Center For Extended Magnetohydrodynamic Modeling

S. C. Jardin for the CEMM consortium

Presentation to the TOPS group

August 20, 2001

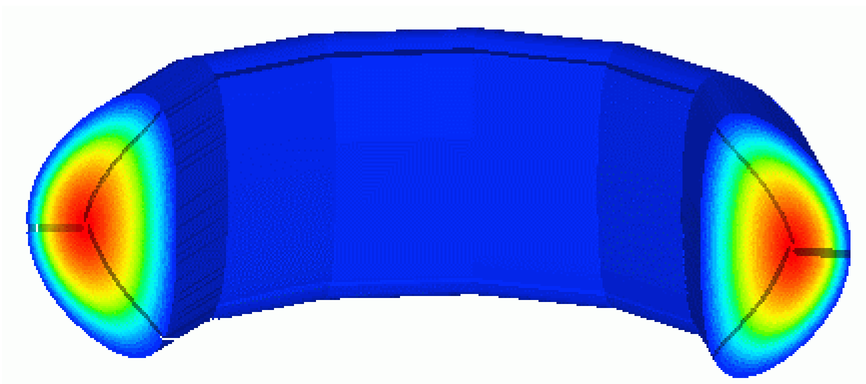
Argonne National Laboratory

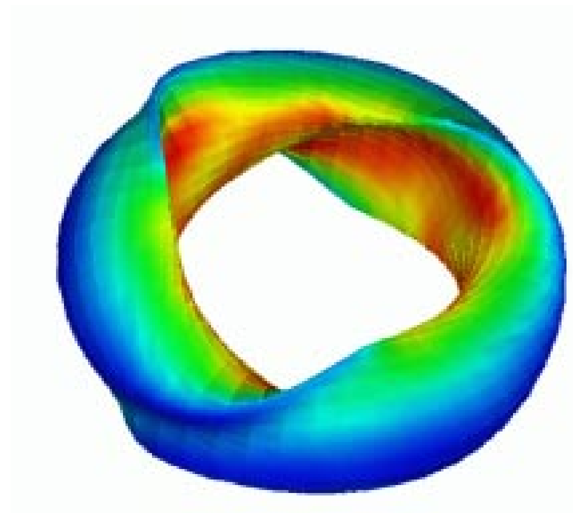
The CEMM Consortium:

- **GA:** D.Schissel
- **LANL:** (T. Gianakon, R. Nebel) ?
- **MIT:** L. Sugiyama
- **NYU:** H. Strauss
- **PPPL:** J. Breslau, G. Fu, S. Hudson, S.Jardin ,W.Park
- **SAIC:** S. Kruger, D. Schnack
- **U. Colorado:** C. Kim, S. Parker
- **U.Texas:** F. Waelbroeck
- **U.Wisconsin:** J. Callen, C. Hegna, C. Sovinec
- **Utah State:** E. Held

Background

"…to develop and deploy predictive computational models for the study of low frequency, long wavelength fluid-like dynamics in the diverse geometries of modern magnetic fusion devices."



