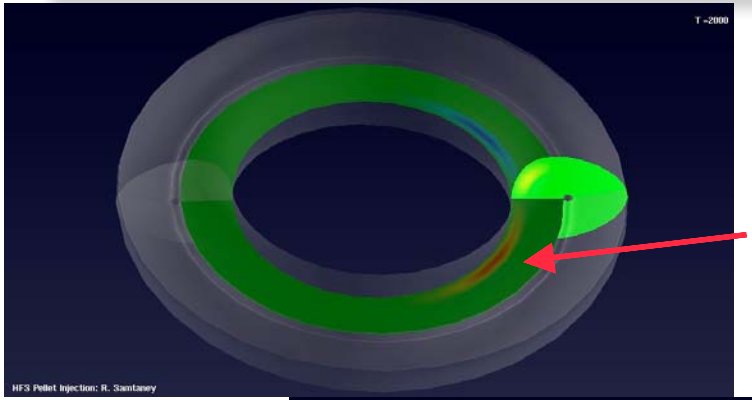


Striations



t=98

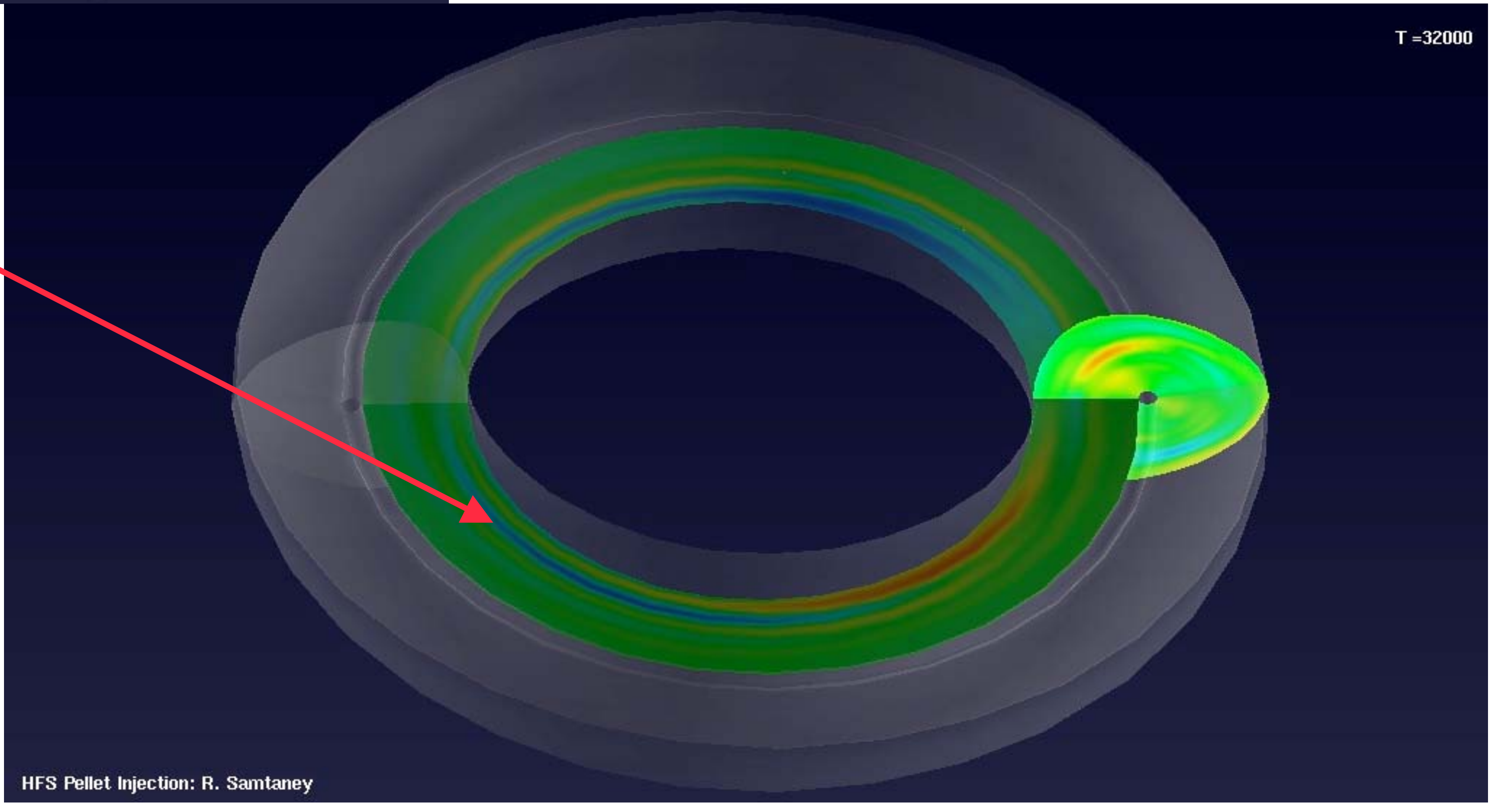
Results - Velocity u_ϕ



Early expansion along field lines

