Energetic Particle Slowing in FRC Edge

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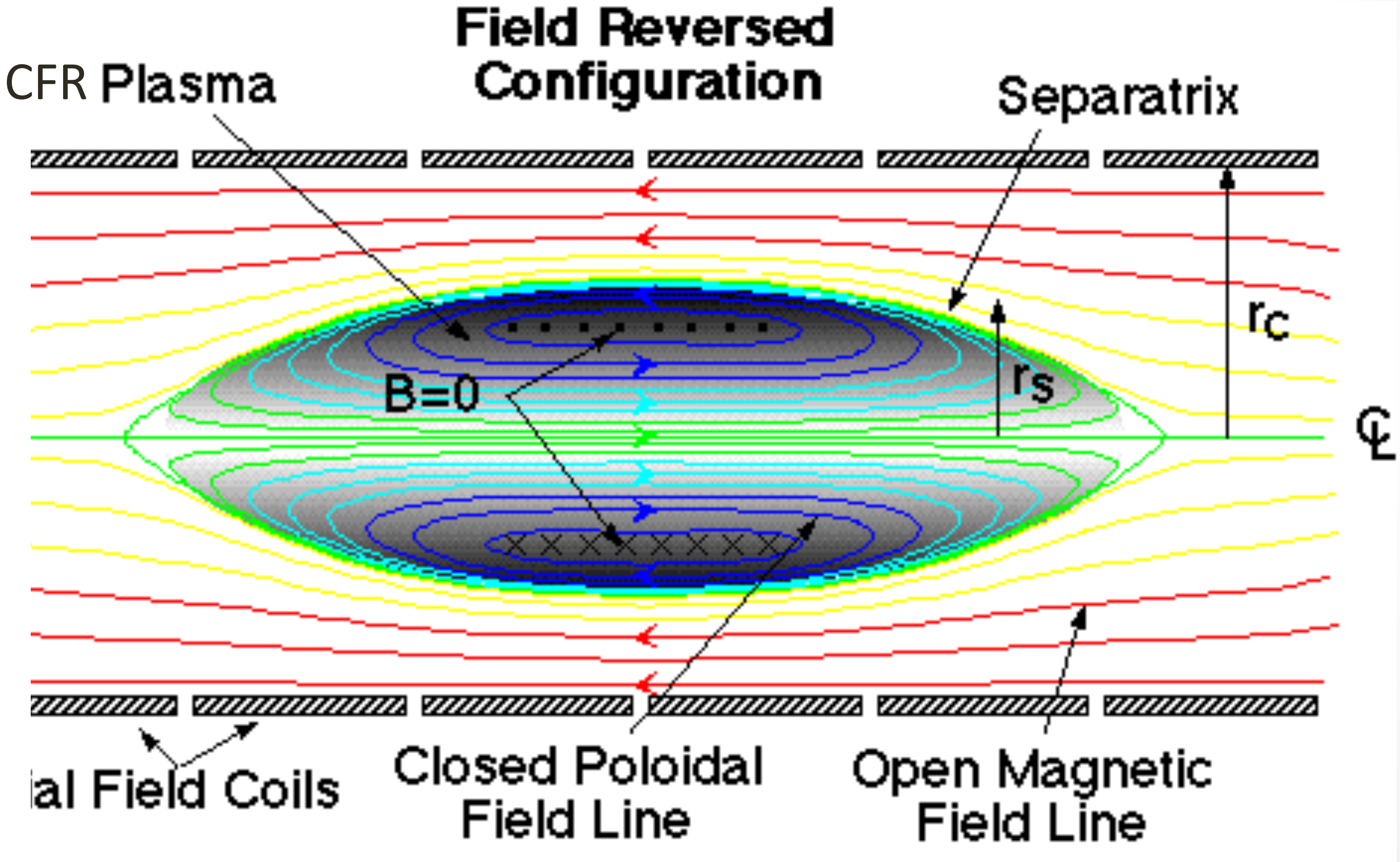
System of Interest: FRC



