Progress Report on LSP Simulations of Magnetic Nozzles and Plasma Detachment

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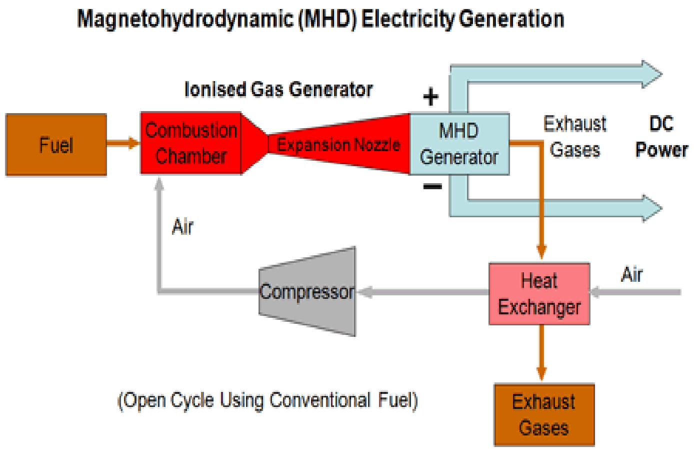
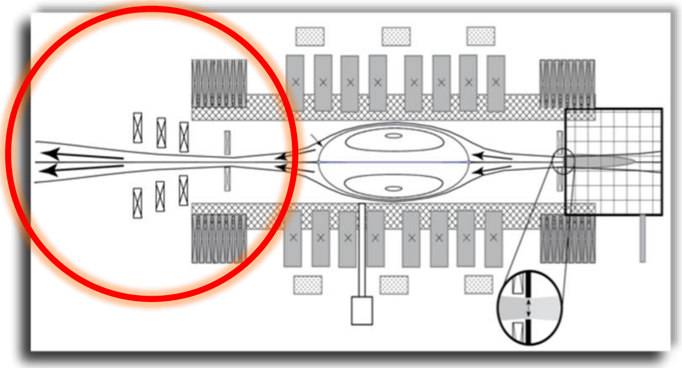
Reasons to Study Magnetic Nozzles

- Converting thermal energy to directed kinetic energy; Valuable for:
  - MHD power generation for FRC power plants
    - High efficiency: energy converts directly to electrical power
  - Electromagnetic propulsion
    - Attractive for certain space missions
    - Princeton Satellite Systems, Direct Fusion Drive
    - VASIMR

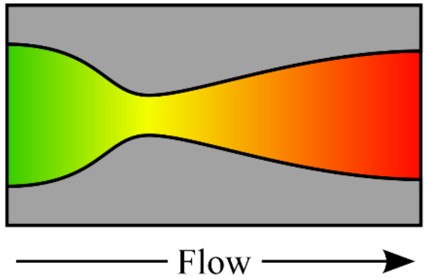


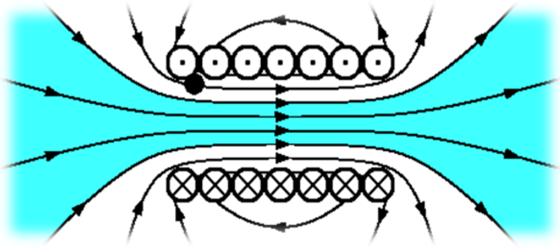
$$\frac{m_p + (1 + f_s)m_f}{m_p + f_s m_f} = e^{\frac{\Delta V}{u_e}}$$



Physical Description

- Similar to physical nozzle, fluid mechanics
  - De Laval nozzle: convergent, throat, divergent
  - Principally interested in divergent portion, where the momentum transfer and detachment occur
  - Mechanical 2D wall surface acts as a surface constraint
  - In contrast, plasma confined on original field lines





