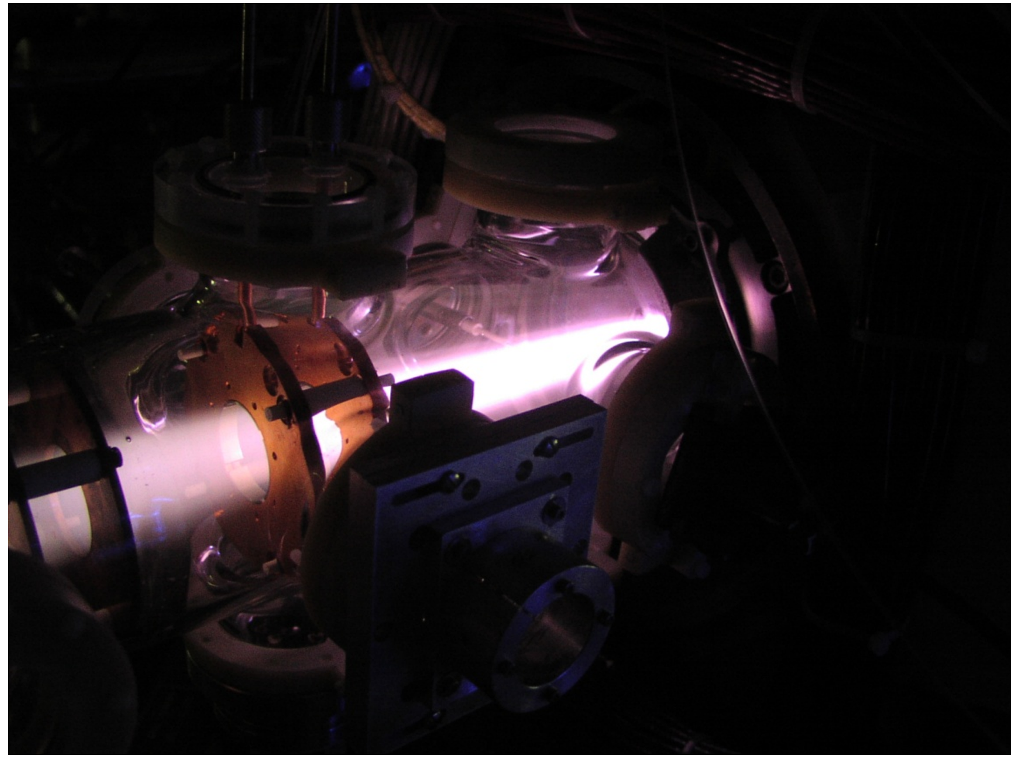

Modeling Plasma Behavior in Magnetized Exhaust Systems: Towards Plasma Detachment

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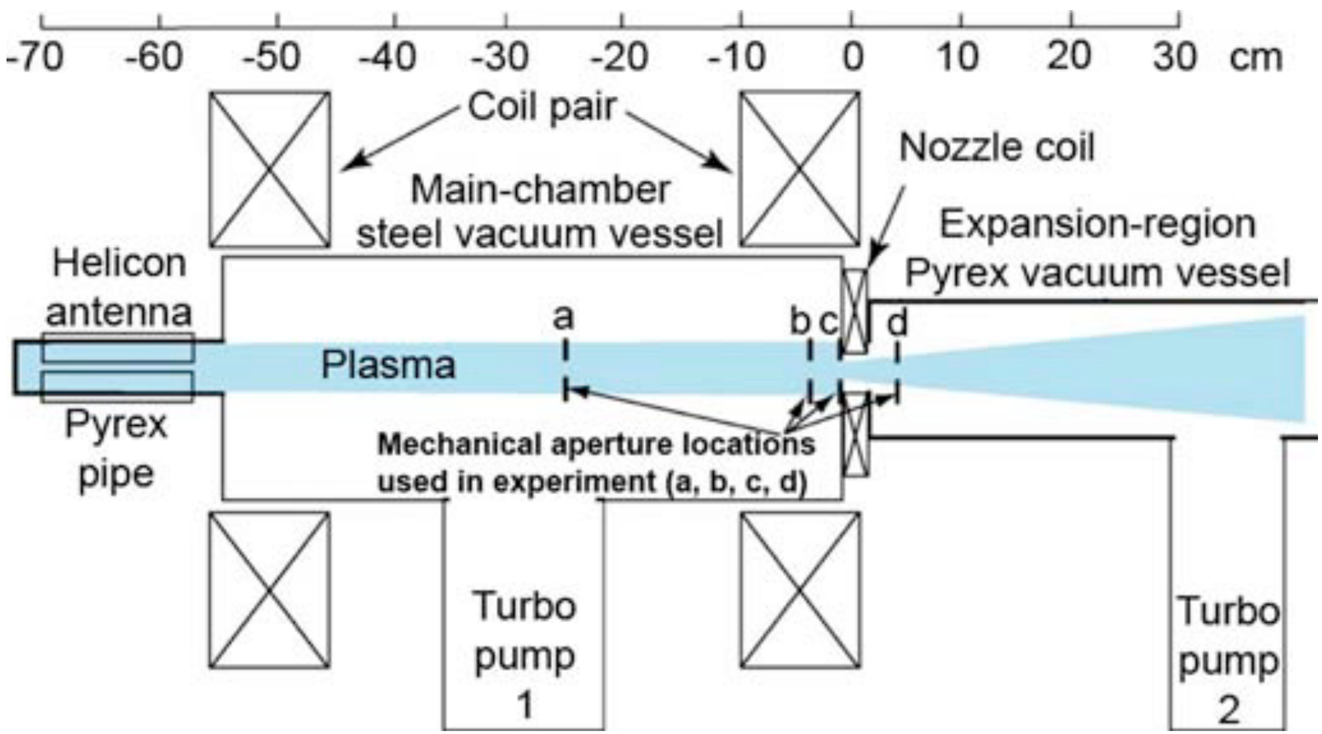
Introduction

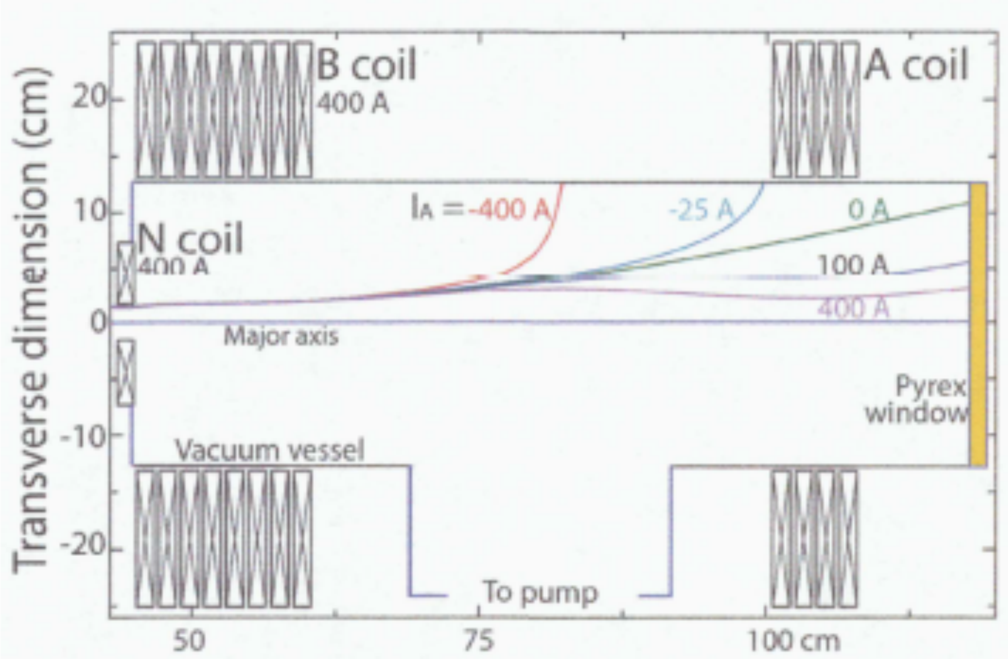
- Cylindrical plasma devices use exhaust mechanisms that propel ions and electrons away from the machines.
- A major problem in such systems is how to most quickly detach the plasma from its magnetic field lines.

A machine with a magnetized expansion region exhaust system. [1]



Goals and Motivations

- To simulate whether plasma detaches when magnetic field line curvature and expansion are increased by the addition of a coil with a reversed current
- Magnetic field to be created in MathCad, an engineering computation software
- Particle simulations to be run on LSP, a Particle-In-Cell program written in C



Modeling the Magnetic Field

- The magnetic field of the expansion region is created by 3 solenoids, two of which enclose the expansion region and one around the nozzle.
- Much of the region is outside or near the end of finite solenoids, so simple models of infinite solenoids are useless, especially since expanding field lines are desired.

- In order to find the magnetic field, a solenoid is assumed to be composed of thin, circular, current-carrying hoops.
- In the following derivations:

B_z *is the axial magnetic field*

B_r *is the radial magnetic field*

I *is the current, assumed to be constant*

J *is the current density, assumed to be constant*

ρ *is the radius of a circular, current − carrying hoop*

l *is the axial position of a circular, current − carrying hoop*

R *is the inner radius of the coil*

d *is the radial thickness of the coil, such that* $R + d$ *is the outer radius of the coil*

L *is the length of the solenoid*

N *is the number of turns in the solenoid*

C *is the axial position of the center of the solenoid*

- The magnetic field components around a thin, circular, current-carrying hoop are: [2]

$$b_z = \frac{B_0}{\pi\sqrt{Q}}\left[E(k)\frac{1-\alpha^2-\beta^2}{Q-4\alpha} + K(k)\right] \qquad b_r = \frac{B_0\gamma}{\pi\sqrt{Q}}\left[E(k)\frac{1+\alpha^2+\beta^2}{Q-4\alpha} - K(k)\right]$$

$$\alpha = \frac{r}{\rho} \qquad Q = (1+\alpha)^2 + \beta^2$$

$$\beta = \frac{z}{\rho} \qquad k = \sqrt{\frac{4\alpha}{Q}}$$

$$\gamma = \frac{z}{r} \qquad B_0 = \frac{I\mu_0}{2\rho}$$

$$K(k) \mathit{is\ the\ complete\ elliptic\ integral\ of\ the\ first\ kind}{:}\, K(k) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}$$

$$E(k) \mathit{is\ the\ complete\ elliptic\ integral\ of\ the\ second\ kind}{:}\, E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1-k^2\sin^2\theta}\, d\theta$$