Motivation

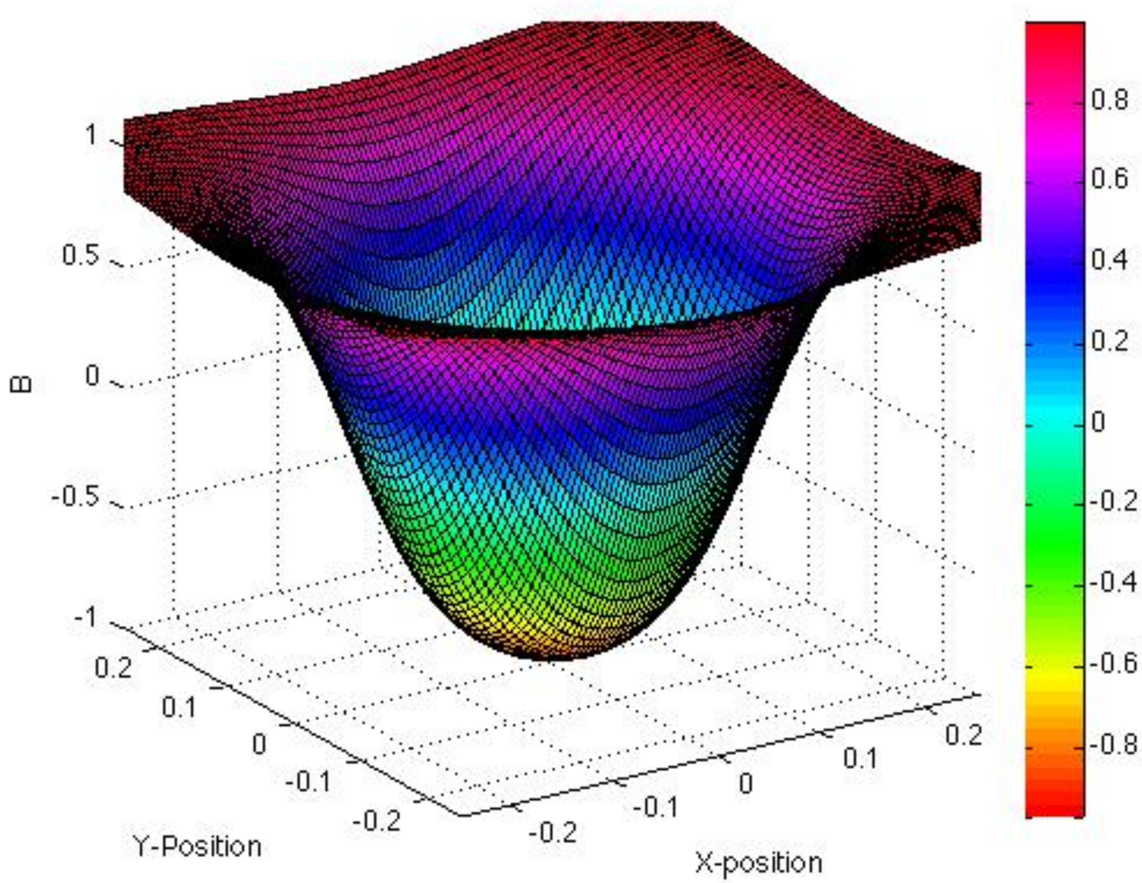
- Previously, trajectory for time-invariant single particle (in FRC) has been analytically solved
  - Studies have not examined the effect of drag on the trajectories
- Goal: to see if accounting for drag in some regime would significantly change the modeled trajectory
  - Initial motivation: hypothesized that drag would produce an outward radial drift
  - Scrape off Layer: region outside the high-temperature plasma core. Coulomb drag is more pronounced
  - Potential applications for ash, energy extraction

Magnetic Field

- Field-Reversed Configuration
  - Polarity of field changes about some nonzero radius, r_0, where the field strength is 0
- Simulated for $z = 0$
  - Magnetic field entirely in z direction
  - Particle simulated for no initial z-velocity: confined in $z = 0$ plane

Magnetic Field (cont.)

- Analytic approximation of magnetic field: $B(r) = B_0 \tanh(\frac{r - r_0}{L})$



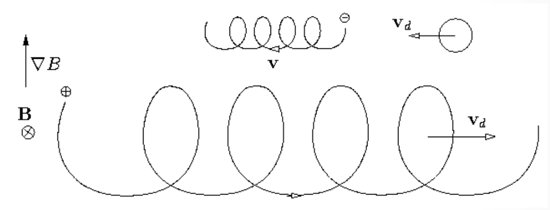
Scrape-Off Layer

- Region outside separatrix ($r > r_s = 0.25$ m)
  - Magnetic field lines are open
- High conductivity along field lines
  - Energy dissipation along open field lines: SOL is cooler than CFR
  - Lower temperatures allow for closer particle interactions: stronger Coulomb forces
  - More pronounced Coulomb drag leads to energy losses
- Drag force linearly proportional to velocity $\vec{F}_d = -k\vec{v}$
- Drag constant dependent upon temperature, plasma density
  - To promote a more rapid understanding of the features of energy loss, k was exaggerated to a larger, uniform value

Expected Behavior

- Drift due to nonuniform magnetic field (grad B drift):

$$\vec{v}_{\nabla B} = \frac{v}{qB}\frac{\vec{B}\times\nabla B}{B^2}$$



- Drift due to drag force:

$$\vec{v}_f = \frac{1}{q}\frac{\vec{F}\times\vec{B}}{B^2}$$

- Fields are discontinuous or nonlinear: difficult to analytically determine their influence and interaction

Dynamical System

- Forces: $\overrightarrow{F}_d = -k\vec{\upsilon}$ $\overrightarrow{F}_B = q\vec{\upsilon} \times \overrightarrow{B}$

$$\ddot{x} = \begin{cases} \dfrac{qB}{m}\dot{y} - k\dot{x} : r \geq r_s \\ \dfrac{qB}{m}\dot{y} : r < r_s \end{cases}$$

$$\ddot{y} = \begin{cases} -\dfrac{qB}{m}\dot{x} - k\dot{y} : r \geq r_s \\ -\dfrac{qB}{m}\dot{x} : r < r_s \end{cases}$$

Parameters

- 2 types of particles simulated- products of fusion reaction:

$$D + He^3 \rightarrow He^4(3.6MeV) + p(14.7MeV)$$

- System conditions:

$$B_0 = 10T$$

$$r_s = 0.25m$$

$$r_0 = 0.17m$$

$$L = 0.08m$$

- Chosen parameters:

$$(x_i, y_i) = (0, 0.26m)$$

$$0 < k < 1$$

Algorithm

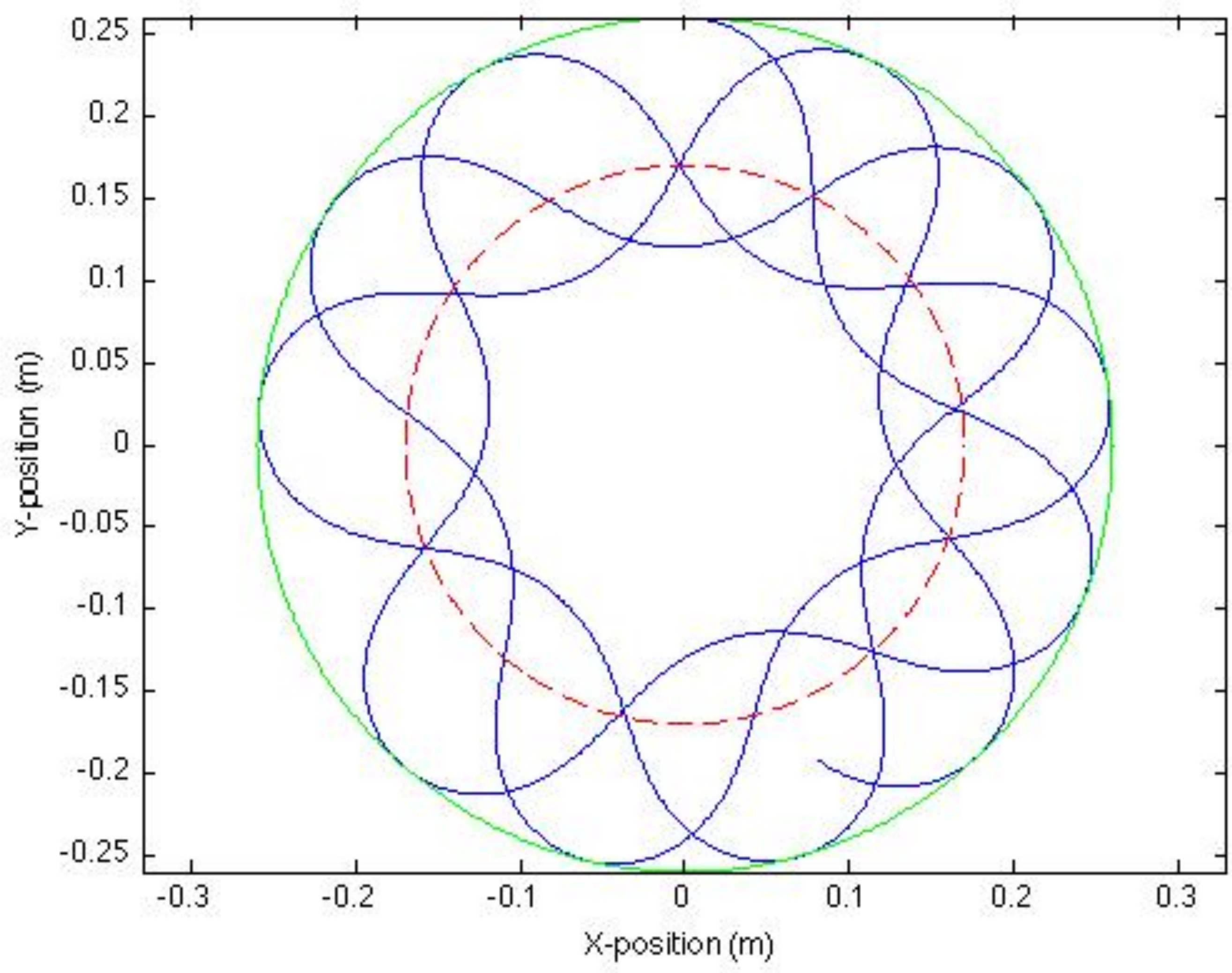
- Coupled, second order, differential equations
  - Reducible to a system of four first-order DEs

- Algorithm based on Runge-Kutta-Fehlberg method (RKF)
  - Adaptive step-size: minimizes computational time while staying within a maximum error bound
  - Adaptive step algorithms use two methods: RKF uses 4^{th}, 5^{th} order