Typical plasma parameters

$T_e = 1 - 100$ eV; $T_i = 0.5 - 10$ eV

$n_i = 10^{10} - 10^{14}$ cm^{-3}

$E_i = 1 - 500$ eV

$B < 5\ kG$

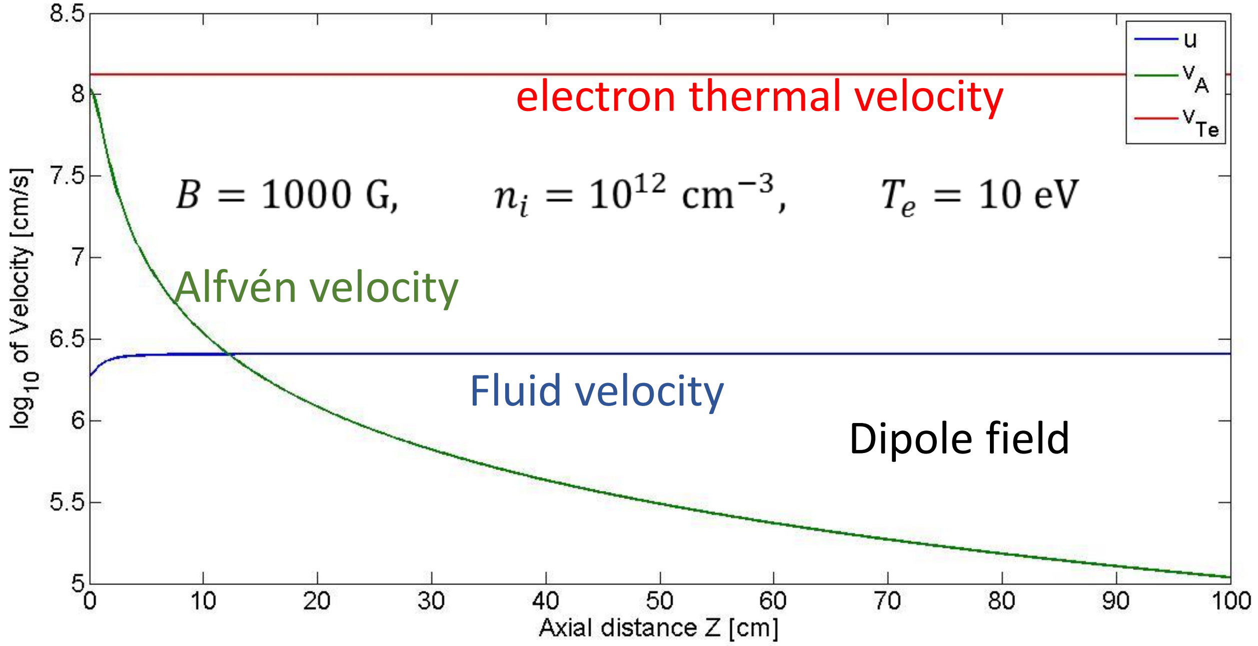
Important velocities

$c_s = 9.79 \times 10^5 (\gamma Z T_e/\mu)^{1/2}$ cm/s

$v_A = 2.18 \times 10^{11} (\mu n_i)^{-1/2} B$ cm/s

$v_{Te} = 4.19 \times 10^7 T_e^{1/2}$ cm/s

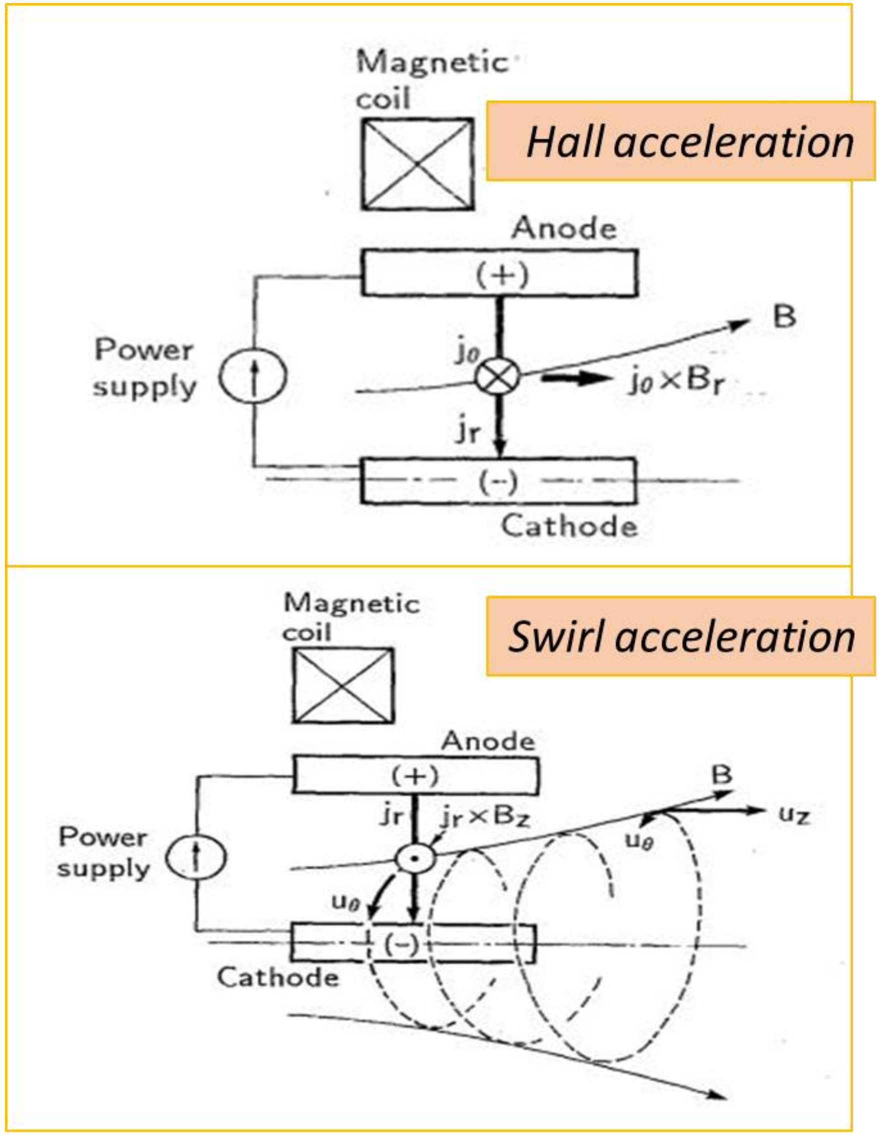
Plot of Important Velocities vs. Axial Distance from Throat



Directed Kinetic Energy

Momentum Exchange, Lorentz force

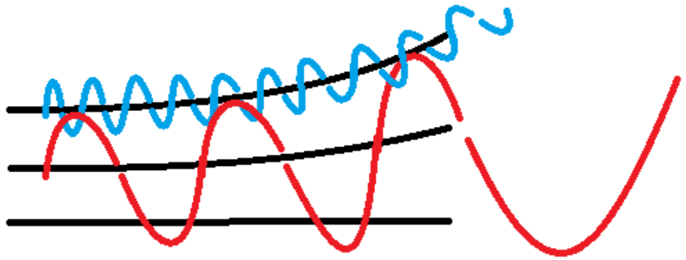
- Momentum transfer via Lorentz interaction
  - Applied external B field acts on currents in plasma
  - Induced B field in plasma acts on current in electromagnets
- Circulating azimuthal current in field
  - Paramagnetic
    - Strengthens field, reduces thrust (drag effect), focuses field
    - *Swirl acceleration*
  - Diamagnetic
    - Increases thrust, weakens field, increases divergence of field
    - *Pressure gradient*
    - *Hall acceleration*



A. Sasoh: Phys. Plasmas 1, 464 (1994)

Methodology: Detachment Parameters

- Detachment mechanisms:
  - **Collisionless**
  - Collisional
  - B field rearrangement



$$r_{L_B}\left|\frac{\nabla B}{B}\right| < 1$$

$$r_{L_A}\left|\frac{\nabla B}{B}\right| > 1$$

- **Collisionless Detachment**
  - Gyroradius larger than B field spatial variation, $r_L\left|\frac{\nabla B}{B}\right| > 1$
    - **Single particle inertial** detachment
    - Hooper, **plasma inertial** detachment, $G = \frac{eB}{m_i}\frac{eB}{m_e}\frac{L^2}{U^2}$

    Simulation: $B = 500\ \mathrm{G}$, $L = r_0 = 1\ \mathrm{cm}$, $U = c_s = 1.38 \times 10^6\ \mathrm{cm/s}$ $\boldsymbol{G = 2.35 \times 10^4}$
  - Plasma no longer confined, $\beta_p > 1$
    - **Collective plasma** detachment
  - Super-Alfvénic plasma, $\beta_f > 1$
    - **Induced field** detachment

E.B.Hooper; Plasma Detachment from a Magnetic Nozzle

Methodology: Building Simulation

- Injection scheme
  - Ion injection
    - $J = qn_i v_i$
      - Fix v_i in units of gamma-beta
      - Fix n_i by setting J
      - Simple case: J constant in time and space
  - Electron injection
    - Child-Langmuir
      - Steady state: maintains quasi-neutrality
- Magnetic Field
  - Solenoid
    - $B_{max} = 800\ \mathrm{G}, \quad B_0 = 500\ \mathrm{G}$
    - $L = 20$ cm; $R = 10$ cm
    - Center $(0,0)$

Injection Parameters

$T_i = 0.5\ \mathrm{eV}$

$n_{i0} = 1.0 \times 10^{10}\ \mathrm{cm}^{-3}$

$u_{i0} = c_s$

$\gamma\beta = \frac{u_{i0}}{c}\left(1 - \frac{u_{i0}^2}{c^2}\right)^{-1/2} = \mathbf{4.3 \times 10^{-4}}$

