#### Beta-induced Alfvén-Acoustic Eigenmodes in NSTX and DIII-D Driven by Beam Ions

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Interpretation of new experimental observations on tokamaks requires low frequency instability studies

Strongly heated plasmas with energetic particles reveal complicated MHD spectra with multiple instabilities DIII-D ECE spectrum



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At low frequencies: 1) plasma  $\beta$  and geodesic curvature are responsible for Alfvén-acoustic mode coupling 2)  $\omega_{*i,e}$  effects become important  $\Rightarrow$  need to be understood

## *Motivation to study low-f instabilities*

• Various \*AE and a new class of instabilities called here Beta-induced Alfvén Acoustic Eigenmode (BAAE) help to study two fundamental MHD waves: Alfvén and acoustic (*Gorelenkov*, APS'06, EPS'07).

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• Energetic Particle (EP) driven low-*f* instabilities lead to radial EP transport

On NSTX:

- three modes form avalanches,
- can be interpreted as BAAEs,
- induce losses  $\sim 13\%$  in this case as inferred from neutron signal.
- SFLIP diagnostic observes complicated beam ion loss dynamic.

### *Motivation:* BAAEs are useful as $q_{min}$ diagnostic

• BAAEs MHD spectroscopy application is confirmed by MSE in NSTX



 $q_{min}$  from BAAEs (high  $\beta$ ) complements  $q_{min}$  from RSAEs (low to medium  $\beta$ )

• \*AEs are expected in burning plasmas, ITER.

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### Shear Alfvén-acoustic continuum capture main effects

- Alfvén/acoustic continuum bounds global modes: D(r) = 0,  $(\partial_r D \partial_r S) \phi = 0$
- In a low- $\beta$ , large aspect ratio plasma, low  $\omega_*$ , (Cheng, Chance, PFI '86):

$$\Omega^2 y + \partial_{\parallel}^2 y + \gamma \beta \sin \theta z = 0 (Alfvenic)$$
(1)

$$\Omega^{2}\left(1+\frac{\gamma\beta}{2}\right)z+\frac{\gamma\beta}{2}\partial_{\parallel}^{2}z +2\Omega^{2}\sin\theta y =0 (acoustic), \qquad (2)$$

where  $\Omega \equiv \omega R/v_A$ ,  $y \equiv \xi_s \varepsilon/q$ ,  $\xi_s \equiv \vec{\xi} \cdot \frac{[\mathbf{B} \times \nabla \psi]}{|\nabla \psi|^2}$  and  $z \equiv \nabla \cdot \vec{\xi}$ ,  $\hat{k}_{\parallel} \equiv i \partial_{\parallel}/R$ . Geodesic curvature coupling: *m* Alfvénic and  $m \pm 1$  acoustic harmonics.



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• uncoupled *Alfvénic* (A)  $\Omega^2 = k_{\parallel}^2 R^2 + \Omega_{GAM}^2$ and *acoustic* (a)  $\Omega^2 = \frac{1}{2} \gamma \beta k_{\parallel}^2 R^2$  branches.

• **GAMs**: 
$$\Omega_{GAM}^2 = \gamma \beta \left(1 + 1/2q^2\right)$$

• modified shear Alfvén branch  $\Omega^2 = k_{\parallel}^2 R^2 / (1 + 2q^2)$ 



\* Winsor'68, Mikhailovski'75,'98, Chu'92, Zonca'96, van der Holst'00, Smolyakov'08

# Alfvén/acoustic coupling in toroidal equilibrium (schematic)

- Alfvén (A) continuum at low frequency:  $\Omega^2 = k_{\parallel \pm 1}^2 R^2$
- Acoustic (a) branch  $\Omega^2 = \gamma \beta k_{\parallel \pm 1}^2 R^2 / 2(1+\delta)$



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Global modes exist in A-a continuum gaps (van der Holst'00)

Lower (below TAE) gaps are due to  $\beta$  and geodesic curvature effects

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#### Two limit cases of the kinetic dispersion for BAAEs

Basic assumptions  $\beta \ll 1$  ( $\delta B_{\parallel}$  is negligible),  $\omega \leq v_s/qR \ll v_A/qR$ . General dispersion: *Zonca et.al. PPCF'96, Mikhailovskii et.al. PI.Phys.Rep'99* 



• phase velocity is different from Alfvénic, depends on  $T_e/T_i$ .

