Nature and Properties of Turbulent Cascades

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What is turbulence?

No exact definition. Loosely, random motion of fluid where many nonlinearly interacting modes are involved.

Plasma in astrophysical systems (solar wind, interstellar medium, galaxy clusters, etc) is typically magnetized and turbulent.

Why is turbulence important?

- solar wind heating
- solar corona heating
- magnetic reconnection, magnetic dynamo action
- magnetic and density structures in the ISM (scintillation of radio sources, star formation in molecular clouds, magnetic structures in Galaxy center, etc)…
- cosmic ray acceleration and scattering
- heat conduction in galaxy clusters
- transport in fusion devices
- etc.

Signatures of turbulence

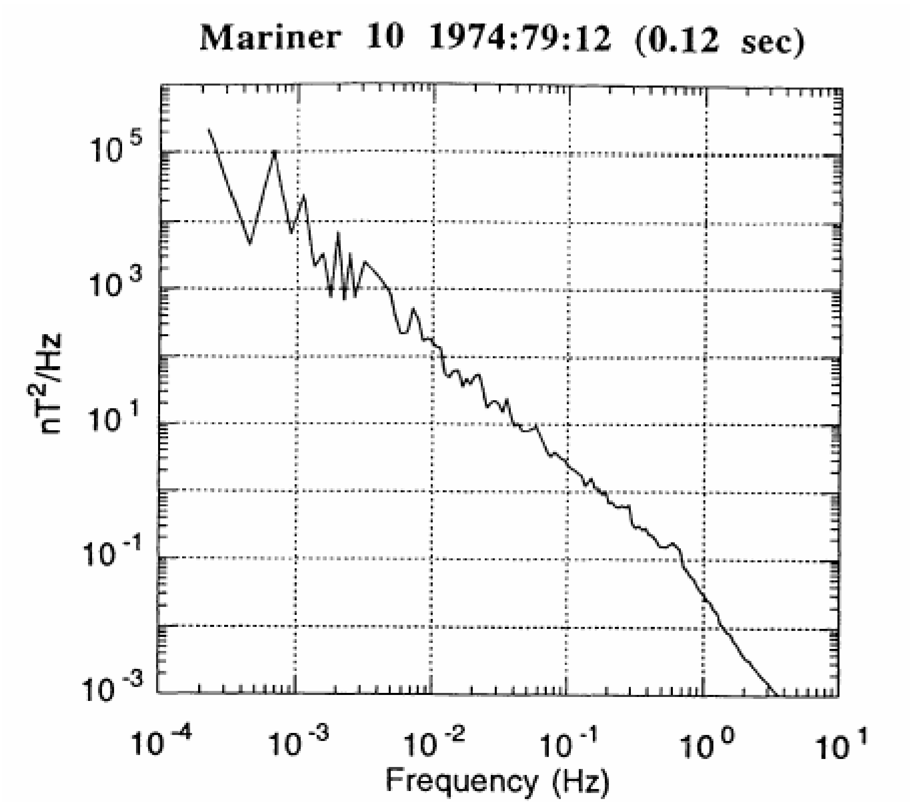
Scaling relations: spectra, structure functions, etc.

Structures: filaments, current sheets, etc.

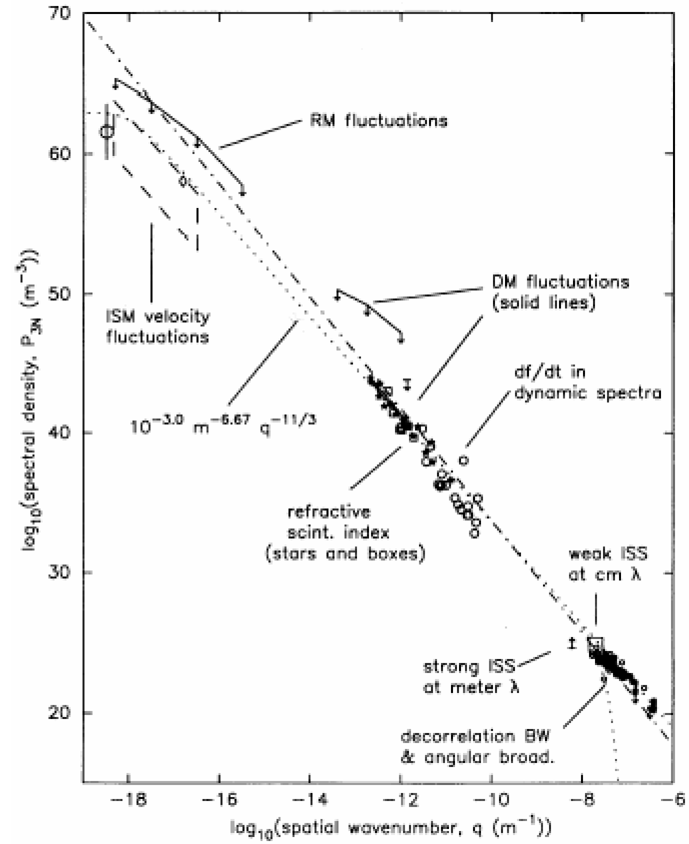
Different turbulent systems have similar signatures…