$J_i = qn_{i0}u_{i0} = \mathbf{0.0022\ A/cm^2}$

$T_e = 2.0\ \mathrm{eV}$

$J_e \propto \frac{V^{3/2}}{d^2}$, computed by LSP

Important velocities at throat, $(r,z) = (0,0)$

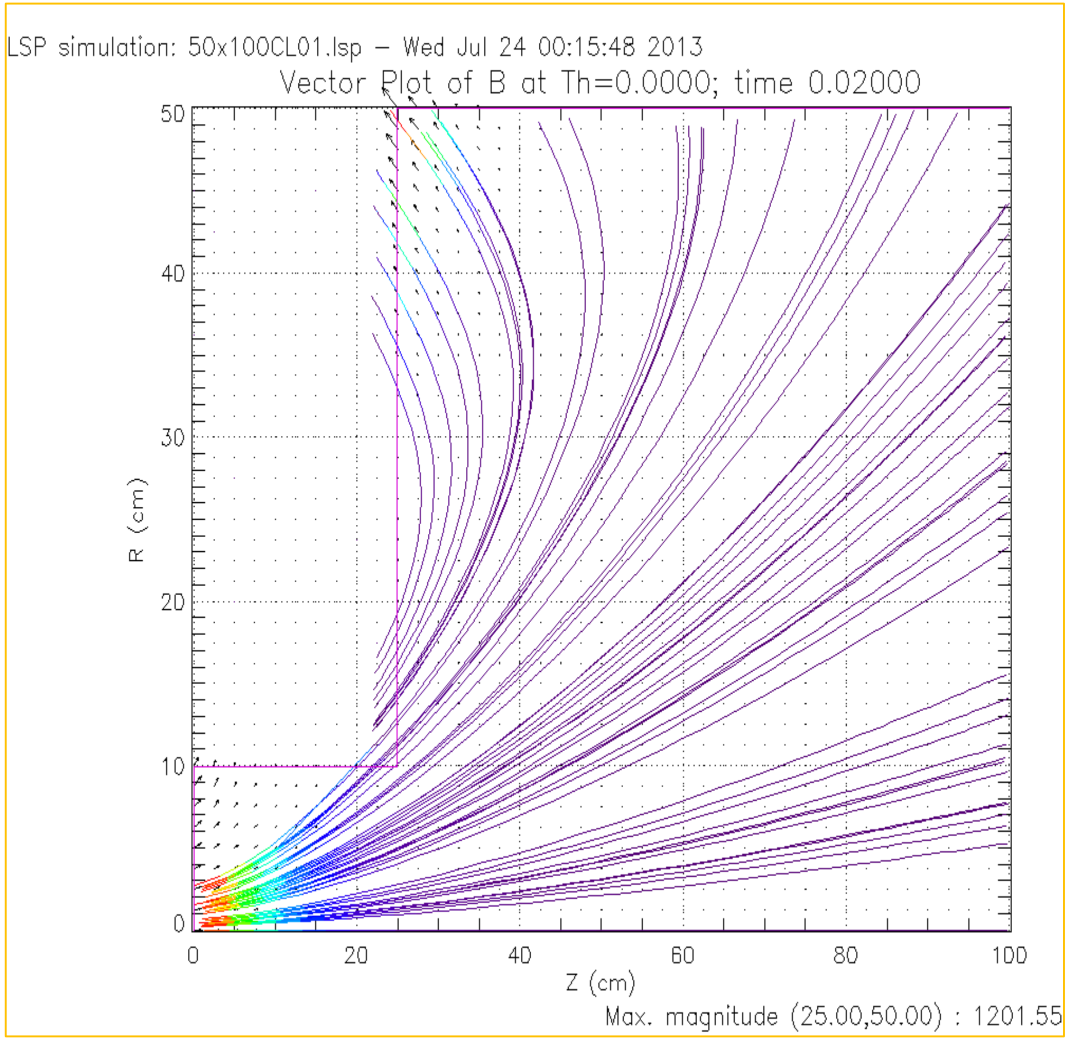
$c_s = 1.38 \times 10^6\ \mathrm{cm/s}$

$v_A = 1.09 \times 10^9\ \mathrm{cm/s}$

$v_e = 5.93 \times 10^7\ \mathrm{cm/s}$

Methodology: Simulation

- P4 postprocessor
  - IDL to visualize data from simulation:
  - History
    - Energy (field, particle, total, net)
    - Global velocity total
    - Number
  - Scalar
    - Species density n_i, n_e
    - Species temperature T_i, T_e
    - Species pressure p_i, p_e
    - Electric potential ϕ
  - Field
    - Magnetic field $\vec{B} \rightarrow B_r,\ B_z$
    - Electric field $\vec{E} \rightarrow E_r,\ E_z,\ E_\phi$
    - Current density $\vec{J} \rightarrow J_\phi$
    - Species velocity $\vec{v}_i,\ \vec{v}_e$



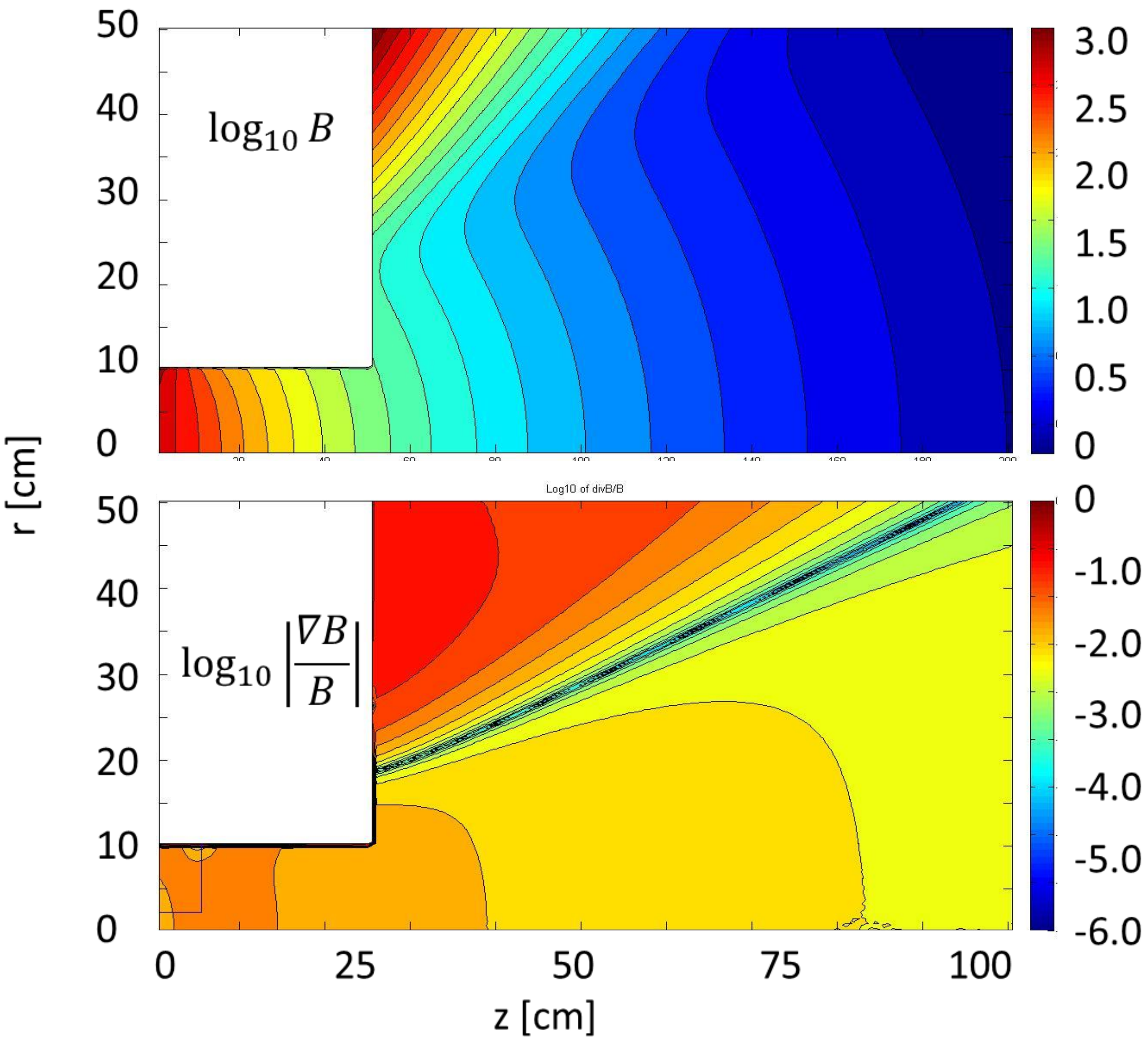
Methodology: Diagnostics

MATLAB: Computing + visualization

- Script and plotting
  - Read P4 data writes into matrices/ *meshgrid* format
  - Plot as contour plots; visualize detachment
- Dimensionless plasma parameters
  - Beta: $\boldsymbol{\beta_p = \frac{nkT}{B^2/2\mu_0}}$
  - Inertial detachment $\boldsymbol{r_L \left|\frac{\nabla B}{B}\right|}$