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# NSTX ( $T_e \simeq T_i$ ) multiple BAAE frequency measurements confirm kinetic dispersion



 $q_{min}$  is from MSE:  $f = f_{BAAE} + nf_{rot}$ , n < 0,  $n = -1 \div -4$ . Applied modified Alfvénic dispersion with rotation  $f_{rot}(q_{min}) = 19 - 23kHz$ ,  $\omega_{*n=1} \simeq 2kHz \ll f_{BAAE}$ Modified Alfvénic wave dispersion agrees with the kinetic dispersion at  $T_i = T_e$ :  $f_{BAAE} = v_A k_{\parallel}/2\pi \sqrt{1 + q_{min}^2(1/2 + \pi/8)}$  vs MHD  $v_A k_{\parallel}/2\pi \sqrt{1 + 2q_{min}^2}$ .

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Kinetics improves and complements MHD framework for BAAE studies: i) proper acoustic wave dispersion, ii) ion Landau damping

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#### Global modes are localized to the extremum points of Alfvén - acoustic continuum



- Core localized and gap BAAEs are found with one dominant poloidal harmonic (Gorelenkov, PLA'07):
  - monotonic q-profile,  $q_0 \sim 1, q_a = 4$ .
  - 1. low shear sweeping BAAE (A):  $\omega \simeq v_A k_{\parallel}/\sqrt{1+2q_{min}^2}|_{r=0}$
  - 2. gap BAAE:  $\Omega_{+} \simeq \sqrt{\gamma \beta/2}/q_{min},$  $\gamma = (T_e + 7T_i/4) / (T_e + T_i)$
- $\nabla \xi$ ,  $m \pm 1$  sidebands are present  $(\sim \xi_{\theta}/a)$ .

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#### How these global BAAE structures compare with experiments?

#### NSTX experiments address BAAE dispersion

- Low density  $n_e \simeq 3 \times 10^{19} m^{-3}$ ,  $P_{NBI} = 2MW$ ,  $E_{NBI} = 90 keV$ .
- 12 channel MSE measures q profile (reversed shear).
  - helps to validate theory.
- Low frequency oscillations (BAAEs) are seen unstable:
  - Characteristic upshift frequency evolution from  $\sim$ zero (plasma frame).
  - Modes are localized to  $q_{min}$  surface.
- High-k diagnostic sees BAAEs at r/a = 0.7 (H.Park, APS'07).
- At t = 0.263  $q_{min} = 3/2 \Rightarrow$  even n $(m = nq_{min})$



#### NOVA: BAAE broadens radially as $q_{min}$ decreases



- BAAE frequency sweeps as *q*-profile relaxes.
  - f does not depend on beta (as expected) near rational  $q_{min}$  (=1.5).
  - $\xi_r$  has one dominant harmonic  $m = nq_{min} = 3$ .
- $f_{BAAE}$  is close to modified Alfvén branch  $f_A = v_A k_{\parallel}/2\pi \sqrt{1+2q_{min}^2}$
- Continuously transforms to gap mode (second harmonic, m = 2)
- BAAEs interact with the continuum.

#### Ultra SXR measures the same radial BAAE broadening



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### ECE measures localized BAAE structure in DIII-D



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- Temperature perturbation:  $\delta T_e = -\vec{\xi} \nabla T_e - (\gamma_e - 1) div\vec{\xi}$
- For low frequencies  $\gamma_e = 1$  $\Rightarrow \delta T_e$  is comparable on LFS and HFS.
- Oscillation amplitude  $\xi_r/a \simeq 3 \times 10^{-3}$ .



- NOVA computes BAAE structure with symmetric amplitudes around *Raxis*.
- Small shift toward the center is due to the gap structure.