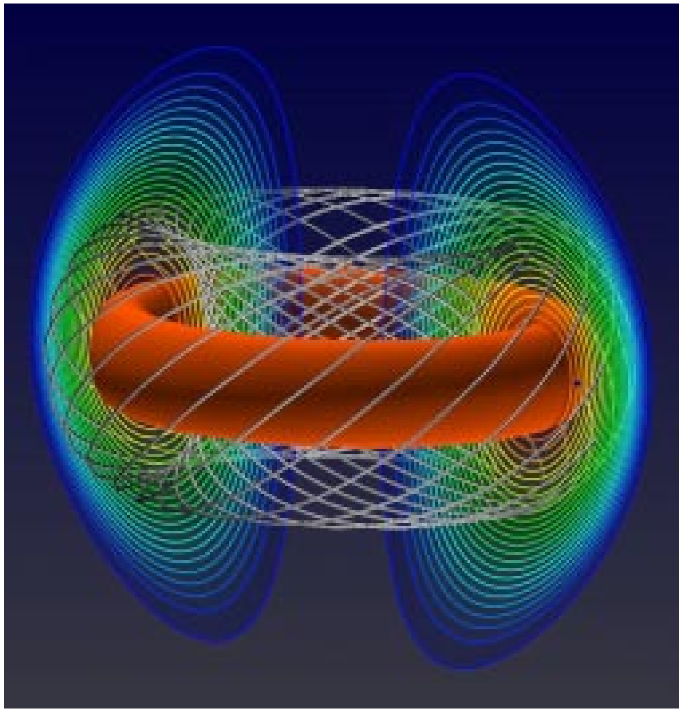


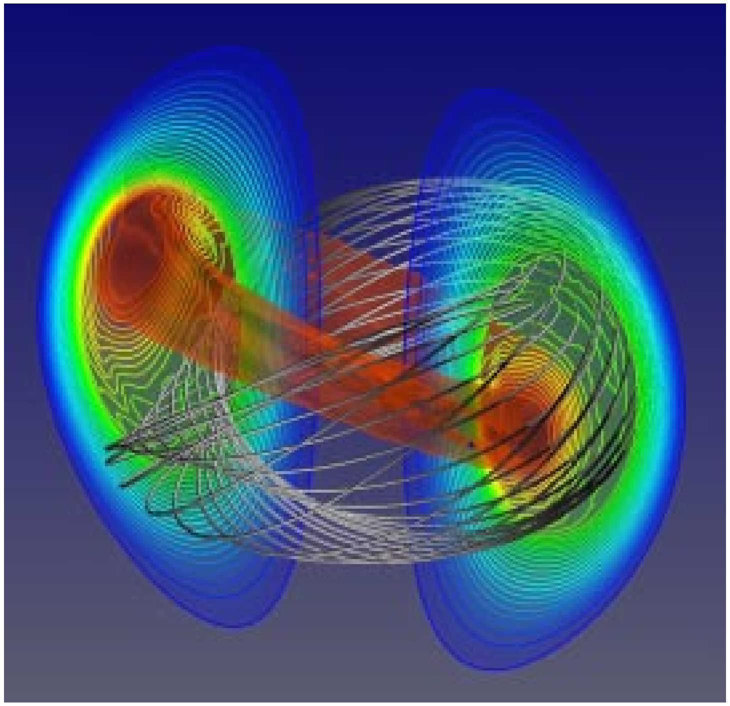
- Extreme separation of time and space scales, and extreme anisotropy

NIMROD and **M3D** codes form basis: build on these assets

Improved physics models and better resolution!

Pressure contours, B-lines, pressure surfaces for "sawtooth" (m=1) instability with $q_o<1$ (peaked current)





Periodic oscillation or discharge termination?

Need to incorporate improved physics models and more realistic physics parameters => **more resolution**

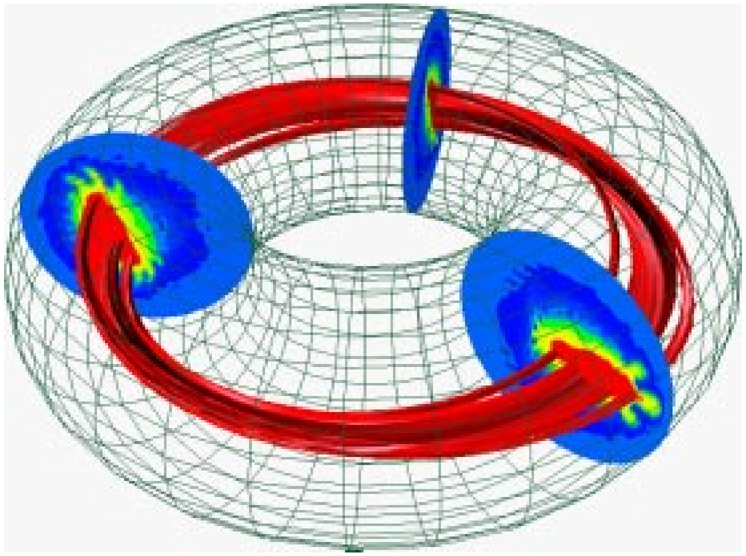
Park

m=1 mode can also destablize short wavelength modes

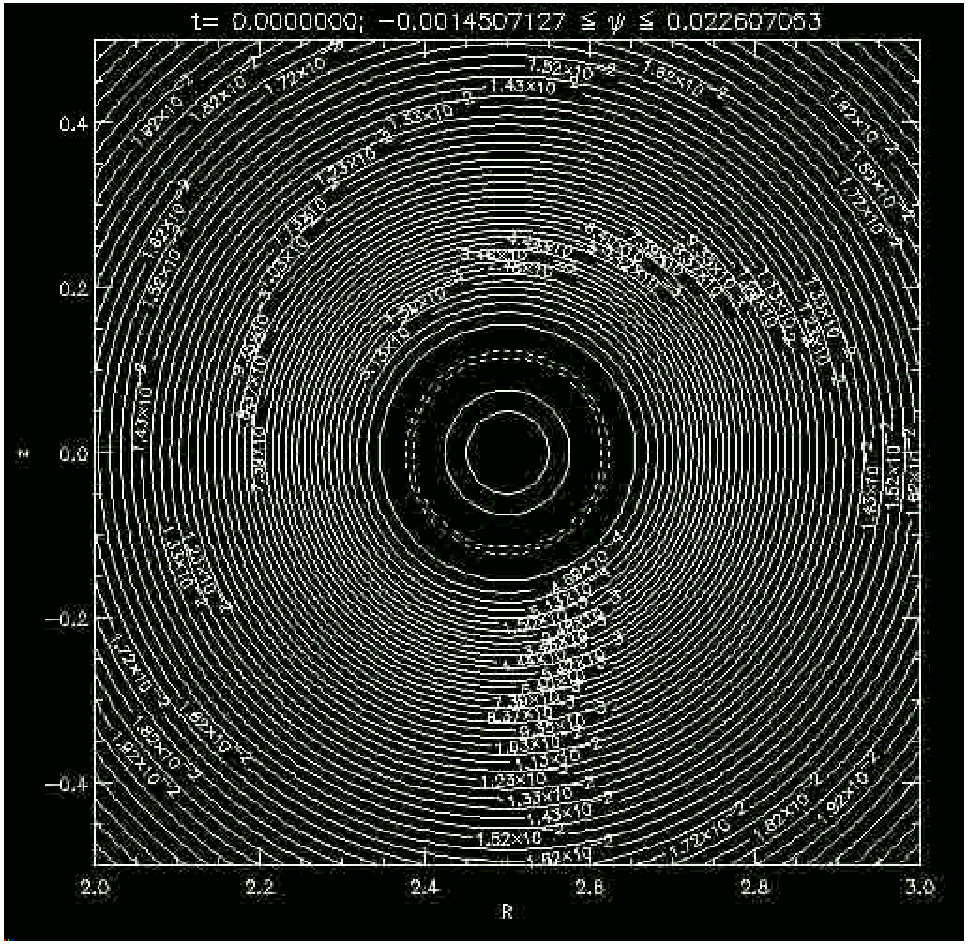
• Toroidally localized moderate-n ballooning mode is driven unstable by a local pressure bulge formed by the m=1 mode.

