Sources, Boundary Conditions, and Parameters in the M3D CDX Studies, and Comparison to NIMROD Parameters

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I. Current sources and boundary conditions

A. Current source term

The toroidal current density in M3D is given by

$$J_\varphi \equiv -\frac{C}{R} = -\frac{1}{R}\left(\Delta^*\psi + \frac{1}{R}\frac{\partial F}{\partial z}\right) = -\frac{1}{R}\left(C_A + \frac{1}{R}\frac{\partial F}{\partial z}\right) \tag{1.1}$$

The parallel version of M3D evolves C directly (rather than ψ); hyperbolic terms are treated explicitly, while the parabolic resistive term is implemented by the implicit solution of the equation

$$\frac{\partial C_A}{\partial t} = \Delta^*\left(\eta(\vec{x})C_A\right) \tag{1.2}$$

That is, discretizing in time, we solve

$$\left(\frac{1}{R}\Delta^* - \frac{1}{\eta R\delta t}\right)\left(\eta C_A^{n+1}\right) = -\frac{C_A^n}{R\delta t} \tag{1.3}$$

However, if there is a current source term (as in the CDX runs), we define C_B to be the value of C_A at time zero, and then apply the resistivity only to deviations from this source, solving

$$\left(\frac{1}{R}\Delta^* - \frac{1}{\eta R\delta t}\right)\left[\eta\left(C_A^{n+1} - C_B\right)\right] = -\frac{C_A^n - C_B}{R\delta t} \tag{1.4}$$

instead at each subsequent time step. This method will drive the same current profile C_B for any resistivity profile η. η may in general evolve in time, though in the CDX case it does not.

For the CDX study, $\eta(x)$ is set up by first computing a Spitzer profile based on the initial temperature distribution:

$$\eta = T^{-3/2}\ln\Lambda \tag{1.5}$$

where the coulomb logarithm is defined as

$$\ln\Lambda \equiv 23 - \ln\left(n^{1/2}T^{-3/2}\right) \tag{1.6}$$

with n in cm^{-3} and T in eV. The resulting profile is then renormalized so that its smallest value, at the peak of the temperature profile is 1.0. Next, a cutoff is applied, restricting the maximum resistivity to 100 times the minimum value to avoid numerical problems caused by sharp gradients at the edge; this is applied smoothly by making the substitution

$$\eta(\vec{x}) \to 1.0 + 99.0\tanh\left(\frac{\eta(\vec{x}) - 1.0}{99.0}\right) \tag{1.7}$$

Finally, this profile is multiplied by an overall normalization of 5.15×10^{-5} (equivalent to S=1.94×10^4 in the center).

B. Loop Voltage

An alternate (or complementary) way to drive a toroidal current in M3D is by applying an emf (loop voltage). This voltage will appear as a boundary condition, and will be chosen to be just that amount required to maintain the current density distribution at its initial (presumed equilibrium) value. Because C is the quantity evolved in the parallel version, this reduces to the simple Dirichlet condition that the toroidal current density on the boundary be equal to its initial value.

By convention, if the loop voltage condition is being used in M3D, then the resistivity profile is initialized as

$$\eta(\vec{x}) = \frac{\eta_0}{C_{A0}(\vec{x})} = \frac{\eta_0}{\Delta^* \psi_0(\vec{x})}$$

for any user-specified scalar resistivity η_0. This ensures that, in the absence of flows, the initial electric field is curl-free, as required for a stationary state. When resistivity is set up this way, the correct boundary condition is

$$C_A(\vec{x})\Big|_{\text{boundary}} = \frac{\eta_0}{\eta(\vec{x})}.$$

II. Toroidal Field

The part of the poloidal current density that is not contained in the flux function ψ is instead contained in the toroidal field variable, defined in M3D as

$$\tilde{I} \equiv RB_\varphi - R_0, \tag{1.8}$$

where the cylindrical coordinate R and plasma major radius R_0 have been normalized to the minor radius, and B_φ has been normalized to its on-axis value. The implicitly advanced diffusive component of the time evolution of $\tilde{I}$ is given by

$$\frac{\partial \tilde{I}}{\partial t} = \nabla_\perp \cdot \left(\eta \nabla_\perp \tilde{I}\right) \tag{1.9}$$

which is handled in the usual way, i.e., inversion of the linear equation

$$\left(\nabla_\perp \cdot \eta \nabla_\perp - \frac{1}{\delta t}\right)\left(\tilde{I}^{n+1} - \tilde{I}_S\right) = -\frac{\left(\tilde{I}^n - \tilde{I}_S\right)}{\delta t} \tag{1.10}$$

where $\tilde{I}_S$ is a source term that will in some cases be set to the initial value $\tilde{I}_0$, but is in the CDX case simply set to zero, as it is not needed so long as the toroidal current and the pressure are supported by source terms.

