Tailoring Materials and Radiation to Explore Cloaking Phenomena

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1 Introduction

Radiation-matter interaction is very important in energy research, especially in the areas of fusion sciences and solar radiation conversion. Research on the subject has moved beyond the question of what physics actually underlies this interaction, to now explore how materials and radiation can be tailored in order to influence and even control this fundamental interaction. One of the areas in which this study is being pursued is that of cloaking. In this area, a primary goal is to create materials that can shape the path and radiation pattern of light in order to make that material appear invisible to an observer. A natural extension to this quest is to consider tailoring the light incident on the cloak in such a way that it can interact with the cloak in order to create a different radiation pattern to make the cloak, and the object once again, visible to the observer. Whereas *cloaking* attempts to control the fundamental radiation-matter interaction by tailoring the material, *decloaking* attempts to control this by tailoring the incident radiation. This research aims to first understand and formulate the recently documented cloaking effect and then to discover if light can be tailored to undermine this effect. The scientific outcome from this research is expected to be a better fundamental understanding of the degree to which radiation-matter interactions can be manipulated ultimately for beneficial energy generation and conversion applications. In addition, there should be implications in broader domains of radiation-matter interactions including bio-system light harvesting and other fields.

2 Cloaking

2.1 Introduction

For centuries, man has sought to achieve invisibility. Legend, folklore, and even artistic work portrays the quest for this power. This is seen in such works as Wagner's *Der Ring des Nibelungen* in which a helmet, Tarnhelm, grants invisibility to its wearer. However, except for in the realm of fantasy and magic, invisiblity seemed, to many, unattainable.

However, recent advances in the technology of what are known as *metamaterials* have made the prospect of cloaking feasible. Metamaterials are materials that are composed of many small unit cells each having specific material properties. One of the types of cells are *Split-Ring Resonators*[1], made up of split rings of metal embedded in a background dielectric material. As the parameters of the ring, such as radius and width, change, the cell exhibits certain desirable material properties. Thus, one can design an entire library of these cell types which all have different material properties. Since each of these cells can exhibit a different index of refraction, a basic question is whether they can be placed in an array such that the material can effectively guide light around it, creating the illusion of invisibility? As researchers have recently discovered, the answer to this question is yes.

2.2 Mathematical Formulation of Cloaking

There are many methods of cloaking currently being explored, but perhaps one of the easiest to conceive of and state mathematically is *cylindrical cloaking*[2]. This configuration makes use of metamaterials to guide light around a central region without interacting with the central region. Thus, if something is placed within the central region, it would be effectively cloaked in all directions. Ideally, light would leave the cloak in exactly the same manner in which it enters the cloak (i.e. the phase information is preserved, etc.) so that the cloak itself could not be seen. This perscription can be attained by the use of a simple coordinate transformation.

To start, the governing *Maxwells equations* are[3]:

$$\nabla \cdot \mathbf{D} = 0 \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B} \tag{3}$$

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t}\mathbf{D} \tag{4}$$

Where **D** is the electric displacement field, **B** is the magnetic field and **E** is the electric field. **D** and **H** are related to **E** and **B** by the following relationships:

$$\mathbf{D} = \epsilon\mathbf{E} + \mathbf{P}$$

$$\mathbf{H} = \frac{1}{\mu}\mathbf{B} + \mathbf{M},$$

where ϵ is the permittivity of the material, **P** is the electric dipole moment per unit volume of the material, μ is the permeability of the material, and **M** is the magnetic dipole moment per unit volume of the material. If we assume the material is linear, with $P = 0$ and $M = 0$, then **D** and **H** are related to **E** and **B** by the following relationships:

$$\mathbf{D} = \epsilon \mathbf{E}$$

$$\mathbf{H} = \frac{1}{\mu}\mathbf{B}.$$

The physical information contained in the solutions of Maxwells equations is coordinate-invariant - that is, it holds regardless of the coordinate system used. Thus, with the goal of guiding light around a circular region, for example, one can take a point in a coordinate system where light travels in straight lines and blow that point up to a circular region in a coordinate system in which light travels in curved lines around that region. To blow a point up to a circular region of radius a and to contain the field-line distortions in an annular region of radius b (the cloak), one can use the coordinate transformation[4]:

$$\begin{aligned} r' &= \frac{b-a}{b}r + a; \\ \theta' &= \theta; \\ z' &= z. \end{aligned} \tag{5}$$

This transformation calls for the following scaled material parameters to preserve the invariance of Maxwell's equations[4]:

$$\begin{aligned} \epsilon_r = \mu_r &= \frac{r-a}{r}; \\ \epsilon_\theta = \mu_\theta &= \frac{r}{r-a}; \\ \epsilon_z = \mu_z &= (\frac{b}{b-a})^2 \frac{r-a}{r}. \end{aligned} \tag{6}$$

If the electric field of the incident radiation is polarized in the z-direction, then we only need to consider ϵ_z, μ_r, and μ_θ.