• This mode steepens nonlinearly in a ribbon like structure driving field line stochasticity and leading to plasma termination.

•The high-beta disruption in record making TFTR discharge has been explained, with good agreement between experiment and simulation.



m=1 mode in hot plasmas is high priority objective

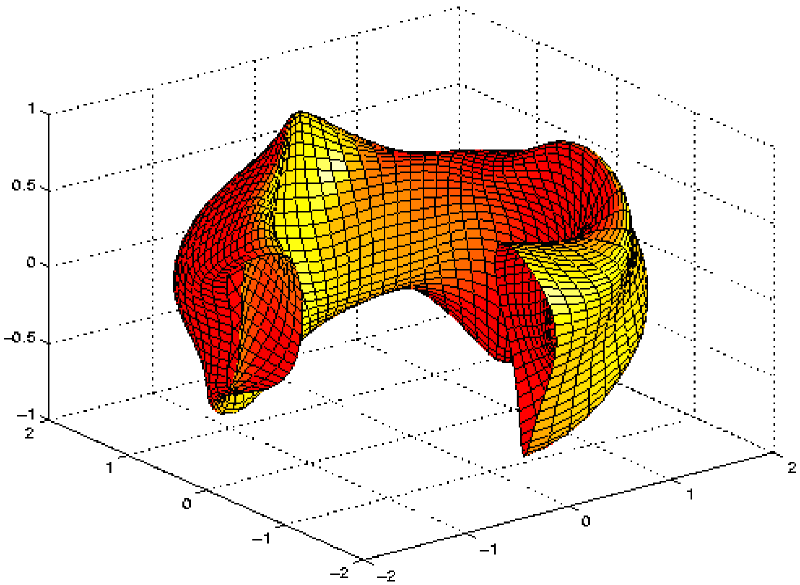


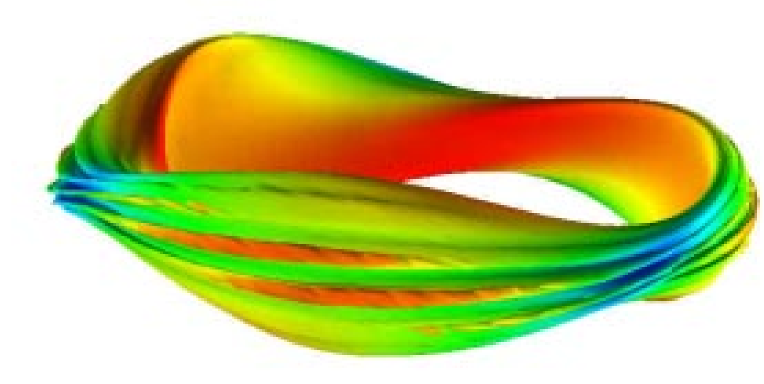
better predictive model of m=1 mode is needed for next step burning plasma

- sensitive to plasma parameters
- multiple time scales and space scales
- new physics comes in with new parameters