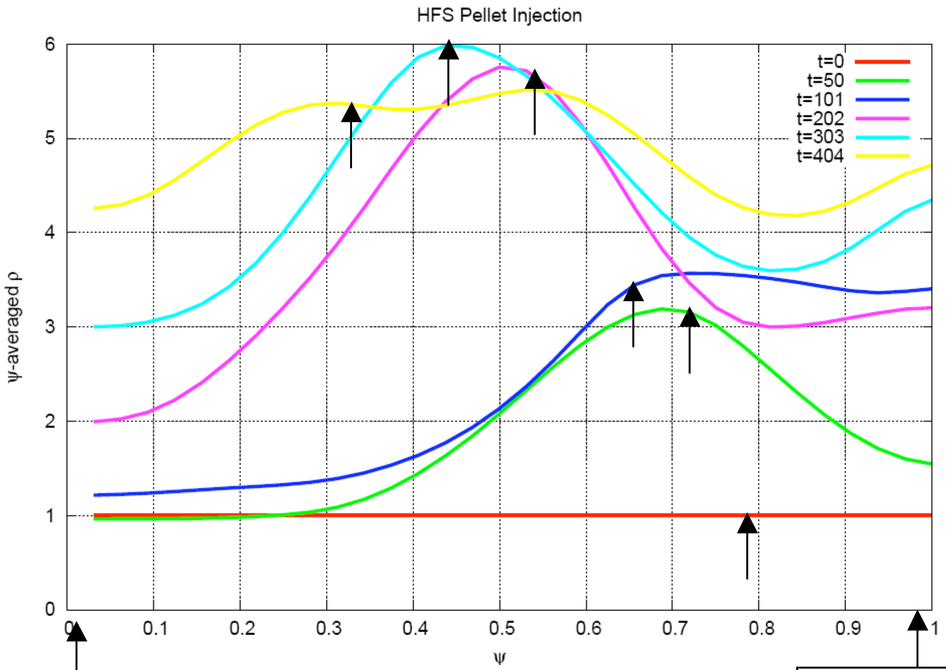
t=6.2

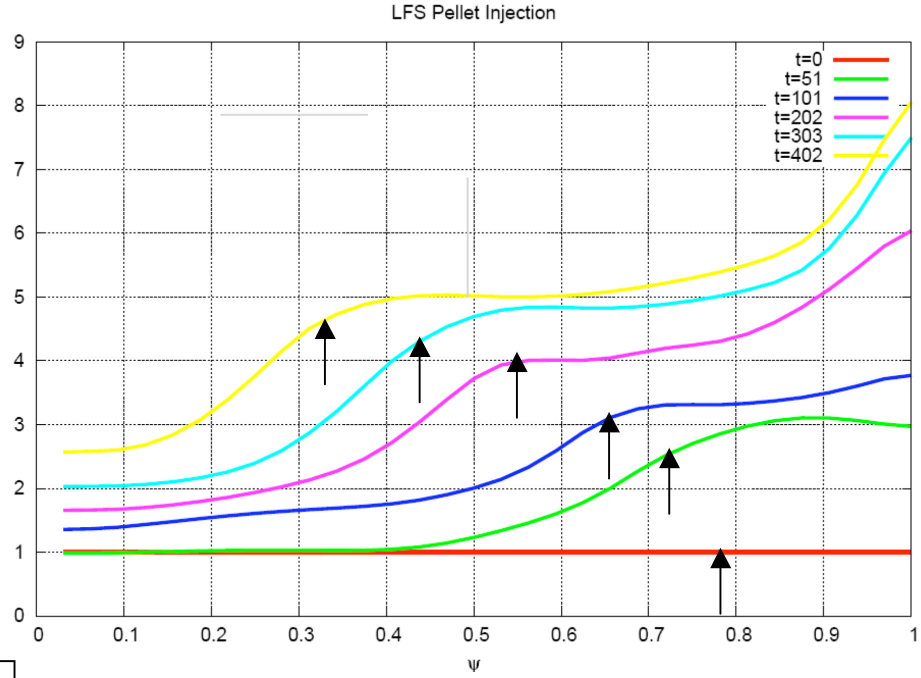
Striations



t=98

HFS vs. LFS - Average Density Profiles





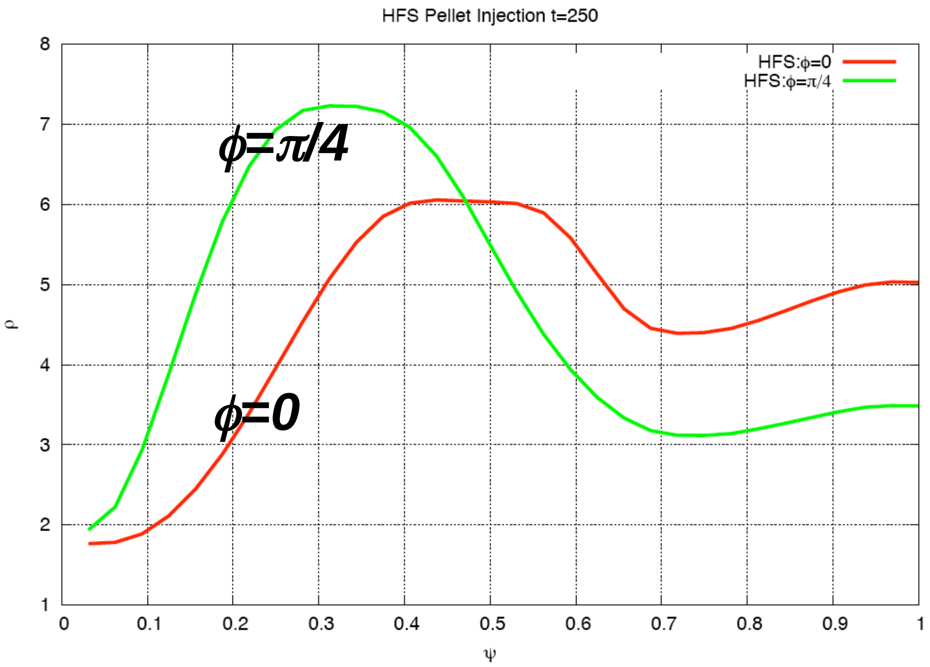