Magnetic turbulence in nature

Energy spectra



Solar wind

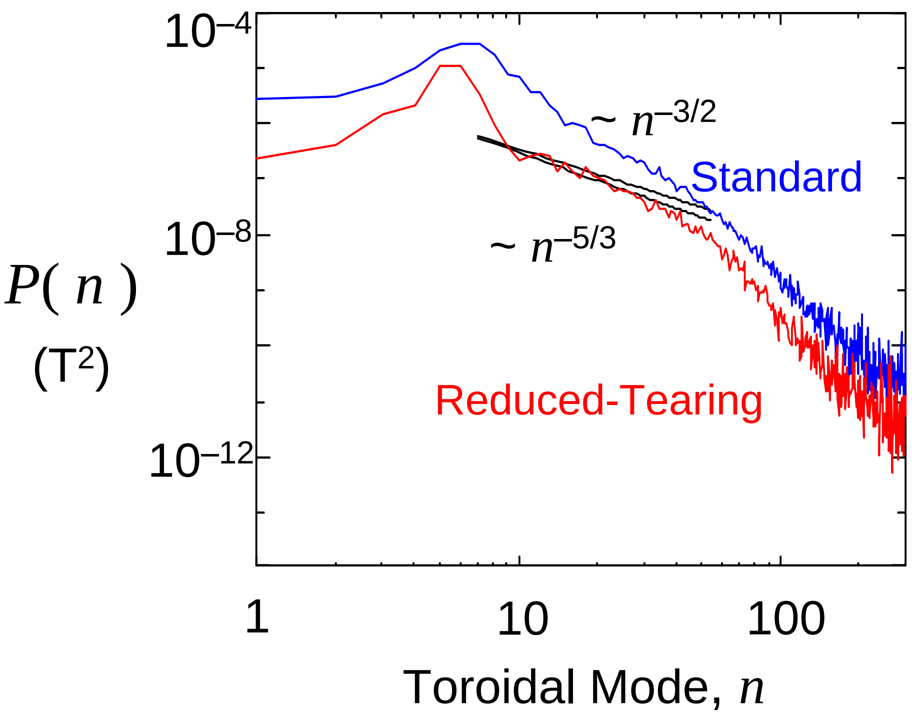


ISM

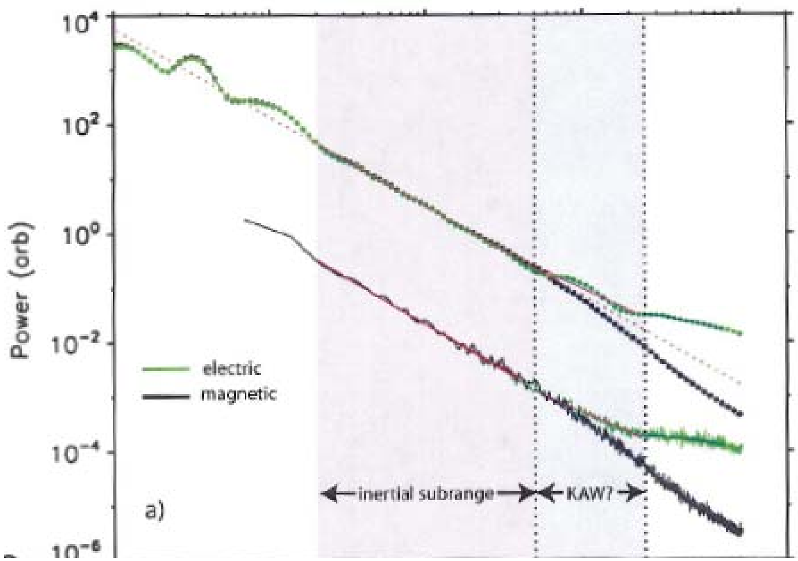
[Goldstein, Roberts, Matthaeus (1995)] [Armstrong, Rickett, Spangler (1995)]

Magnetic turbulence in nature

Energy spectra



(MST experiment)