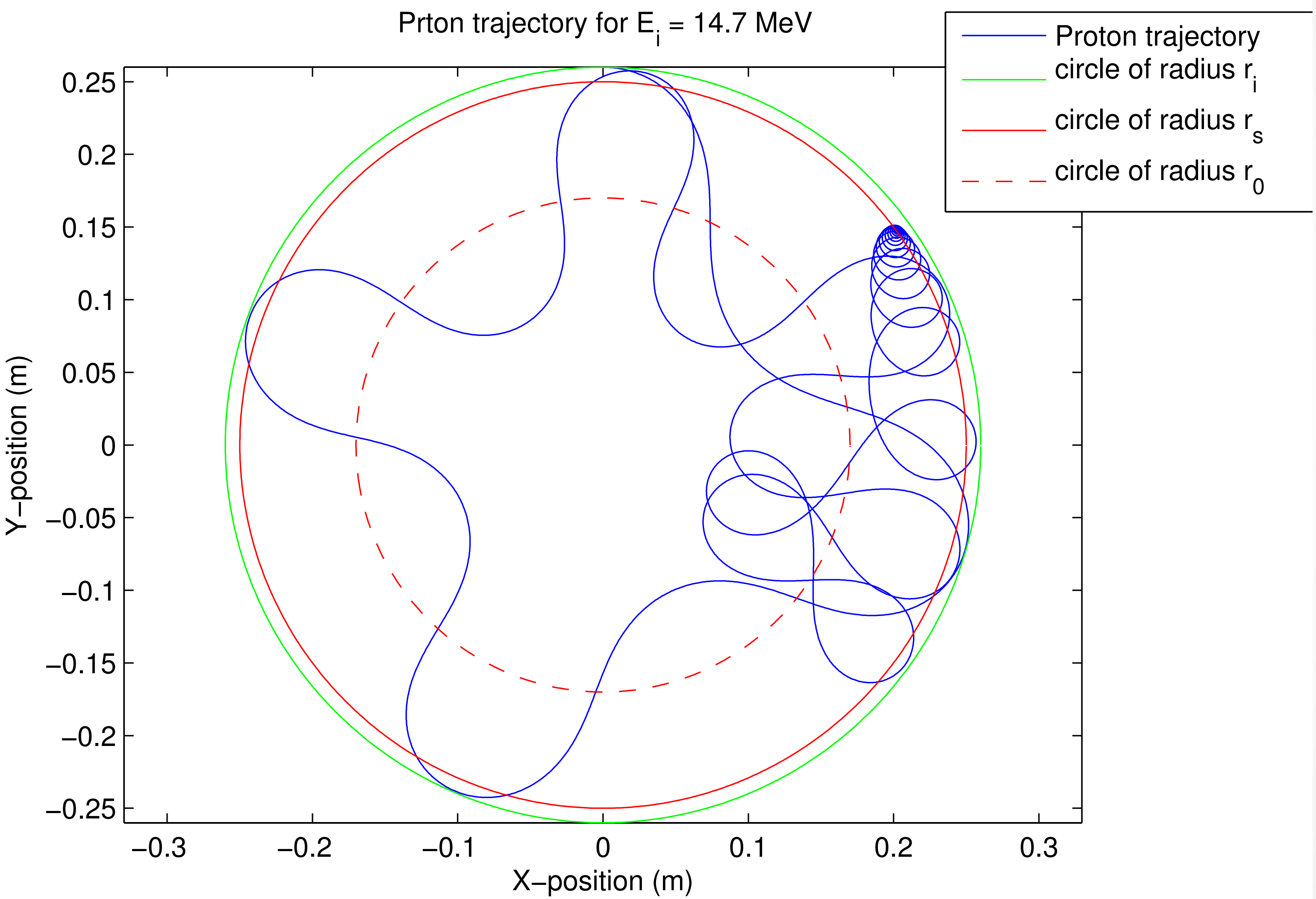
Simulations

- Particles simulated with initial position in z = 0 plane, in the SOL
- All initial energy is considered to be kinetic energy, in the $-\phi$ direction
- Simulated 14.7 MeV protons, 3.6 MeV He-4 ions, for different drag constants