Core

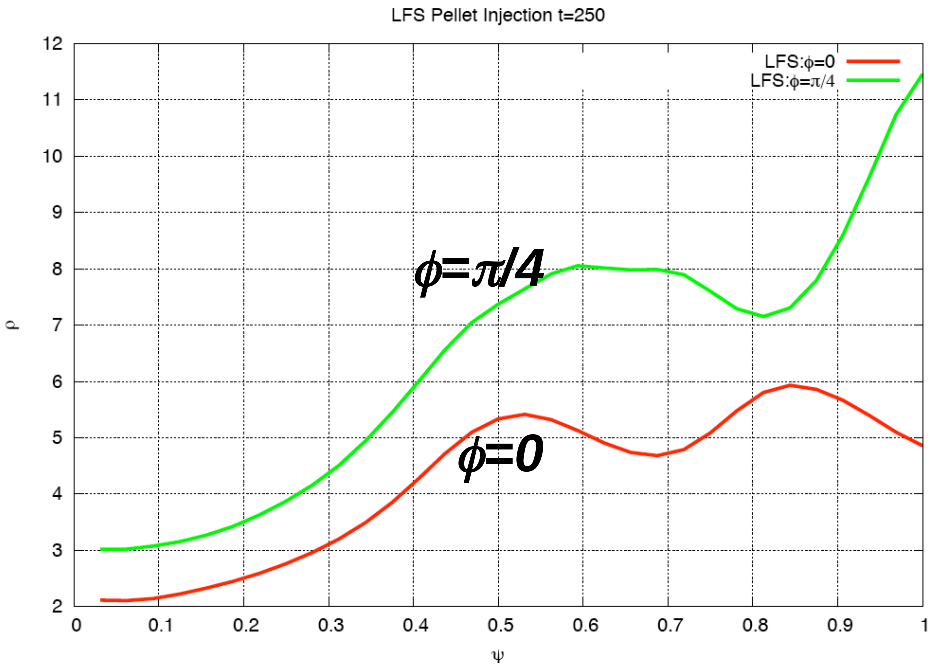
Edge

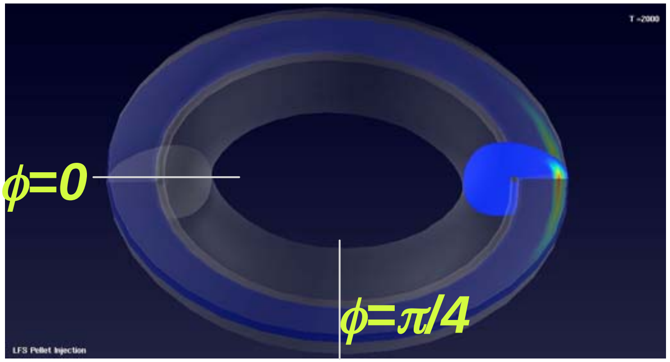
HFS Pellet injection shows better core fueling than LFS