    $\boldsymbol{G = \frac{eB}{m_i}\frac{eB}{m_e}\frac{L^2}{U^2}}$

- Others
  - Breizman: $\boldsymbol{\beta_f = \frac{nmU^2}{B^2/2\mu_0}}$
  - E field development: $\boldsymbol{\vec{E}(r, z)}$
  - $\vec{v}$ and $\frac{v_r}{v_z}$, $\vec{B}$ and B_r/B_z: $\boldsymbol{\left(\frac{v_z}{v_r}\right)\left(\frac{B_z}{B_r}\right)^{-1}}$
  - Azimuthal $\vec{J}$, Lorentz: $2\pi r J_\phi B_r$

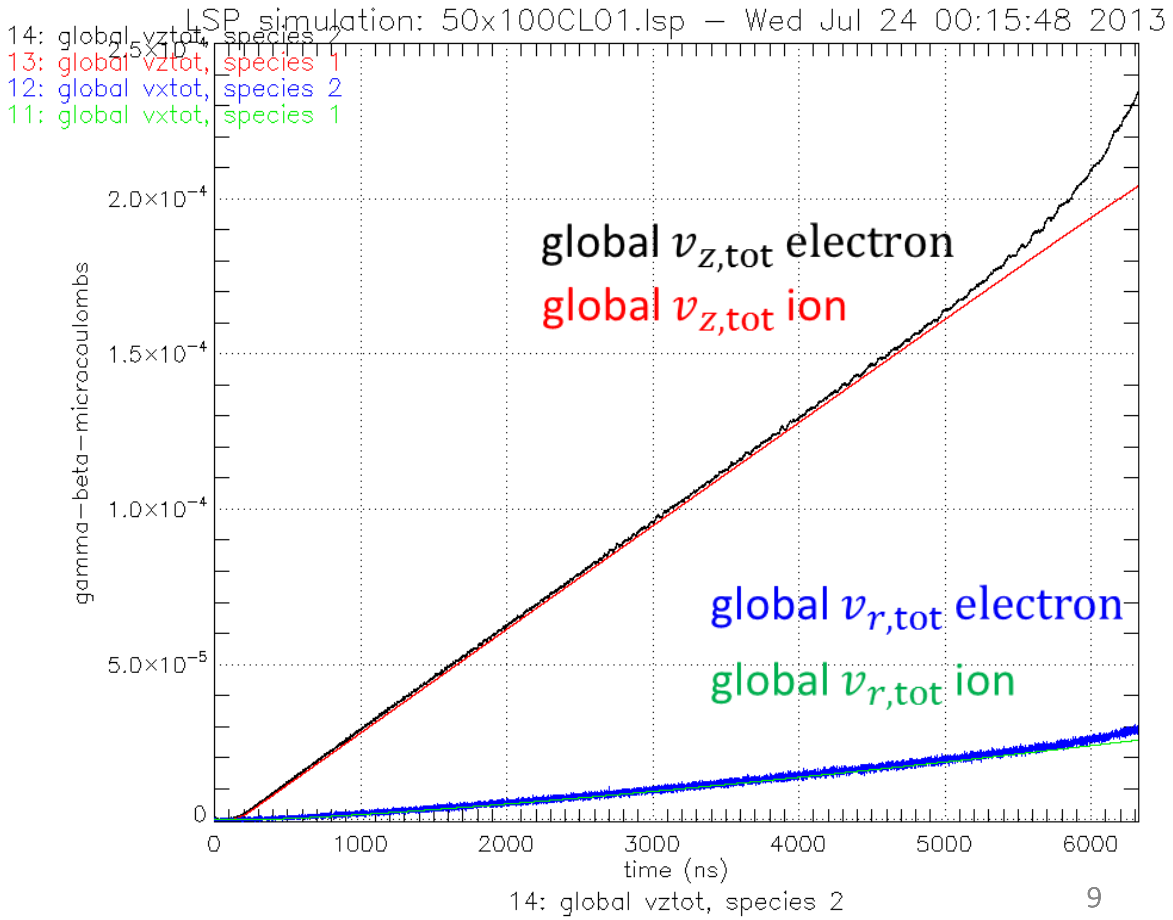


Results: System Development

- System parameters

$n_i = 10^{10}\ \text{cm}^{-3}, \quad T_i = 0.5\ \text{eV}, \quad T_e = 2.0\ \text{eV}, \quad B_{max} = 800\ \text{G}, \quad c_s = 1.4 \times 10^6\ \text{cm/s}$

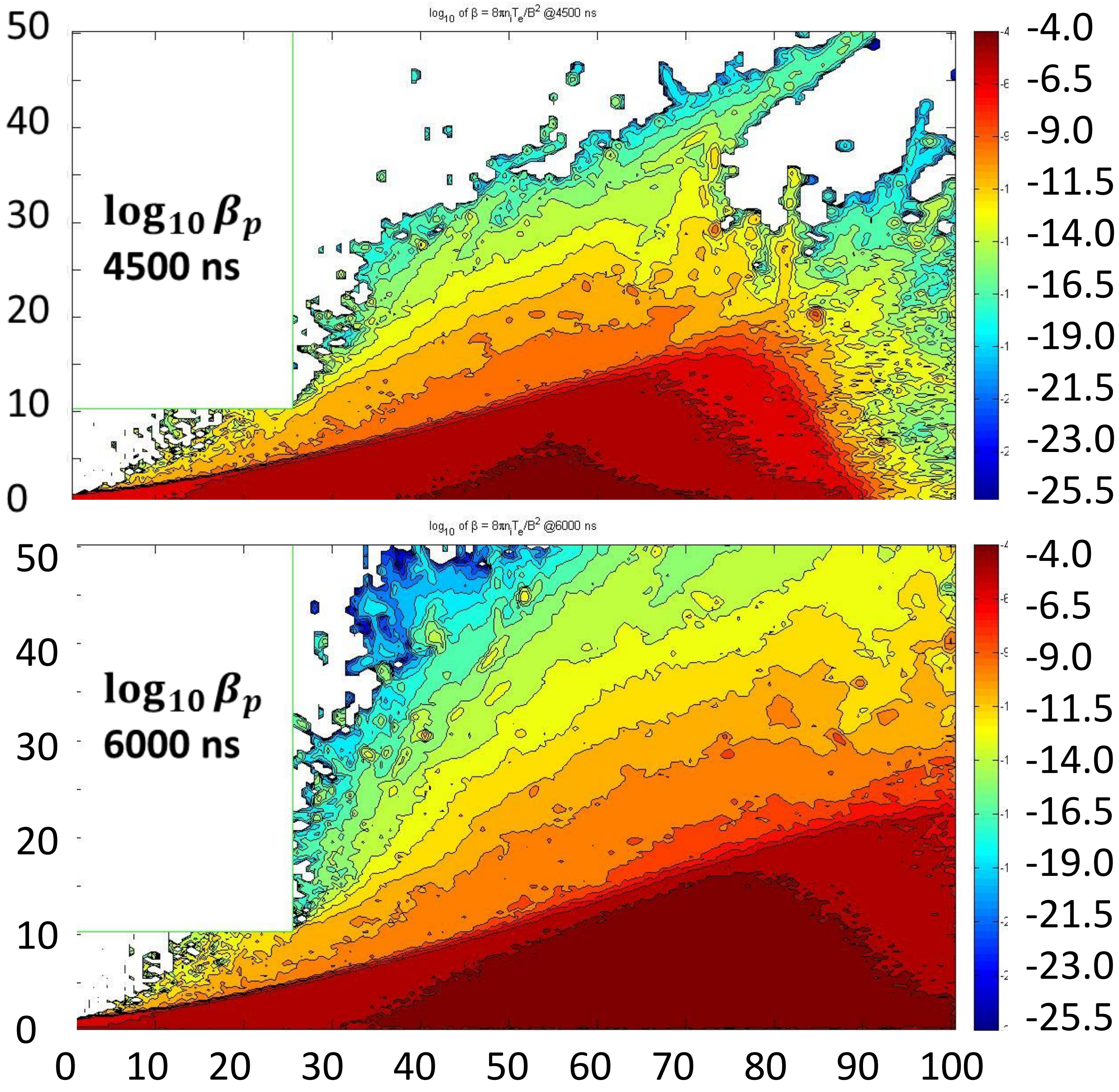
- Energy
  - Particle $E_{part} = \sum_j E_j$
  - Field $E_{flds} = \iiint_{system} \frac{1}{2}\left(\epsilon E^2 + \frac{B^2}{\mu}\right) dV$
  - Total $E_{tot} = E_{part} + E_{flds}$
  - Net $E_{net} = E_{tot} - E_{meas}$
- V Total
  - $\boldsymbol{N v_{z,av}}$
  - $\boldsymbol{N v_{r,av}}$
- Number
  - Number of macroparticles
    - Subject to particle collapse
    - Quasineutrality