- To integrate the hoops, the magnetic field of a single hoop must be redefined:

$$b_z(z-l,r,\rho) = \frac{\mu_0 I}{2\rho\pi\sqrt{Q}}\left[E(k)\frac{1-\alpha^2-\beta^2}{Q-4\alpha}+K(k)\right]$$

$$db_z(z-l,r,\rho) = \frac{\mu_0 dI}{2\rho\pi\sqrt{Q}}\left[E(k)\frac{1-\alpha^2-\beta^2}{Q-4\alpha}+K(k)\right]$$

$$JdA = dI, dA = dl d\rho$$

$$db_z(z-l,r,\rho) = \frac{\mu_0 J}{2\rho\pi\sqrt{Q}}\left[E(k)\frac{1-\alpha^2-\beta^2}{Q-4\alpha}+K(k)\right] dl d\rho$$

$$JA = I, A = \frac{d}{N}L$$

$$J = \frac{IN}{Ld}$$

$$db_z(z-l,r,\rho) = \frac{\mu_0 IN}{2Ld\rho\pi\sqrt{Q}}\left[E(k)\frac{1-\alpha^2-\beta^2}{Q-4\alpha}+K(k)\right] dl d\rho$$

- The axial magnetic field of a solenoid can then be found by defining and then integrating the magnetic field density of the hoops, M_z.

$$M_z(z-l,r,\rho) = \frac{\mu_0 I N}{2Ld\rho\pi\sqrt{Q}}\left[E(k)\frac{1-\alpha^2-\beta^2}{Q-4\alpha} + K(k)\right]$$

$$B_z(z,x) = \int_{R}^{R+d}\int_{-\frac{L}{2}}^{\frac{L}{2}} M_z(z-l,r,\rho)\,dl\,d\rho$$

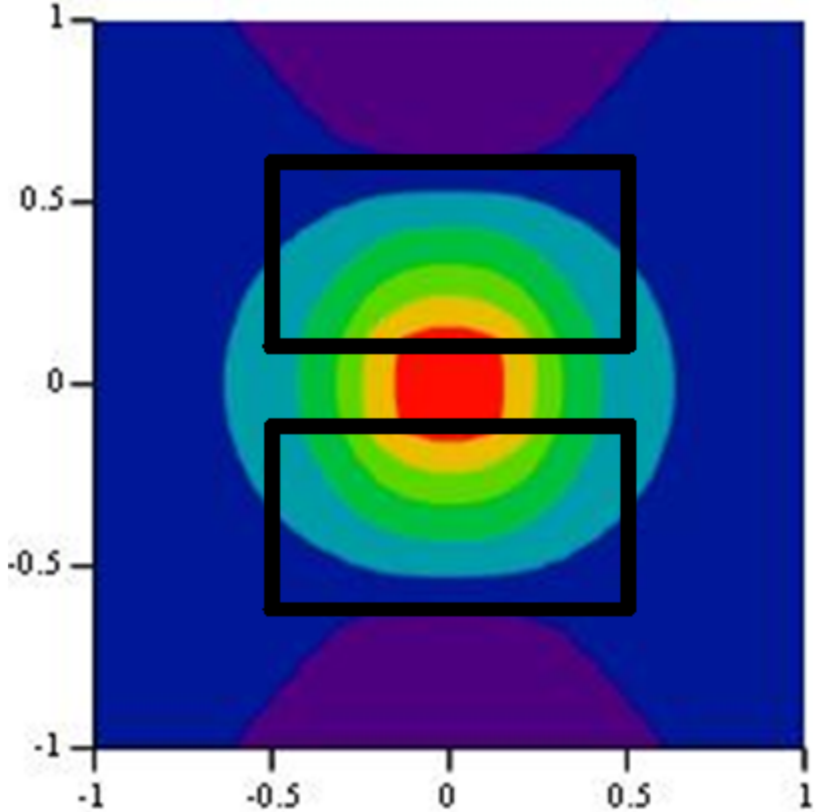
- Using a similar procedure, the radial magnetic field is defined as follows:

$$M_r(z-l,r,\rho) = \frac{\mu_0 I N \gamma}{2Ld\rho\pi\sqrt{Q}}\left[E(k)\frac{1+\alpha^2+\beta^2}{Q-4\alpha} - K(k)\right]$$

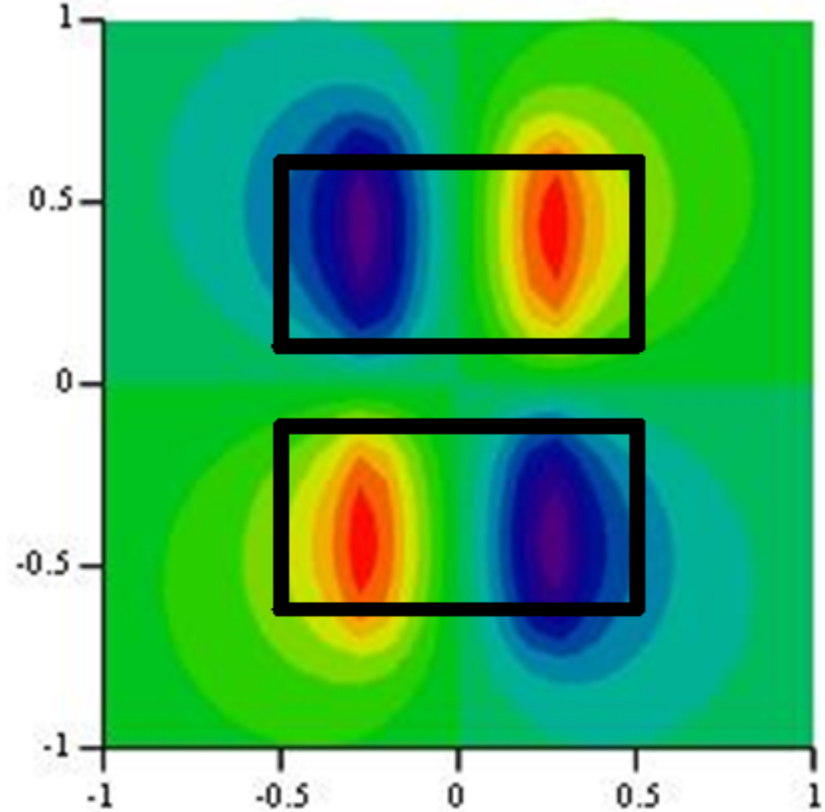
$$B_r(z,x) = \int_{R}^{R+d}\int_{-\frac{L}{2}}^{\frac{L}{2}} M_r(z-l,r,\rho)\,dl\,d\rho$$

Magnetic Field of a Thick Finite Solenoid

L=.5m, R=.1m, d=.6m, N=10, I=400A



Magnitude of Bz in the Plane

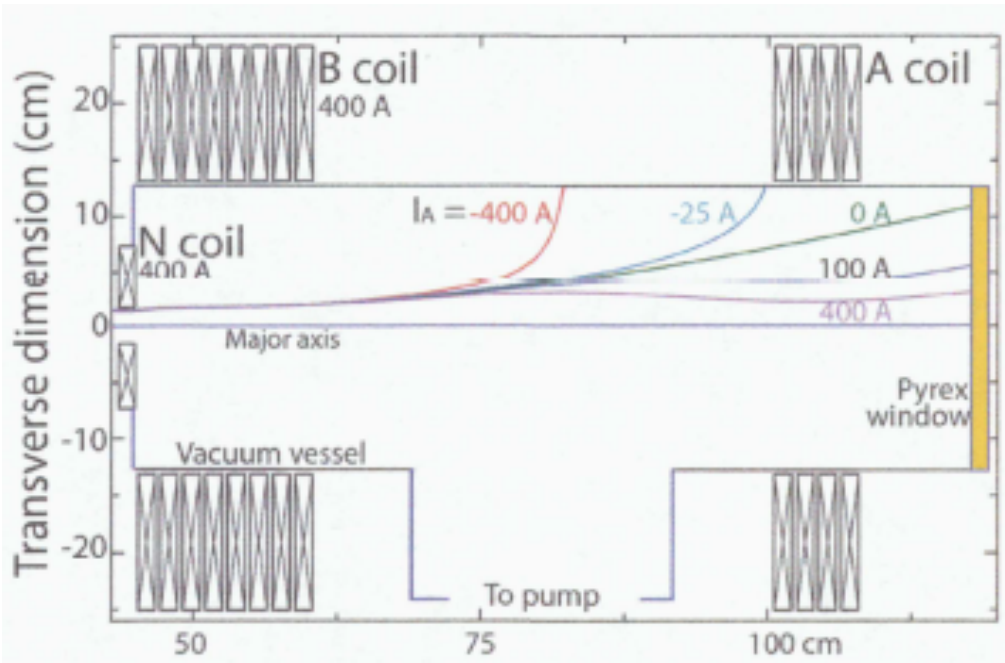


Magnitude of Br in the Plane

Magnetic Field-Expansion Region

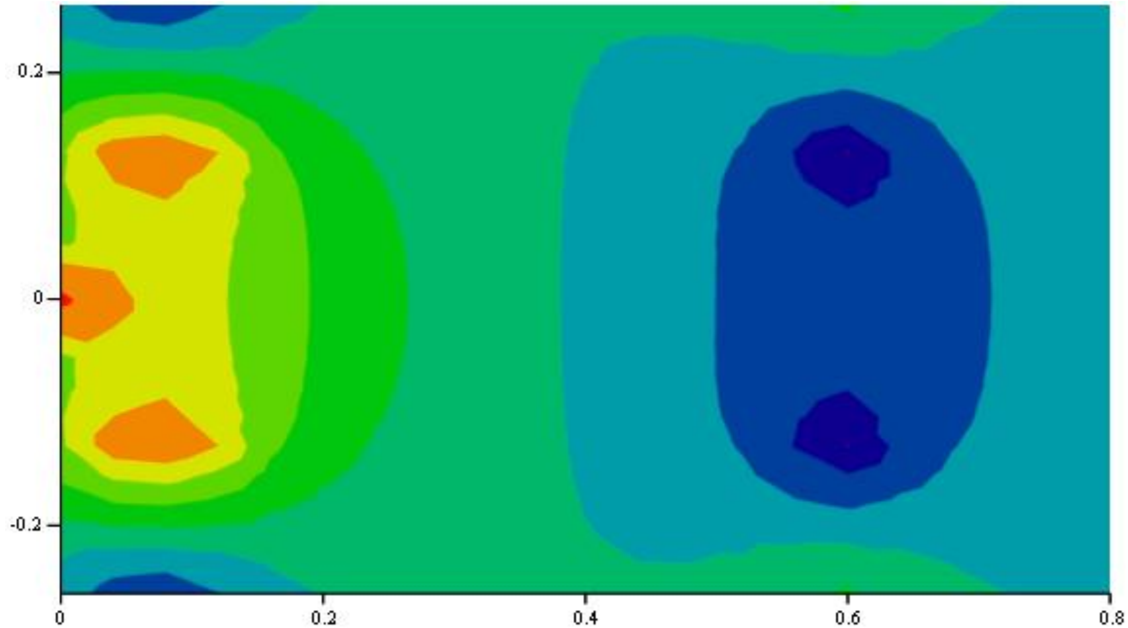
- The expansion region of the simulated device is made up of 3 different coils spaced along the z-axis. The magnetic field of a single solenoid at axial position C is given by:

$$B_z(z,x) = \int_{R}^{R+d} \int_{-\frac{L}{2}}^{\frac{L}{2}} M_z(z-l-C, r, \rho)\, dl\, d\rho$$

