Using Single-Particle Motion Simulation to Optimize Coil Parameters for Inducing Autoresonant Heating in the PFRC

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1. Abstract

The heating of ions confined in a field-reversed configuration (FRC) equilibrium magnetic geometry subject to a small-amplitude, odd-parity rotating magnetic field (RMF) has previously been observed in single-particle Hamiltonian simulations. We consider a form of the autoresonance method to provide added heating capabilities. Two mirror coils encircling the FRC were added near the X-points of the FRC, co-axial with the major axis; these may be used to add oscillating components, primarily to the axial field, stiffening or relaxing the field, shortening or lengthening the x-point distance. Various parameters of the simulations were modified, including the positions of the coils along the axis, the amplitude and frequency of the oscillations, as well as other FRC parameters to determine whether autoresonant heating is a feasible method for increasing ion heating. Rapid heating was observed when the frequency of the oscillating mirror coils were in the ion gyrofrequency range, followed by loss of confinement of the ion.

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2. Introduction

2.1 Background information about PFRC

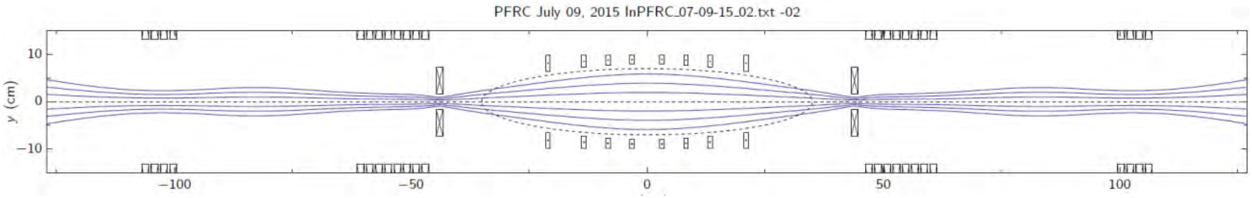


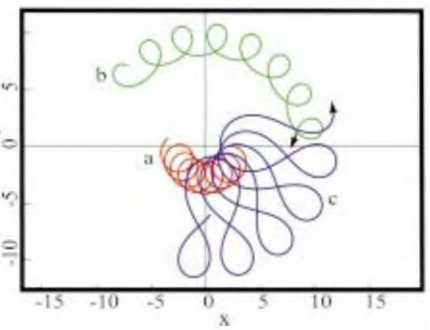
FIG 1 (color). This figure shows the shape of the PFRC, as well as the axial magnetic field lines, and the locations of the simulated coils

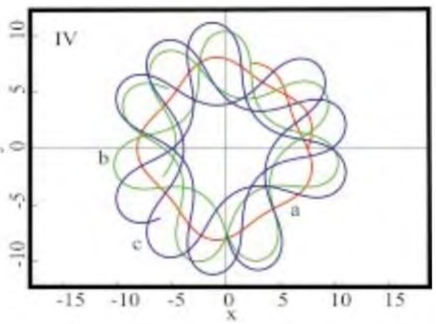
The Princeton Field Reverse Configuration (PFRC) experiment is a toroidal magnetic plasma confinement device that has several unique properties relevant to a magnetic fusion reactor design. An FRC reactor's simply connected topology with no internal conductor and no toroidal magnetic field allows the reactor to be made very compact, and often one or two order of magnitudes smaller in diameter than for a tokamak. [1]

Dynamics of a single particle in the FRC is described by the Hamiltonian [2]

$$H = V_0\{\pi_\rho^2 + \pi_\zeta^2 + \frac{1}{\rho^2}[\pi_\phi - \rho^2(1-\rho^2-\zeta^2)]^{\,2}. \qquad (1)$$

Particles confined in an FRC can undergo 3 main types of orbits: cyclotron, betatron, and figure-8.





[5]

FIG 2 (color). a and b are both cyclotron orbits, while c is a figure-8 orbit

FIG 3 (color). a, b, and c all represent betatron orbits at progressively higher energies

In the $\zeta = 0$ plane, the cyclotron, betatron, and figure-8 orbits can all be modeled by a particle in the potential [2]

$$V(\rho) \approx \frac{V_0}{\rho^2}[\pi_\phi - \rho^2(1-\rho^2)]^2. \tag{2}$$

V can be a single or double potential well, depending on the value of π_ϕ, with one or two minima $\rho_{1,2}$, with cyclotron motion in one of the wells. Expanding the potential near the well ρ_1 to the fourth order gives [2]

$$V(\rho) \approx V_0[a_0 + a_1(\rho-\rho_1) + a_2(\rho-\rho_1)^2 + a_3(\rho-\rho_1)^3 + a_4(\rho-\rho_1)^4], \tag{3}$$

with the dimensionless coefficients $a_n = \frac{1}{V_0}\frac{1}{n!}\frac{d^n V}{d\rho^n}|_{\rho=\rho_1}$.

The linear frequency at this minimum is $\omega_0 = \sqrt{V''(\rho_1)}$. [2]

Defining $x = \rho - \rho_1$ allows the potential to become [2]

$$V(x) = {\omega_0}^2\left(\frac{x^2}{2} + \lambda\frac{x^3}{3} + \beta\frac{x^4}{4}\right), \tag{4}$$

where $\lambda = \frac{3a_3V_0}{{\omega_0}^2}$ and $\beta = \frac{4a_4V_0}{{\omega_0}^2}$.

2.2 Background information about autoresonance

Plasmas exhibit properties of strongly nonlinear oscillators, which causes problems when attempting to heat it because a plasma's frequency will tend to drift with increasing oscillation amplitude. [3] Without autoresonance, it would be very difficult, if not impossible, to drive a nonlinear oscillator to very high frequencies without an automated feedback system that adjusts the driving frequency to match the drift in the oscillator's resonance frequency. When a plasma's oscillations are driven to a high amplitude, its resonance frequency decreases, and a non-chirped drive will increase the plasma's oscillation amplitude for a while until the plasma's oscillations slip out of resonance, causing the amplitude to fall to zero again before the process repeats. [4] Autoresonance is a phenomenon whereby a nonlinear oscillator that is linearly driven will tend to stay in resonance with the linearly changing drive frequency. [4]

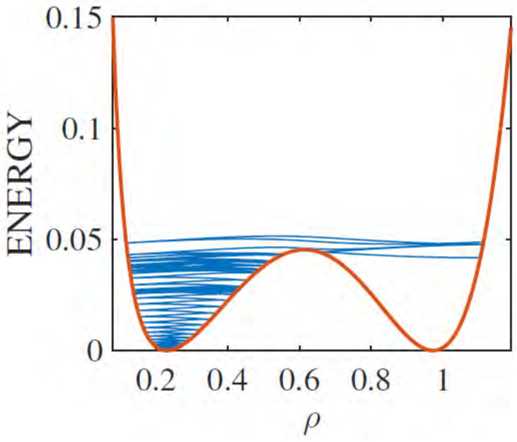


FIG 4. The predicted means of autoresonance in the PFRC is by accelerating ions confined in potential wells with oscillating magnetic mirror fields at the ends of the PFRC along the z-axis. [2]