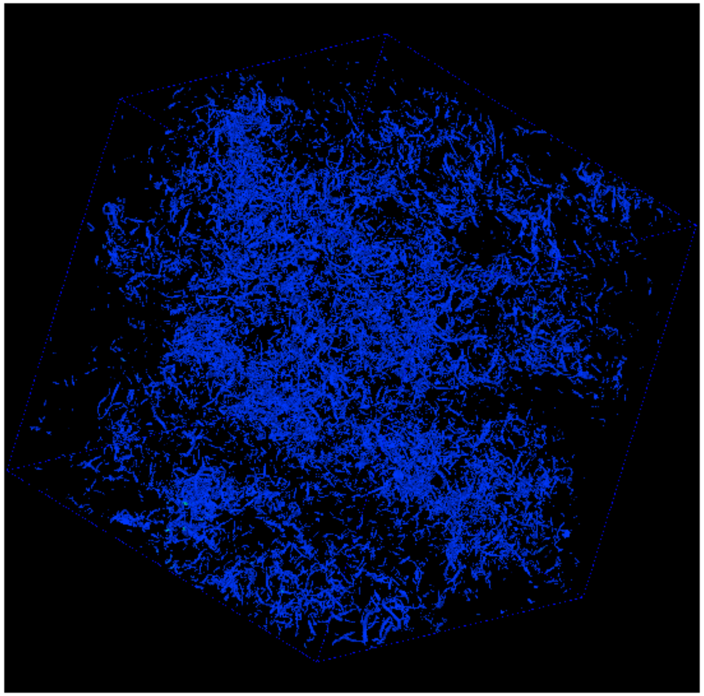
Solar wind (V, B)
Bale et al 2005

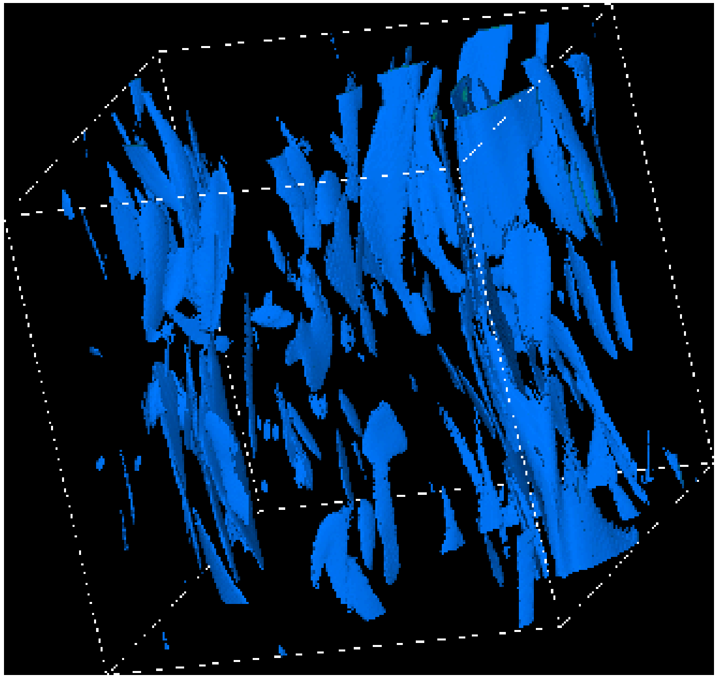
Magnetic turbulence in nature

structures

Neutral fluid, B=0

MHD, B>0





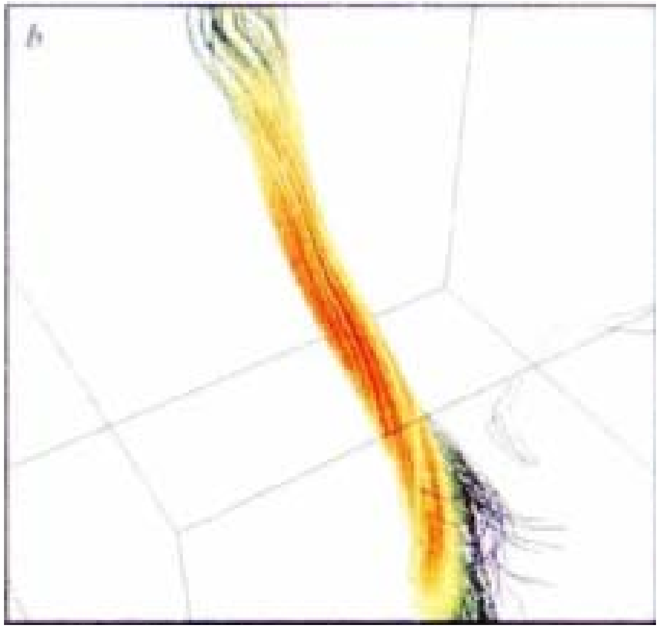
Filaments

"Ribbons" stretched along B

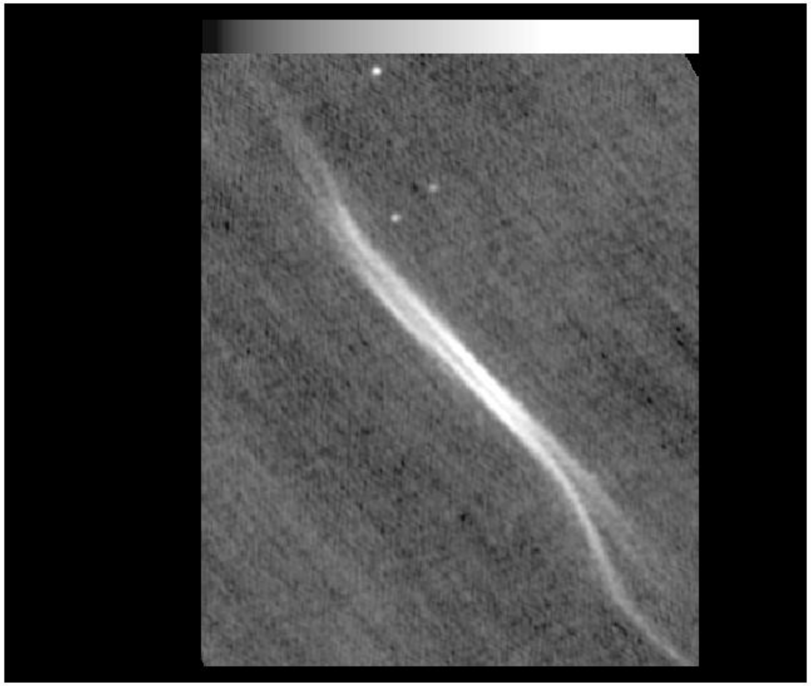
Biskamp & Muller (2000)

Magnetic turbulence in nature

structures



Vorticity filament in fluid turbulence (numerics) She et al 1990



Nonthermal radio filament in the Galactic center. Yusef-Zadeh et al

Magnetic turbulence in nature: major problems

1. Do observed power-law spectra imply that turbulence is present?
-- not always... E.g., discontinuous, shock-dominated non-turbulent fields produce energy spectrum $E(k) \sim k^{-2}$, same as weak MHD turbulence.

2. Do turbulent systems observed in nature exhibit universal behavior?
-- not necessarily. E.g., magnetic and velocity fluctuations in the solar wind have different spectra at different distances from the Sun. Two-fluid effects, compressibility effects, kinetic effects, etc. typically invalidate one-fluid incompressible MHD description.

3. Does MHD turbulence have universal regimes (similar to hydrodynamic turbulence, that is, independent of driving, dissipation, etc)?
-- possibly yes, as is seen from recent high-resolution numerical simulations and analytic studies.

Observations, laboratory, analytical, and numerical studies are required to answer these questions.