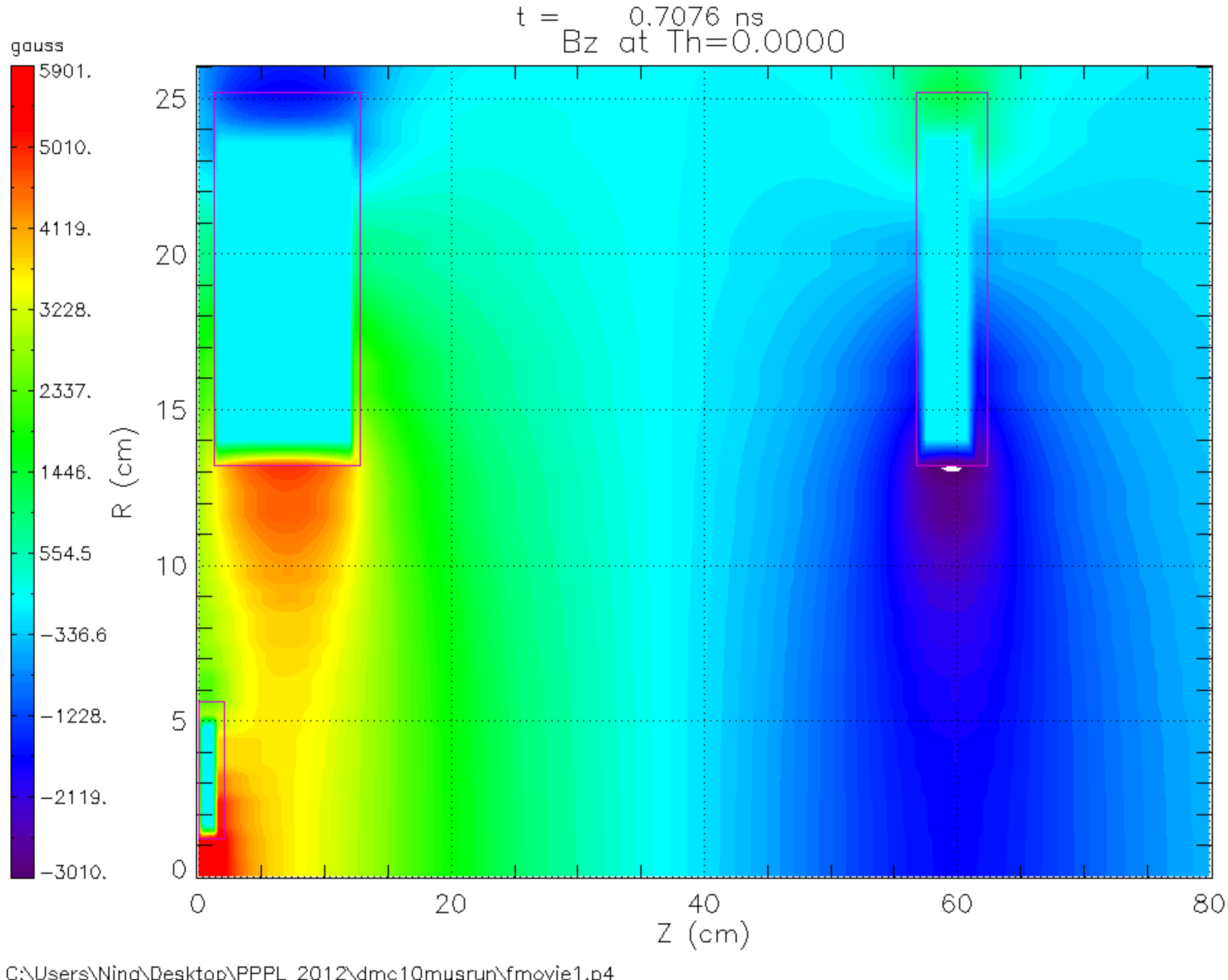


Coil Parameters and Expansion Field

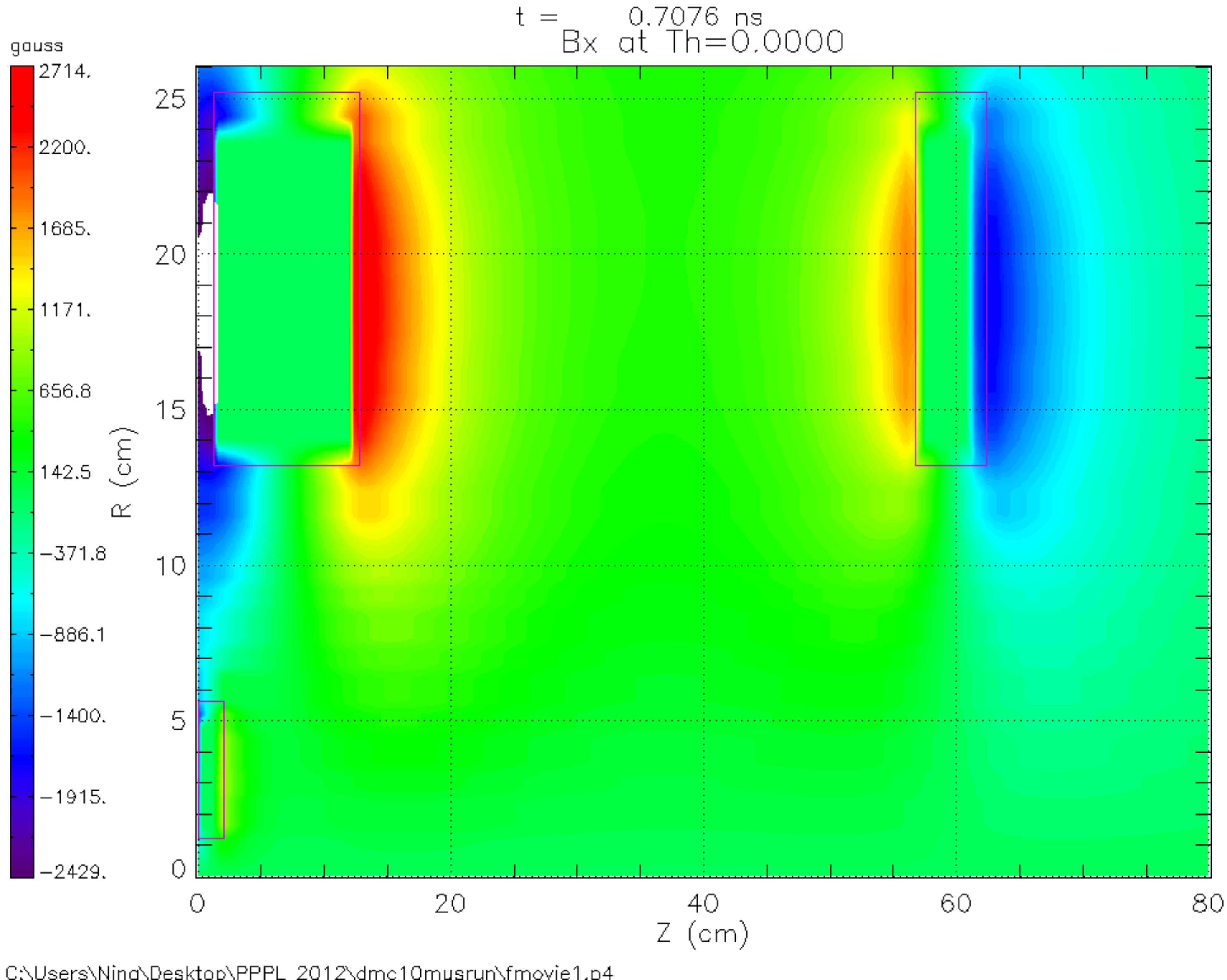
	Nozzle Coil (N)	Coil B	Reverse Coil (A)
Current (I)	400 A	400 A	-400 A
Length (L)	1.9 cm	11.5 cm	5.8 cm
Thickness (d)	4.5 cm	11.9cm	11.9cm
Radius (R)	1.3 cm	13.0 cm	13.0 cm
Number of Turns (N)	36	264	132
Axial Position (C)	.95 cm	7.0cm	59.5cm



Axial Magnetic Field



Radial Magnetic Field



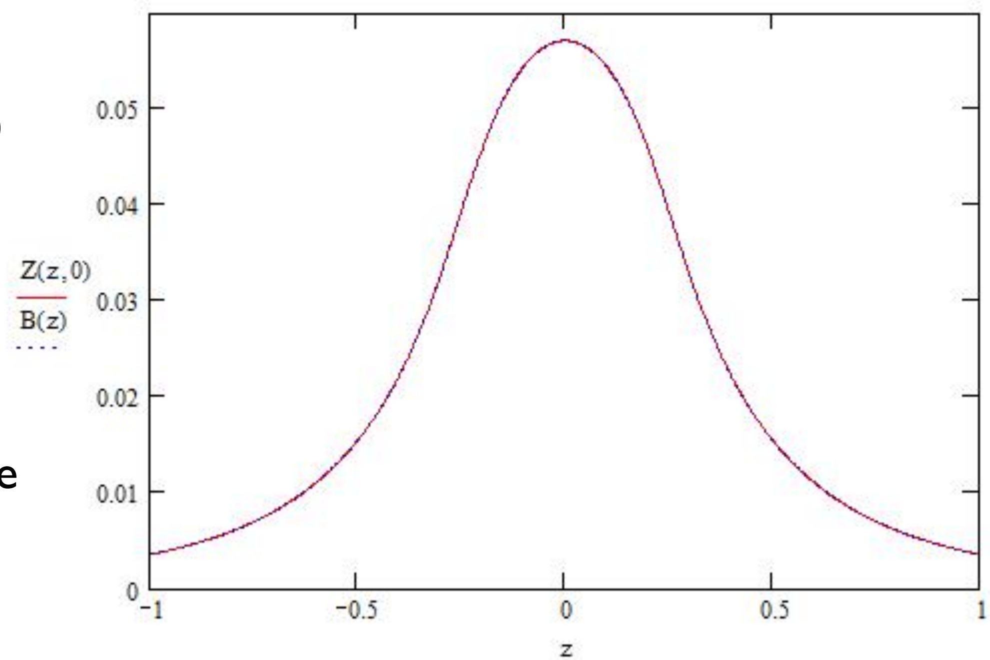
Advantages of the Model

- Very easy to change parameters, including the addition of more solenoids both on and off the axis
- Compares well with on-axis solution for axial field [3]

$$B_z(z) = \frac{\mu_0 IN}{4L}\left[(L-2z)\log\left(\sqrt{L^2-4Lz+4(\rho^2+z^2)}+2\rho\right)+(L+2z)\log\left(\sqrt{L^2+4Lz+4(\rho^2+z^2)}+2\rho\right)\right]\Bigg|_{R}^{R+d}$$