To observe autoresonance in this system, we initialize a particle at one of the minima, then apply the driving potential [2]

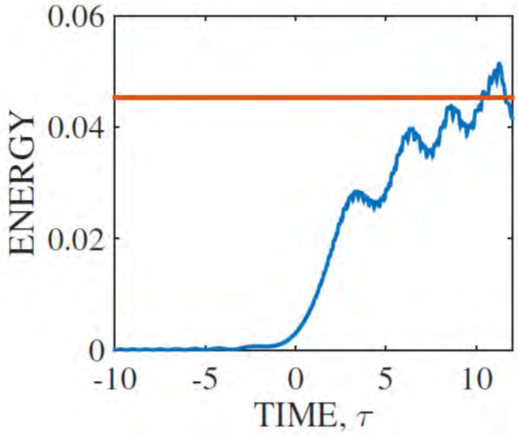
$$V_d(x,t) = A\rho \cos \omega_d \ , \tag{5}$$

where the driving frequency $\omega_d = \omega_0 - \alpha t$ and passes through the linear frequency at $t = 0$. The driving force is a small, with $A \ll V_0$, and the chirp is slow, with $\frac{\alpha}{\omega^2} \ll 1$. However, a threshold phenomenon occurs where A must be larger than the critical value [2]

$$A_{cr} \approx 0.82\, \alpha^{3/4} \gamma^{-1/2}, \tag{6}$$

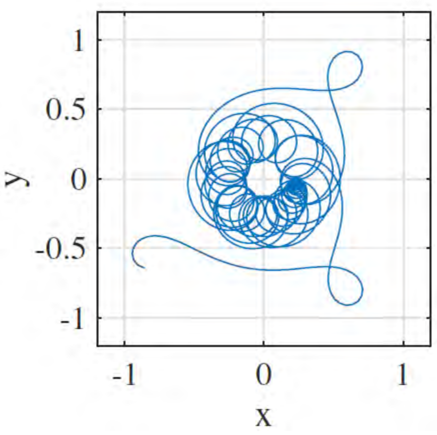
where $\gamma = \frac{3}{8}\beta - \frac{5}{12}\lambda^2$.

Theoretically, the mirror fields should oscillate in sync with the particle's motion along the z-axis. [2] As the particle oscillates back and forth along the z-axis, the mirror fields should be pinching in towards the particles as the particles reflect off of them, imparting additional energy to the particle on each reflection. This is shown graphically as a particle initialized at the bottom of a double potential well that gains energy by reflecting off the sides of the well until it escapes the well and crosses the separatrix.



[2]

FIG 5. Based on numerical analysis of autoresonance in the FRC, theory suggests that to achieve autoresonance in the FRC, the driving potential must be greater than some minimum threshold. [2] Once this threshold is surpassed, the plasma oscillations should grow without bound, regardless of the driving amplitude until the particle begins to undergo chaotic motion. [4] [2]



[2]

FIG 6. A particle trapped inside the separatrix of a double potential well has a cyclotron orbit with increasing gyroradius as it gains energy, and eventually crosses the separatrix and enters a figure-8 orbit.

2.3 Motivation for studying autoresonant heating in the PFRC

Currently, the best known method of achieving ion heating in the PFRC is by applying an odd parity Rotating Magnetic Field (RMF) about the midplane of the PFRC. This paper investigates the possibility of an alternative or additional method of heating which could achieve fusion-relevant temperatures using mirror coils, which is an existing and proven technology that could easily be added to the current PFRC. Creating an oscillating mirror field then is a simple matter of passing an alternating current through the mirror coils at some frequency with some chirp rate. Using this method of heating plasma in the PFRC can possibly lower the construction and operating costs of the PFRC, since creating the RMF is much more difficult than simply install two pre-fabricated coils. It can also be used in conjunction with the RMF to achieve even higher temperatures in the PFRC.

3. Method

3.1 List of FRC parameters

The PFRC simulation is defined by several parameters, several of which were modified in the simulations, and several were kept constant.

Separatrix radius: the radius of the FRC separatrix was kept at a constant 7 cm throughout the simulations.

Kappa: the ratio of length to diameter of the FRC, and this was varied from 2 to 6 in steps of 1, but was kept at a constant 5 for all other runs.

Mirror coil location: the mirror coils were kept at a constant 40 cm from the center of the FRC along the z-axis

Confining magnetic field strength: the confining magnetic field strength was varied from 100 G to 1100 G in steps of 100 G, but was kept at a constant 500 G for all other runs.

Mirror field strength: the mirror field strength was varied from 20 G to 320 G in steps of 20 G, but was kept at a constant 100 G for all other runs.

Frequency of mirror field oscillations: the mirror field oscillation omega frequency was varied from 10^5 rad s^{-1} to 1.174×10^7 rad s^{-1} in multiples of 1.1

Chirp rate of mirror field oscillations: the linear chirp rate of mirror field oscillations was varied from 1 rad s^{-2} to 1.174×10^4 rad s^{-2} in multiple of 1.1, and then again from -1 rad s^{-2} to -117.4 rad s^{-2} in multiples of 1.1

Initial ion energy: the initial ion energy was varied from 1ev to 256 ev in multiples of 2, but was kept at a constant 3 ev for all other runs.

Initial phi angle: due to axial symmetrical around the z-axis, the initial phi angle is irrelevant.

Initial theta angle: the initial angle of the particle in the r-z plane was varied from 0 to 360 degrees in steps of 10 degrees, but was kept at 0 for all other runs.

Initial Z-axis Position: the initial position of the ion along the z-axis was varied from to 0.0 to 0.55 times the separatrix radius (in the z-directions) in steps of 0.05, but was kept at 0.2 for all other runs

Initial radius: the initial radius of the particle was varied from 0.1 times the separatrix radius to 1.2 times the separatrix radius in steps of 0.1, but was kept at a constant 1.1 for all other runs

3.2 The RMF code

All of the simulations were conducted using an RMF code written by Dr. Alan Glasser of the University of Washington, which simulated the trajectory of a single Hamiltonian particle confined in an FRC. The code uses the vector potentials of magnetic fields in a simulated FRC to calculate the potentials and trajectories of confined particles. Oscillating current was passed through filamentary mirror coils in the simulation, and various parameters of the FRC, such as the amplitude of the magnetic field strength as well as the frequency of the oscillations were varied. We observed the maximum kinetic energy as well as confinement time of an ion confined in an FRC across different ranges of frequencies, and compared the results with theoretical predictions to determine if autoresonance was achieved, and whether autroresonance could be a feasible method of heating in the PFRC with parameters comparable to what's achievable in the actual experiment.

Although the code is known as the "RMF code", it is actually a single particle Hamiltonian code, and the RMF was not simulated in any run.

4. Results

In the following graphical results of batch runs of the RMF code, the line colors range from yellow to green to purple, in order of increasing magnitude, to indicate the variable not displayed on the horizontal axis.