MHD turbulence cascades: search for universality

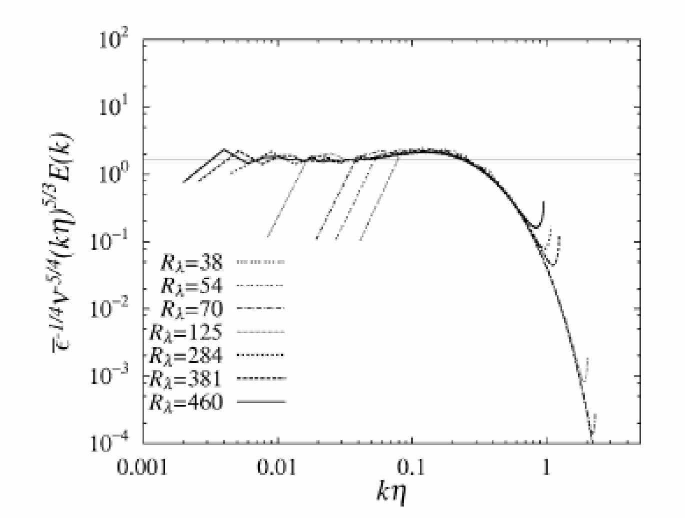


Fig. 1. Scaled energy spectra, $\bar{\epsilon}^{-1/4}\nu^{-5/4}(k\eta)^{5/3}E(k)$. The inertial range is between $0.007 \leqslant k\eta \leqslant 0.04$. $K = 1.64 \pm 0.04$. The horizontal line indicates $K = 1.64$.

Gotoh 2002

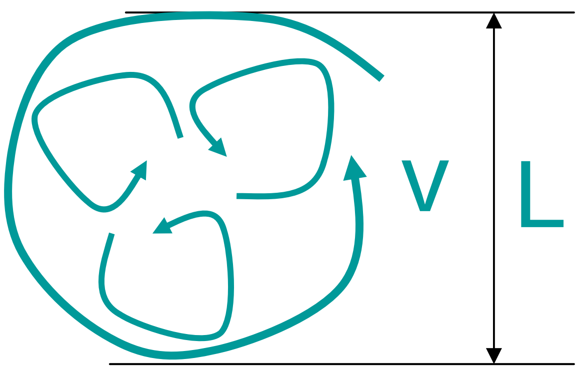
Hydrodynamic turbulence is universal. Has same spectrum (close to -5/3) independent of large-scale driving and small-scale dissipation

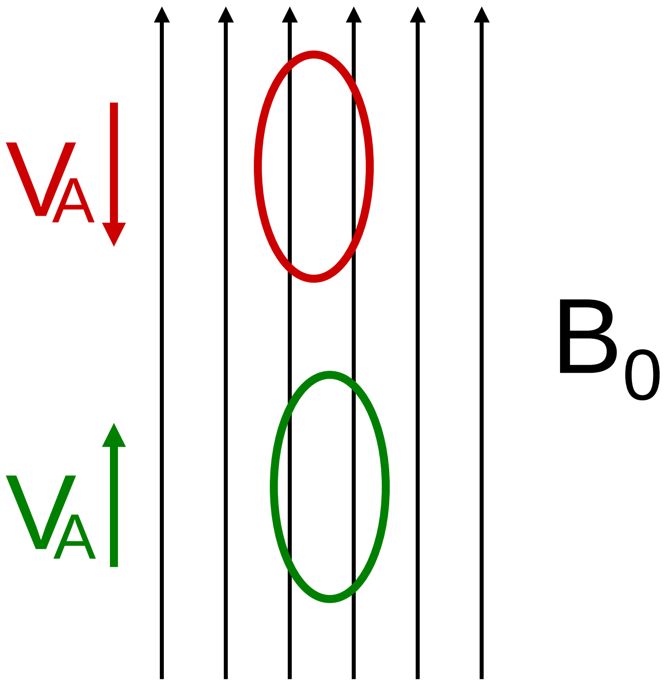
Are there universal regimes of magnetized turbulence?
What regimes can be efficiently studied (analytically, numerically, etc)?

Nature of MHD turbulence cascades

HD turbulence:
interaction of eddies

MHD turbulence:
interaction of wave packets
moving with Alfven velocities



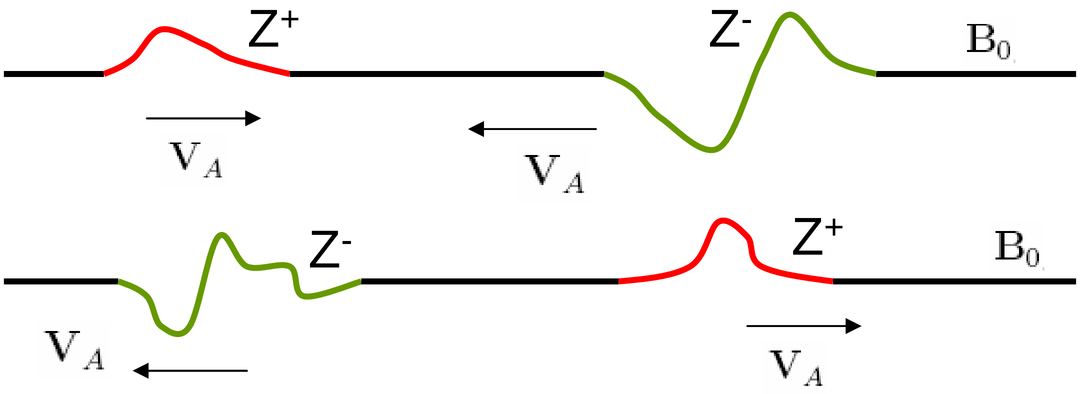


Nature of MHD turbulence: Alfvenic cascade

$$\partial \mathbf{z}^{\pm} \mp (\mathbf{v}_A \cdot \nabla)\mathbf{z}^{\pm} + (\mathbf{z}^{\mp} \cdot \nabla)\mathbf{z}^{\pm} = -\nabla P + \frac{1}{Re}\nabla^2 \mathbf{z}^{\pm} + \mathbf{f}^{\pm}$$