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## Drift frequencies strongly modify BAAE dispersion

$$\begin{aligned} \frac{k_0^2 \omega}{\Omega^2 \delta \left(\omega - \omega_{*Ti}\right)} &\simeq 1 + q^2 \left[ \frac{1}{2} + \frac{\pi \tau}{4} \frac{\left(\omega - \omega_{*Ti}/2\right)^2}{\omega \left(1 + \tau\right) \left(\omega - \omega_{*Ti}\right)} \right] \\ &+ \left[ \omega - \frac{3\omega_{*Ti}}{2} - \tau \omega_{*Te} \frac{\omega - \omega_{*Ti}/2}{\omega \left(1 + \tau\right)} \right] \frac{iq^2 \sqrt{\pi} e^{-\xi_s^2 \tau/2}}{\xi_s \sqrt{2\tau} \left(\omega - \omega_{*Ti}\right)} \end{aligned}$$

*Mikhailovskii*,*PI*.*Phys*.*Rep*.'99 (without  $\omega_{*e}$ ), Kolesnichenko, IAEA'08 (2 fluid MHD), Lauber, ibid.

- Ion branch at  $k_{\parallel} = 0$ :  $\omega \simeq \omega_{*pi}$  KBM
- Electron branch at  $k_{\parallel} = 0$ :  $\omega \simeq \omega_{*pe}$ .
- Drive is due to ITG (at higher frequencies Afvénic ITG, Zonca, PoP'99)
- BAAEs can be destabilized by  $\omega_{*Ti}$  (ITG).

# Ion and electron drift branches converge to BAAEs away from rational surface $(|k_{\parallel}|\uparrow)$



- Both ion and electron branches are unstable at  $k_{\parallel} = 0$ but global mode structure and trapped electrons maybe stabilizing (most unstable solutions are shown)
- Inside BAAE gap damping rate is strongly reduced to  $\gamma/\omega \sim -10\%$  from  $\sim -25\%$  without  $\omega_{*Ti,e}$  effects.
- NSTX  $\omega_{*Ti} \ll \omega_{BAAE}$ , DIII-D  $\omega_{*Ti} = 0.9\Omega_+$  for n = 8.

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KBM ( $\omega_{*pi}$ ) can transform to gap BAAE if  $q_{min}$  decreases

#### DIIID observations are consistent with gap BAAE excitation



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#### For the global gap modes ideal MHD can be applied

# MHD and kinetics compute measured BAAE frequency patterns



- Numerically (NOVA) global BAAEs are found inside Alfvén-acoustic continuum gaps not as sweeping modes
  - maybe due to strong  $\beta$  profile variation and shear effects
  - modes interacting with the continuum are not resolved (strong damping)
  - kinetic theory renormalization gives similar frequencies
  - $q_{min}$  comes from MSE, MHD spectroscopy (RSAE, sawtooth)
- There are uncertainties in *n* numbers.

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#### Flattening of beam ion profiles in DIIID when BAAEs are observed

Classical (TRANSP) simulations and Fast Ion  $D_{\alpha}$  spectroscopy diagnostic (FIDA) show beam ion depletion when BAAEs are excited.



(for FIDA in NSTX see M.Podestà, GI1.00001)

- BAAEs are excited to large amplitudes,  $\xi_r/a = 3 \times 10^{-3}$ .
- Just RSAE/TAEs can not explain beam ion depletion (*Heidbrink,PRL'07*) ⇒ BAAEs may help to explain EP transport.

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- Key elements of the analysis/comparison are:
  - frequency spectrum: trends (JET, NSTX, DIII-D), absolute values (NSTX),
  - mode structure (NSTX, DIII-D), localization (NSTX, DIII-D),
  - need polarization measurements/analysis, such as  $\delta T_e$  vs  $\delta n_e$ .

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- BAAE effects on the plasma
  - radial transport of beam ions on NSTX, DIII-D,
  - thermal ions can also be effected,
  - energy channeling from beam ions directly to thermal ions ( $\alpha$ -channeling, Fisch, PRL'93; hot-ion mode, Zakharov, LiWall).

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- Future work:
  - cross machine experiments/comparison,
  - numerical studies with kinetics and global mode structure,
  - stability with fast ions.

## Summary

- 1. Low frequency Alfvén Acoustic global Eigenmodes (BAAEs) are observed and studied **using MHD**.
- 2. BAAE dispersion is strongly modified in **kinetic theory.**
- 3. BAAEs induce beam ion transport in NSTX and DIII-D.
- 4. Both RSAEs and BAAEs can be used for  $q_{min}$  diagnostic.

#### BAAEs can induce fast ion losses in NSTX





- Fast ions are lost in a broad pitch angle range  $v_{\parallel}/v$
- Multiple *n* instabilities form avalanches
- Neutron flux is dropped by 13%