Results β_p

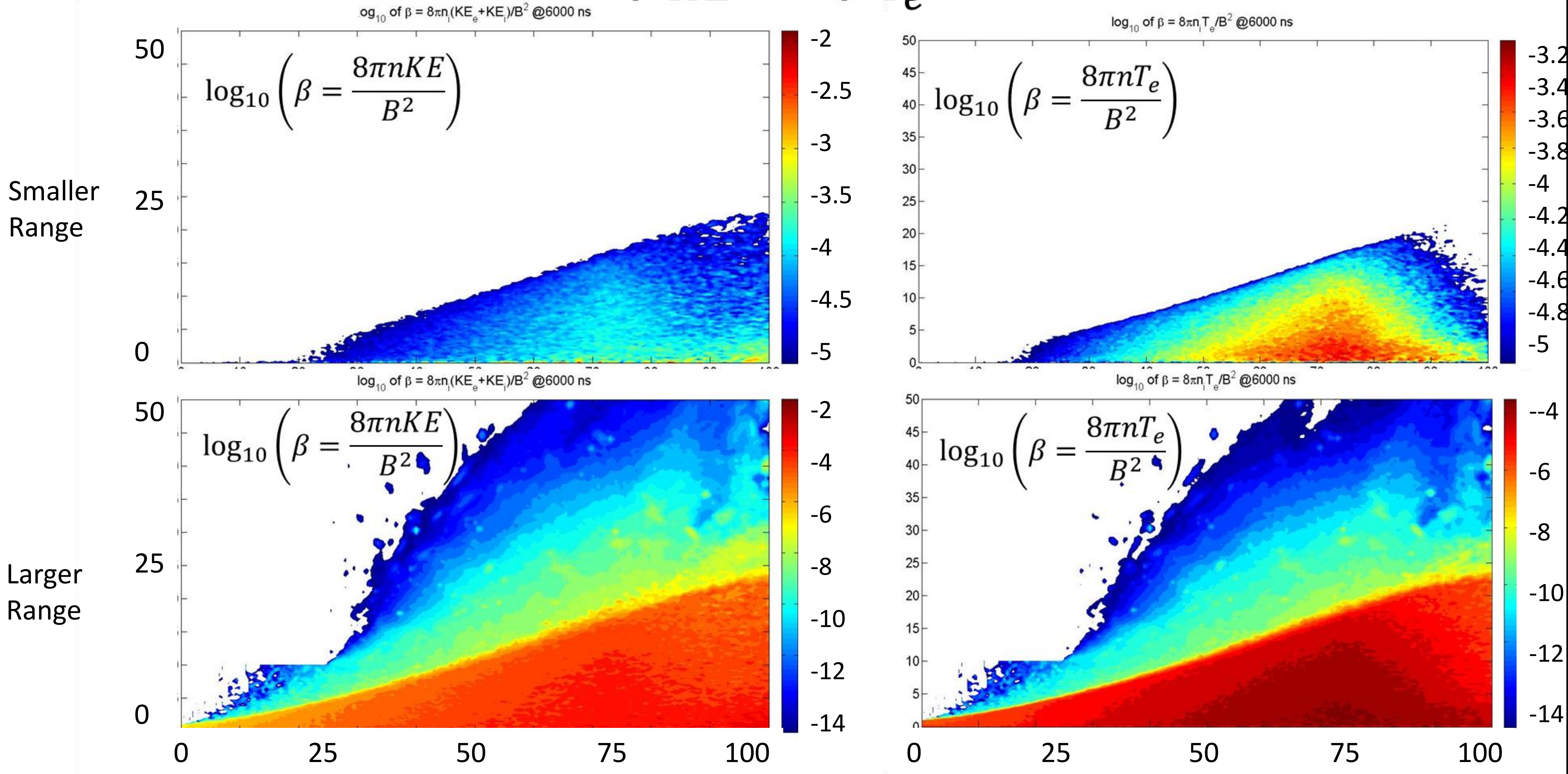
$$\beta_p = \frac{nkT}{B^2/2\mu_0} = 4.03 \times 10^{-11}\, n_i T_e B^{-2}$$

- Motivation:
  - Assess plasma confinement locally, $\sim$10 cm scale, rather than system-wide average
- Caveats:
  - β_p not meaningful for single particle
  - Focus on collective characteristics, e.g. contour levels
- Observations
  - Maximum $\log_{10}(\beta_p) \approx -3.5$
  - Large gradient with change in density n_i – envelope for plume
  - Development of pocket of maximum β_p near axis