Breslau

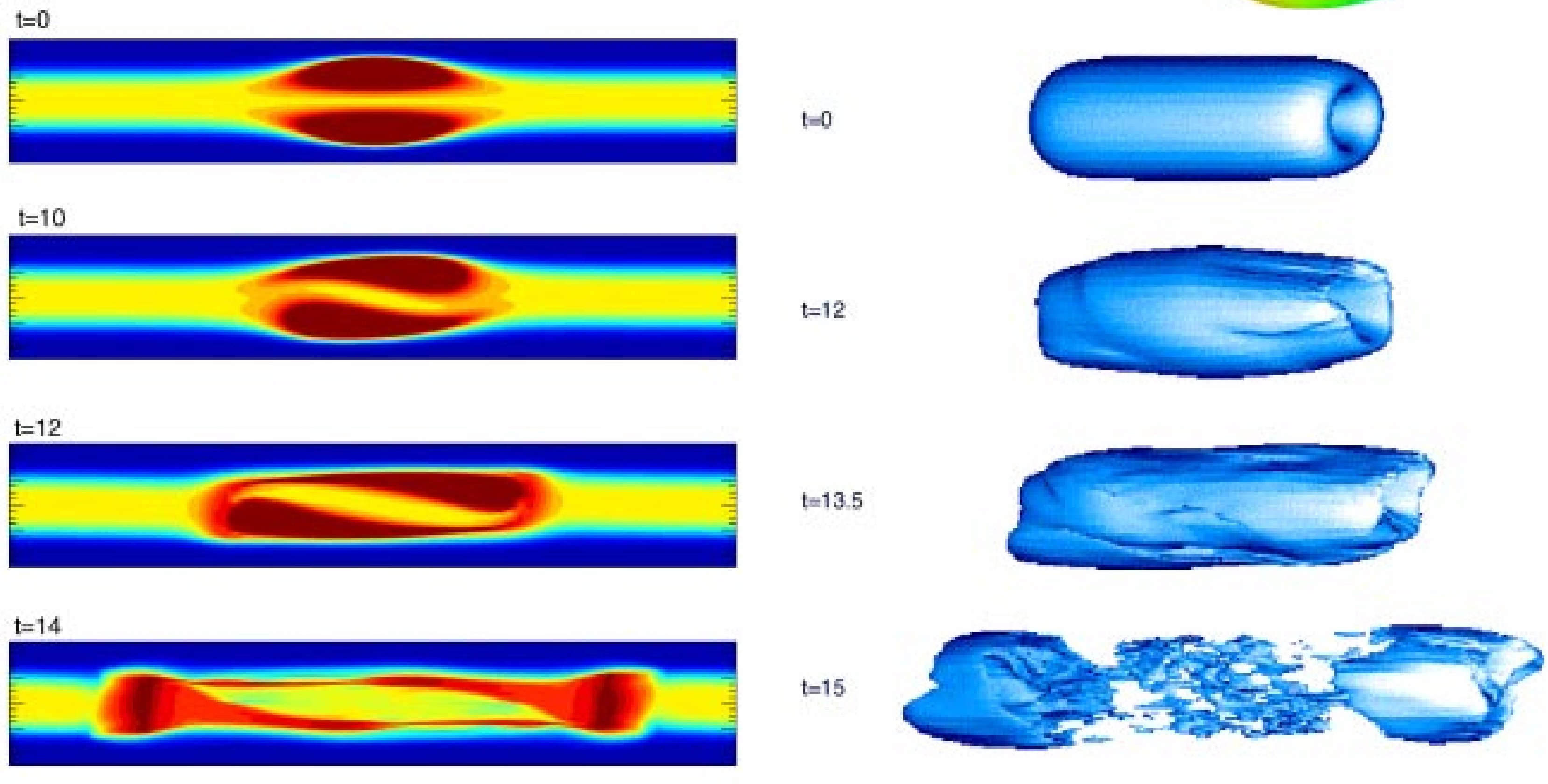
Quasi-Axisymmetric Stellarator NCSX





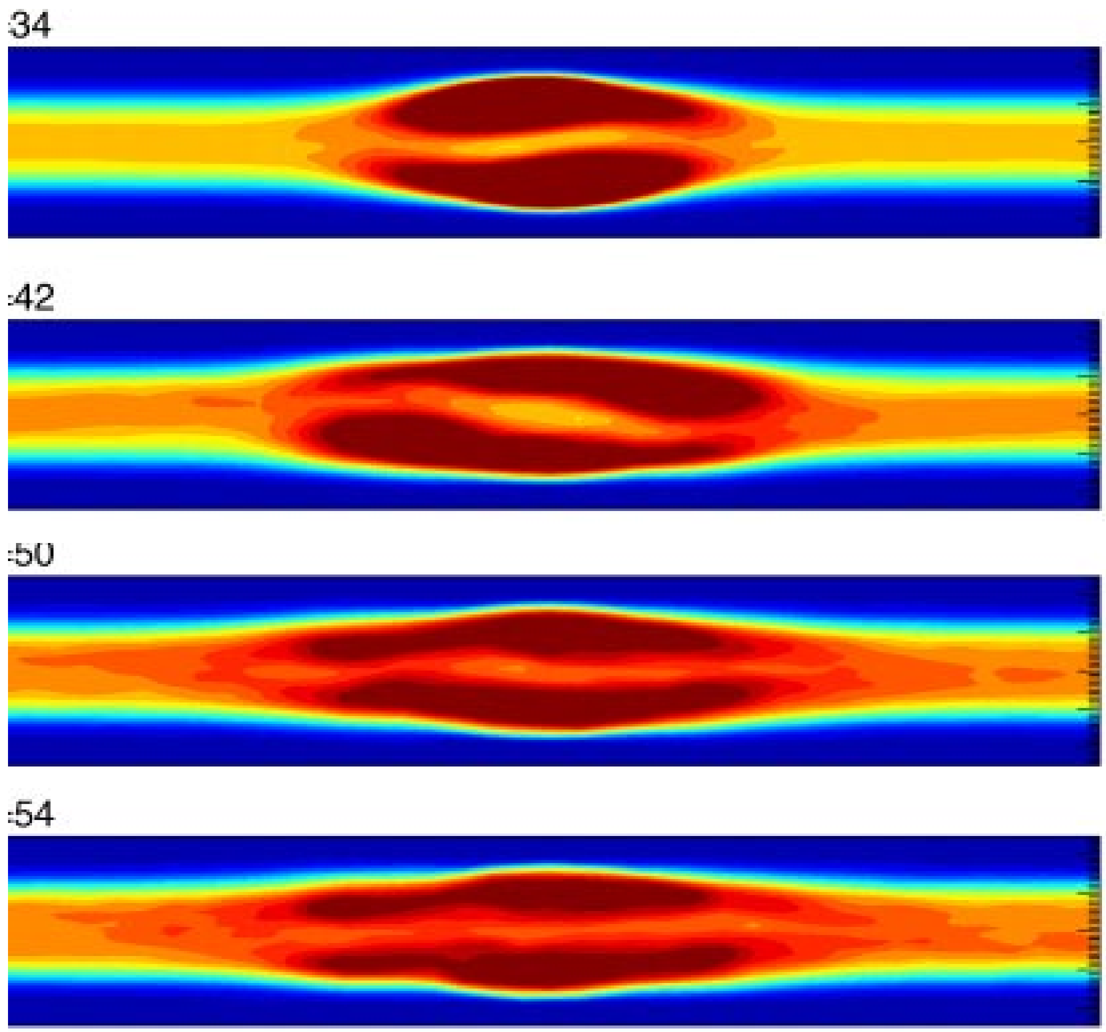
- “Twisted” outer surface formed by 3D coil set
- Ballooning mode develops nonlinearly when the design pressure is exceeded…consequence ?

