On the Physics of Onset of Nonlinear Disruptive Instabilities

D.P. Brennan[1], J.M. Finn[2], D.A. Gates[3], S.E. Kruger[4] and C.R. Sovinec[5]

[1]Princeton University, Princeton, NJ
[2]Los Alamos National Laboratory, Los Alamos, NM
[3]Princeton Plasma Physics Laboratory, Princeton, NJ
[4]Tech-X Corporation, Boulder, CO
[5]University of Wisconsin, Madison, WI

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Nonlinear Simulations Find Faster-Than-Exponential Growth: Scaling with Heating Rate in Good Agreement with Theory

- Impose heating source proportional to equilibrium pressure profile

$$\frac{\partial P}{\partial t} = \quad ..... \quad + \gamma_H P_{eq} \quad \Rightarrow \quad \beta_N = \beta_{Nc}(1+\gamma_H t)$$

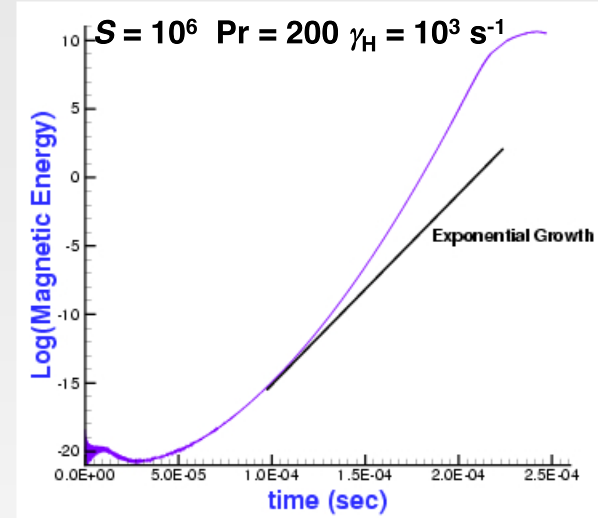
- Follow nonlinear evolution through heating, destabilization, and saturation
- Simulation starts below marginal stability point in beta
- Simulation results with different heating rates are well fit by $\xi \sim \exp[(t\text{-}t_0)/\tau]^{3/2}$
- Time constant scales as

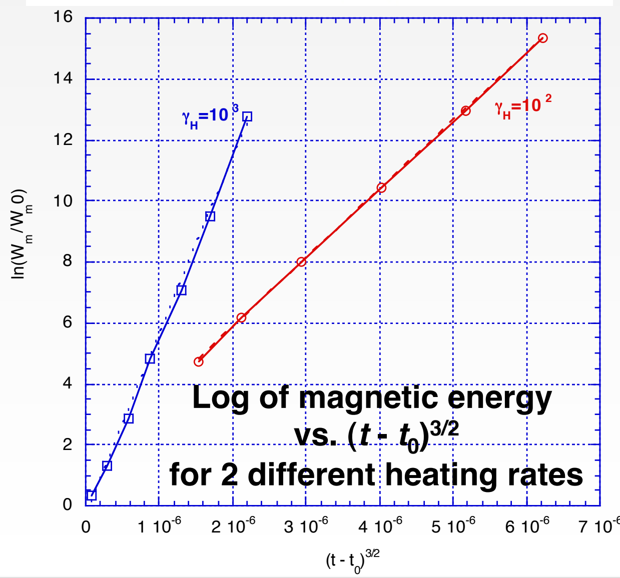
$$\tau \sim \gamma_{MHD}^{-0.72} \gamma_H^{-0.28}$$

- **Compare with theory:**

$$\tau = (3/2)^{2/3} \hat{\gamma}_{MHD}^{-2/3} \gamma_h^{-1/3}$$

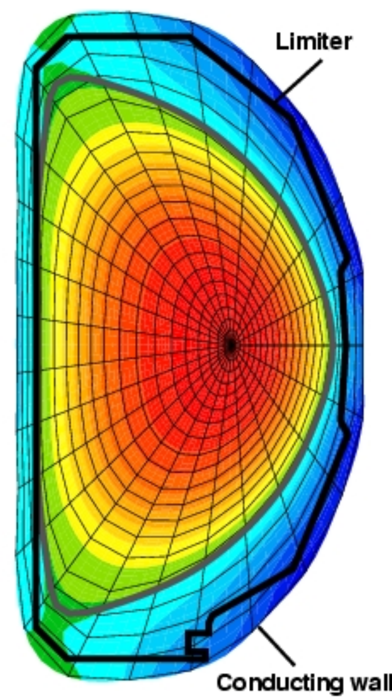
- Discrepancy possibly due to non-ideal effects