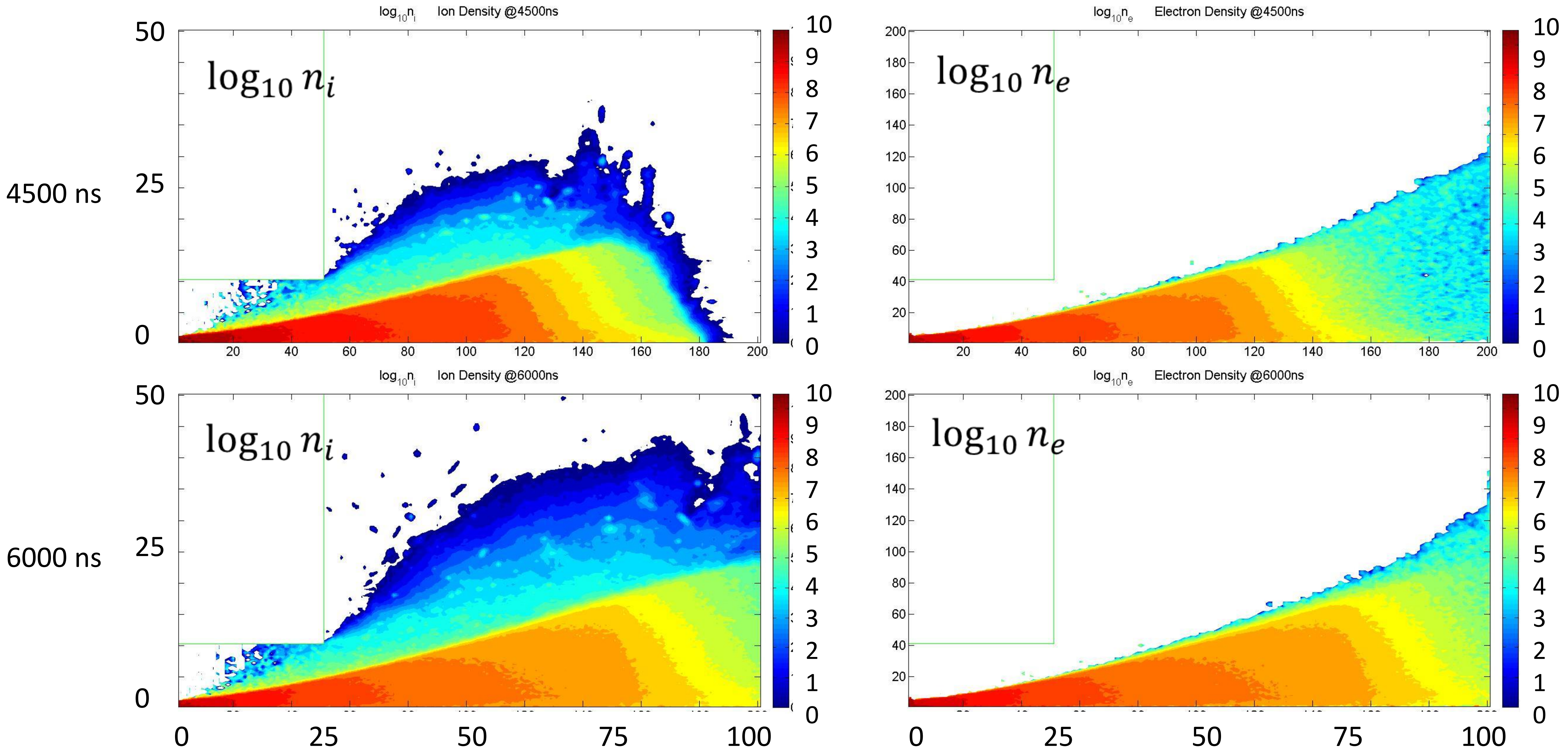
Results

β_{KE} vs. β_{T_e}



Results

n_i vs. n_e



Results $r_L \left|\frac{\nabla B}{B}\right|$

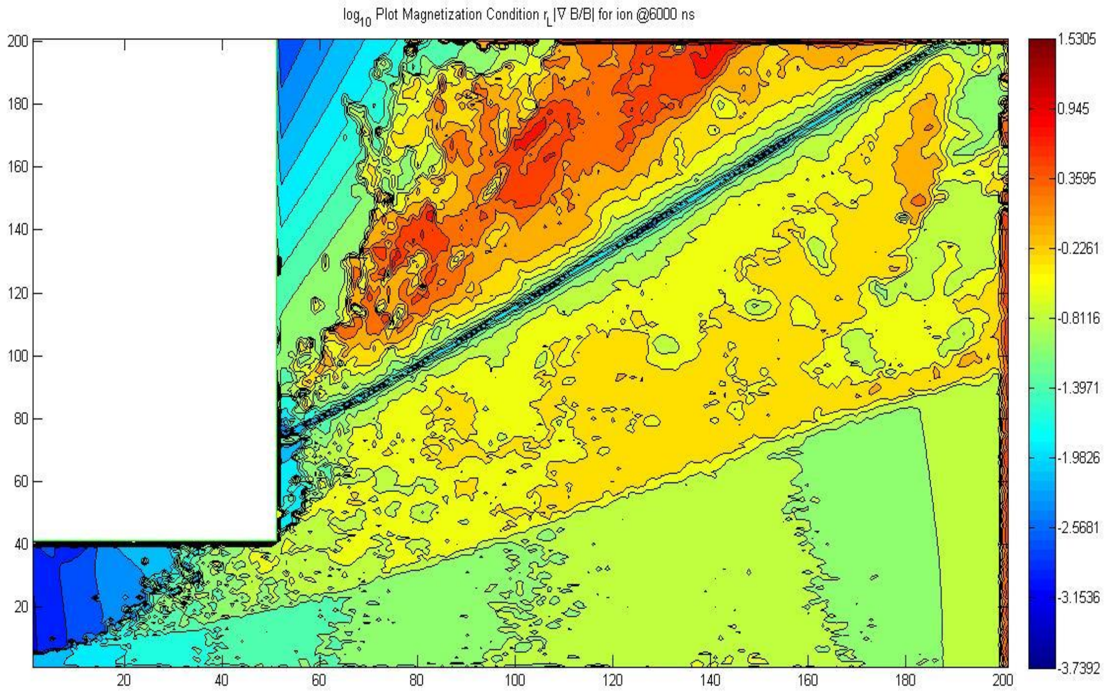
PPPL

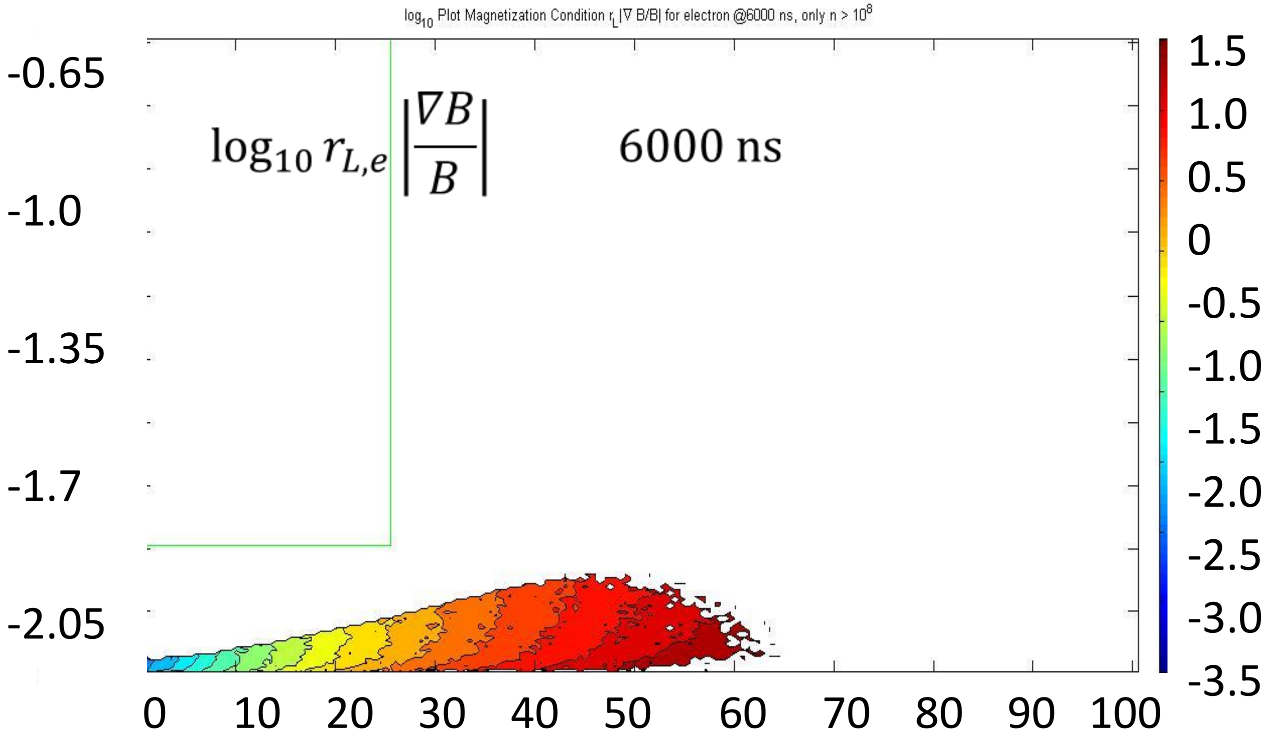
- Motivation:
  - Investigate localized adiabaticity of plasma throughout nozzle
- Gyroradii of both ion and electron species
  - Plot restricted to densities $n_e, n_i \geq 10^8$