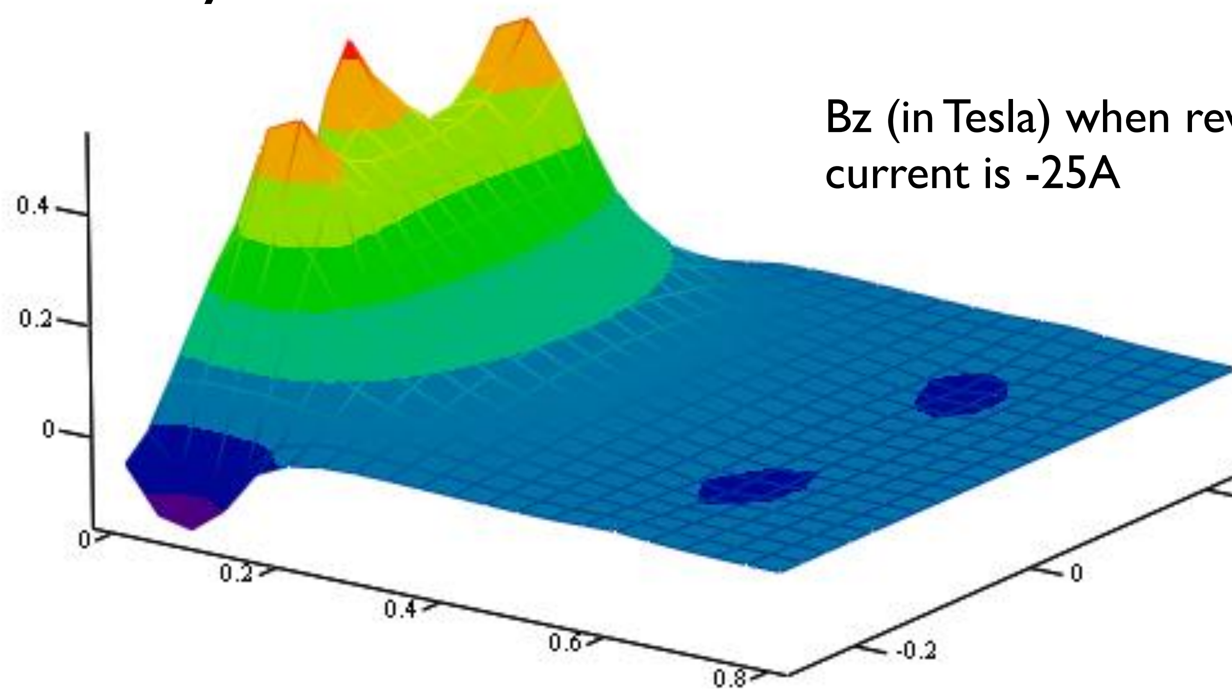
Z(z,0) is the Bz given by the model when x=0

B(z) is the equation for Bz on-axis given above



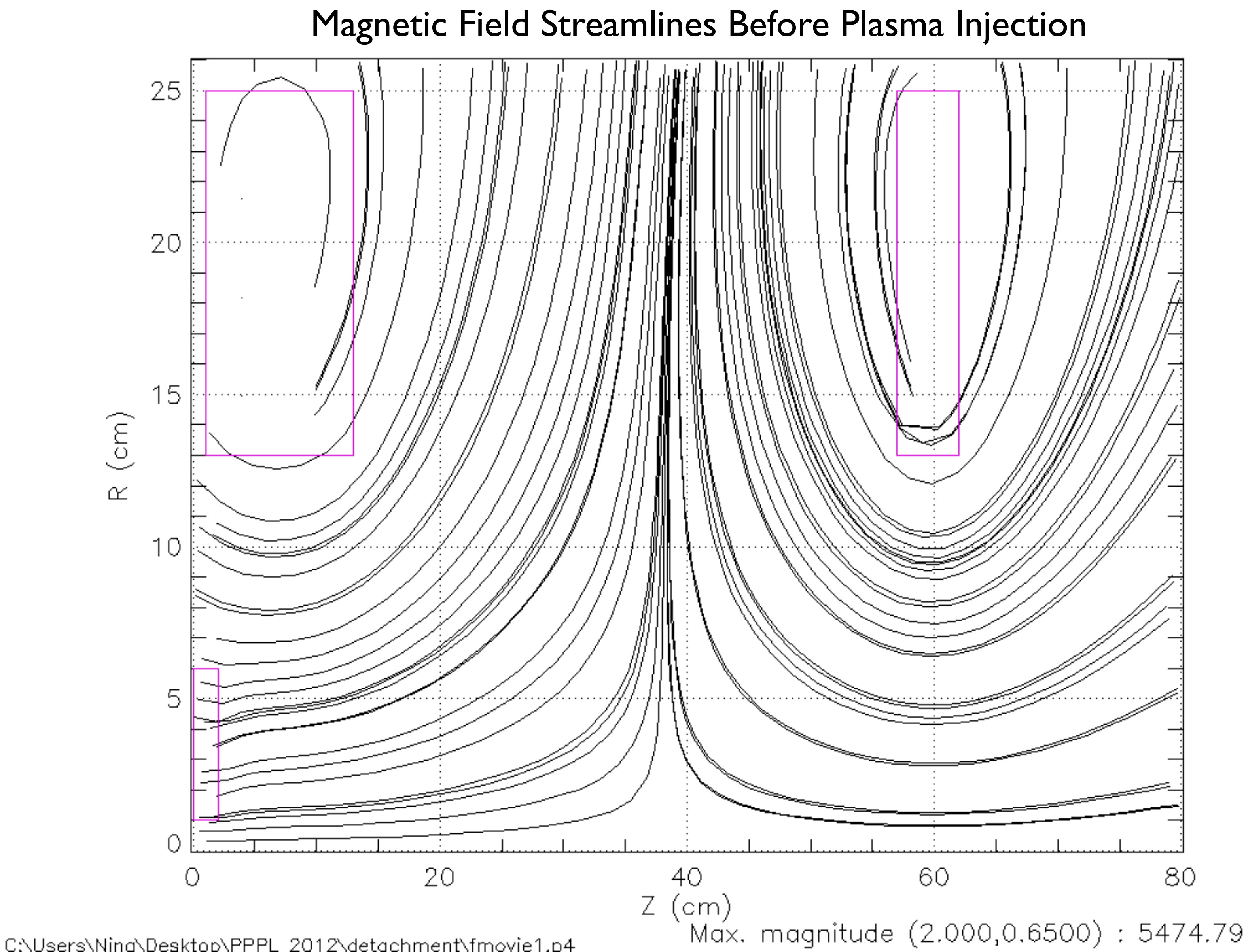
Limitations of the Model

- Difficult to verify model accuracy off-axis
- High allowable integration error in MathCad (up to 10 G)
- Cannot process radial field when r=0m due to division by zero, although by symmetry it can be seen that the radial field on-axis is always 0 G



Bz (in Tesla) when reverse current is -25A

The Detachment Problem



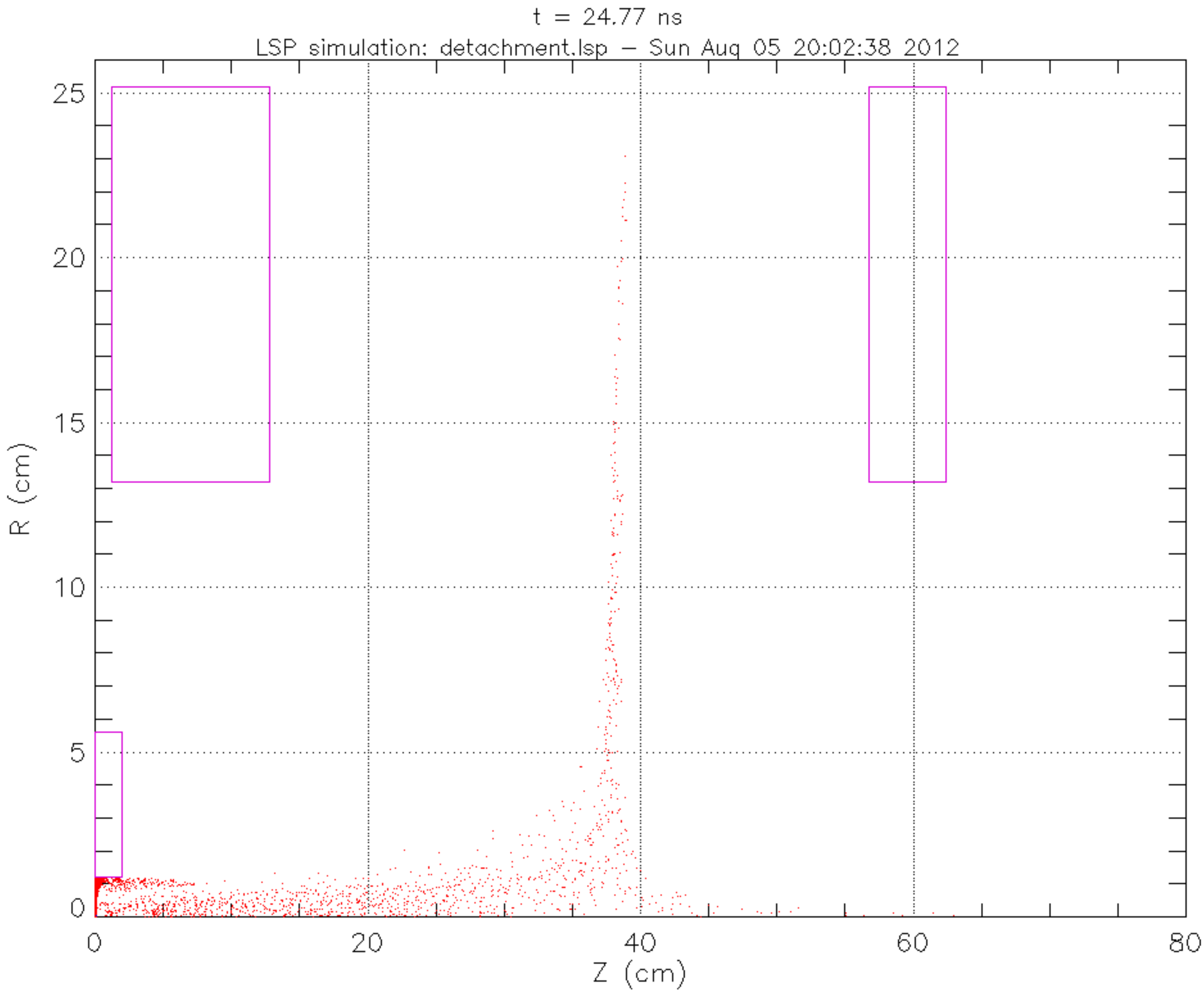
Theory

- In an ideal MHD scenario, plasma detachment has been theoretically predicted to occur when the local kinetic energy of the plasma is greater than the local magnetic field energy. [4]
- LSP can be used to see whether this in fact occurs. Using a coil with a reverse current will create an area of very low magnetic field energy, so the kinetic energy of plasma and the curvature of the magnetic fields lines will be much higher.
- In addition, the more energetic electrons will move out of the nozzle ahead of the protons, creating an electric field in the positive Z-direction, further accelerating the protons in the plasma. [1]