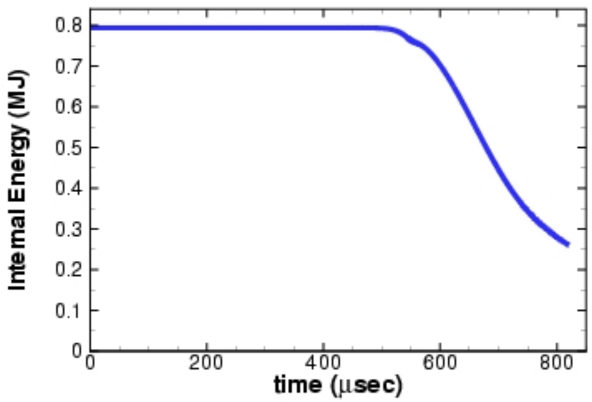


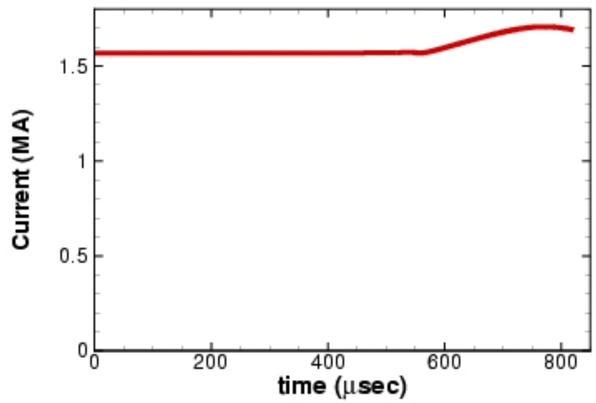
Issues of boundary conditions were understood in early free-boundary simulations – remains a subject of intense debate and focus

- Boundary conditions are applied at the vacuum vessel, NOT the experiment limiter.
  - Vacuum vessel is conductor
  - Limiter is an insulator
- This is accurate for magnetic field:
  - B_n=constant at conducting wall
  - B_n can evolve at graphite limiter
- No boundary conditions are applied at limiter for velocity, temperatures, or density:
  - This allows fluxes of mass and heat through limiter
  - Normal heat flux is computed at limiter boundary
- From Kruger PP 2004:
  "… applying the natural boundary conditions at the limiter would give no density flux across the limiter, which is less physical than our current method of applying the natural boundary conditions at the vacuum vessel and allowing a mass flux across the limiter."
- Large viscosity added in region between limiter and vacuum vessel to make Neumann-like B.C.
  - More accurate models can be implemented.



NIMROD Simulation of DIII-D shot 87009 shows quantitative features of tokamak disruptions