$$r_L = \frac{v_T}{\omega_c} = \frac{m v_T}{qB}$$

$\boldsymbol{r_{L,i} \approx 10 r_{L,e}}$ Given $m_i \approx 2 \times 10^3 m_e$, then $v_{\perp,i} \approx 10^{-2} v_{\perp,e}$?

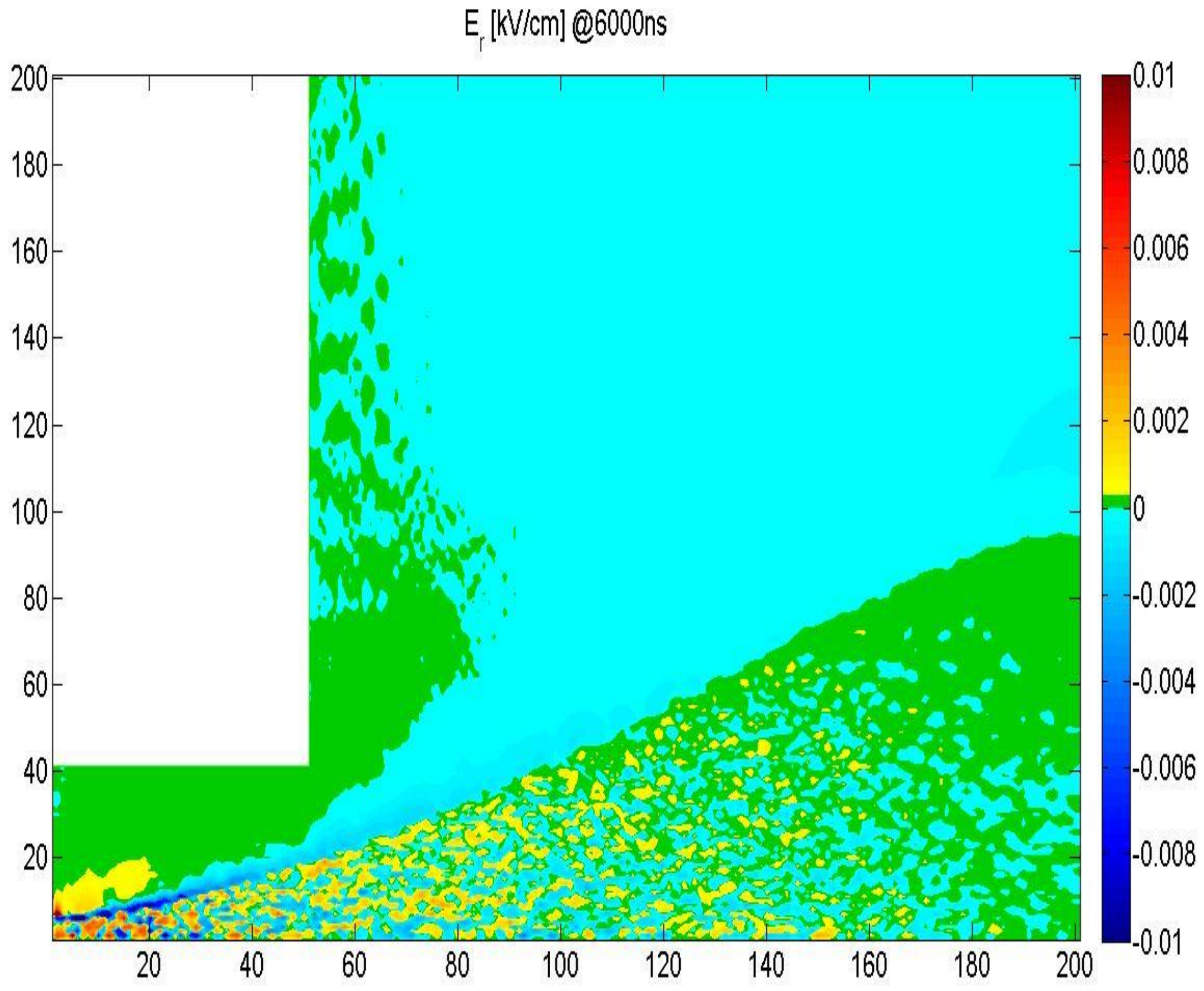
$T_i = \frac{1}{4} T_e$, and $v_i \propto \sqrt{\frac{T_i}{m_i}} \approx \sqrt{\frac{\frac{1}{4}}{2\times10^3}\frac{T_e}{m_e}} \approx \frac{1}{90}\sqrt{\frac{T_e}{m_E}}$ $\therefore \boldsymbol{v_{\perp,i} \approx 10^{-2} v_{\perp,e}}$