4.1 Results of overall trends in the relationships between parameters

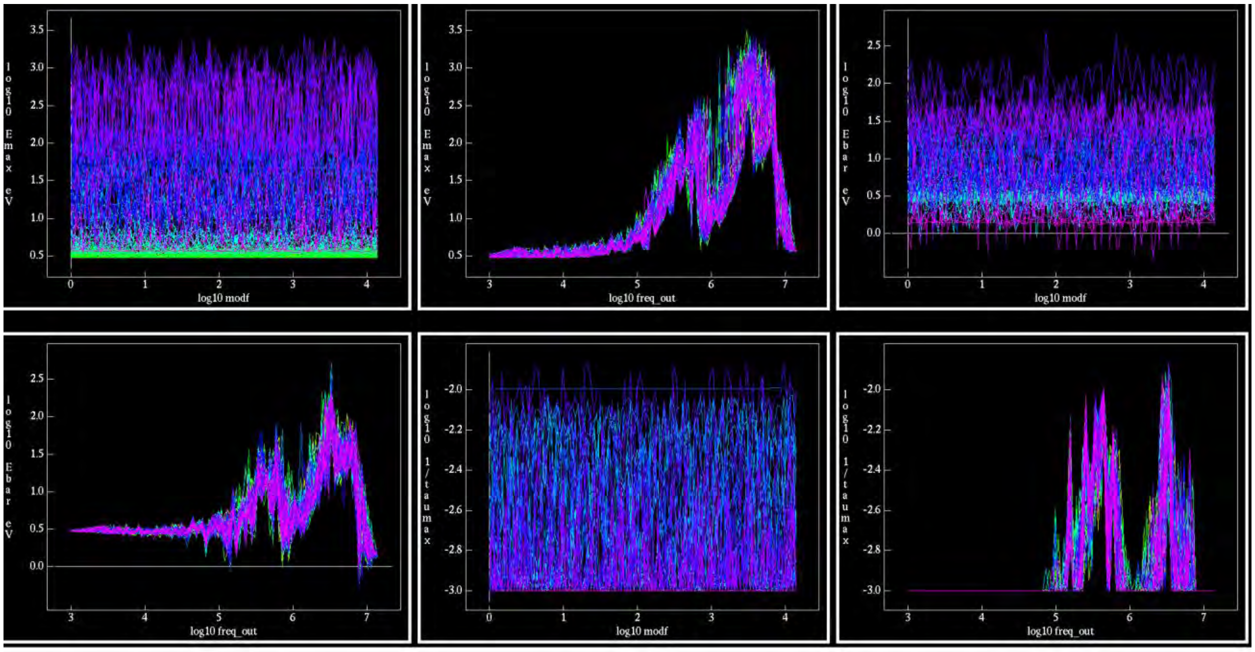
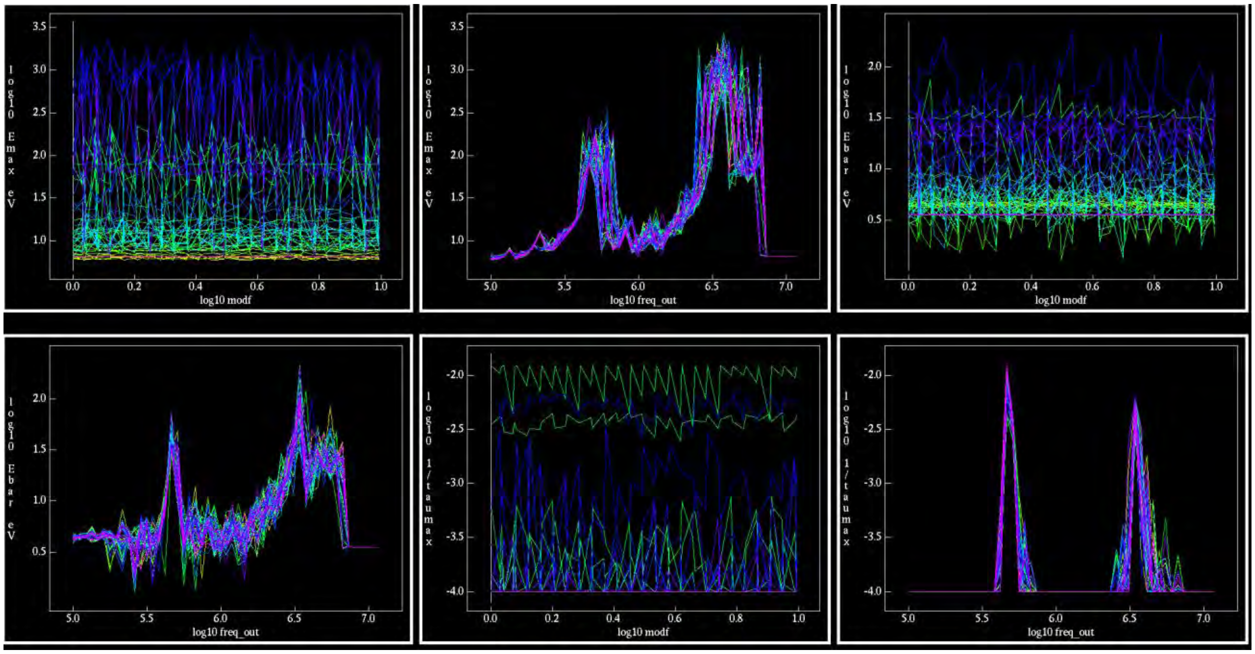


FIG 7-12. These graphs of the mirror coil omega frequency vs chirp rate show that maximum energy achieved definitely showed a double peak at around 5.8×10^5 rad s^{-1} and 3.5×10^6 rad s^{-1}, and the maximum energy peaked, there was a corresponding decrease in confinement time. However, it appears that the chirp rate has no effect on either the confinement time or the maximum energy achieved.

Ignoring the valley at around 6.1×10^6 rad s^{-1}, there appears to be a power law that governs the relationship between the frequency of the mirror field oscillations and the maximum energy achieved, with $E_{max} \propto {\omega_d}^2$ between the ranges of around 9×10^4 rad s^{-1} and 5×10^6 rad s^{-1}.