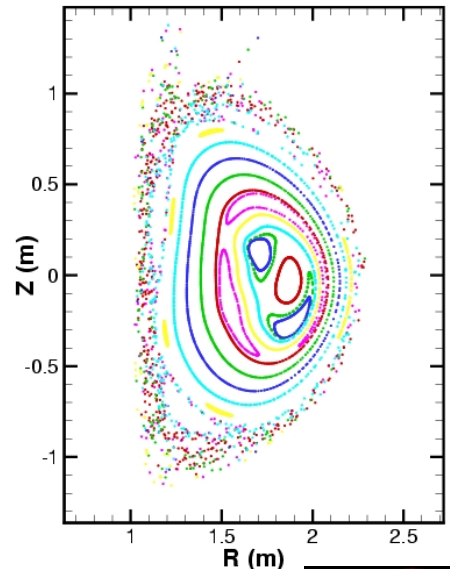




Plasma produces current spike as internal inductance changes

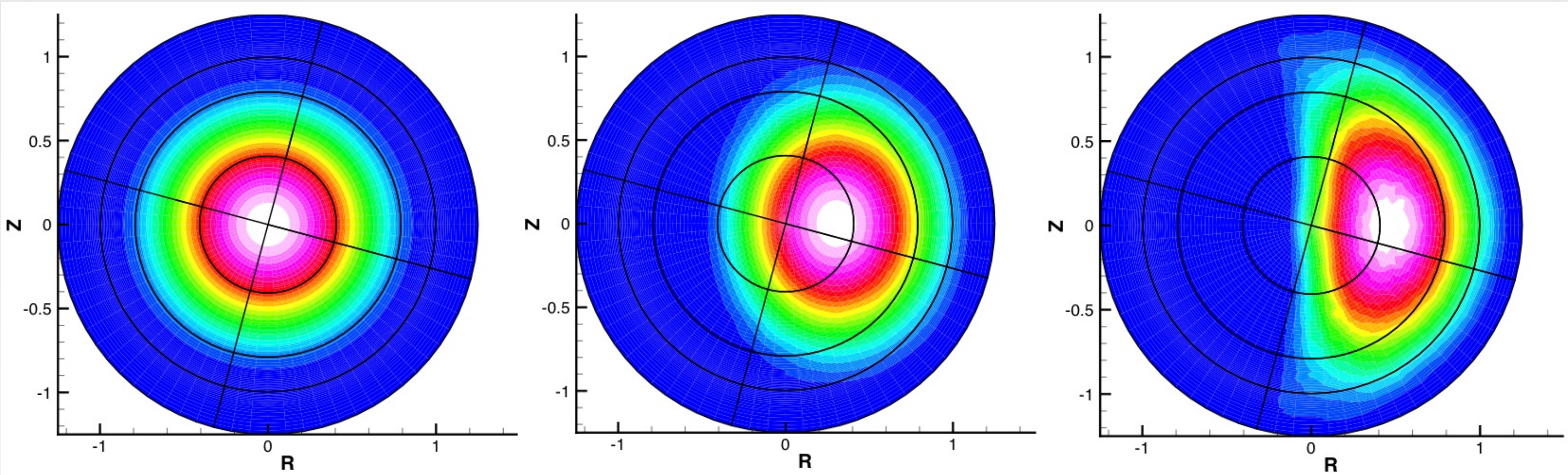
Plasma loses 60% of ~~magnetic~~ energy in ~200 microseconds. Agreement within 20%.

Time scale set by heat transport in complex topological field as the magnetic field becomes stochastic



Similar levels of agreement from Izzo, Whyte

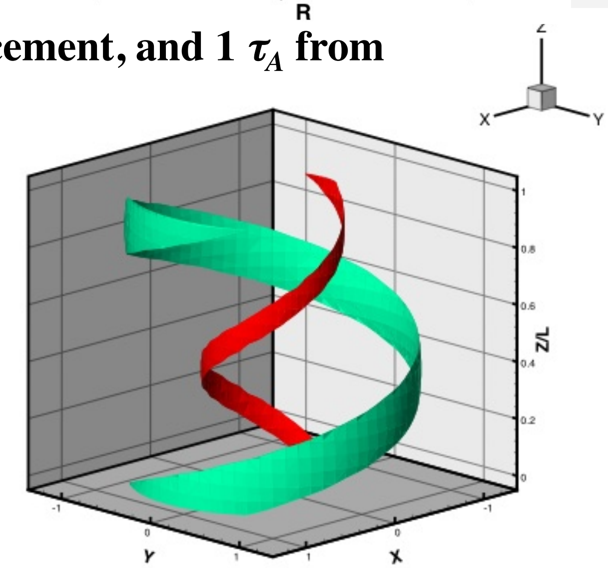
The drag term has not been used in the NIMROD computation; the plasma column distorts into the wall.



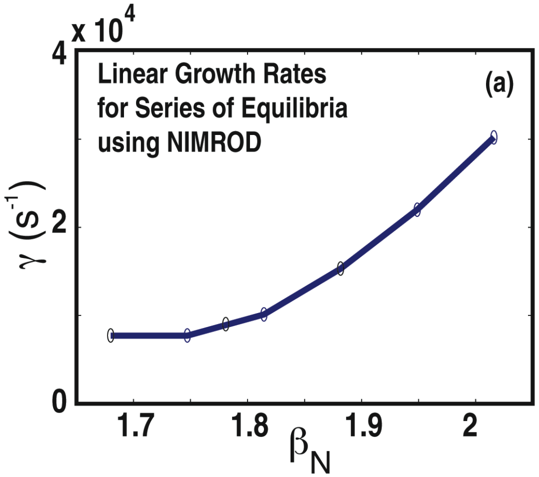
Contours of constant pressure at t=0, 3 τ_A from maximum displacement, and 1 τ_A from maximum displacement.