Arrows indicate average pellet location

HFS vs. LFS: Instantaneous Density Profiles

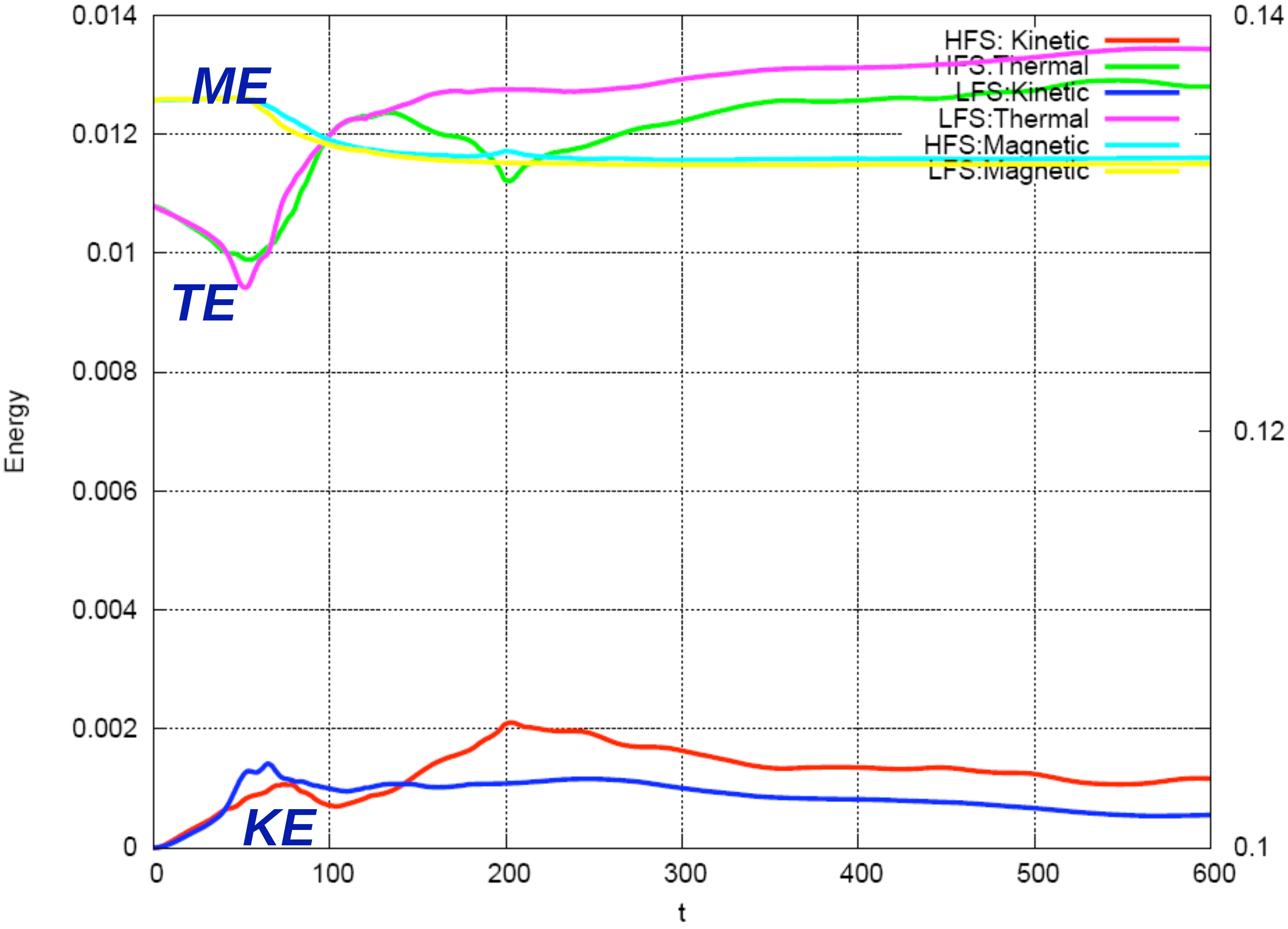






Radially outward shift in both cases indicates higher fueling effectiveness for HFS

Results – Energy budget



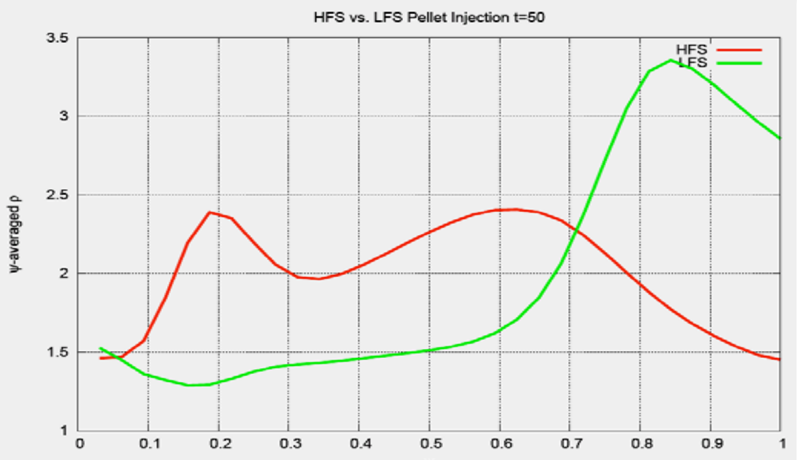
For 0<t<120: Rapid redistribution of thermal energy by electrons