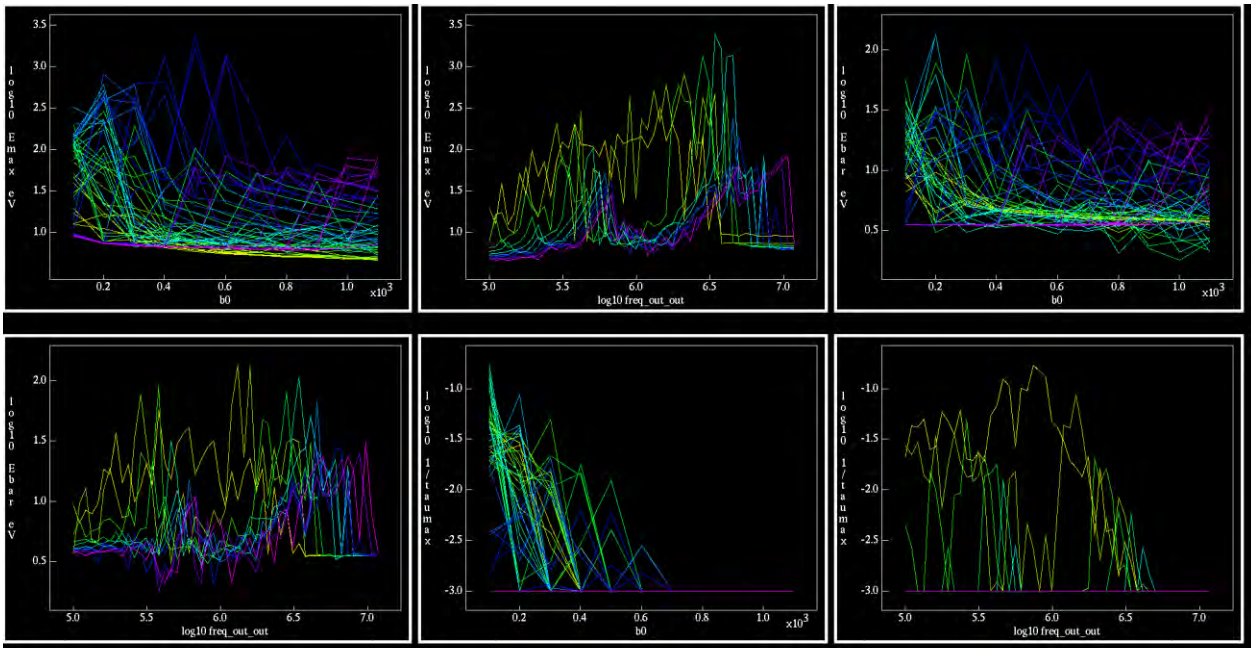


FIG 13-18. Repeating the simulation with a negative simulation confirmed the previous results that neither maximum energy achieved nor confinement time is related to the chirp rate.

FIG 19-24. The optimal strength of the confining magnetic field to maximize ion heating appears to be between 400 G and 600 G. As expected, stronger confining magnetic fields, like those above 700 G, appear to not only impede heating, but also increase confinement times. Interestingly, at somewhat lower confining magnetic fields, the minimum in maximum temperature around 10^6 rad s^{-1} all but disappears.

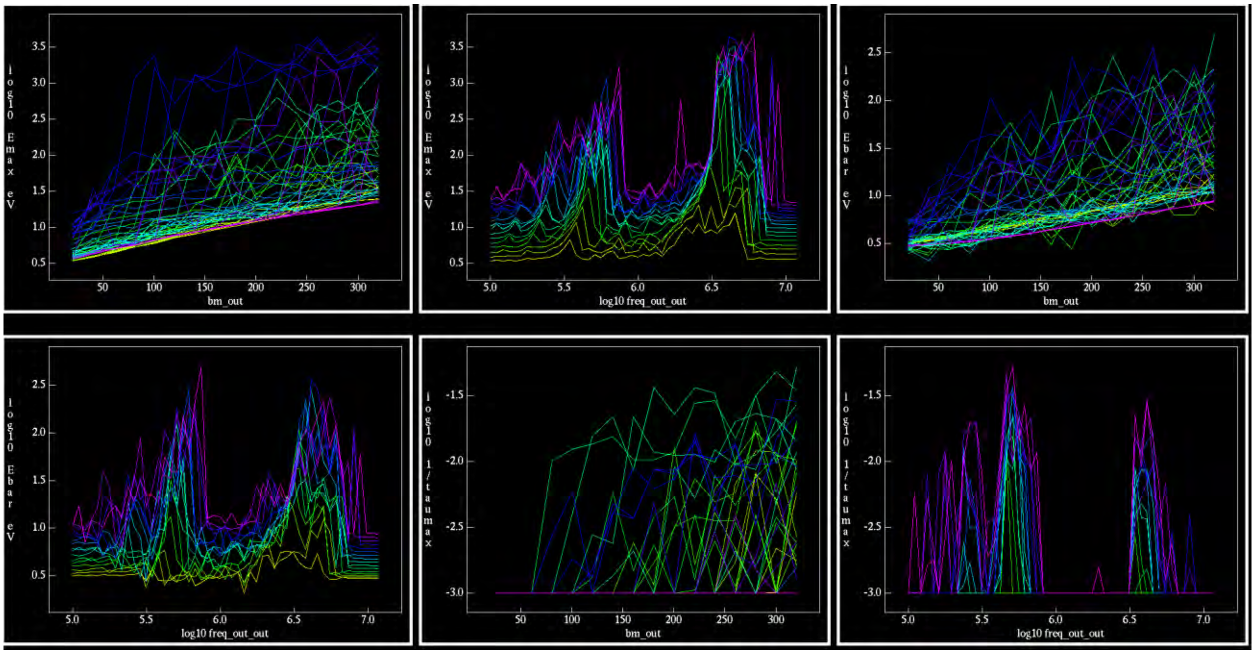


FIG 25-30. As expected, increasing the strength (amplitude) of the oscillating magnetic mirror field increases particle heating, and correspondingly decrease confinement time.

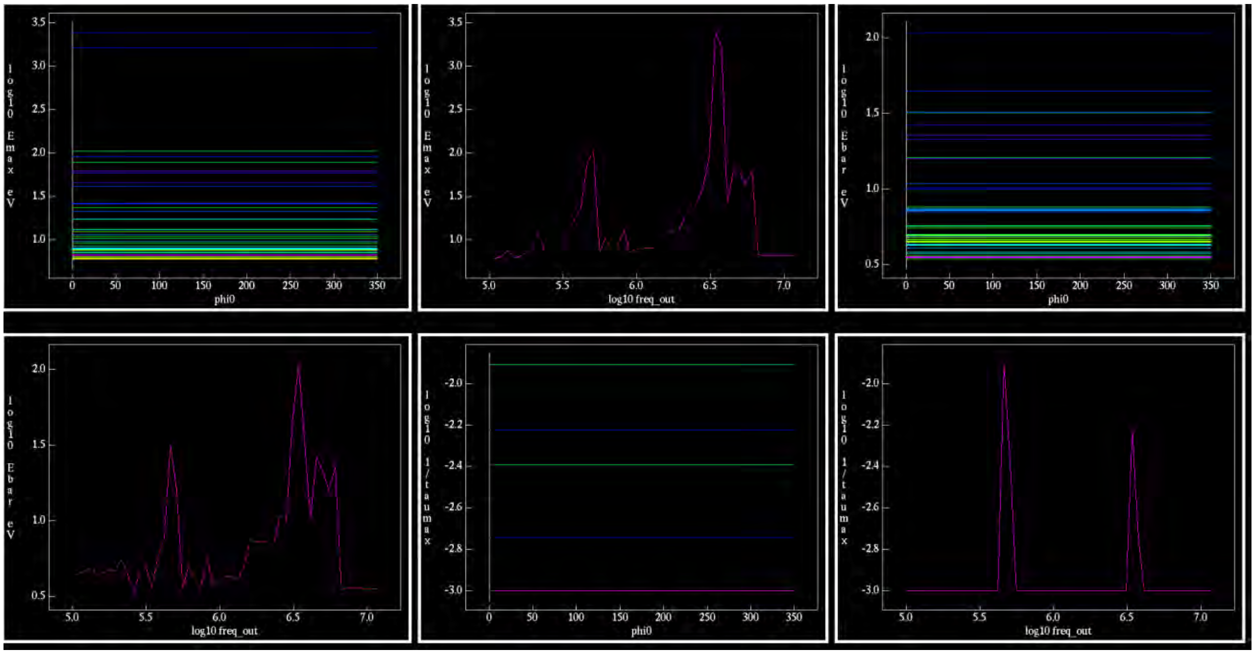


FIG 31-36. Due to symmetry in the phi angle of the FRC, initial phi angle has absolutely no effect on maximum temperature, which is confirmed by these graphs.