Initial LSP Simulation-Parameters

Parameter	Quantity
Plasma Injection Velocity	Mach 1
Plasma Density	10^9/cm^3
Electron Energy	5 eV
Ion Energy	1 eV
Grid Area	80 cm x 26 cm
Number of Cells	6750
Time	4 microseconds

simulations conducted in axial-symmetric cylindrical coordinates



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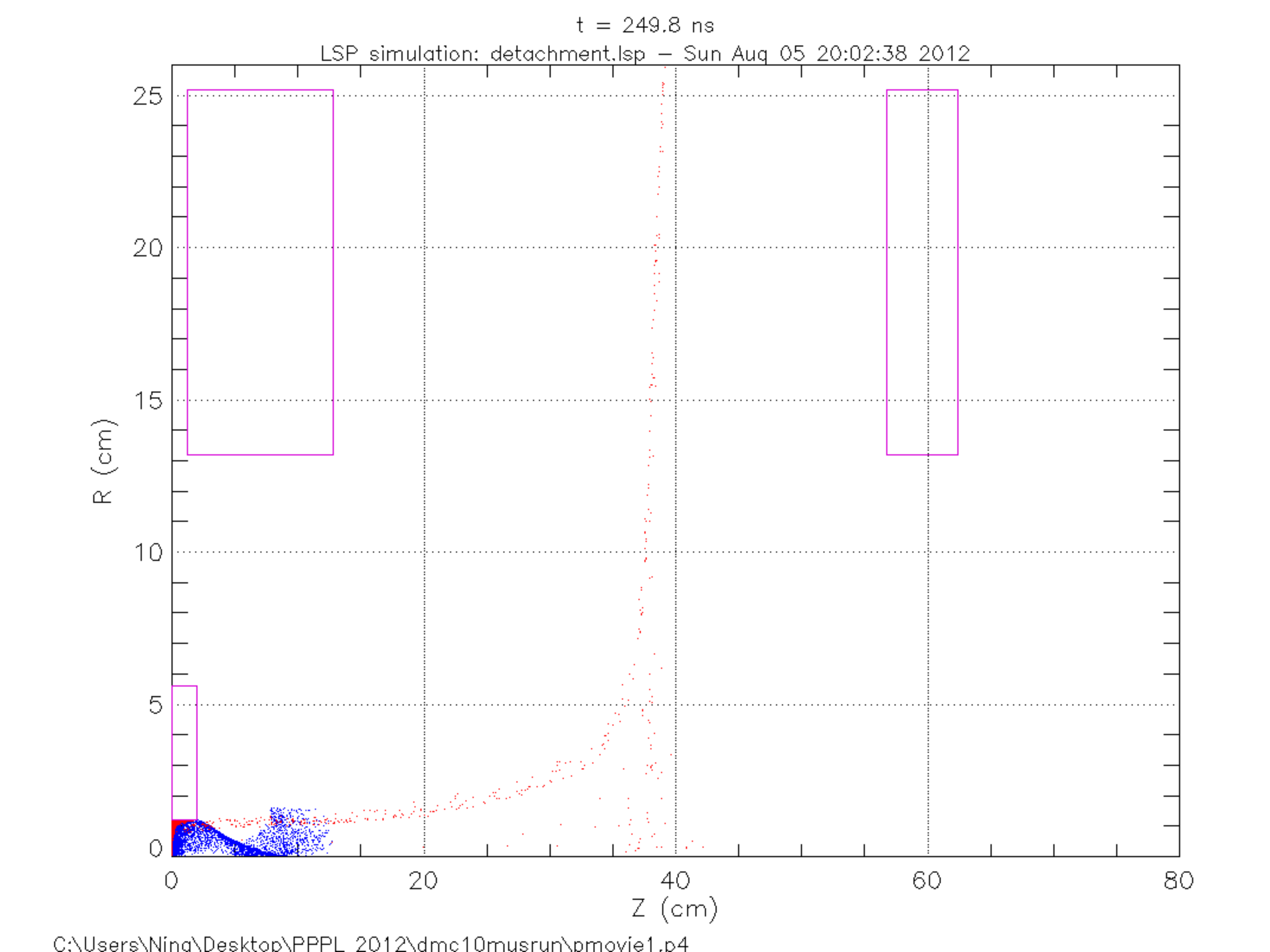

t = 249.8 ns
LSP simulation: detachment.lsp - Sun Aug 05 20:02:38 2012
R (cm)
Z (cm)
0
5
10
15
20
25
0
20
40
60
80
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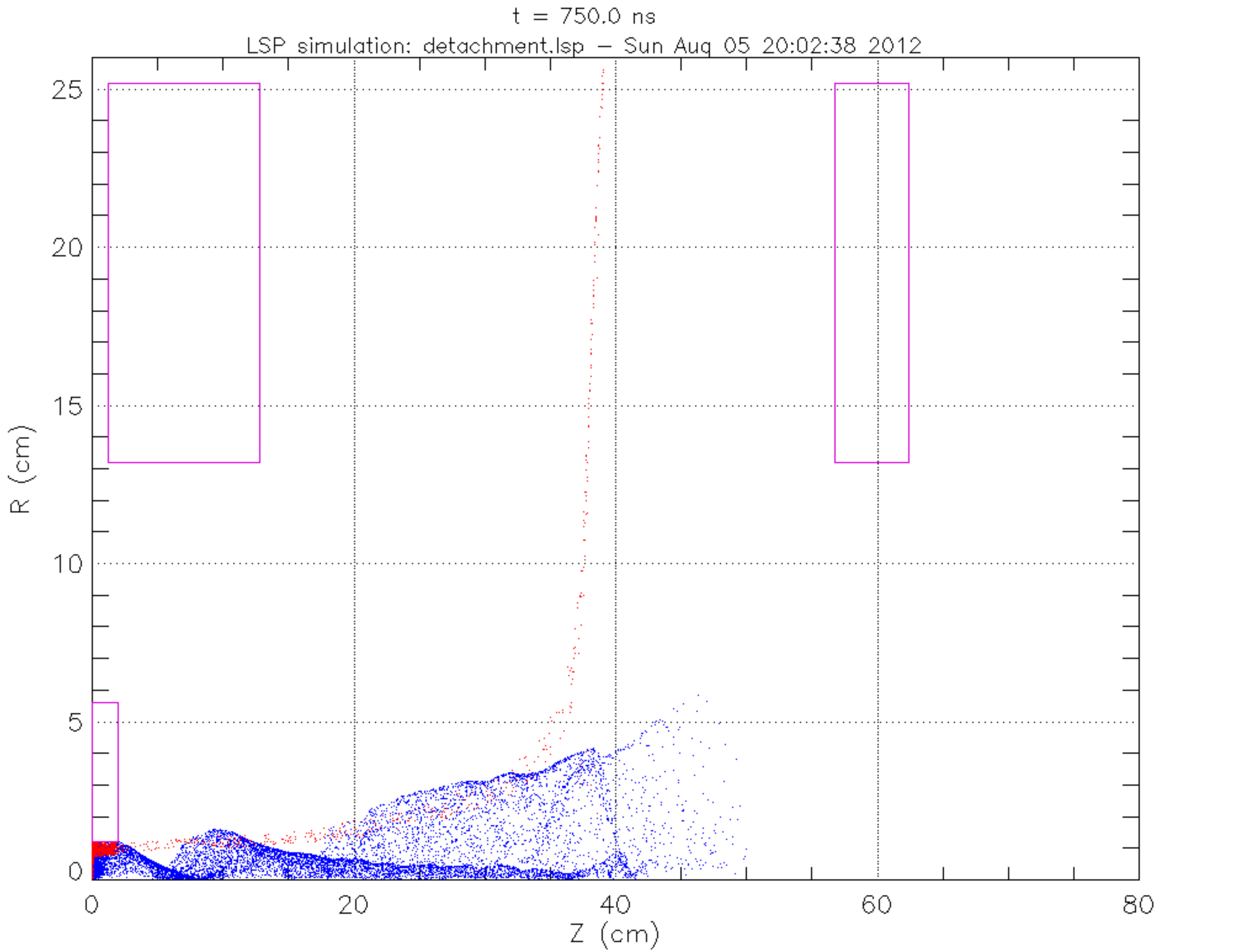