Simple Trajectories in FRC

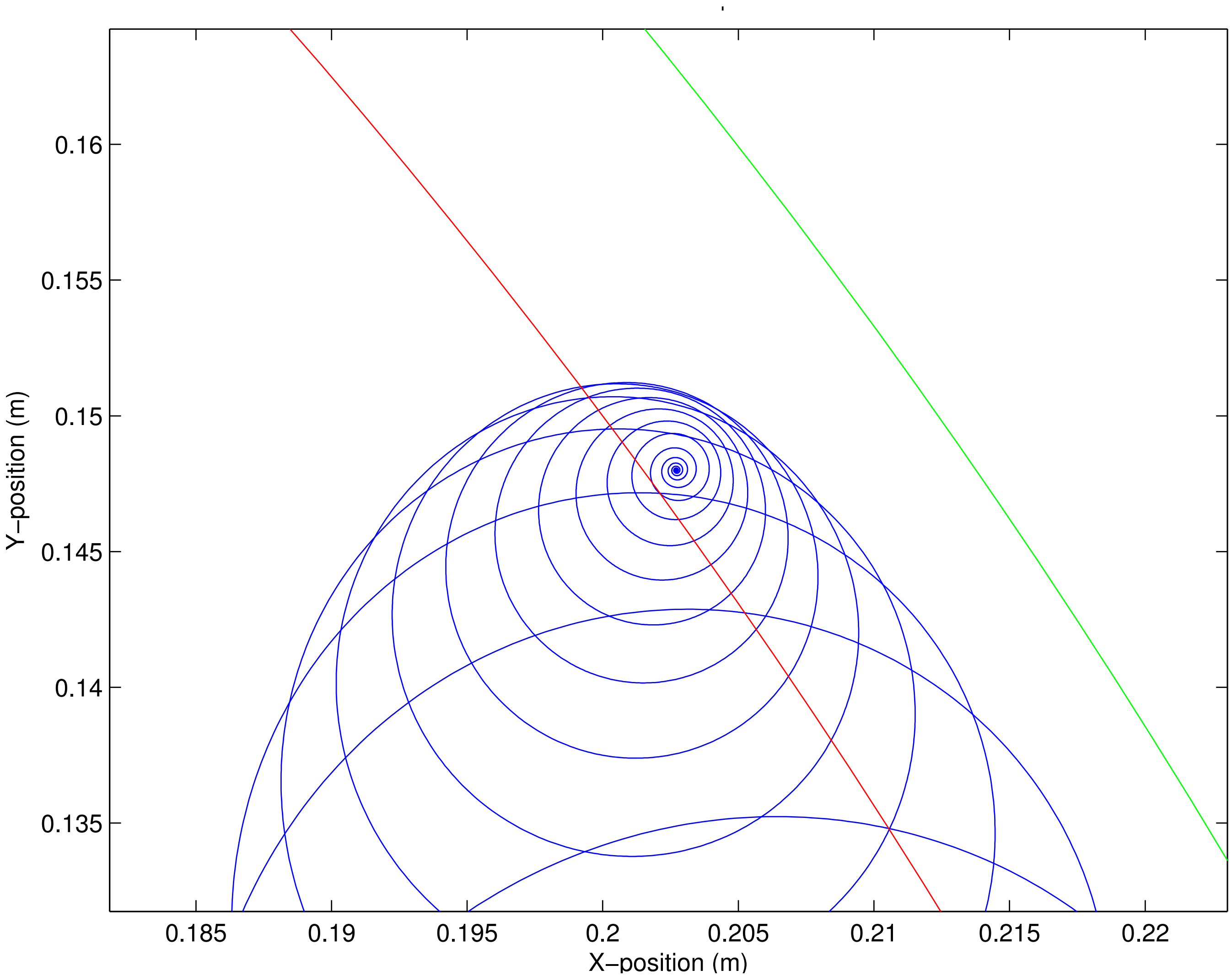


Proton: k = 0.7



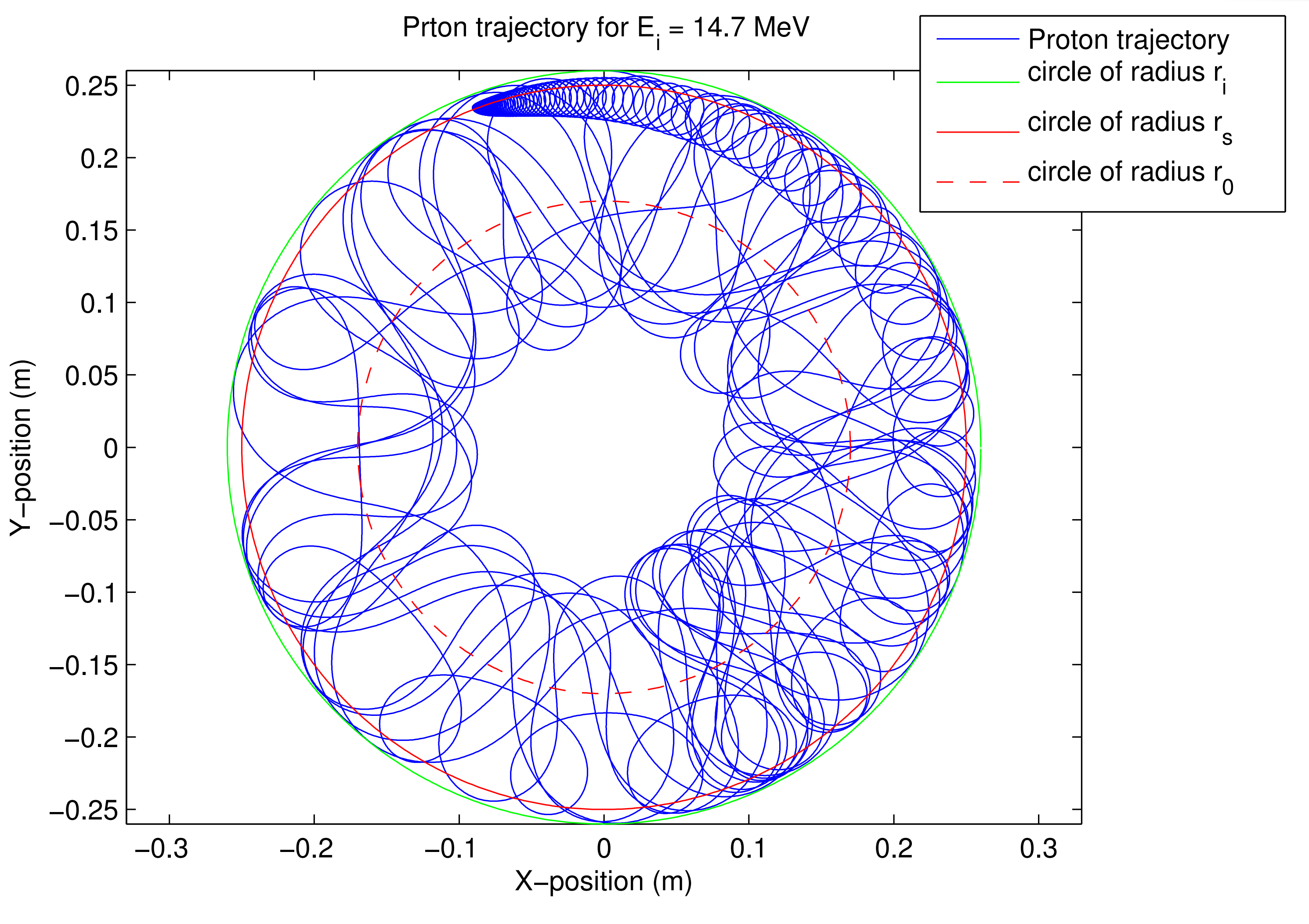
Prton trajectory for E_i = 14.7 MeV
Proton trajectory
circle of radius r_i
circle of radius r_s
circle of radius r_0
Y−position (m)
X−position (m)
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.25
−0.3
−0.2
−0.1
0
0.1
0.2
0.3

Final behavior:

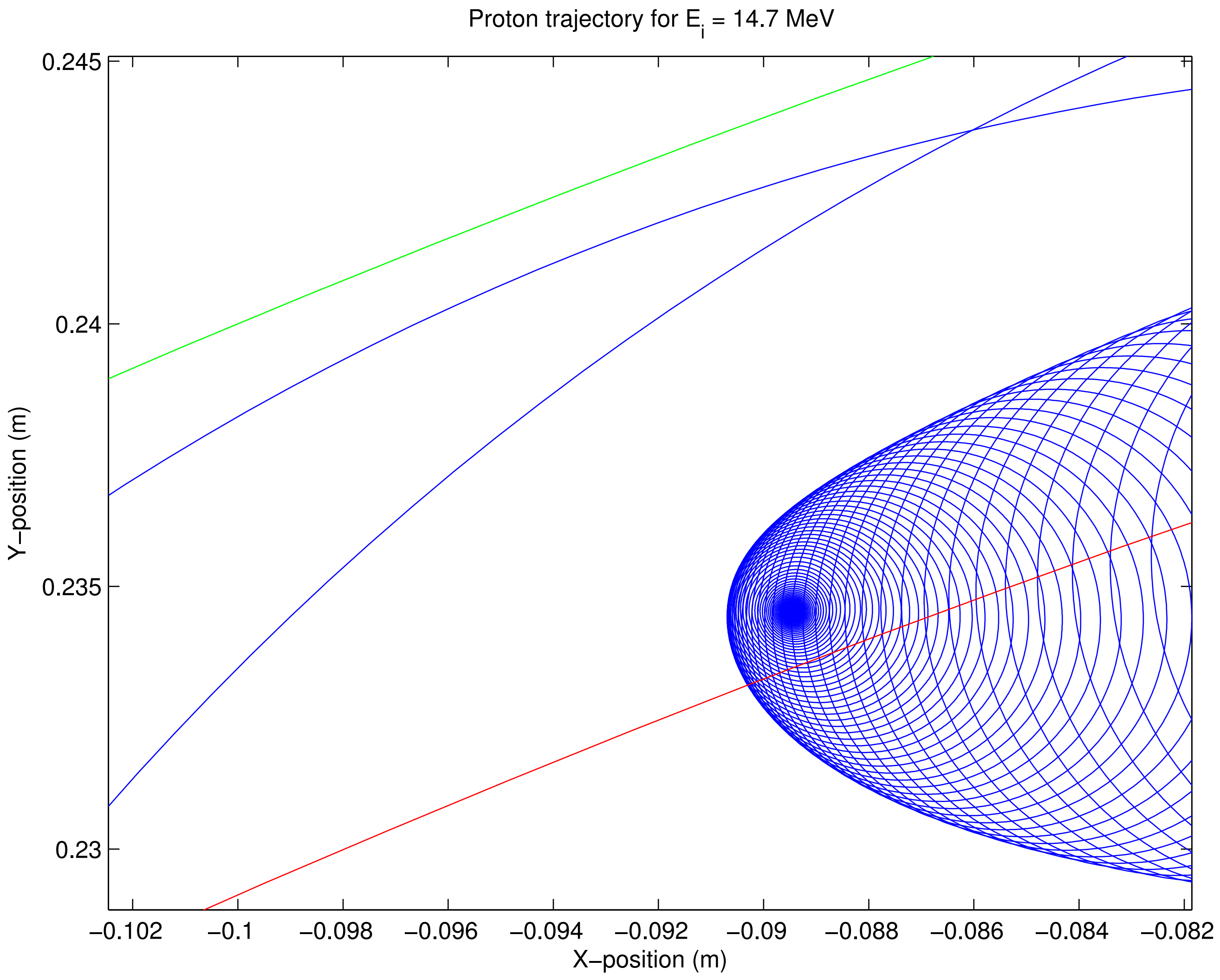


Proton: k = 0.1

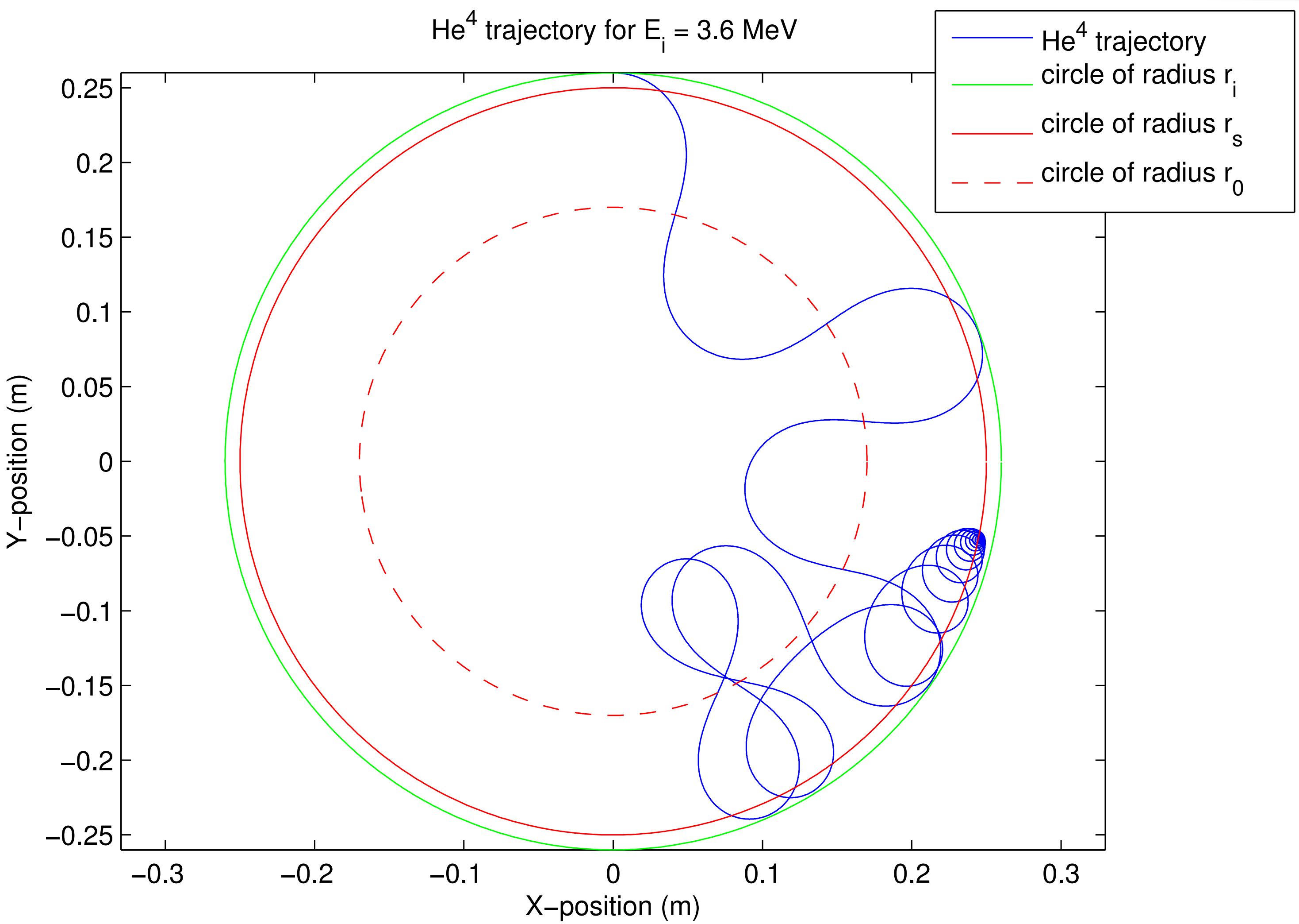
- Behavior not fundamentally changed by magnitude of drag!