Resistive MHD simulations of Field Reversed Configuration (FRC)



Simple "MHD" model of FRC shows they should be <u>unstable</u>

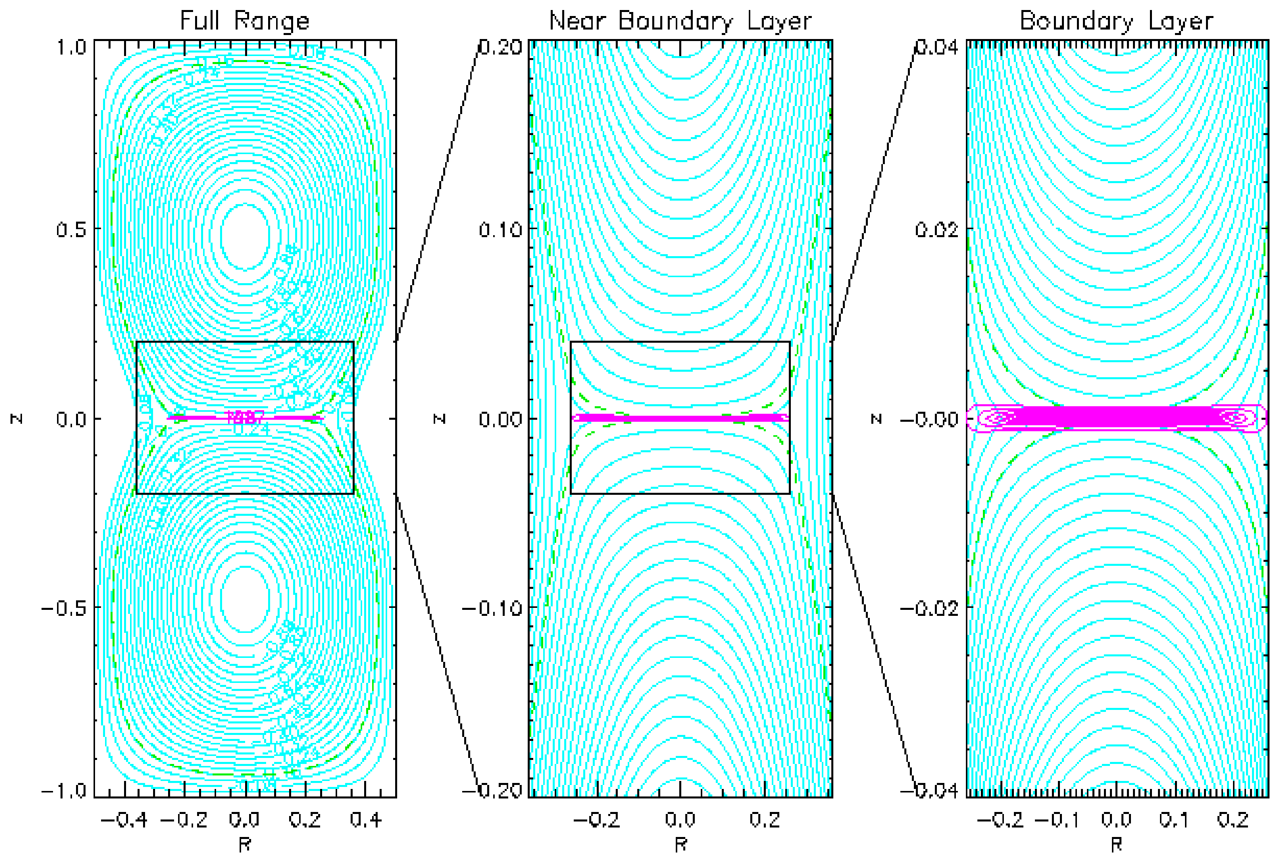
Fully kinetic-ion simulation shows much different behavior



- Simulation starts out with linear growing instability with reduced growth rate
- However, instability saturates nonlinearly!
- Shows remarkable agreement with experimental data
- Illustrates importance of correct plasma model

Model problem: merging spheromaks with full 2-fluid MHD equations, high-resolution