Ideal system conserves the Elsasser energies

$$E^{+} = \int (\mathbf{z}^{+})^2\, d^3x \qquad E = \frac{1}{2}\int (v^2 + b^2) d^3x$$
$$\equiv$$
$$E^{-} = \int (\mathbf{z}^{-})^2\, d^3x \qquad H^C = \int (\mathbf{v}\cdot\mathbf{b}) d^3x$$



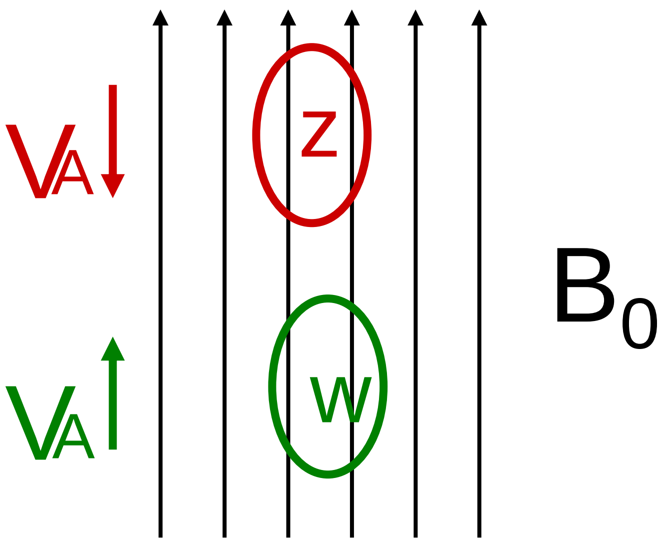
E+ and E- are independently conserved in ideal MHD.

Iroshnikov-Kraichnan picture

Nature of MHD turbulence: Search for universality

$$\partial_t \mathbf{z} + (\mathbf{V}_A \cdot \nabla)\mathbf{z} + (\mathbf{w} \cdot \nabla)\mathbf{z} = -\nabla P,$$
$$\partial_t \mathbf{w} - (\mathbf{V}_A \cdot \nabla)\mathbf{w} + (\mathbf{z} \cdot \nabla)\mathbf{w} = -\nabla P,$$

Limiting cases of MHD turbulence –
Four universal regimes



	$z \sim w$ balanced	$z(\gg,\ll)w$ Imbalanced, cross-helical
$k_\parallel V_A \gg k_\perp(z,w)$ weak turb	√	√ ?
$k_\parallel V_A \sim k_\perp(z,w)$ strong turb	√	√ ?

Universal regimes: Computational challenges

Incompressible MHD	$z \sim w$ balanced	$z(\gg, \ll)w$ imbalanced, cross-helical
$k_{\parallel} V_A \gg k_{\perp}(z,w)$ weak turb	√	√ ?
$k_{\parallel} V_A \sim k_{\perp}(z,w)$ strong turb	√	√ ?

$$\partial_t \mathbf{z} + (\mathbf{V}_A \cdot \nabla)\mathbf{z} + (\mathbf{w} \cdot \nabla)\mathbf{z} = -\nabla P,$$
$$\partial_t \mathbf{w} - (\mathbf{V}_A \cdot \nabla)\mathbf{w} + (\mathbf{z} \cdot \nabla)\mathbf{w} = -\nabla P,$$

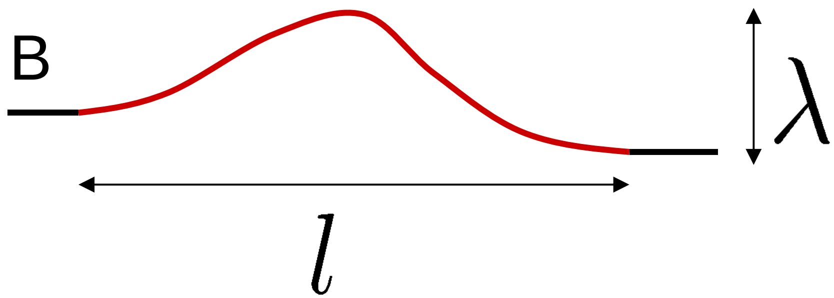
Pseudo-spectral MHD codes, reduced MHD codes. Parallelized codes, run on supercomputers

Numerical study of these regimes is more challenging than the study of hydrodynamic turbulence.

State-of-the-art simulations can have resolution 2048^2 in the guide-field-perpendicular direction. Require thousands of processors, and millions of CPU hours.

Strong MHD turbulence

Anisotropy of “eddies”



$$\partial_t \mathbf{z} + (\mathbf{V}_A \cdot \nabla)\mathbf{z} + (\mathbf{w} \cdot \nabla)\mathbf{z} = -\nabla P,$$
$$\partial_t \mathbf{w} - (\mathbf{V}_A \cdot \nabla)\mathbf{w} + (\mathbf{z} \cdot \nabla)\mathbf{w} = -\nabla P,$$