III. Temperature/Pressure Sources

If `idench.ge.1` in the input file, indicating that the density is evolving, as it is in the CDX cases, then the energy equation solved by M3D is for the temperature; otherwise the pressure equation is solved. In either case, the diffusive term in the equation is handled implicitly by routine `pkkks`(), which applies the diffusion to the deviation from a source term: p_0 for the pressure equation or p_0/ρ_0 for the temperature:

$$\frac{\partial}{\partial t}\left(T - p_0/\rho_0\right) = \kappa_\perp \nabla_\perp^2 \left(T - p_0/\rho_0\right)$$
$$\rightarrow \left(\nabla_\perp^2 - \frac{1}{\kappa_\perp \delta t}\right)\left(T^{n+1} - p_0/\rho_0\right) = -\frac{\left(T^n - p_0/\rho_0\right)}{\kappa_\perp \delta t} \tag{1.11}$$

In the CDX study, I am using `pkkk`$\equiv \kappa_\perp =$`9.09e-4`, which was determined to correspond to a dimensional value of ~200 m^2/s. There is no ohmic (or viscous) heating in this study, so this is the only term tending to restore the temperature peak after a sawtooth crash.

IV. Density Source

In the presence of a nonzero density diffusion coefficient (as in the CDX cases), diffusion is applied only to the departure of the density from its initial value. This diffusion, of course, does not appear in the fluid equations but is added to improve numerical stability. Alternately, it could be considered to arise from sub-gridscale turbulence that is not handled directly by the code.

After explicitly advecting and compressing the density via the continuity equation, M3D implicitly applies the diffusion operator:

$$\frac{\partial}{\partial t}\left(\rho - \rho_0\right) = D\nabla_\perp^2 \left(\rho - \rho_0\right)$$
$$\rightarrow \left(\nabla_\perp^2 - \frac{1}{D\delta t}\right)\left(\rho^{n+1} - \rho_0\right) = -\frac{\left(\rho^n - \rho_0\right)}{D\delta t} \tag{1.12}$$

In the CDX study, I have D = `pdissf=1.0e-3`.

V. Other Parameters

As in the original plan, I am using a constant, uniform scalar viscosity in the momentum equation, equal to ten times the minimum resistivity in dimensionless form, i.e., μ = 5.15×10^{-4} everywhere.

My toroidal resolution is 24 planes for the original, low-res study, allowing me to resolve modes $0 \le n \le 10$; and 48 planes for the high-res follow-up, resolving modes $0 \le n \le 22$. Both use 79 zones in the radial direction with a poloidal symmetry of three, giving them 9244 vertices and 18,252 triangles per plane. Linear elements are used throughout.

The initial condition for the nonlinear study, as mutually agreed during an early discussion between the M3D and NIMROD teams, was to begin with the equilibrium plus a perturbation corresponding to the linear n=1 eigenmode at an amplitude such that the ratio of the maximum B_p in the n =1 to the maximum B_T in the n =0 piece was 10^{-4}.

VI. NIMROD Parameters

NIMROD is following modes $0 \le n \le 41$, evidently with 30 by 30 rectangular elements of higher than first order. These runs have S = 2.06×10^4, slightly higher than mine. The

NIMROD run, unlike the M3D run, included both ohmic heating and time-evolving resistivity. The Prandtl number (10) appears to be the same, though it is unclear whether viscosity is evolving in NIMROD as well. $\kappa_\perp$ is also the same, with $\kappa_\parallel$ presumed "equivalent" (10^8 for NIMROD; $V_{Te} = 6\ V_A$ for M3D).

The NIMROD run was stabilized with a density diffusion coefficient of 1000 m^2/s, with a note in Dalton's presentation saying that 50 m^2/s produced the same results. The effective value in M3D is approximately 220 m^2/s, so this seems unlikely to be a source of disagreement in the results.

According to Carl's note on Ohmic current drive in NIMROD, the resistive part of the magnetic field equation is

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \left(\eta \vec{J} - \vec{E}_{SS} \right) = -\nabla \times \left(\eta \vec{J} - \eta_{SS} \vec{J}_{SS} \right) \tag{1.13}$$

which is identical to the current source formulation in M3D except for the evolving η profile.

Dalton also claims a value of q_0 = 0.98 versus q_0 = 0.92 for my case, which if true indicates that we're no longer starting from the same equilibrium (CDX run06, time11). His run also does not appear to be using the same convention for the initial perturbation, but begins with all mode energies in the 10^{-13} range.