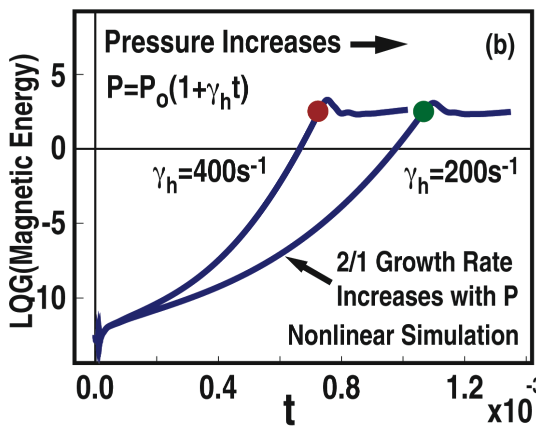
• Computationally, this case 'exercises' NIMROD's ability to advect sharp fronts in n and to solve linear systems with very strong variation in coefficients over the periodic coordinate.

Helical surface currents flow parallel to J_0 along the phase of the column that moves inward and anti-parallel along the phase that moves outward

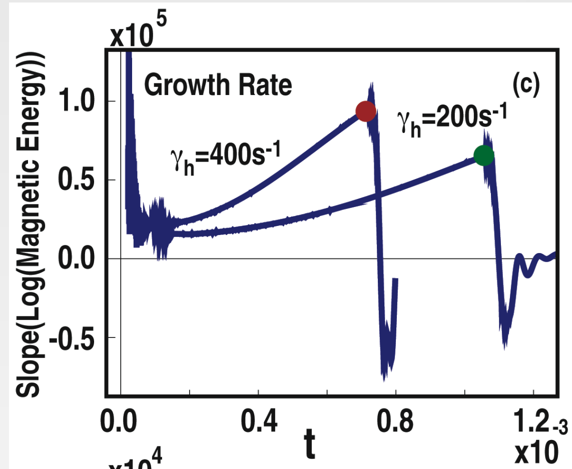


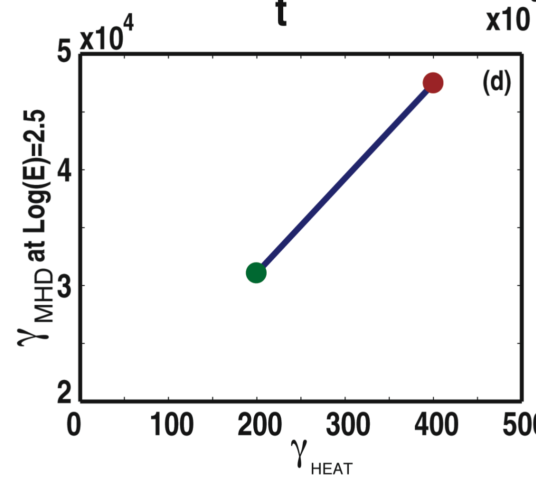
Nonlinear Dependence of β(w) and γ(w) on Heating Rate Captured by NIMROD and Rutherford Modeling

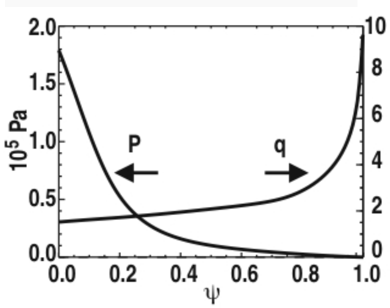


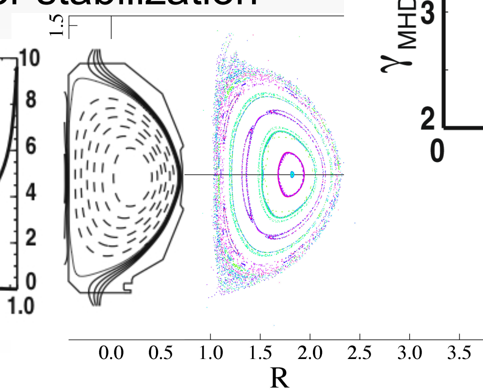


- Timescales: $\tau_R >> \tau_H > \tau_{MHD}$
- Thermal anisotropy: temperature flattening, fast parallel transport included
- Neoclassical and Polarization Effects necessary for NTM threshold and low heating rate limit: not yet in simulations.
- For low heating rates, island growth rate decreases at minimum detected size, reducing ECCD power required for stabilization