A problem with this formulation, however, is that as r approaches a, μ_θ approaches ∞. To prevent this, one can fabricate the cloak with a new set of parameters that preserve the coordinate-invariance (provided we are only considering ϵ_z, μ_r, and μ_θ) and avoid infinite material parameters[2]:

$$\begin{aligned} \epsilon_z &= (\frac{b}{b-a})^2 \\ \mu_r &= (\frac{r-a}{r})^2 \\ \mu_\theta &= 1. \end{aligned} \tag{7}$$

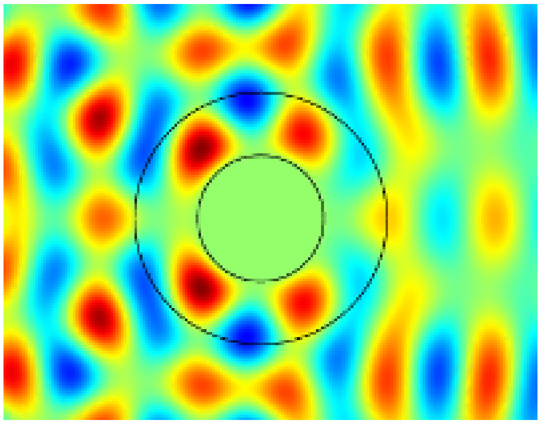

Figure 1: Light incident on a circular object without a cloak surrounding it.

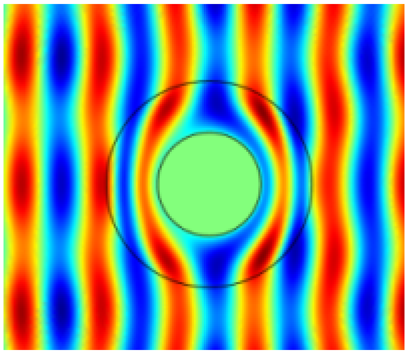

Figure 2: Light incident on a circular object with a cloak surrounding it.

2.3 Computer Formulation of Cloaking

In order to model the cloaking effect, we used the software COMSOL Multiphysics and its RF Module. The cloak was modeled in 2-dimensions by an annulus with material properties given by (7). At the interior of the cloak was placed a perfect electric conductor. We modeled light with transverse-electric waves incident on the cloak from the left. In order to model the infinite space in which the cloak exists, we made the left and right side of the computational domain perfectly matched layers, and we made the top and bottom of the computational domain perfect magnetic conductors. We then ran the simulation with light of different wave-lengths and obtained similar results as obtained by others who have modeled the cloaking effect. Figures 1 and 2 show a simulation of light incident on an object without a cloak and with a cloak respectively. The electric field intensity is plotted.

3 Decloaking Proposal

3.1 Introduction

Since the cloaking effect is now well-defined and we can successfully model it, our attention turns to decloaking. Our approach will be to introduce many different and intelligently shaped electromagnetic pulses on the cloak to find one that successfully allows the object to be seen. We plan to attain this optimal pulse with the use of genetic algorithms which will hopefully attain the solution quickly.

3.2 Shaped Electromagnetic Pulses

As the cloaking phenomenon relies on carefully tuned material parameters, we conjecture that, with carefully tuned radiation, a cloaked object can be decloaked. The key to accomplishing this is through the use of shaped electromagnetic pulses. These are pulses whose bandwidth has been divided into pieces and the relative intensity and phase of each piece has been tuned. This yields radiation with a very distinct profile, different from a normal gaussian pulse shape, and different from regular monochromatic radiation. Hopefully some very specific pulse will be able to detect the presence of a cloaked object.

The bandwidth of the pulse can be divided into discrete pieces, and the intensity and phase of each piece can be tuned over a set of discrete values. However, it is not trivial to find a specifically shaped pulse from this set that can detect a cloaked object. In order to determine a shaped pulse that fits our needs, we will turn to optimization algorithms.

3.3 Non-Linearity

One might be skeptical, and with good reason, as to the capability of shaped electromagnetic pulses to detect a cloaked object. After all, a shaped pulse is a Fourier sum of electromagnetic radiation at certain wavelengths, all of which would presumably fall within the bandwidth of the cloak. Since the final pulse is merely a linear combination of monochromatic light of different wavelengths, the cloak should function as a cloak with the final pulse as well, provided the phase and intensity at each frequency are preserved. Then, to achieve uncloaking, we must venture into the non-linear realm.

In order to do this, the energy of the pulse must be increased. If it is increased past a certain threshold, then the material properties of the cloak will be dependent on the electric and magnetic fields in the pulse. This will then make the material properties dependent on the applied field. If the dependence is appropriate and the pulse shape correct, then the cloaking effect should be broken, and the cloaked object could be detected. Attaining this result is the primary goal of the research ahead.

4 Conclusion

So far, progress on this project has been limited to understanding and formulating the cloaking phenomenon. Much work has been done to read the literature on cloaking and to understand the mathematical theory behind the effect. With this, a working computer model was produced. In the coming months, work will be done to study decloaking. Specifically, we will try to simulate interactions between shaped electromagnetic pulses in the non-linear regime and a cloak in order to determine if a cloaked object can be detected.

5 References

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