$$\boxed{V_A/l \sim \delta b_\lambda/\lambda}$$

Critical Balance

$$l \gg \lambda$$

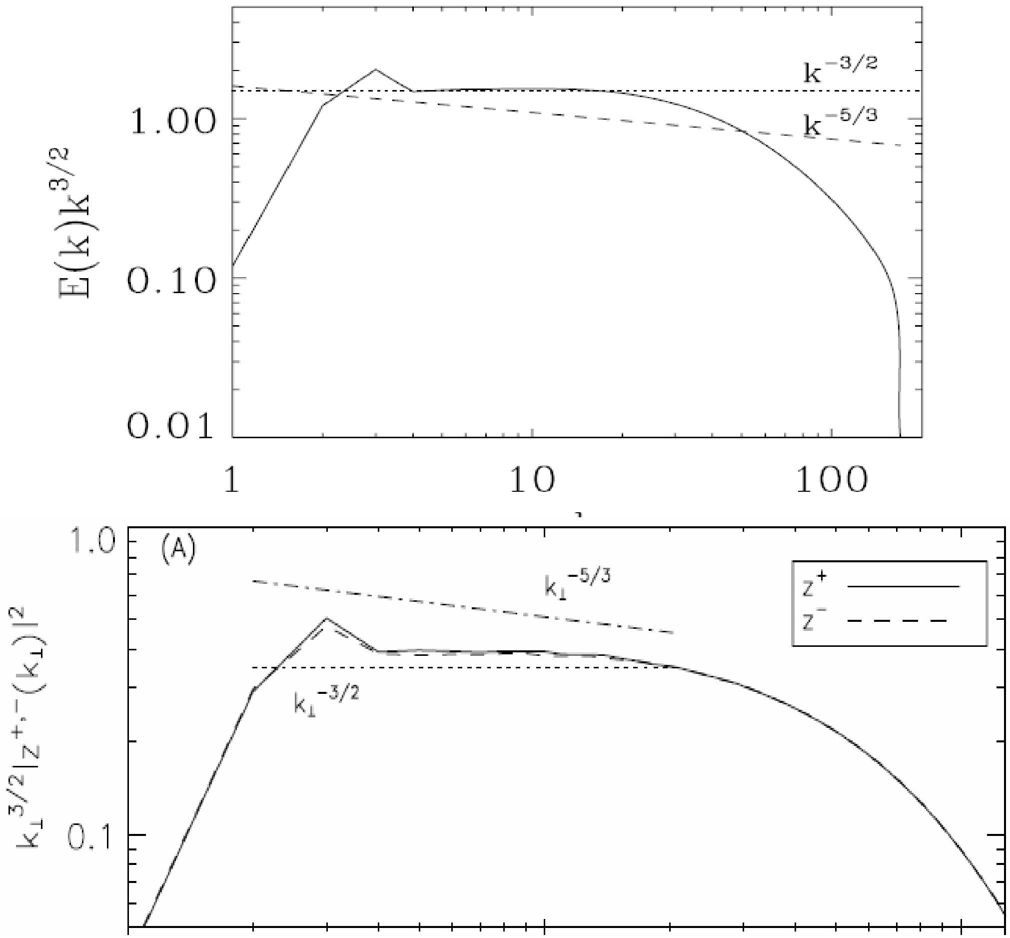
[Goldreich & Sridhar 1995]

Shear Alfvén waves
dominate the cascade:

$$\delta \mathbf{w}_\lambda,\ \delta \mathbf{z}_\lambda \perp \mathbf{B}_0$$

Energy spectrum: $\boxed{E(k_\perp) \propto k_\perp^{-5/3}}$

Spectrum of strong MHD turbulence



Spectrum $E(k_\perp) \propto k_\perp^{-3/2}$

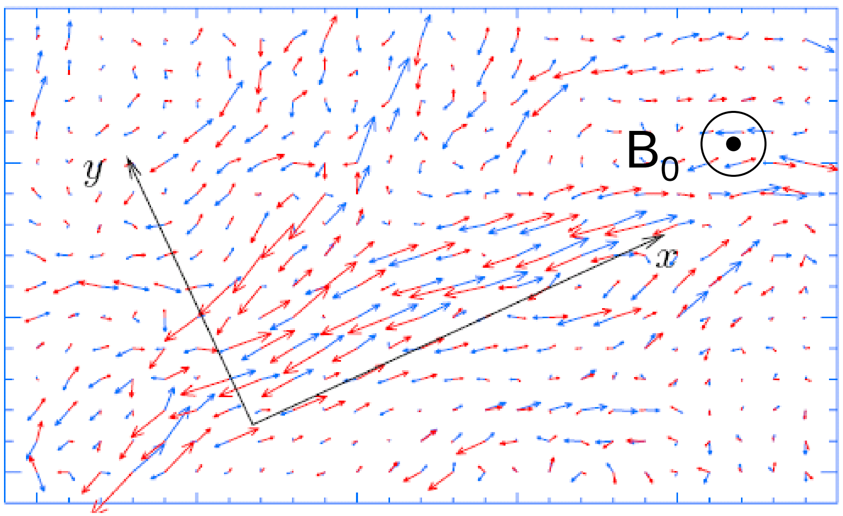
Note the difference with the analytic prediction -5/3

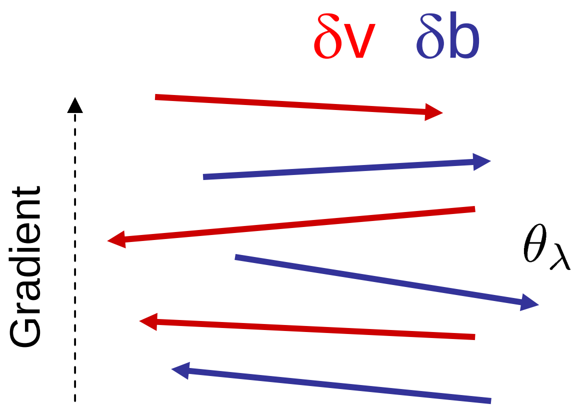
Mason et al 2008, Perez& SB 2008

Possible explanation of the -3/2 spectrum

Dynamic Alignment theory

Fluctuations $\delta \mathbf{v}_\lambda$ and $\delta \mathbf{b}_\lambda$ become spontaneously aligned in the field-perpendicular plane within angle θ_λ