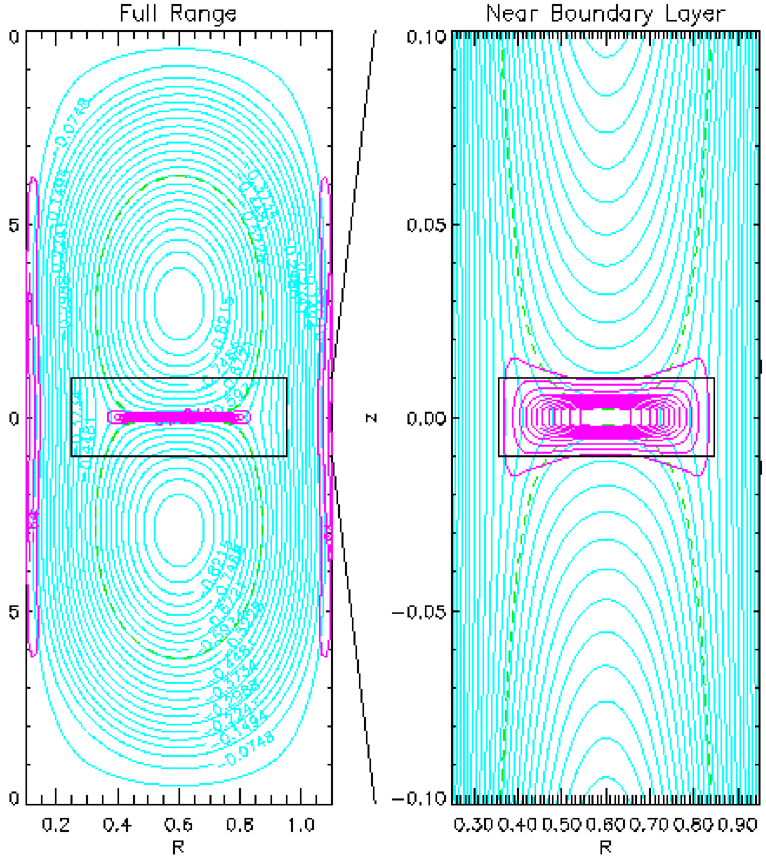
$\eta = 10^{-5}$



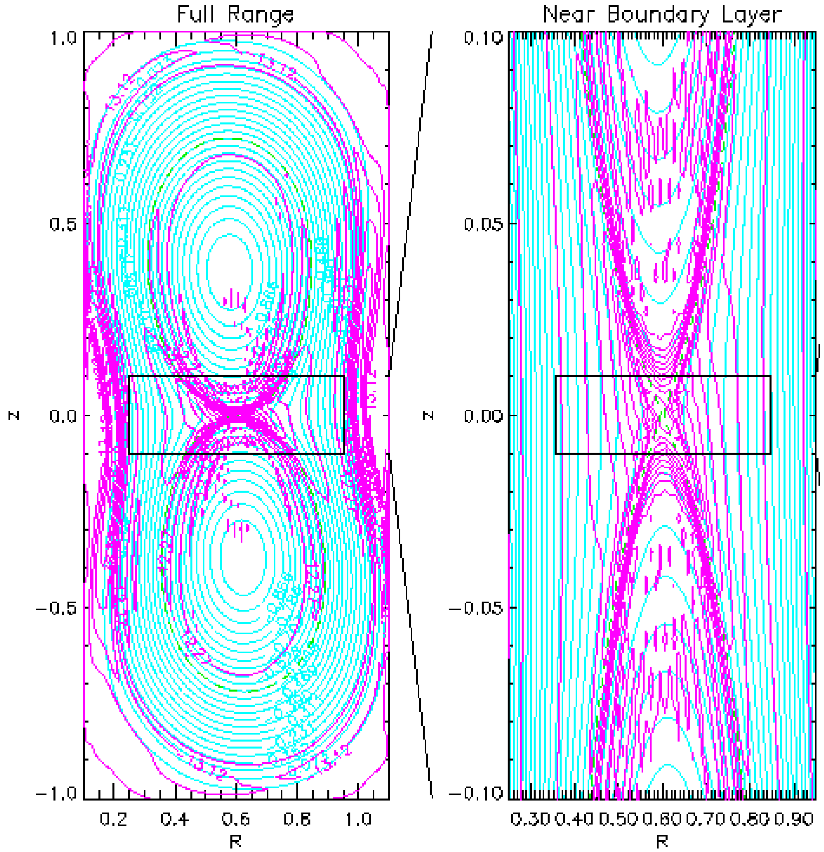
- Variable resolution grid allows resolution of disparate space scales.
- note: cyan: flux purple: current

Breslau

$\chi = 0$ (resistive MHD)



$\chi = 0.2$ (2-fluid MHD)



More complete physics (two-fluid) can change the reconnection rate and the qualitative nature of the reconnection physics

Plasma Models: XMHD

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$\vec{E} + \vec{V} \times \vec{B} = \eta \vec{J} + \frac{1}{ne}\left[\vec{J} \times \vec{B} - \nabla \bullet P_e\right]$$

$$\mu_0 \vec{J} = \nabla \times \vec{B}$$

$$P = pI + \Pi$$

$$\rho\left(\frac{\partial \vec{V}}{\partial t} + \vec{V} \bullet \nabla \vec{V}\right) = \nabla \bullet P + \vec{J} \times \vec{B} + \mu \nabla^2 \vec{V}$$

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \vec{V}) = S_M$$

$$\frac{3}{2}\frac{\partial p}{\partial t} + \nabla \bullet \left(\vec{q} + \frac{5}{2} P \bullet \vec{V}\right) = \vec{J} \bullet \vec{E} + S_E$$

$$\frac{3}{2}\frac{\partial p_e}{\partial t} + \nabla \bullet \left(\vec{q}_e + \frac{5}{2} P_e \bullet \vec{V}_e\right) = \vec{J} \bullet \vec{E} + S_E$$