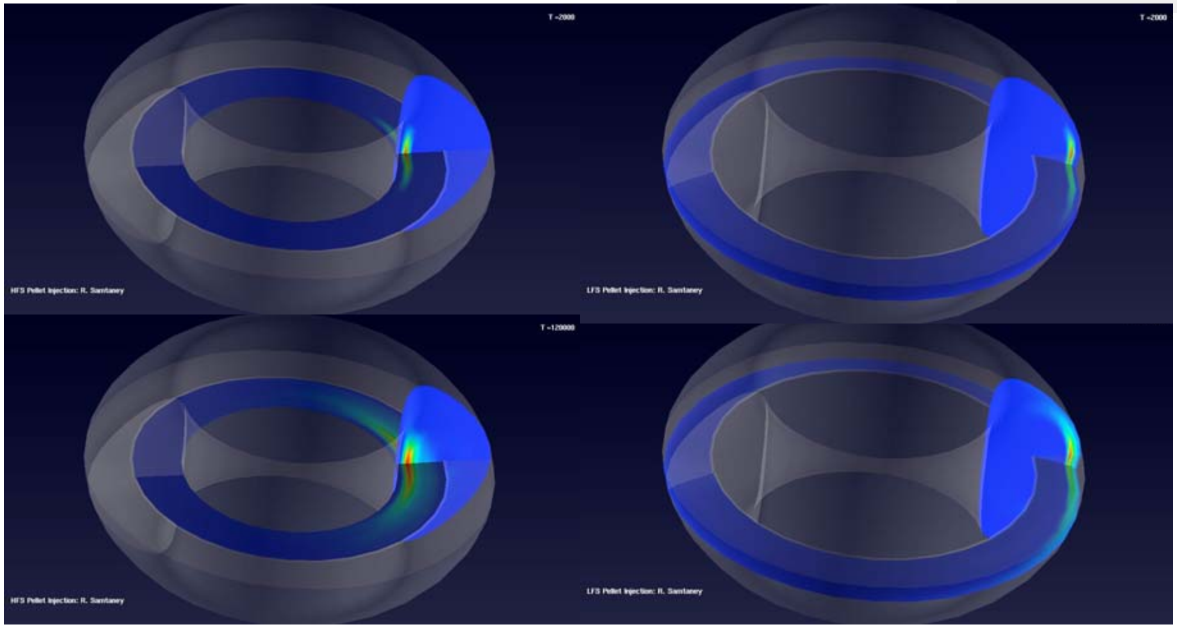
Kinetic energy increases at expense of thermal energy

Thermal energy increases due to reconnection

Results (“DIII-D”): HFS vs. LFS

- B_T= 1T
- $T_{e\infty}$=4-6Kev
- n_o=1.5 × 10^{19}/m^3
- β=0.036
- R_0=1.7m a=0.55m
- Pellet: radius r_p=1mm, velocity v_p=1000m/s