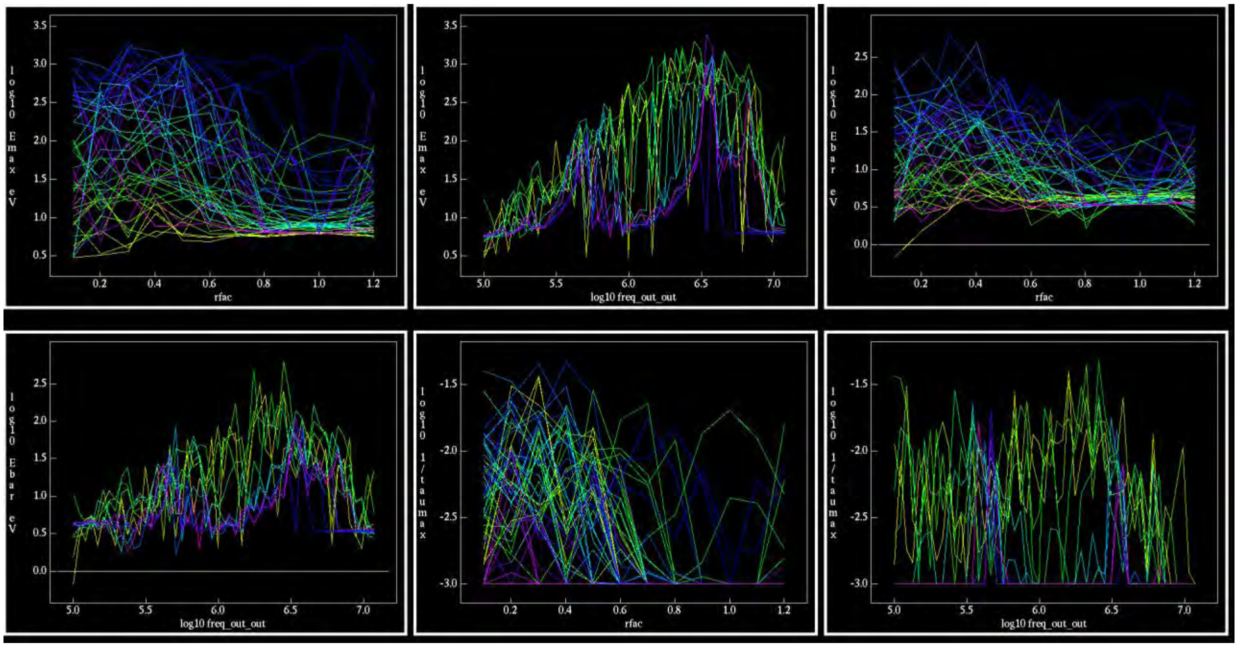


FIG 37-42. Greater heating and lower confinement times were both associated with an initial radius around 0.4. There is also no minimum in the maximum energy achieved at 10^6 rad s^{-1} at that radius, which shows a much more consistent relationship between oscillation frequency and maximum energy achieved.

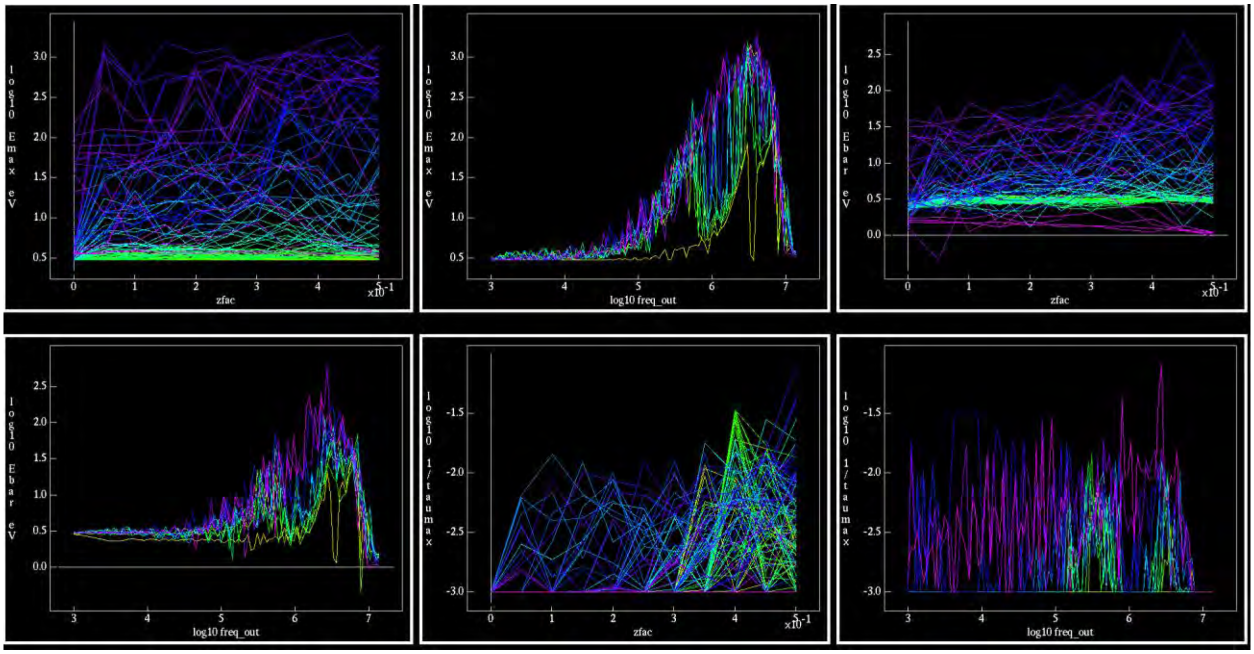


FIG 43-48. It appears that particles farther from the midplane of the FRC seem to consistenly achieve higher temperatures and suffer from lower confinemnet times. As with the radius, increasing the initial z-distance from the midplane also avoids the trough of maximum energy at 10^6 rad s^{-1}.