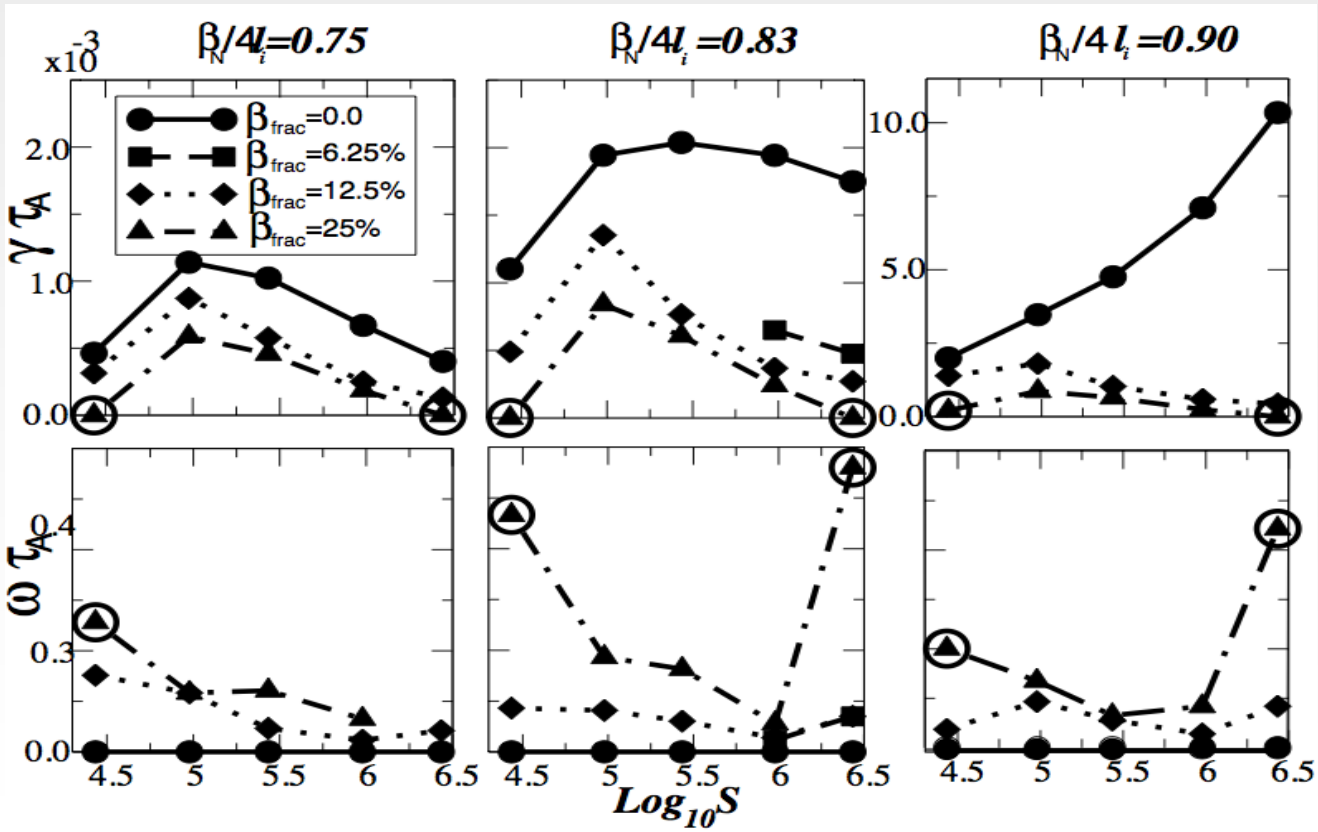


Energetic Particles have been shown to be important to the stability of the m>1 resistive and ideal modes

Energetic particles included into these cases cause significant damping and stabilization to the 2/1 modes, for slowing-down distributions similar to experiment
-> can be critical to modeling the threshold to nonlinear disruptive instability

