Adding a Thin Rectangular Resistive Wall to M3D-C^1

June 12, 2008

We want to add a thin resistive wall to the M3D-C1 code. Phase 1 is to incorporate this as a boundary condition on a *rectangular boundary*. Phase 2 will extend this to an arbitrary shaped boundary.

M3D-C^1 uses the following variables to represent both the magnetic field and the vector potential (see Appendix): f, F, ψ. (The first two are related by the definition: $F \equiv R_0 + R^2 \nabla \bullet \nabla_\perp f$, where R_0 is a constant related to the vacuum TF). In the linear mode, these are all 2D *(R,Z)* complex numbers that get multiplied by $e^{in\varphi}$.

At each mesh point, each of the scalar variables is represented by 6 quantities corresponding to the variable and its first 5 derivatives with respect to *R* and *Z*. Thus, for ψ, we have at each mesh point the following: $(\psi, \psi_R, \psi_Z, \psi_{RR}, \psi_{RZ}, \psi_{ZZ})$

Boundary values do not need to be supplied for each of these 6 quantities for each variable, only for the appropriate number. For example, for a Dirichlet boundary condition on ψ, we would prescribe only the value of ψ on the boundary nodes. The other 5 quantities associated with this variable at the boundary node, $\psi_R, \psi_Z, \psi_{RR}, \psi_{RZ}, \psi_{ZZ}$, get determined by the Galerkin method as for the interior points. Similarly, for a Neumann boundary condition at vertical boundary (z=const), we would specify only ψ_R and $\psi, \psi_Z, \psi_{RR}, \psi_{RZ}, \psi_{ZZ}$ would get determined by the Galerkin method.

Question: Can we use Green's function techniques and the thin wall approximation to come up with appropriate relations between the M3D-C^1 magnetic variables that can be used in a linear time-dependent calculation as a thin resistive wall boundary condition? The immediate interior of the wall would be high resistivity plasma, and the exterior would be a vacuum region extending to infinity.

Appendix: Vacuum equations:

I. Preliminaries:

Starting from the definition: $\mathbf{A} = R^2 \nabla\varphi \times \nabla f + \psi \nabla\varphi - R_0 \ln R\hat{Z}$, we can compute

$$\mathbf{B} = \nabla \times \mathbf{A} = \nabla\psi \times \nabla\varphi - \nabla_\perp f' + F\nabla\varphi = \nabla\psi \times \nabla\varphi - \nabla f' + F^* \nabla\varphi \tag{H-1}$$

$$\mathbf{J} \equiv \nabla \times \mathbf{B} = \nabla F^* \times \nabla\varphi + \frac{1}{R^2}\nabla_\perp \psi' - \Delta^* \psi \nabla\varphi \tag{H-2}$$

Here, we have defined: $F \equiv R_0 + R^2 \nabla \bullet \nabla_\perp f$, $F^* \equiv R_0 + R^2 \nabla^2 f = F + f''$

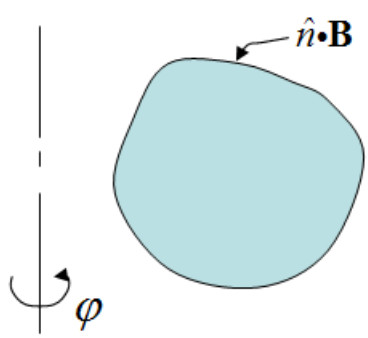
From (H-2), and the condition **J** = 0, we have the following conditions in Vacuum:

$$\Delta^* \psi = 0 \tag{H-3}$$

$$\nabla_\perp F^* = -\nabla\varphi \times \nabla_\perp \psi' \tag{H-4}$$

From (H-4), we get the condition

$$\nabla_\perp^2 F^* = 0 \tag{H-5}$$



II. Solving for the vacuum solution in M3D with prescribed boundary conditions:

Say we have an axisymmetric interface surface defined by $\phi(R,Z) = 0$, with normal vector $\hat{n} = \nabla\phi / |\nabla\phi|$. In order to match the normal component of the field at the surface requires that we define boundary conditions on the two functions ψ and f' such that:

$$\hat{n}\bullet\mathbf{B} = \hat{n}\bullet\nabla\psi \times \nabla\varphi - \hat{n}\bullet\nabla_\perp f' \tag{H-6}$$

(1) Choose ψ on boundary such that the solubility constraint associated with (H-6) is satisfied. i.e.:

$$\oint \hat{n}\bullet\mathbf{B} d\varphi = \oint \hat{n}\bullet\nabla\psi \times \nabla\varphi d\varphi \tag{H-7}$$

Note that this only imposes a condition on the axisymmetric (n=0) part of ψ. There is a freedom regarding how much of the non-axisymmetric part of $\hat{n}\bullet\mathbf{B}$ to represent in the first term on the right in (H-6), and how much to represent in the second term.

(2) Solve $\Delta^*\psi = 0$ using the boundary condition given in (1)

(3) Solve $\nabla_\perp^2 F^* = 0$ subject to Neumann BC: $\hat{n}\bullet\nabla_\perp F^* = -\hat{n}\bullet\nabla\varphi\times\nabla_\perp\psi'$

(4) Solve $\nabla^2 f = R^{-2}(F^* - R_0)$ using Neumann BC: $\hat{n}\bullet\nabla_\perp f' = -\hat{n}\bullet\mathbf{B} + \hat{n}\bullet\nabla\psi\times\nabla\varphi$ while enforcing the solubility condition:

$$\int dS\hat{n}\bullet\nabla f = \int dVR^{-2}(F^* - R_0) = \int dS(-\hat{n}\bullet\mathbf{B} + \hat{n}\bullet\nabla\psi\times\nabla\varphi)$$

with

$$R_0 = \frac{\int dVR^{-2}F^* + \int dS(\hat{n}\bullet\mathbf{B} - \hat{n}\bullet\nabla\psi\times\nabla\varphi)}{\int dVR^{-2}}$$

III. On the freedom in (H-6) and relation to the scalar potential:

Note that if the axisymmetric part of (H-6) is zero, we could set ψ to zero everywhere, and use f' to satisfy the boundary condition. Then, both ψ and F^* are zero everywhere, and we need only solve $\nabla^2 f = 0$. This is equivalent to Morrell's scalar potential χ if we identify $-f' \to \chi$. In Morrell's notation, we would then have:

$$\mathbf{B} = \nabla\chi \tag{H-8}$$

$$\nabla^2\chi = 0 \tag{H-9}$$