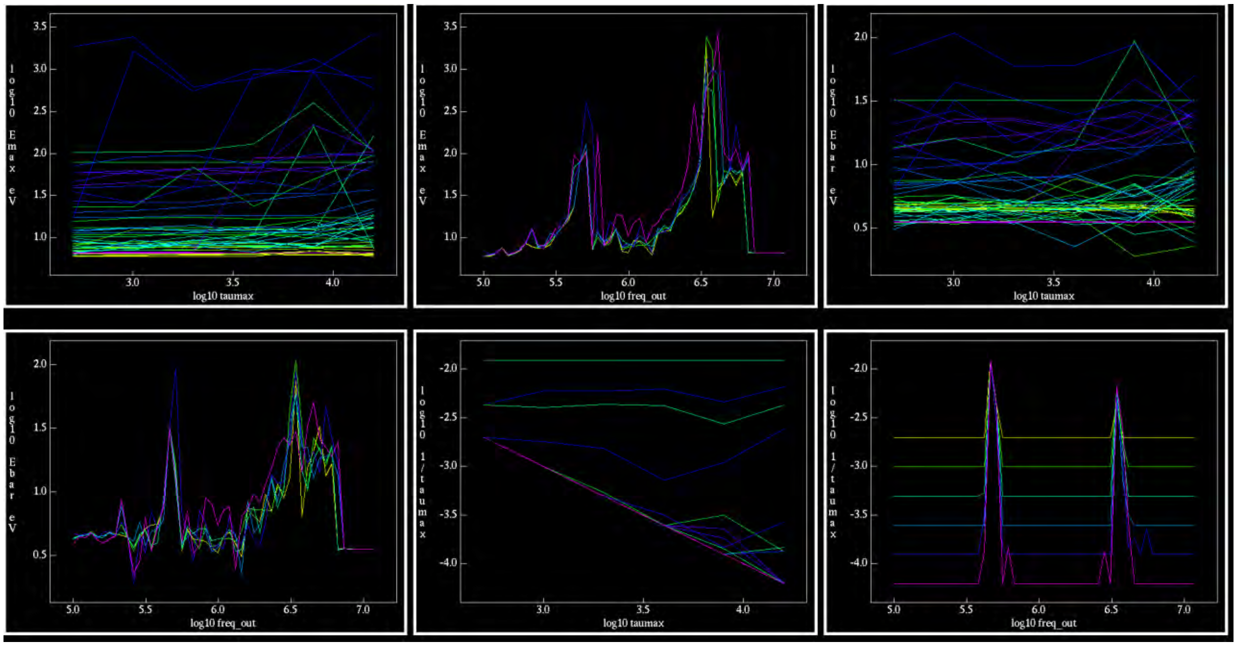


FIG 49-54. There does not seem to be a significant relationship between the maximum energy achieved and the simluation length, most likely due to the fact that confinement time and maximum particle temperature appear to be negatively correlated, which means particles do not take very long to heat up, and once they're hot, they tend to escape quickly.

4.2 Results of overall trends

We found heating of particles by several orders of magnitude, depending on the initial conditions of the particle, such as its initial kinetic energy, position, and trajectory. There were two notable peaks in maximum energy achieved when varying all parameters except for those that varied the initial positions of the ions. In cases where the initial position of the ion did not result in a minimum of maximum energy achieved at around a mirror field oscillation omega frequency of 10^6 rad s^{-1}, the degree of plasma heating appeared to steadily rise, followed by a sharp fall as the coil oscillation frequency approached and surpassed the ion gyrofrequency.

When the drive frequency ω_d was just below the ion gyrofrequency range ω_0, approximately 0.02 to 0.8 times ω_0, rapid heating was observed, followed by a rapid transition to chaotic particle motion leading to the particle escaping confinement.

4.3 Results of specific, important runs

4.3.1 Run with no oscillation in mirror field or heating of ions

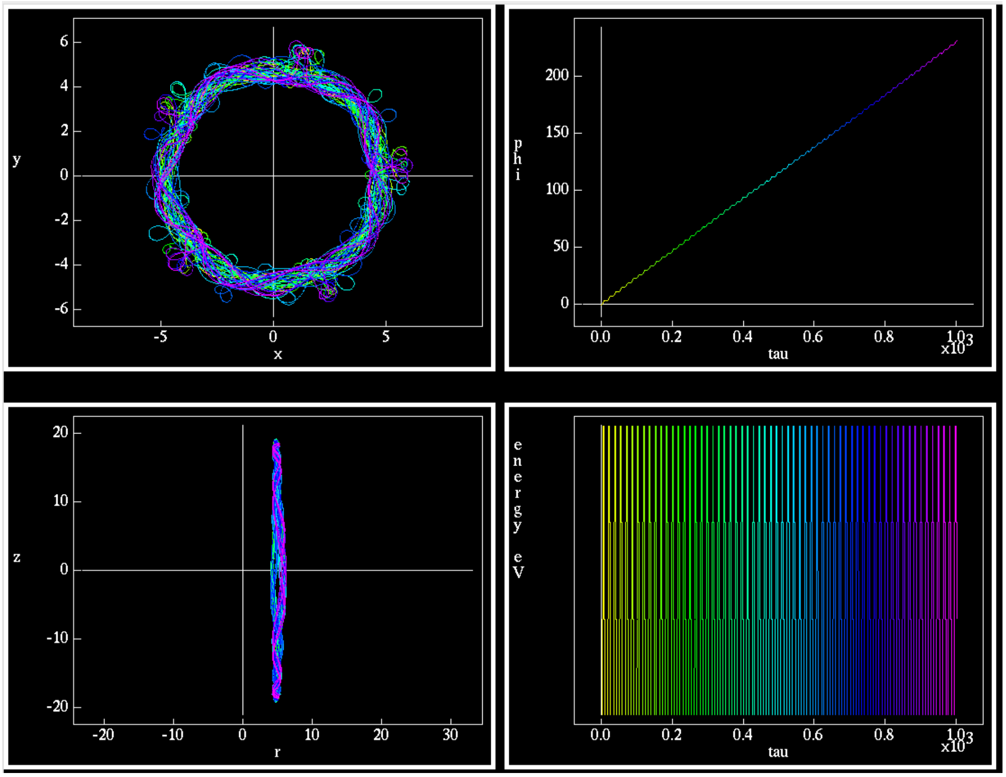


FIG 55-58. In the control case, where no oscillating mirror field was applied, the ion remained in a constant betatron orbit and kept a constant 3 ev of energy with no acceleration, as seen in the linear tau vs phi graph. The particle had a very small gyroradius, as shown in the z vs r graph.