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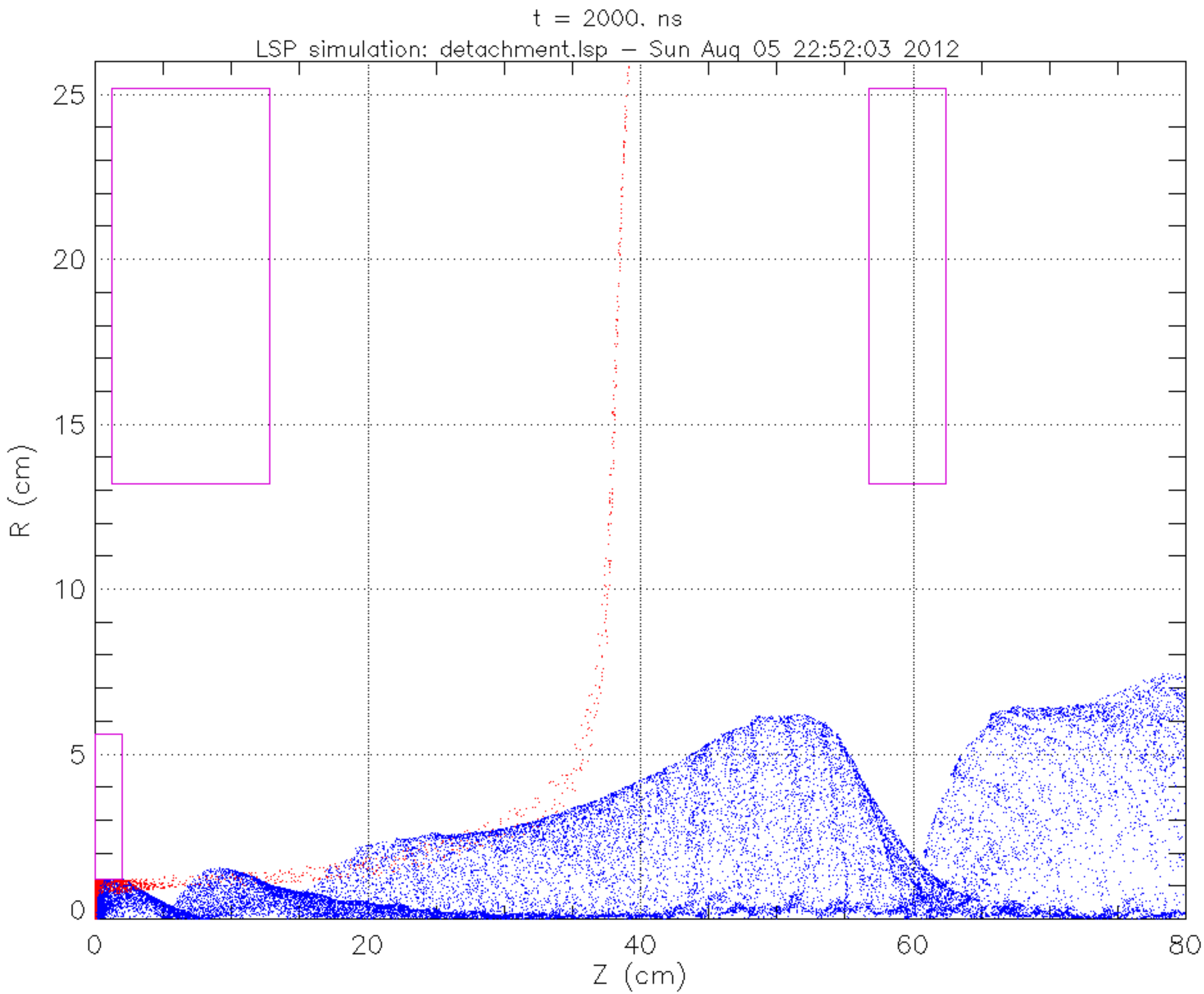

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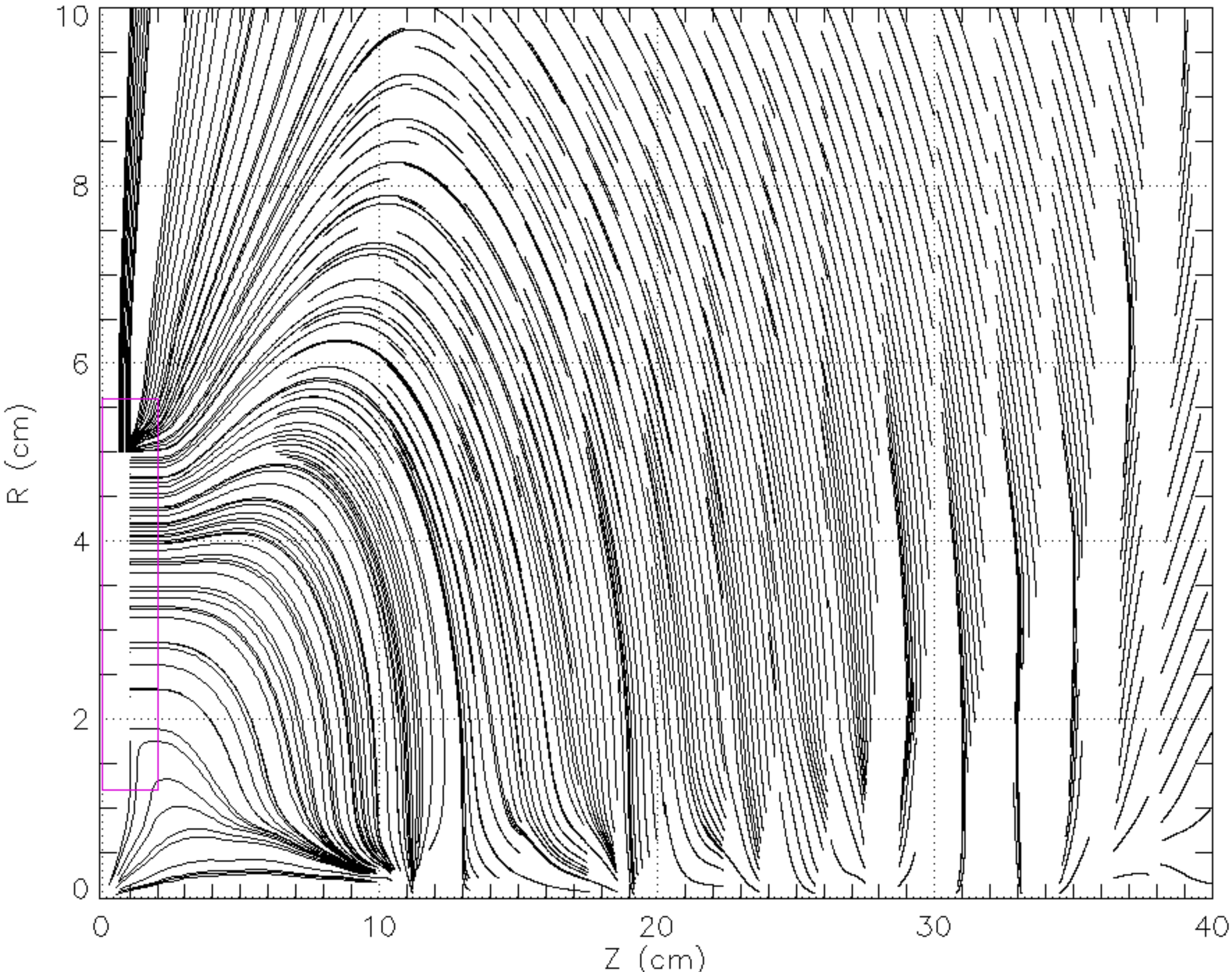
LSP simulation: detachment.lsp — Sun Aug 05 22:52:03 2012

t = 99.77 ns

Vector Plot of Ex at Th=0.0000



Max. magnitude (0.0000,1.000) : 2.34757

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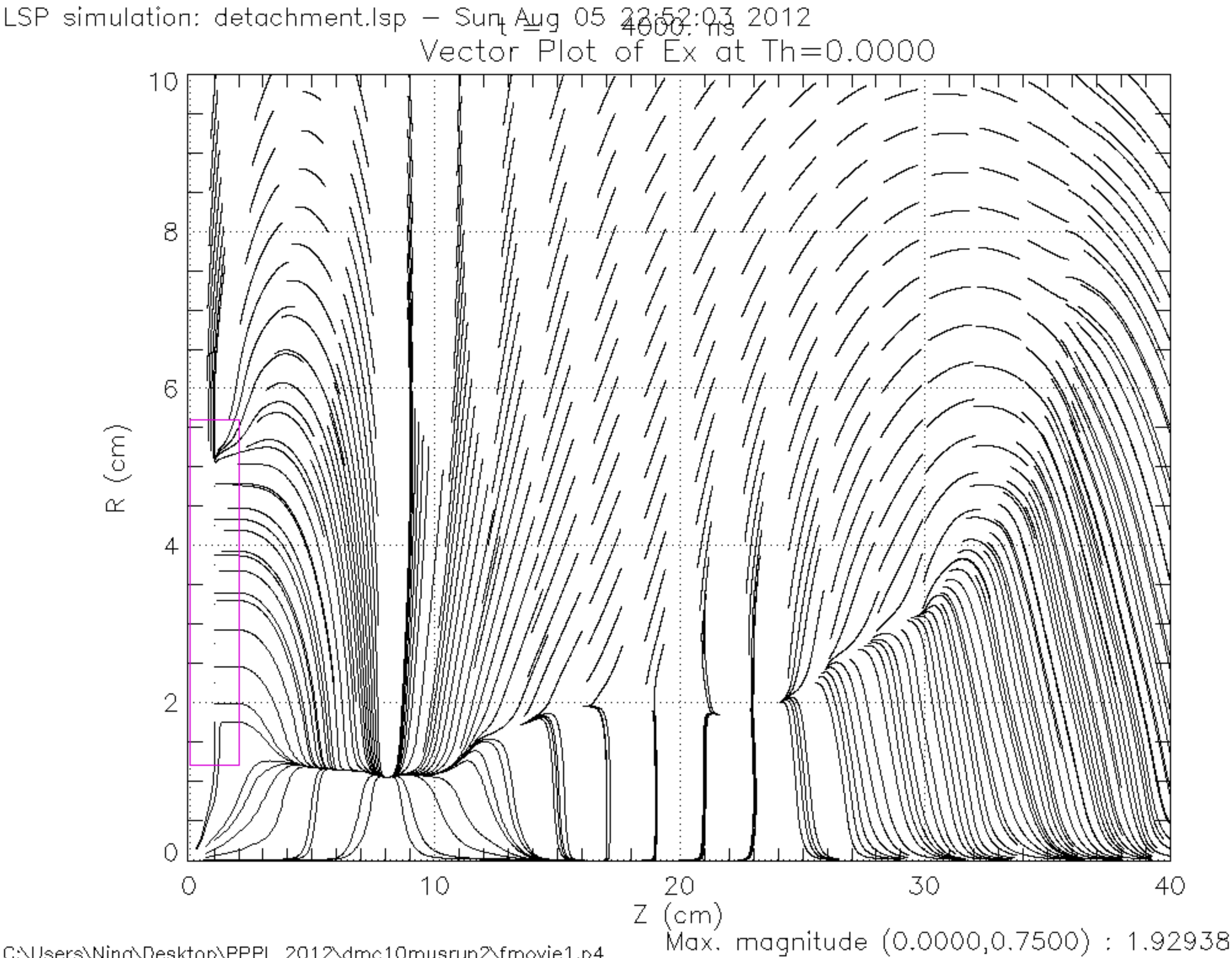
LSP simulation: detachment.lsp — Sun Aug 05 22:52:03 2012
t = 4000. ns
Vector Plot of Ex at Th=0.0000
R (cm)
Z (cm)
0
2
4
6
8
10
0
10
20
30
40
Max. magnitude (0.0000,0.7500) : 1.92938
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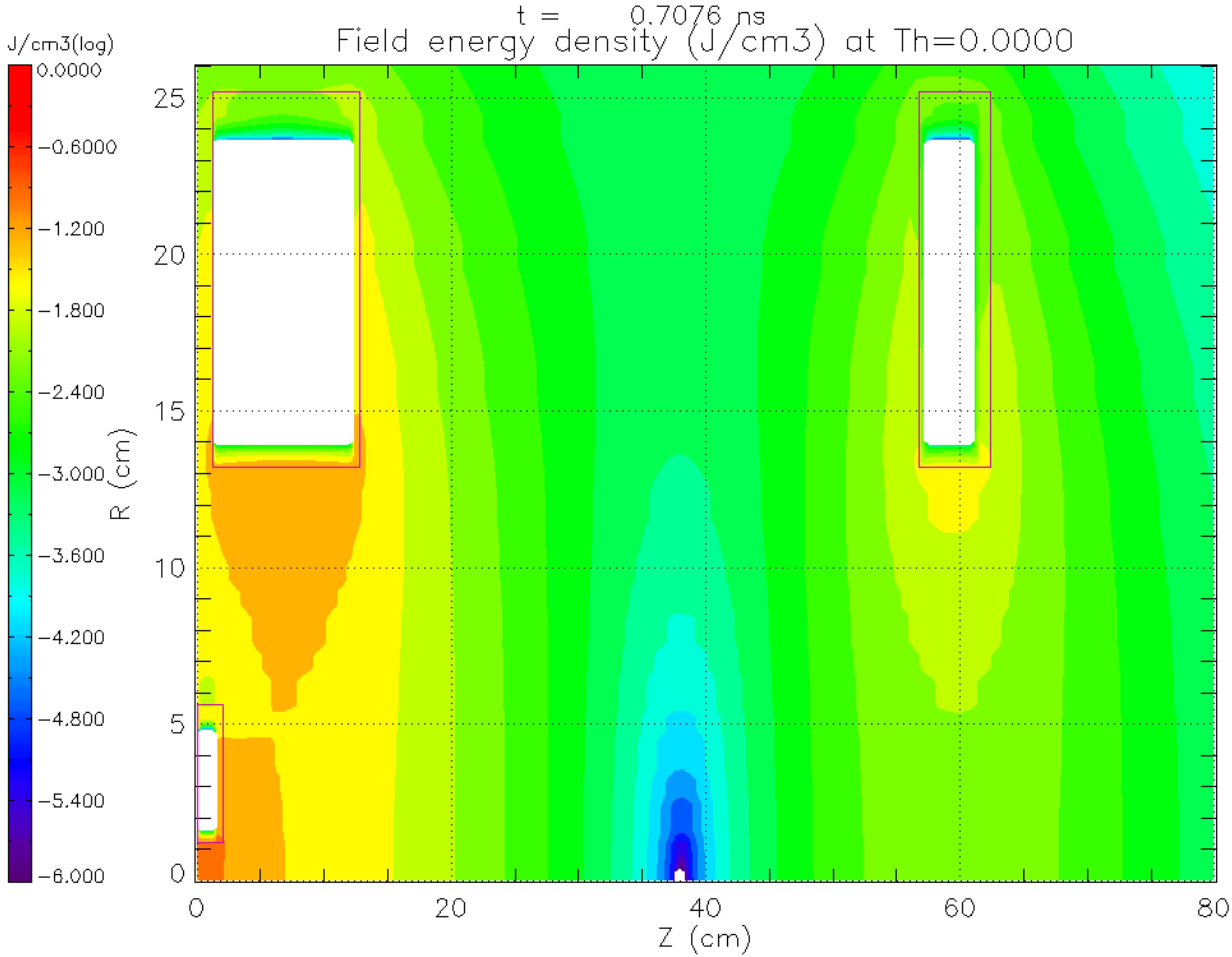