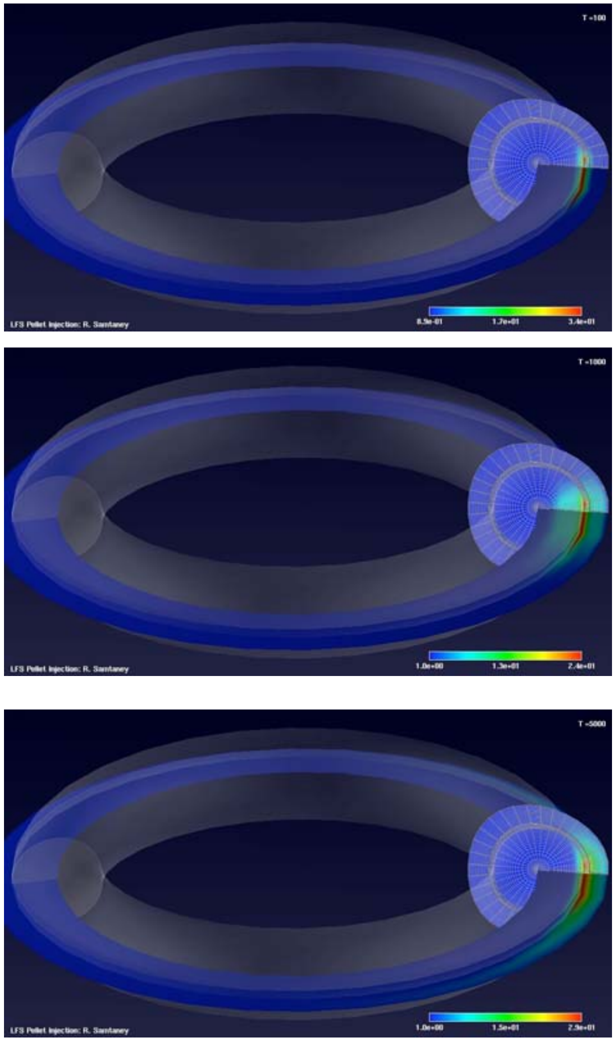


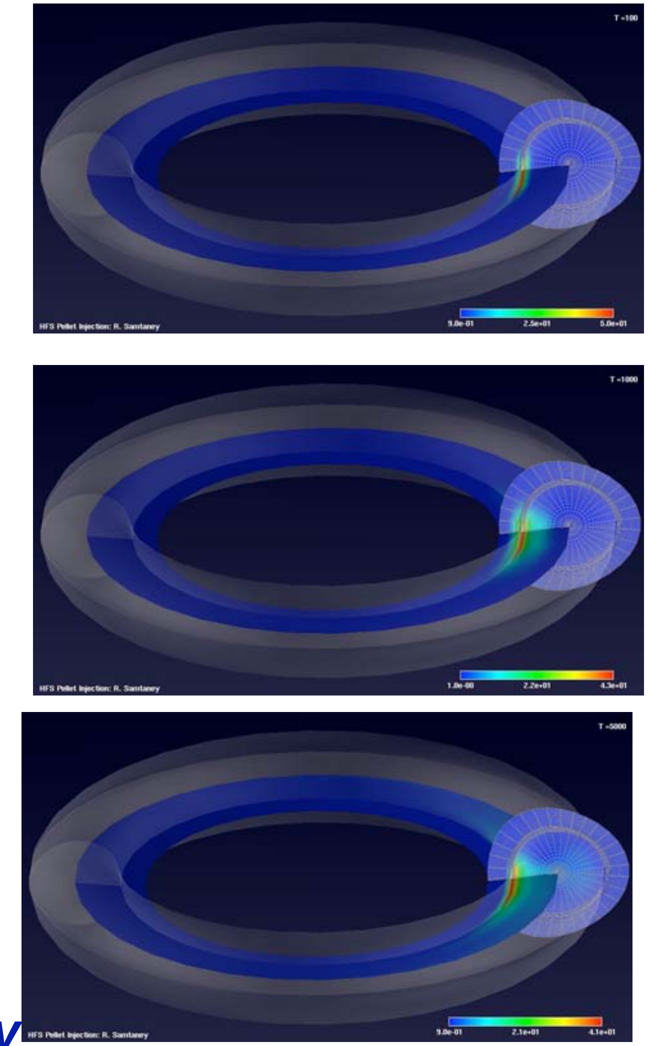


HFS ***LFS***

Larger core fueling for HFS than LFS

Pellet Injection: LFS/HFS Launch

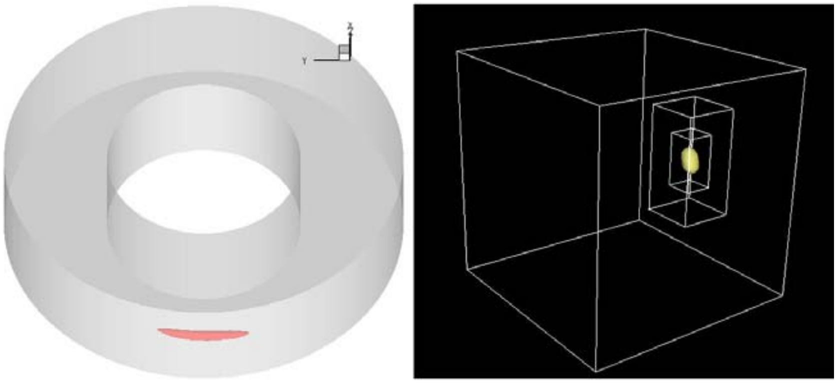




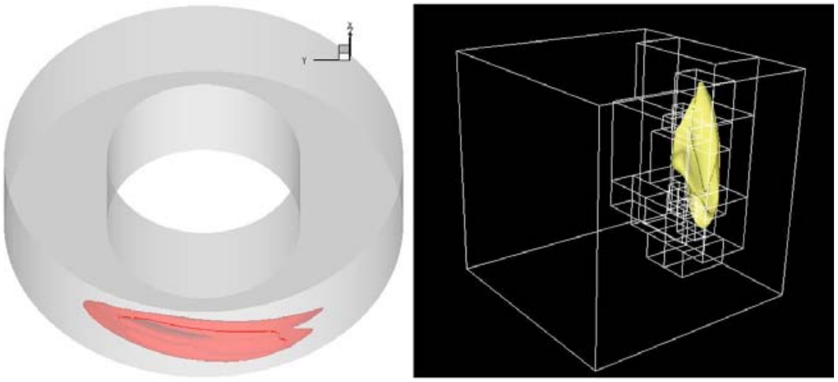
Density

Instantaneous temp equilibration on flux surfaces

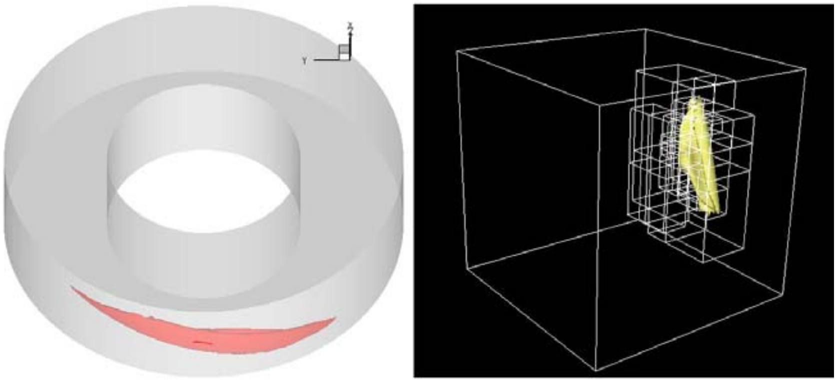
Pellet Injection: LFS Launch



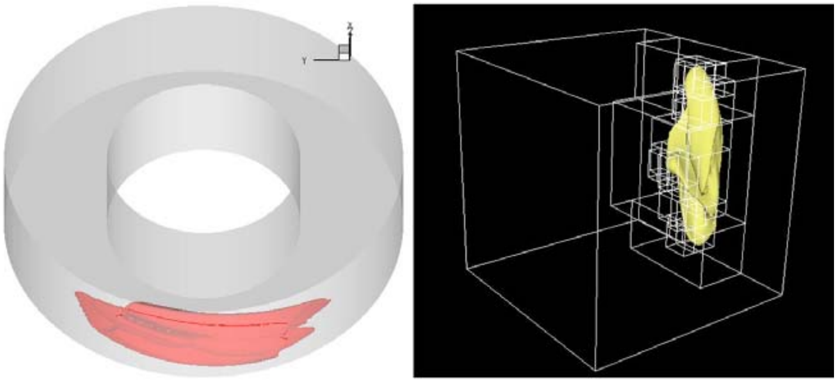
t=2



t=40

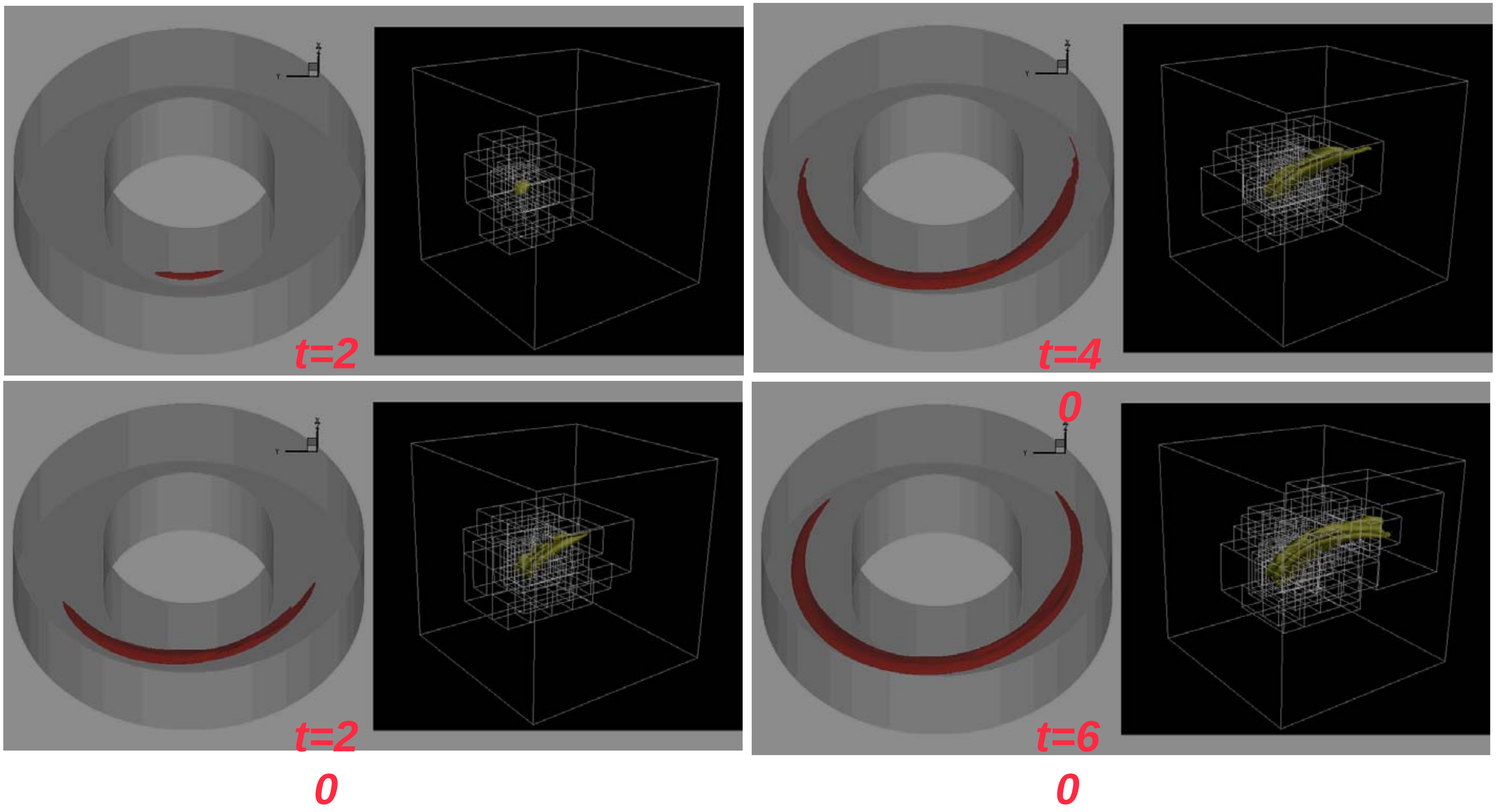


t=20



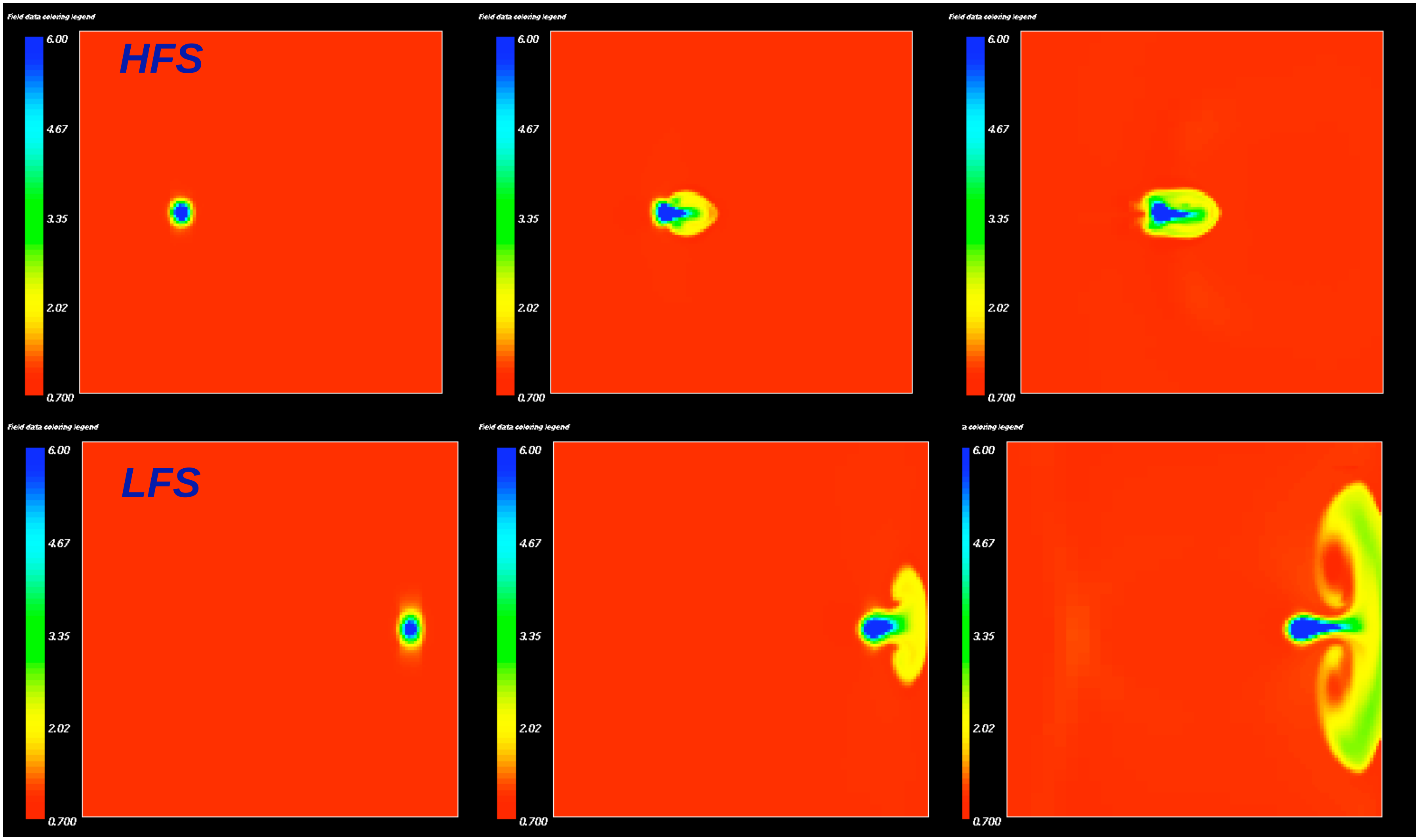
t=60

Pellet Injection: HFS Launch



Note: Left (right) side of frame shows physical (computational) space.

Pellet Injection: HFS vs. LFS



Anomalous transport across flux surfaces

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Conclusion and Future Plan

- Preliminary results presented from an AMR MHD code utilizing flux tube geometry
  - *Physics of non-local electron heat flux included in the simulations*
  - *HFS vs. LFS pellet launches*
    - *HFS core fueling is more effective than LFS*
    - *Outward radial shift due to ∇B, toroidal curvature effects*
    - *Simulation results are consistent with previous studies, and qualitatively consistent with experimental observations*
  - *Numerical method is upwind, conservative and preserves the solenoidal property of the magnetic field*
- AMR is a practical necessity to simulate pellet injection in a tokamak with detailed local physics
- Future work
  - *Refinement of the models ("atomic physics"- ionization, dissociation)*
    - *R. Samulyak (BNL), P.Parks (GA), Postdeadline poster session*
  - *Higher resolution simulations for DIII-D*
    - *Validation against DIII-D experiments*
  - *Predictions for ITER*
    - *Vertical launches (HFS is hard to achieve for ITER)*