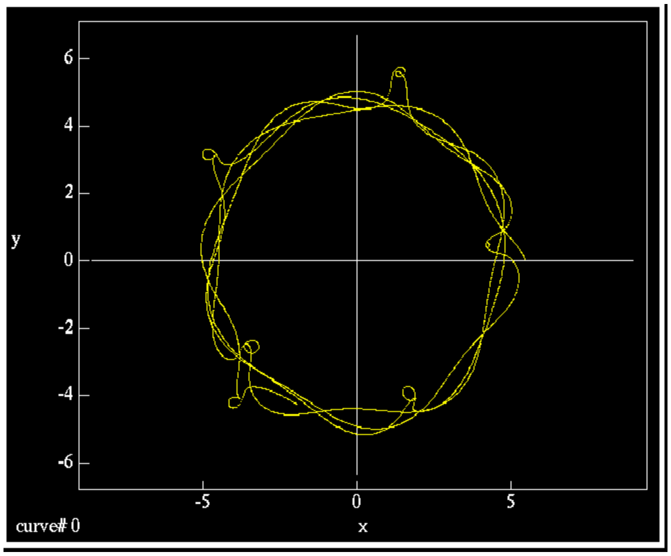


FIG 59. The particle remained in a betatron orbit, which indicates they were in a raised single potential well. [5]

4.3.2 Run with oscillating mirror field exhibiting clear and sustained heating of ions

One case which shows clear, sustained heating to 120 ev while keeping the ion confined for at least a thousand gyroperiods is a run which used all the default parameters, with no chirping and the omega driving frequency set at 3.300×10^6 rad s^{-1}. The ion omega gyrofrequency for this run was 5.075×10^6 rad s^{-1}. The colors of the lines indicate time in gyroperiods, with yellow, green, blue, purple, and pink representing increasing times in all the graphs.

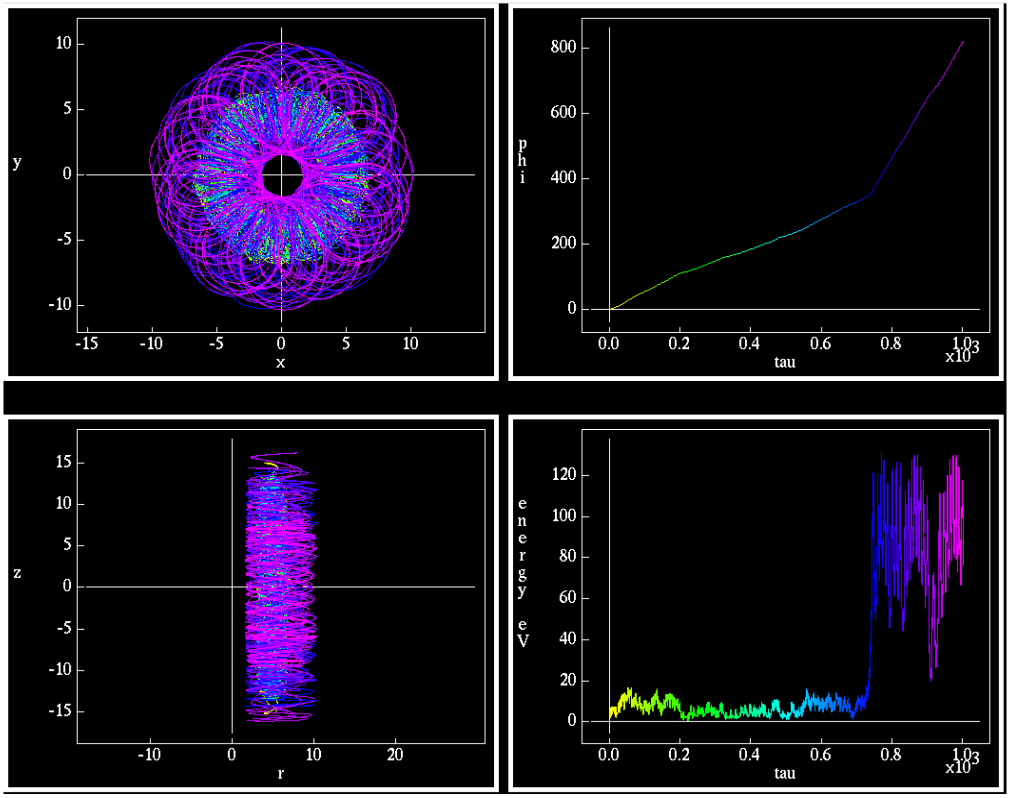


FIG 60-63. As the x vs y graph shows, the particle remained trapped inside the separatrix of 7 cm for the first approximate 750 gyroperiods, which is supported by the tau vs phi graph. Then the particle quickly gains energy and breaks out of the separatrix and settles into another stable orbit. This can also be seen with the linear tau vs phi line, which kinks sharply at around 750 tau, indicating a rapid increase in energy, yet the linearity on both sides of the kink indicates a stable orbit in which the particle in rotating around in the phi direction at a constant rate.

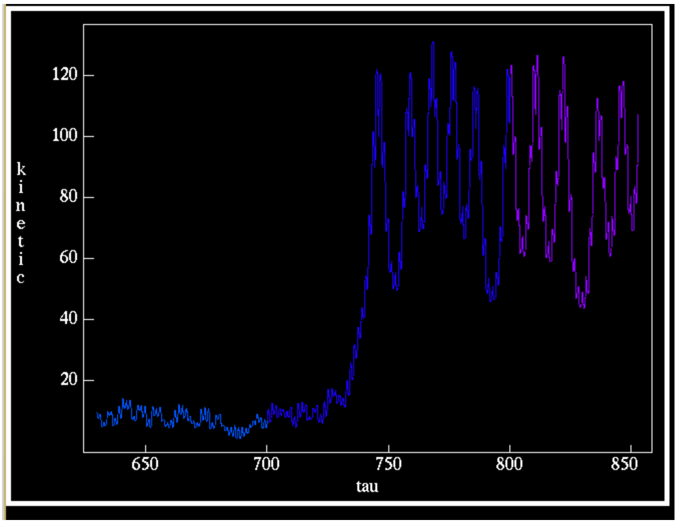


FIG 64. The pattern of rapid heating followed by chaotic motion seems to agree with predictions of patterns of autoresonant heating.

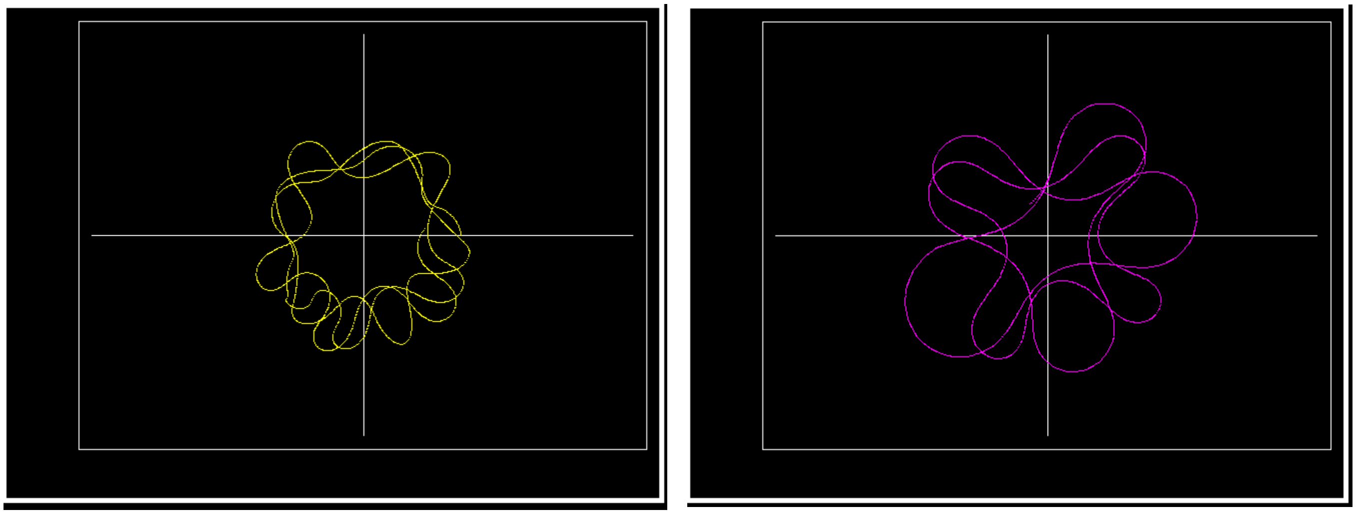


FIG 65-66. Bothe the low energy orbit inside the separatrix and the high energy orbits crossing the separatrix are both of the betatron type, indicating that the particle is trapped in a raised single potential well.