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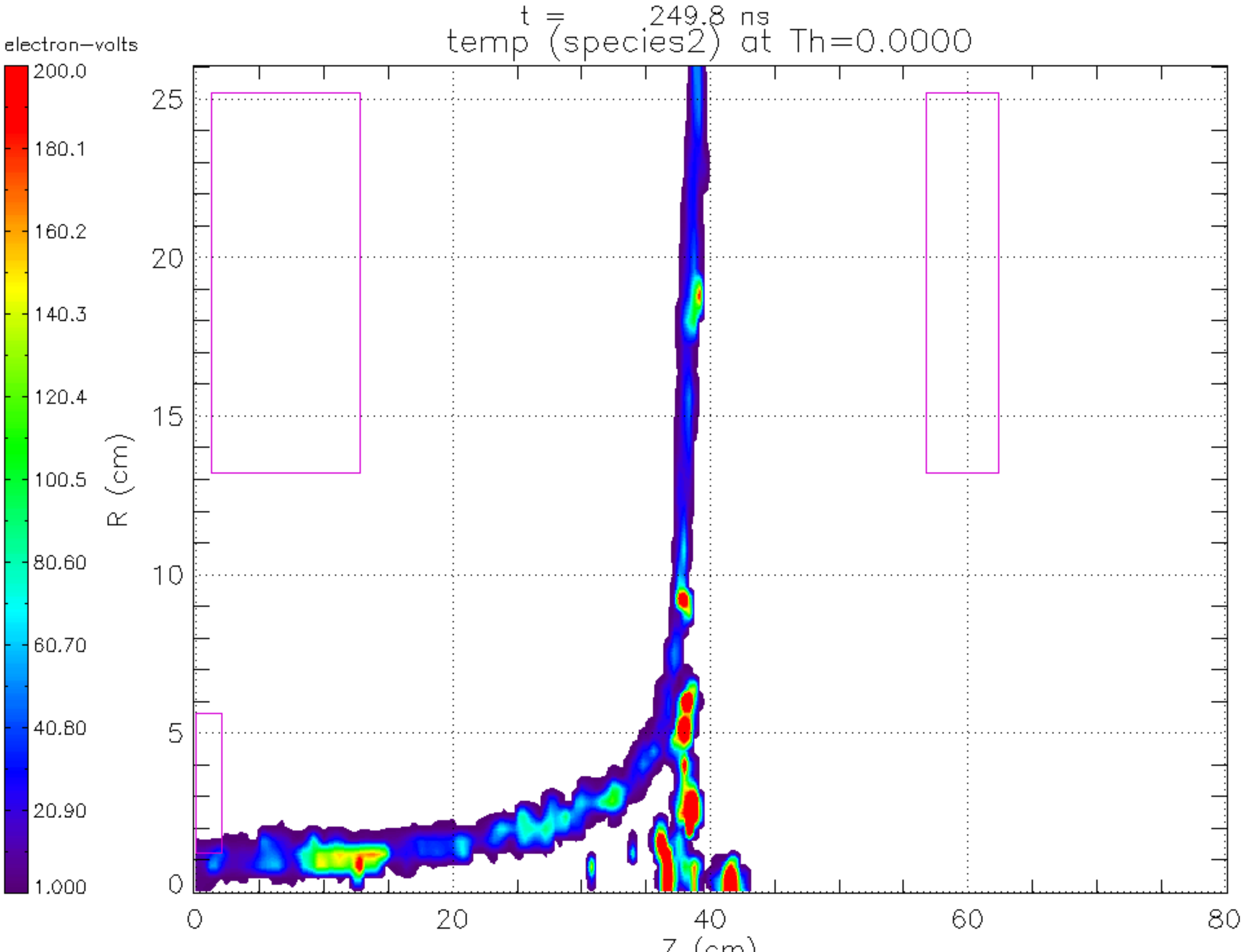

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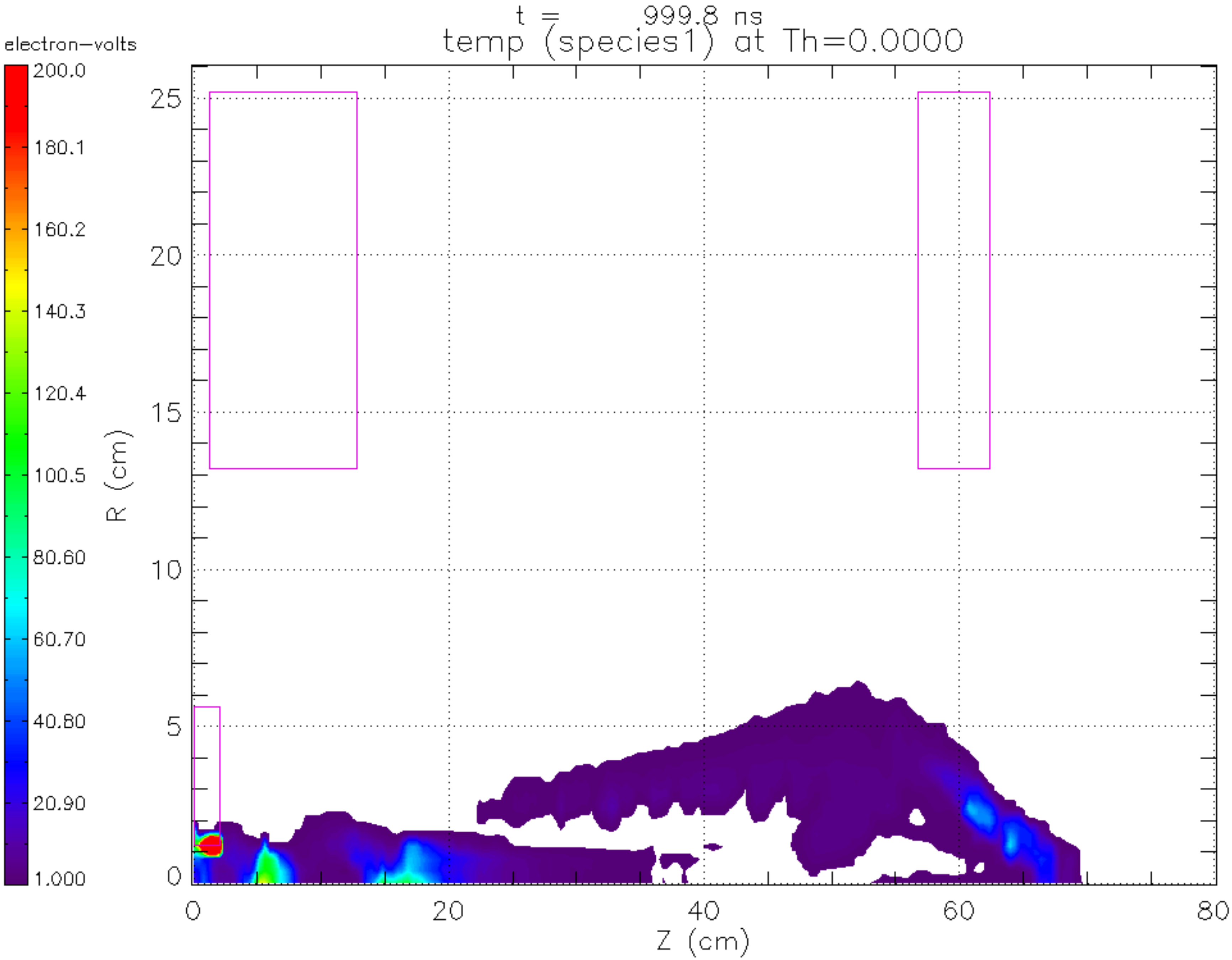