Nonlinear interaction is depleted ⇐ $(\mathbf{z} \cdot \nabla)\mathbf{w} \sim \theta_\lambda \delta v_\lambda^2/\lambda$

S.B. ApJ 626 (2005) L37, Perez & S.B. PRL (2009)

Unbalanced MHD turbulence

(imbalanced, cross-helical…)

Imbalance means that cross-helicity is nonzero:

$$H^C = \int (\mathbf{v}\cdot\mathbf{b}) d^3x = \frac{1}{4}(E^+ - E^-) \neq 0$$

Or, equivalently, the energies of waves traveling in opposite directions along the guide field are not equal. This is a very common situation in nature:

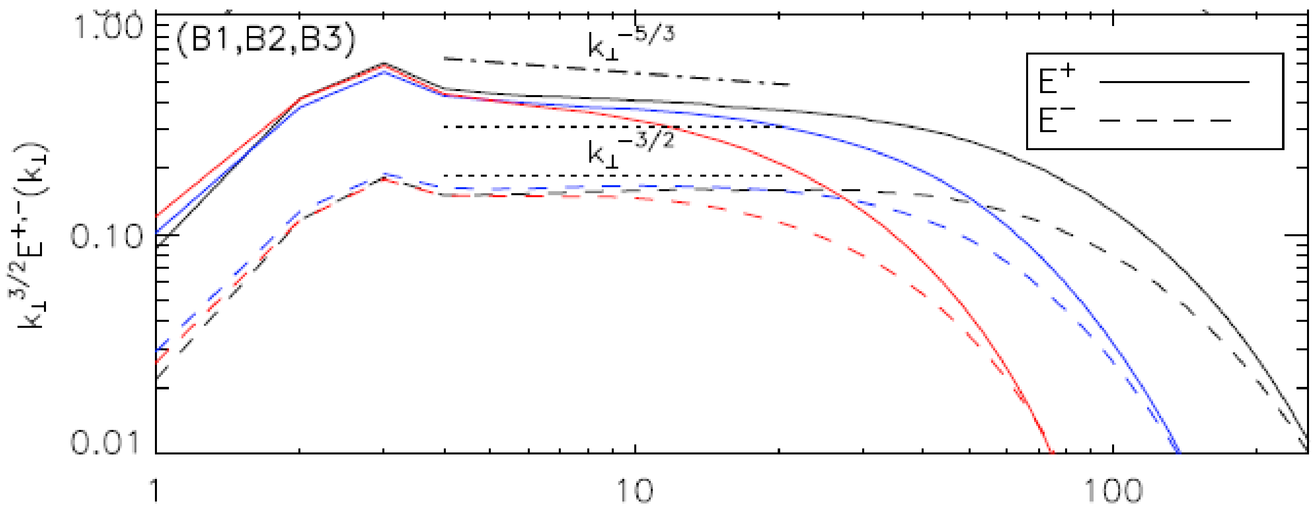
• Solar wind: more Alfven waves travel out of the sun than toward the sun

• Interstellar medium: MHD turbulence is driven by spatially localized sources

• Even when balanced overall, MHD turbulence is always locally unbalanced– it creates patches of positive and negative cross-helicity.

[Lithwick & Goldreich (2003); Ng et al (2003); Rappazzo et al (2007); Chandran (2008); Beresnyak & Lazarian (2008); Matthaeus et al (2008); Perez & SB (2009)]

Strong Imbalanced MHD turbulence: Numerics



Spectra of E^+ and E^- for different magnetic Reynolds numbers, up to Rm~5000

Perez & SB 2010

Strong MHD turbulence: summary of numerical results

- In balanced MHD turbulence, both spectra have same amplitudes and spectra

$$E^+(k_\perp) \sim E^-(k_\perp) \sim k_\perp^{-3/2}$$

- In imbalanced MHD turbulence, the spectra E^+ and E^- have different amplitudes, but same scaling:

$$E^+(k_\perp) \propto E^-(k_\perp) \propto k_\perp^{-3/2}$$

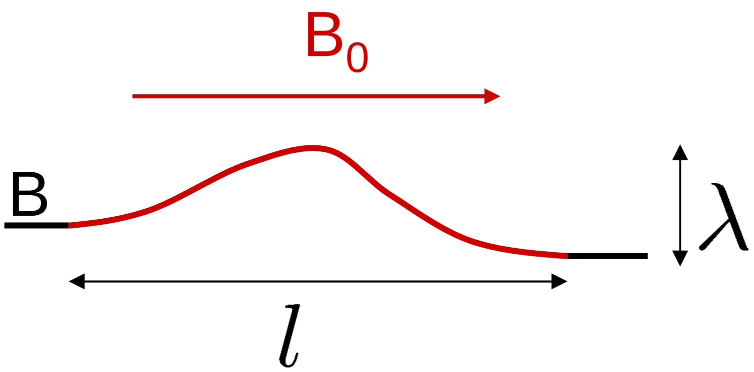
- MHD turbulence is always locally imbalanced, at all scales. This hierarchical structure is a fundamental property of MHD turbulence, which results from cross-helicity conservation.

Puzzles, unsolved questions

Turbulence spectra in the solar wind are close to $E(k_\perp) \propto k_\perp^{-5/3}$ while numerical simulations consistently produce -3/2.
(e.g., Bale et al 2005, Muller & Grappin 2005).
This discrepancy is one of the principal unsolved questions. Is incompressible homogeneous MHD turbulence a bad model for the solar wind?