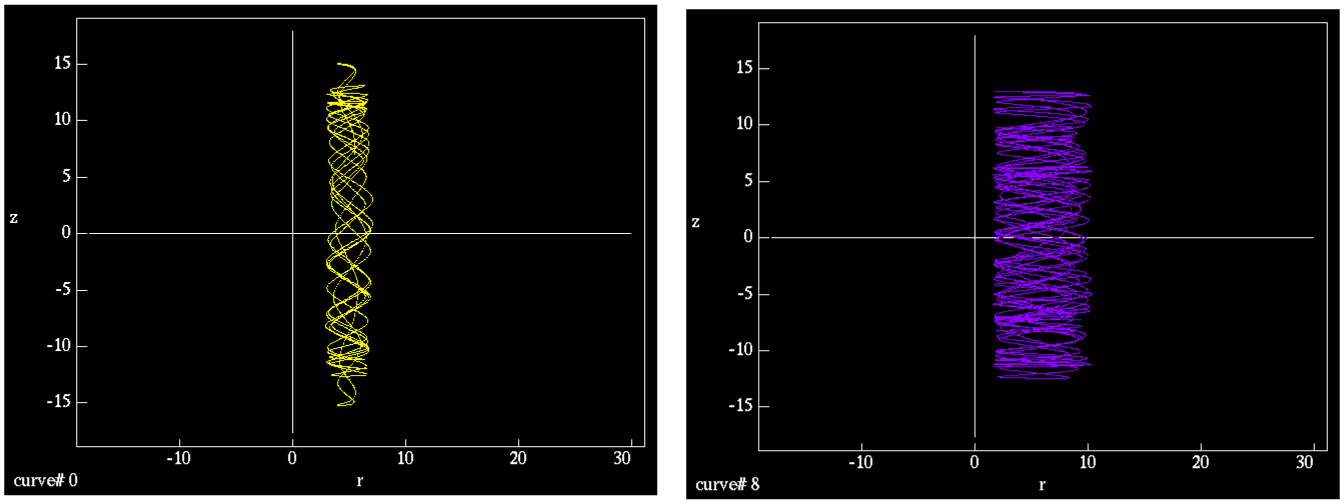


FIG 67-68. The high energy orbits are characterized by a large gyroradius, as seen in the large amplitude of r-direction oscillations in the purple orbit at higher tau times, as well as a shorter orbit in the z-direction.

5. Analysis

5.1 Comparison of theoretical autoresonant heating and experimental results

The theoretical example provided earlier in this paper examined a particle in a double potential well in a cyclotron orbit, whereas the particles studied in these simulations have all been in raised single potential wells with a betatron orbit.

There appears to be an upper limit on the temperature of ions in the FRC where they can still remain in large, stable orbits. Once the ions are heated to above 200 ev, they inevitably enter chaotic motion and quickly escape confinement. There appears to be a power law that governs the relationship between the frequency of the mirror field oscillations and the maximum energy achieved, with $E_{max} \propto {\omega_d}^2$ when There appears to be a power law that governs the relationship between the frequency of the mirror field oscillations and the maximum energy achieved, with $E_{max} \propto {\omega_d}^2$ when $0.02\ \omega_0 < \omega_d < 0.8\ \omega_0$.

Whether or not autoresonance was actually the source of heating was unclear. The particle's motion seems to suggest it is "bouncing" up the potential well and experiencing heating, an increased gyroradius, crossing the separatrix, and the shape of the energy vs time graph all seem consistent with autoresonance, but there are several problem with that explanation.

In all theoretical models, autoresonance requires the oscillation frequency to do a chirp sweep because autoresonance can only match frequency to a linearly changing frequency, not a constant one. Also, the frequency of oscillations seems too high, because the theory of autoresonance in the FRC requires the oscillations to match the frequency of the particle travelling back and forth along the z-axis, which should be orders of magnitude lower than the gyrofrequency. Finally, autoresonance without damping should allow any driving potential above the threshold to cause the amplitude of the driven wave to grow without bound, but in the simulations, clearly, a higher amplitude magnetic mirror field cause significantly more heating. More study is certainly needed into this heating phenomenon to determine whether autoresonance is in fact responsible, or if it's another cause entirely.

6. Discussion

6.1 Possible sources of error

The RMF code is a simulation that attempts to model the motion of a particle inside and FRC. As such, it must make many approximations, from its numerical approximations of elliptical integrals to the filamentary mirror coils to all the small random factors inside the FRC that cannot be account for.

Furthermore, since this code only simulates a single particle, a method of heating that's viable for a single particle may not be viable in a real experiment due to collisions, turbulence, etc.

6.2 Applicability for experimental use in the PFRC

Filamentary mirror coils are ideal for simulation problems because their fields are easy to compute, but to experimentally implement such a coil in the PFRC would be impractical from an engineering standpoint, as the current that would have to flow across a single coil to generate the necessary magnetic field strengths would not be feasible.

If this method of heating does in fact work, however, then there is the very real possibility that oscillating mirror field can be used in addition to, or instead of, a rotating magnetic field to heat particles in the FRC.

6.3 Direction of future research on autoresonant heating in PFRC

In order to actually implement this method of heating, the problem of explosive heating followed by rapid loss of confinement of particles must be addressed. More computational research is required to understand this method of heating in the FRC. Using a particle-in-cell code that can simulate multiple particles and their interactions inside the FRC would also be very beneficial, and can provide a much more detailed and realistic understanding of whether this method of heating is truly feasible. Finally, studying the effects of using this method of heating in conjuction with the RMF should also yield some promising results.

7. Conclusion

Heating of ions confined in a simulated FRC from 3 ev to over 100 ev using oscillating magnetic mirror fields was observed. There appears to be a power law that governs the relationship between the frequency of the mirror field oscillations and the maximum energy achieved, with $E_{max} \propto {\omega_d}^2$ when $0.02\ \omega_0 < \omega_d < 0.8\ \omega_0$. Rapid heating within Whether this is evidence of autoresonant heating of particles could not be determined from the data collected, but the observed form of heating does not seem to completely agree with the theoretical model of autoresonant heating in plasmas. More research will be needed to investigate the cause of the observed heating phenomenon.

Citations:

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