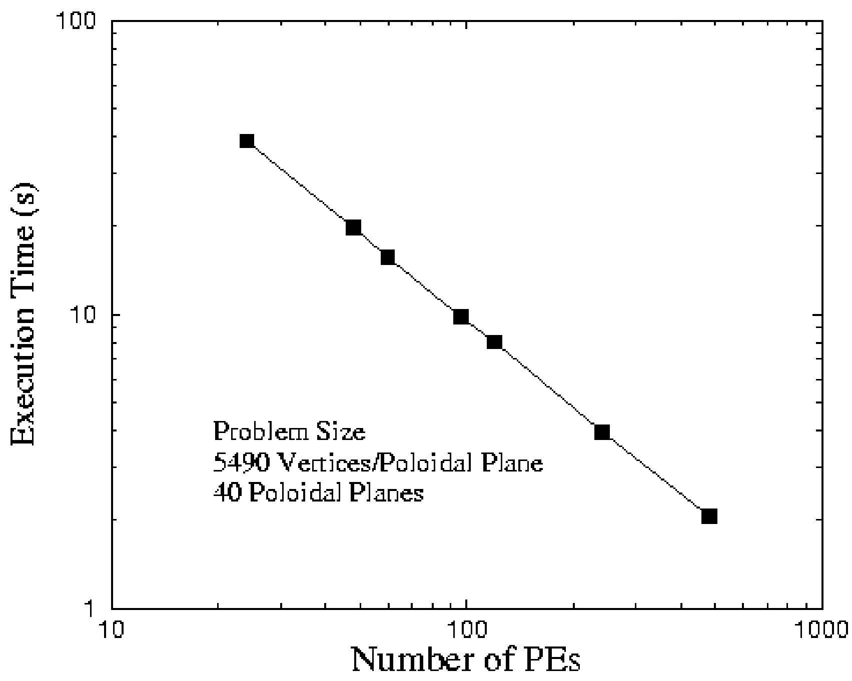
Two-fluid XMHD: define closure relations for Π_i, Π_e, q_i, q_e

Hybrid particle/fluid XMHD: model ions with kinetic equations, electrons either fluid or by drift-kinetic equation

Simulation Codes:

NIMROD: semi-implicit time integration, 2D quad and triangular finite elements+ pseudospectral, grid packing, AZTEC, MPI

M3D: quasi-implicit time integration, stream-function/potential representation, 3D Mesh, PETSc, MPI



M3D: Considerations for a Scalar Representation

- Allow accurate description of plasma motion with $\nabla \bullet (\mathbf{V}_\perp / R^2) = 0$
- Isolate fast wave in small number of variables for implicit solution
- Separate variables for representation of $\mathbf{B_P}$ and $\mathbf{B_T}$ to enable efficient implicit solution of field diffusion
- Accurate representation of $\nabla \bullet \mathbf{B} = 0$
- Avoid repetitive solution of 3D elliptic equations

$$\vec{V} = R^2 \nabla U \times \nabla \phi + \nabla_\perp \chi + v_\varphi R \nabla \phi$$

$$U,\ \chi,\ v_\varphi$$

$$\psi,\ f$$

$$\vec{A} = \psi \nabla \phi + R \nabla f \times \nabla \phi$$

$$p,\ \rho$$

$$(note: this\ \ implies\ gauge\ \ \nabla_\perp \bullet \vec{A}_\perp \equiv \nabla \bullet \left[R \left(\nabla \phi \times \vec{A} \right) \times \nabla \phi \right] = 0)$$

$$\frac{\partial Z}{\partial t} = -I\Delta^{*}\underline{I} - \Delta^{*}\underline{p} + \frac{\mu}{\rho}\nabla^{2}\underline{Z}\ldots$$

$$\frac{\partial I}{\partial t} = -I\underline{Z} + \eta\Delta^{*}\underline{I}\ldots$$

$$\frac{\partial p}{\partial t} = -\gamma p\underline{Z}\ldots$$

$$\frac{\partial C}{\partial t} = \eta\Delta^{*}\underline{C} + \ldots$$

$$\frac{\partial W}{\partial t} = \frac{\mu}{\rho}\nabla^{2}\underline{W} + \ldots$$

$$\frac{\partial v_{\varphi}}{\partial t} = \frac{\mu}{\rho}\nabla^{2}\underline{v_{\varphi}}\ldots$$

$$\frac{\partial d}{\partial t} = \ldots$$

$$\Delta^{*}\chi = Z$$

$$\Delta^{\dagger}U = W$$

$$\nabla_{\perp}^{2}\Phi = \ldots$$

$$\nabla_{\perp}^{2} f = -I/R$$

$$\Delta^{*}\psi = C$$

Each Time Step:

3 *coupled implicit time advance equations*

3 *uncoupled implicit time advance equations*

1 *explicit time advance*

5 *elliptic solves...but all 2D*

M3D Discretization

- Toroidal geometry is discretized by a set of regularly-spaced poloidal sections

- The mesh is unstructured only in the poloidal sections

- Code uses 2D triangular linear finite elements in the poloidal section $\lambda_i(\mathbf{r})$

- Scalar variables χ, ψ are expanded in basis function, eg

$$\psi(\mathbf{r},t) = \sum \psi_i(t)\, \lambda_i(\mathbf{r})$$

- Equations are discretized using Galerkin method

$$\int \lambda_i(\mathbf{r}) \; [\text{equation}] \; d^2x$$

- The toroidal derivatives are calculated directly on the structured toroidal grids.