As the fraction of β that the energetic particles make up, β_{frac}, increases to experimental values, modes are strongly damped.
DIII-D~0.16
JET~0.3
Varies with discharge

Nonlinear evolution also strongly affected



Detailed studies suggest both trapped and passing particles are responsible for stability properties

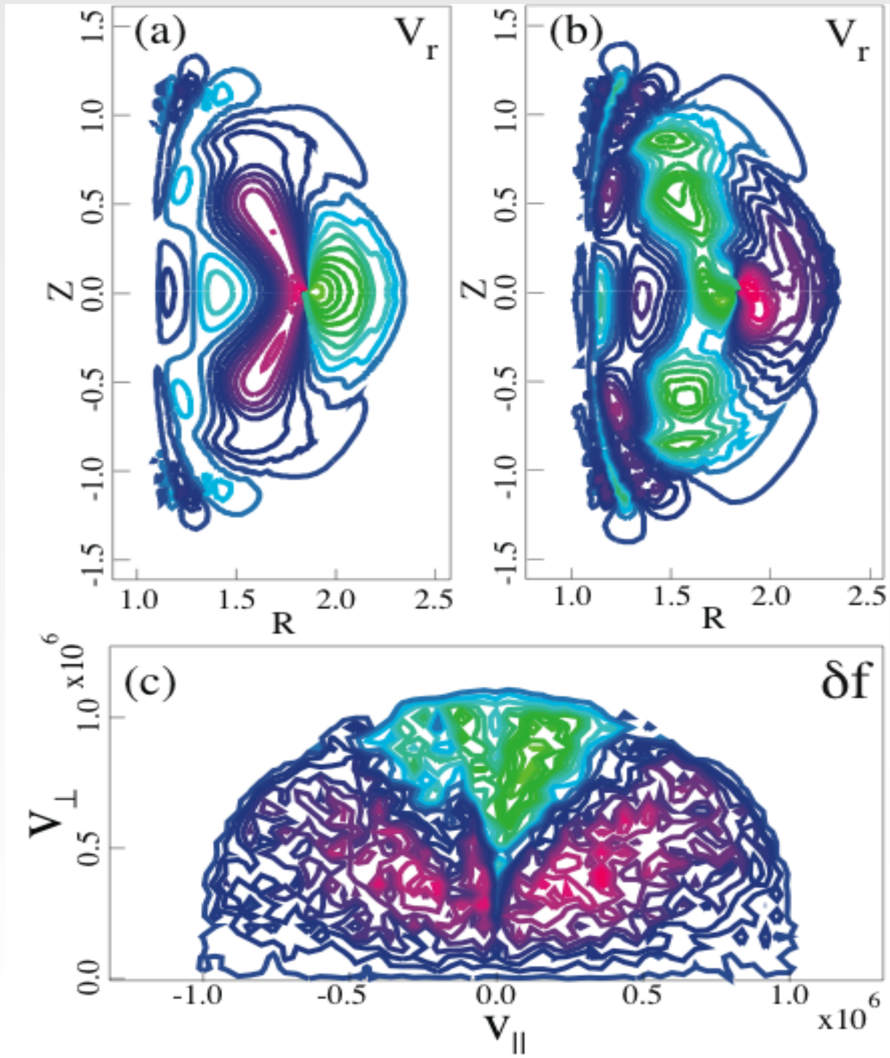
Damped and stabilized 2/1 mode conjectured to be responsible for qualitative differences between experiments (eg. DIII-D, vs JET).

R. Takahashi, D. Brennan, C.C. Kim, PRL 2009

Understanding effect in NSTX currently underway.

Simple Rutherford Modeling of Energetic Particle Effects not Viable, need Kinetic - MHD model such as NIMROD or M3D.

The Eigenfunction of V_r, the n=1 Spatial Projection of δf in Phase Space Shows Trapped Cone, but also significant passing activity, non-trivial interaction for realistic cases.