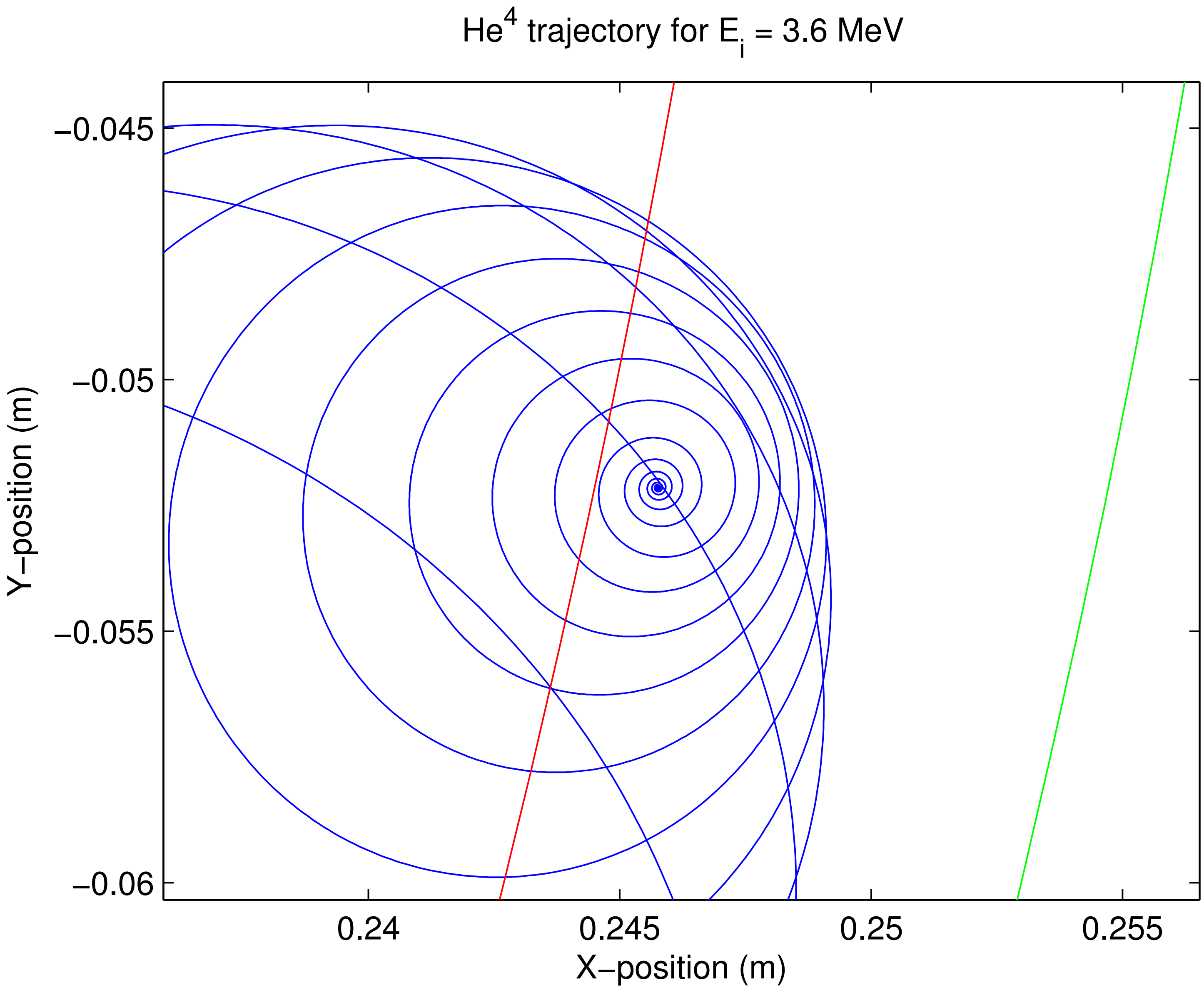
Final behavior: again, fundamentally independent of drag



Helium: k = 0.7: fundamentally similar to proton



Final Behavior: similar to proton



Observations

- Trajectory transitions from:
  - betatron orbit (highest energy)
  - figure-8 orbit (intermediate energy)
  - cyclotron orbit (lowest energy)

- If particle passes through SOL, it ends up in SOL
  - Final radial position less than initial radial position

- This behavior is fundamentally independent of drag, observed for both particles
  - Drag magnitude only changes the timescale of change

Why?

- This behavior is a product of the energy loss, but how can we better understand it?
  - Intuitively, since energy is only lost in the SOL, if the particle has enough energy to leave the SOL, it will have enough energy to return to the SOL

- Hamiltonian perspective is helpful
  - Invariants in Hamiltonian would allow for a view of effective potential barriers
  - Even with drag, one can still consider how the formerly-invariant effective potential varies with time

Hamiltonian (without drag)

- Hamiltonian: $$H = \frac{1}{2m}\left[p_r^2 + p_z^2 + \frac{1}{r^2}(p_\phi^2 - \frac{q}{c}\psi)^2\right]$$

- Magnetic flux function: $$\psi(r,z) = rA_\phi = \psi_0(\frac{r^2}{r_s^2})(1 - \frac{r^2}{r_s^2} - \frac{z^2}{z_s^2})$$