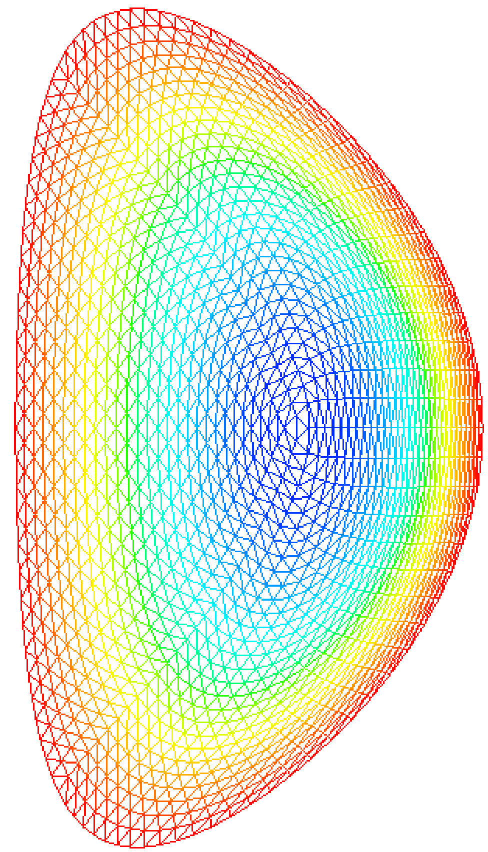
Domain Decomposition

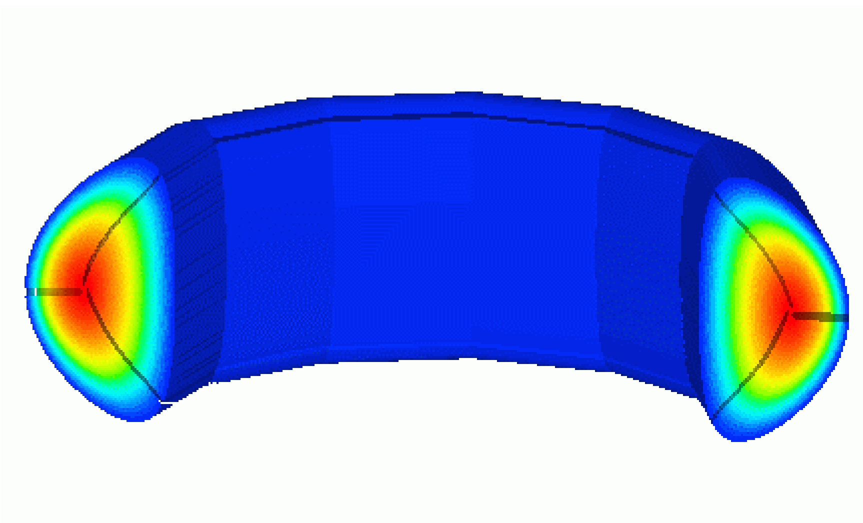
Toroidal geometry is sliced into a set of poloidal planes

Poloidal plane further partitioned into equal area patches

One or more poloidal patches assigned to each processor

gives excellent load balance





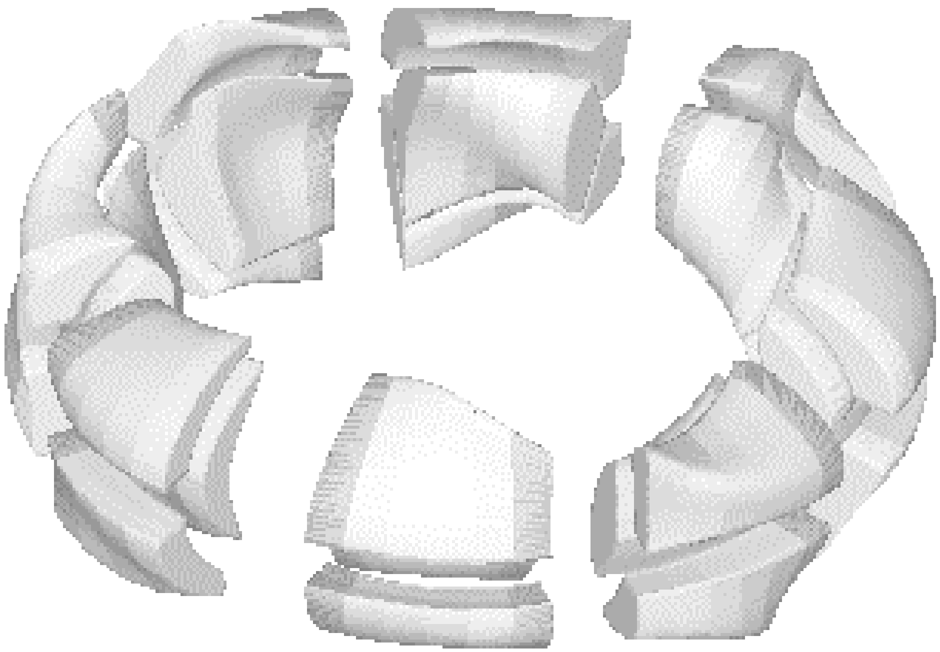


Illustration of 3D Domain Decomposition used in stellarator simulation

Utilization of PETSc

- The parallel data layout is done in the framework of PETSc
- PETSc provides Krylov accelerated iterative solvers
- PETSc provides overlapped Schwarz preconditioner
- Impact of PETSc
    - expedite the code development cycle
    - impose discipline and helps produce a compact code
    - The price is the effort to learn a pseudo-language
- Overall, we are happy with PETSc and would recommend it.

CEMM Interests in ISIC centers

Incorporation of “standard” grid generation and discretization libraries into M3D (and possibly NIMROD)

Higher order and mixed type elements

Explore combining potential and field advance equations

Extend the sparse matrix solvers in PETSc in several ways that will improve the efficiency of M3D

- Develop multilevel solvers for stiff PDE systems
- Take better advantage of previous timestep solutions
- Refinements in implementation to improve cache utilization

Implement and evaluate adaptive mesh refinement (AMR) for reconnection and localized instability growth

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