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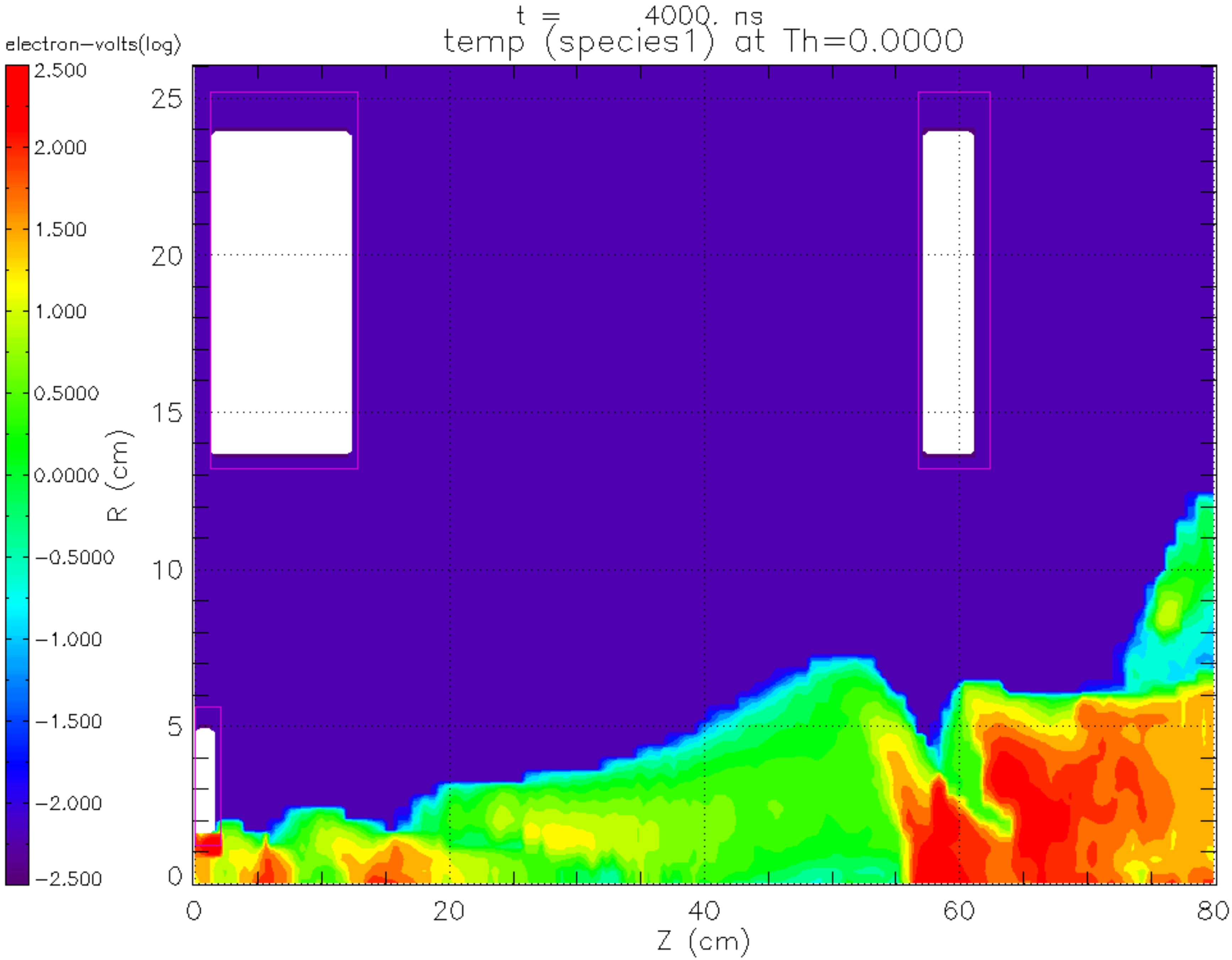

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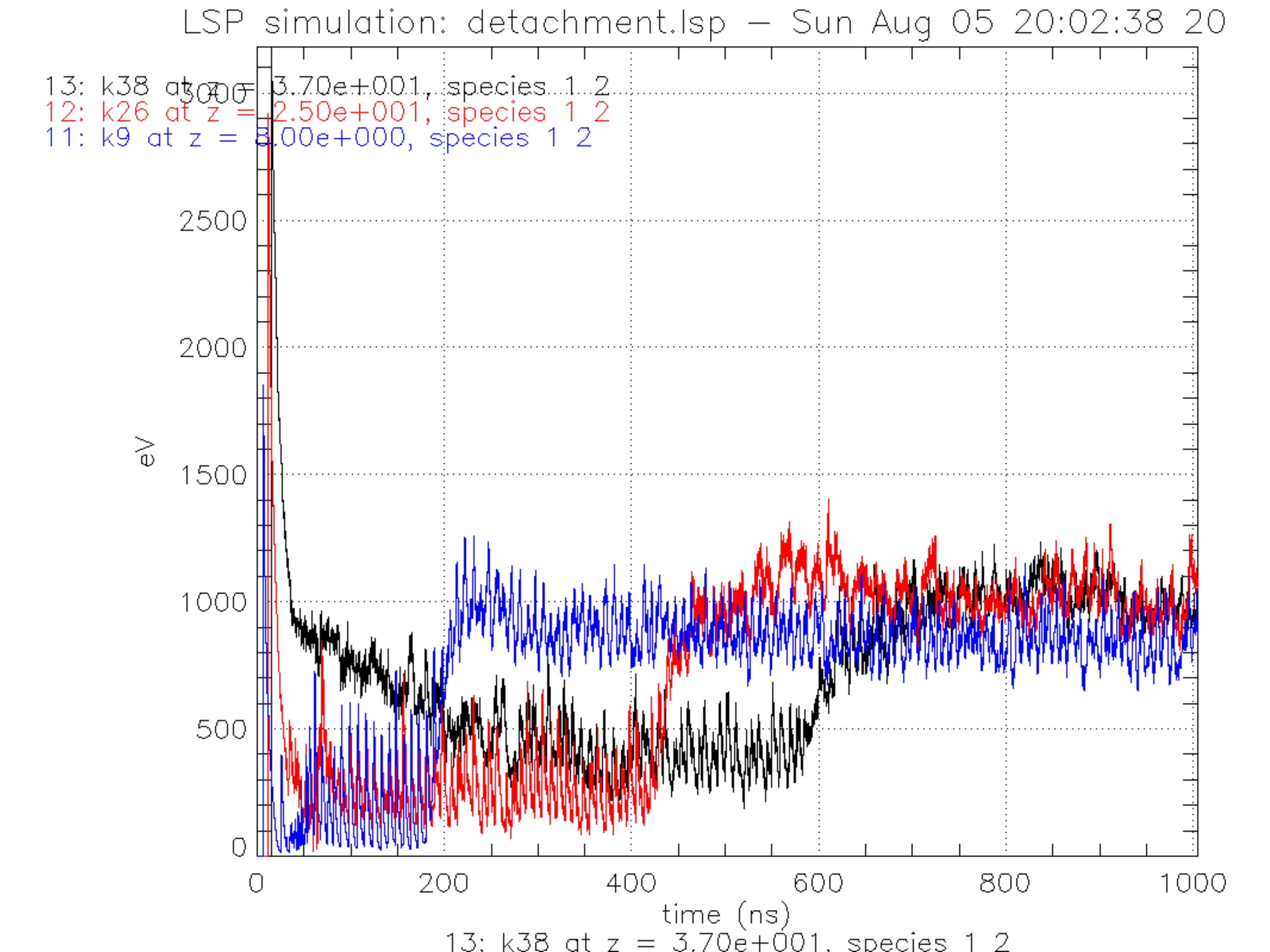

LSP simulation: detachment.lsp — Sun Aug 05 20:02:38 20
13: k38 at z = 3.70e+001, species 1 2
12: k26 at z = 2.50e+001, species 1 2
11: k9 at z = 8.00e+000, species 1 2
eV
3000
2500
2000
1500
1000
500
0
0
200
400
600
800
1000
time (ns)
13: k38 at z = 3.70e+001, species 1 2

Concerns in LSP Simulations Thus Far

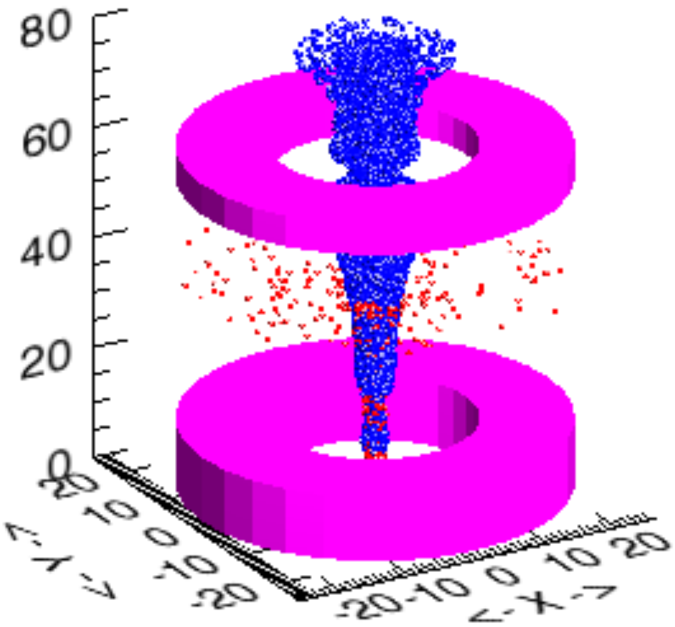
- No electrons present on-axis after 100 nanoseconds (lack of quasi-neutrality)
- Very high ion energy gain (1 eV to 300 eV)
- Presence of standing ion waves
- Direction and strength of electric field created by plasma

Continuing Investigation

- Magnetic Field Model
  - Using magnetic vector potential ($\boldsymbol{B} = \nabla \times \boldsymbol{A}$) for modeling
  - Using a program other than MathCad that would allow for more accurate integration
- LSP
  - Using multiple processors to compute using smaller time-steps, more super-particles, and higher grid resolutions for consistency with theory
  - Increasing the accuracy of the magnetic field
- Parameters
  - Experimenting with different current strengths in order to change field geometry
  - Defining parameters: curvature and expansion ratio
  - Comparing simulation results with theory

References

- [1] A. B. Sefkow and S. A. Cohen, Phys. Plasmas **16**, 053501 (2009)
- [2] Stewart, Joseph V. *Intermediate Electromagnetic Theory*. London: World Scientific Publishing Co. Pte. Ltd., 2001. 275-276. Print.
- [3] Stratton, Julius Adams. *Electromagnetic Theory*. York, PA: McGraw-Hill Book Company, Inc., 1941. 233. Print.
- [4] A. V. Arefiev and B. N. Breizman, Phys. Plasmas **12**, 043504 (2005)



Thank You