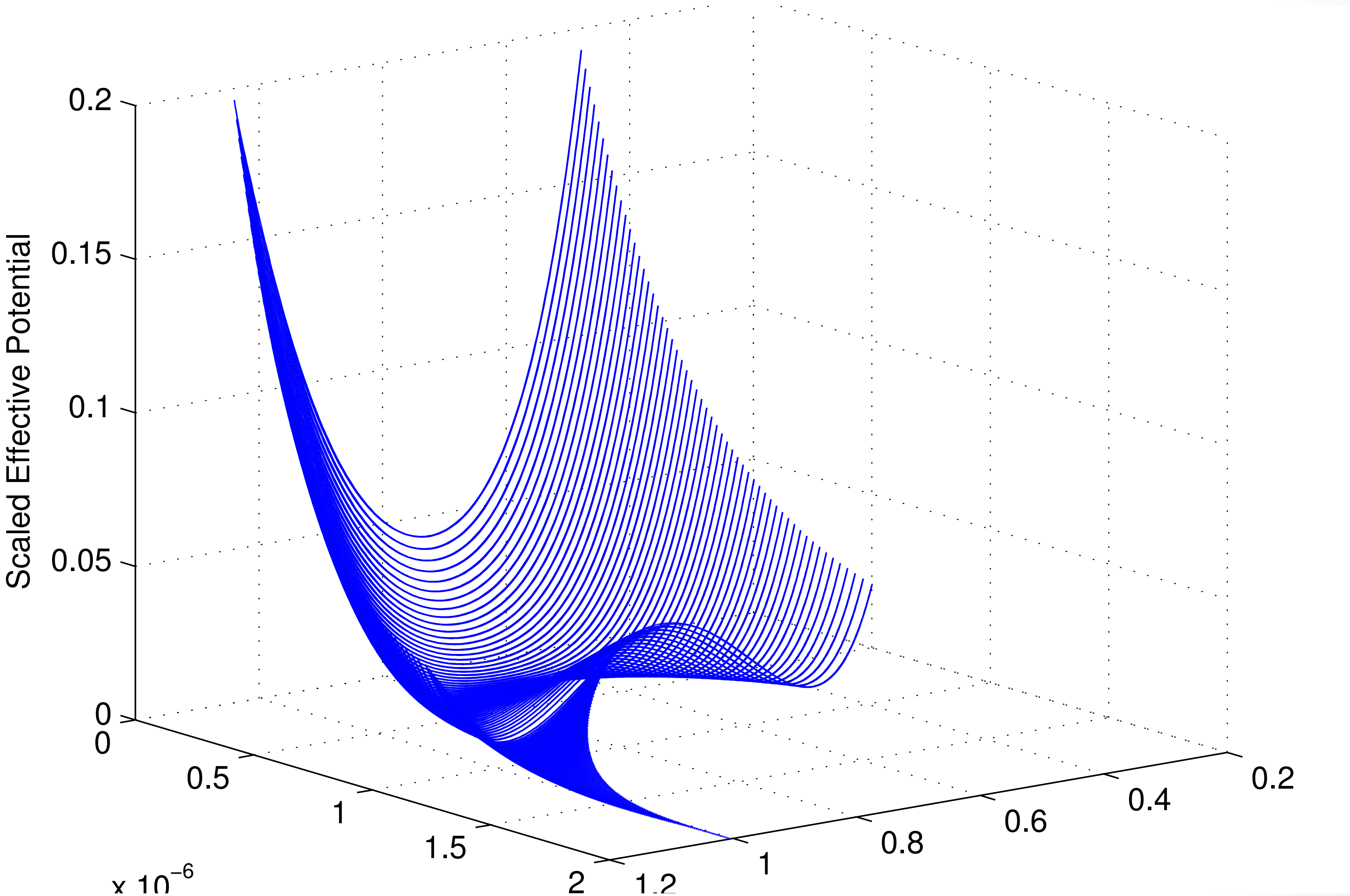
- In drag-free system, angular momentum is conserved
  - Effective potential:

$$V_{eff} = \frac{1}{2mr^2}(p_\phi^2 - \frac{q}{c}\psi)^2$$

Evolution of Effective Potential

- For case with drag, angular momentum not conserved in SOL
  - Leads to a change in effective potential
- Mapping effective potential vs. time can show how the effective potential barrier changes as energy is lost
  - Can be used to understand why orbit trajectory changes
- Landsman et al: transition from single-well potential to double-well potential indicates a transition from betatron to figure-8 trajectories

Effective potential of proton: k = 0.1



Significance of Effective Potential Change

- Early energy loss (in SOL) lowers the minima of the single-well potential, until it approaches 0 and transitions to a double-well

- Further energy loss: double-well potential persists
  - Particle eventually has insufficient energy to transition from the higher-radius well to the lower-radius well
  - Damped oscillation in higher-radius well corresponds to decaying cyclotron orbit (confined to higher-radius region)

Explanation of Final Behavior

- Particle ultimately ends up in SOL, but why?

$$V_{eff} = \frac{1}{2mr^2}(p_\phi^2 - \frac{q}{c}\psi)^2$$

$$\psi(r, z=0) = \psi_0(\frac{r^2}{r_s^2})(1 - \frac{r^2}{r_s^2})$$

- For lower energy particles, there is an effective potential minima at radii near, or greater than, the separatrix radius
  - As angular momentum approaches zero, the effective potential minima approaches the separatrix radius
  - Explains why particle ends up in SOL, at radii less than the initial radius

Conclusion

- If an energetic particle is “swirling” in the SOL, at these energy ranges, then even as it loses energy, it will end up in the SOL
- Helpful for ash extraction- a fundamental problem for fusion reactors
  - Ash displaces energetic products: detrimental effect upon power output
- Potential applications for energetic particle exhaust
  - Rocket propulsion
  - Energy extraction