Numerical simulations of imbalanced MHD turbulence suggest that the spectra E+ and E- have different amplitudes, but same scaling: $E^+(k_\perp) \propto E^-(k_\perp) \propto k_\perp^{-3/2}$, which agrees with some theories, numerics, and observations (e.g., Perez & SB 2009, Podesta & Bhattacharjee 2009), and contradicts others (e.g., Beresnyak & Lazarian 2008, Chandran 2008). There has been no satisfying resolution to this contradiction.

Weak MHD turbulence (wave turbulence)



Shear Alfvén waves
dominate the cascade:

$$\mathbf{z}^{+}_{\lambda}, \mathbf{z}^{-}_{\lambda} \perp \mathbf{B}_0$$

MHD equations in Elsasser variables

$$\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}$$

$$\partial_t \mathbf{z}^{+} - (\mathbf{v}_A \cdot \nabla)\mathbf{z}^{+} + (\mathbf{z}^{-} \cdot \nabla)\mathbf{z}^{+} = -\nabla P$$

$$\underbrace{\partial_t \mathbf{z}^{-} + (\mathbf{v}_A \cdot \nabla)\mathbf{z}^{-}}_{V_A/l \gg b_\lambda/\lambda} + (\mathbf{z}^{+} \cdot \nabla)\mathbf{z}^{-} = -\nabla P$$

Weak turbulence

Weak MHD turbulence: Phenomenology

Three-wave interaction of shear-Alfven waves

$$\omega(k) = |k_z| v_A$$

$$\begin{cases} \omega(k) = \omega(k_1) + \omega(k_2) \\ \mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2 \end{cases}$$

Only counter-propagating waves interact, therefore, k_{1z} and k_{2z} should have opposite signs.

$\Downarrow$

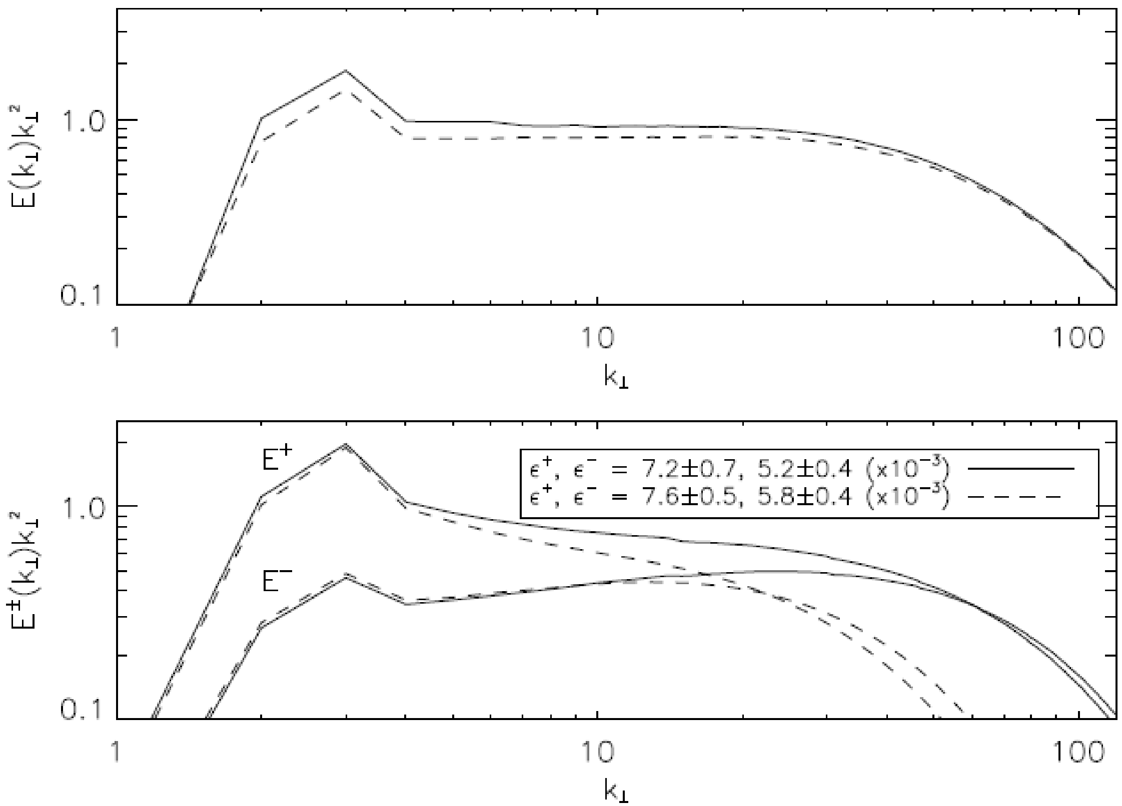
Either $k_{1z} = 0$ or $k_{2z} = 0$

Wave interactions change $k_\perp$ but not k_z

At large $k_\perp$: $E(k_z, k_\perp) \propto g(k_z) k_\perp^{-\beta}$

Montgomery & Turner 1981, Shebalin et al 1983

Unbalanced weak MHD turbulence: Numerical results



Balanced

$$E(k_\perp) \propto k_\perp^{-2}$$

Unbalanced

SB & Perez 2009

The spectra are essentially different from strong MHD turbulence. In the unbalanced case, they contradict analytical theory, e.g., Galtier et al 2000

Directions of research

- Observation/laboratory opportunities: identify the regimes of turbulence. Measurements of the guide-field parallel and perpendicular spectra, measurements of v- and b-spectra, measurements of the aligment effects between v and b vectors, etc.

- Analytic work: Need to develop a good theory for unbalanced (cross-helical) MHD turbulence. Need to reconcile theories and to correctly interpret the results of numerical simulations.

- Numerical work: much effort should be devoted to unbalanced turbulence (currently only few numerical results are available). Significantly higher numerical resolution (compared to balanced case) may be required.