Results $\vec{E}$

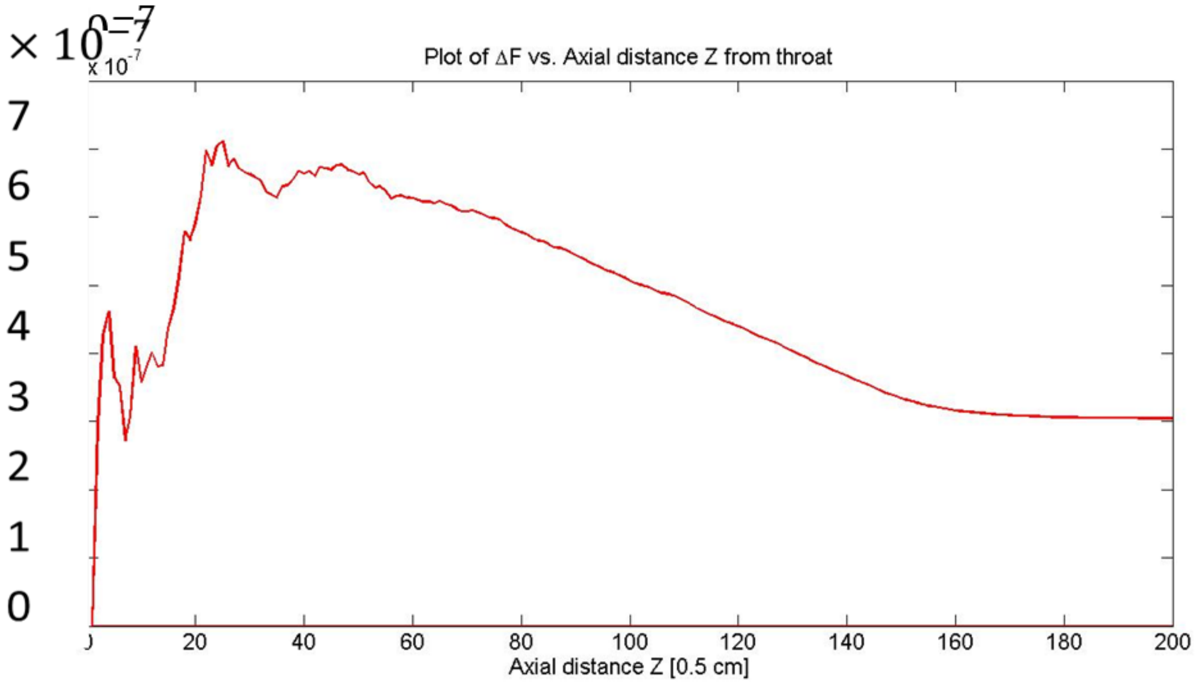
- Motivation:
  - Radial motion of ions, generates E?
  - Formation of $E \times B$ drift in plume
- Observations
  - Contour plots
    - E_r present at magnitudes of $\approx 10^{-2}$ kV/cm
    - Responsible for azimuthal effects?

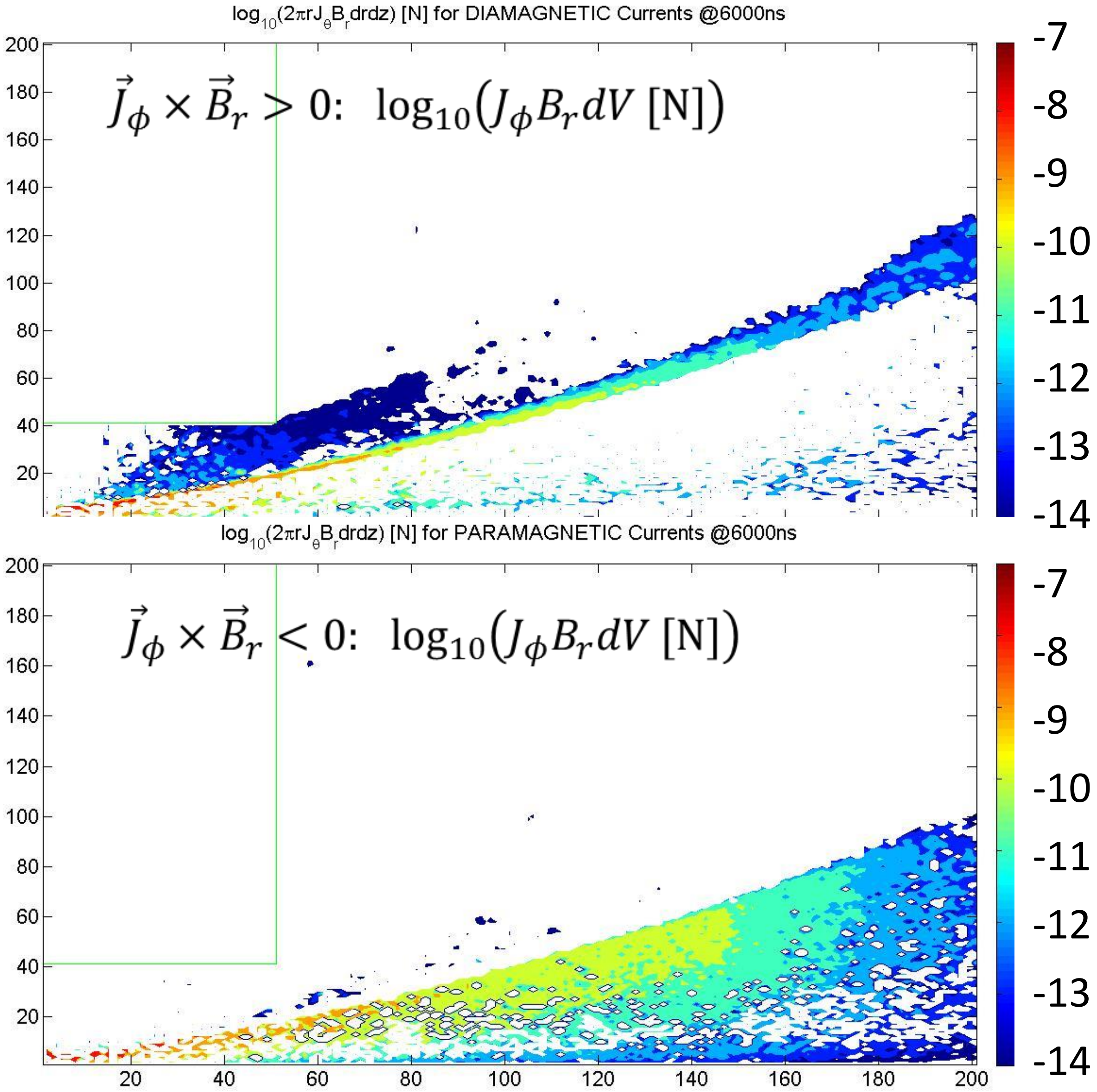


Results J_ϕ and ΔF

- Motivation
  - Momentum transfer by applied $\vec{B}$ and plasma $\vec{J}$
  - Find $\vec{J}$, and thereby ΔF
- $\vec{J}$ output by LSP; MATLAB computation
- Observations
  - J_ϕ found to be *diamagnetic* at boundary of high density plume, *paramagnetic* within high density plume

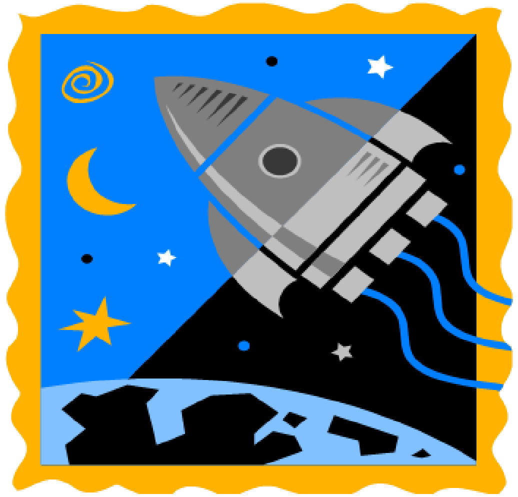
    $$\boldsymbol{\Delta F_{z=100,J} = -6.4181 \times 10^{-7}\ \mathrm{N}}$$
  - Suggests induced B fields have net *drag* effect





Conclusions

- Plume development
  - Confinement within magnetic streamlines
- Separation of ions from electrons
  - Electrons well confined, per above
  - Ions spread radially
- Electric field
  - Potential difference from charge separation, per above
- Azimuthal current
  - Force on plasma plume, $J \times B$



Ongoing/Future Work

- EPPDyL, independent work
  Professor Edgar Choueiri, Justin Little, Matthew Feldman
- Mikhail Khodak's ongoing work (coming up next)

Acknowledgements

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