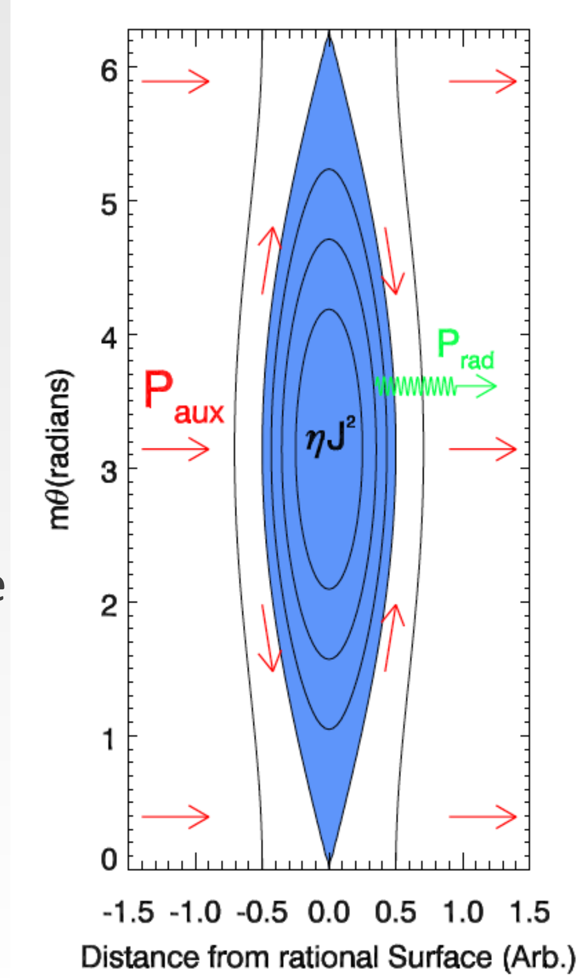


Radiation driven islands are cooled by magnetic insulation, increasing resistivity -> increased drive

- The island is magnetically insulated from it's surroundings, cooling the island
- Lower temperature leads to increased resistivity, enhancing helical island current
- The island then grows causing the process to continue.

$$P_{rad} < \eta J^2 \rightarrow n_e < \sqrt{\frac{m_e}{e^2 E_{eff}} \frac{\nu_{ei}}{\nu_{(eZ)_{eff}}}} J$$

- **Radiation drive can change sign –** eg. when island cools
- Auxiliary power is shunted around the island by paralle conduction, consistent with density limit being independent of heating power
- Quantity in square root is nearly independent of temperature*
- Reminiscent of the Greenwald limit



* F. W. Perkins and R. A. Hulse, Phys. Fluids **28** (1985) 1837.
P. H. Rebut and M. Hugon, Plasma Physics and Controlled Nuclear Fusion Research 1984 (Proc. 10th Int. Conf. London, 1984), Vol. 2, IAEA, Vienna, 197, (1985).

Modeling Active Stability Control Show Stable Windows of Operation – Simple Example: RFP with no flow

Control model

$$\tilde{B}_r(r_c) = -G\tilde{B}_r(r_w) + K\tilde{B}_\phi(r_w)$$

For resistive wall and fixed plasma resistivity, four limits defined. With two walls: six limits.

Stable window with feedback for λ above resistive limit.

Possible to stabilize near ideal plasma, ideal wall limit.

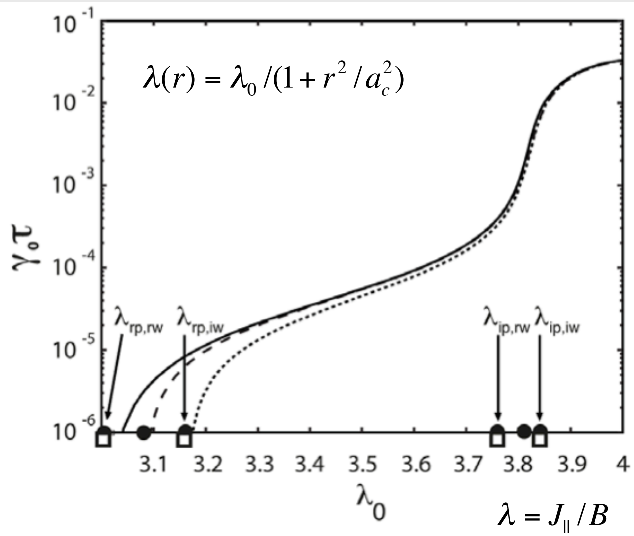
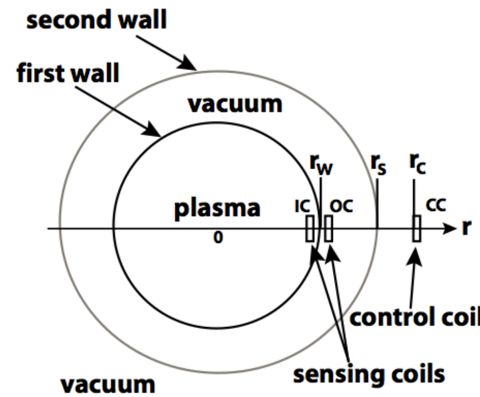
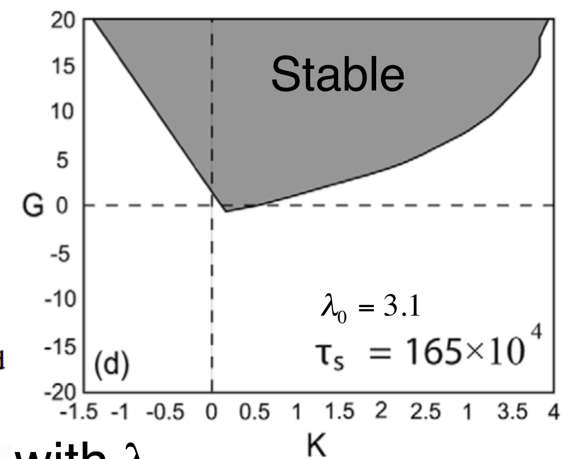


Figure 12. Growth rates ($m = 1$, $n = 8$) for a resistive inner wall and outer shell ((rp,rw,rs)—solid curve), a resistive inner wall and ideal outer shell ((rp,rw,is)—long-dashed curve), and an ideal inner wall ((rp,iw,*)—short-dashed curve). One-wall stability thresholds ((rp,rw) and (rp,iw), quoted in section 3.2) are denoted by open squares and two-wall thresholds are represented by solid disks. The corresponding marginal stability points for ideal plasma are extapolated to zero plasma resistivity. Parameters are $\eta = 10^{-7}$, and $\nu_{\rm kin} = 2 \times 10^{-5}$. See table 1 for the numerical values of the two-wall limits.





Stable window varies in shape and size significantly with λ_0

Sassenberg, Richardson, Brennan, Finn, "Control of MHD modes in RFPs with normal and tangential magnetic field sensing and two resistive walls", Plasma Phys. Contr. Fusion 55, 084002 (2013).
Richardson, Finn, Delzanno, "Control of ideal and resistive MHD modes in RFPs with a resistive wall", PoP 17, 112511 (2010).
Finn, "Control of MHD modes with a resistive wall above the wall stabilization limit", PoP 13, 082504 (2006).

Summary and Looking Forward

What are the foci for additional physics in the core plasma?

- Two-fluid terms: Do we have to resolve the electron skin depth locally? If so, it will be impossible within our lifetimes
- Models for anisotropic heat conduction are inadequate
  - Using Braginskii even in collisionless or semi-collisional regimes
- More accurate heat flux: In progress.
- Energetic particle effects can be critical – nonlinear simulation current focus
- Flow and flow shear – not mentioned but important
- Radiation driven island theory provides a testable quantitative prediction of the density limit -> possible solution to long standing puzzle
  - Theory predicts exponentially growing islands with a sudden robust onset condition
  - Consistent with a robust density limit and observed rabidly growing 2/1 tearing mode
  - Need to directly verify local power balance in experiments and include in sims

Summary and Looking Forward

What are the foci for additional physics in the boundary?

- An external kink in cylindrical geometry demonstrates nonlinear free-boundary capability.
  - With a drag term, this case will provide a nonlinear benchmark.
  - The solver is the primary computational challenge.
- Nonlinear evolution across stability boundary to disruption has been simulated, but there is much more to do to address slow approach quantitatively.
- Resistive wall is crucial to modeling disruptions -> not discussed here, but nearly complete in NIMROD
- Active Feedback Control modeling will add a new layer of necessary complexity in understanding experimental outcome in these cases.

- Others?

Theory of Non-Axisymmetric Vertical Displacement Events

Richard Fitzpatrick

Institute for Fusion Studies

University of Texas at Austin

Austin TX

Effect of $n = 1$ Mode

- The $n = 1$ mode also requires moderation to prevent it from growing on Alfvenic timescale. This leads to significant increase in vertical force exerted between vacuum vessel and plasma, and, hence, in halo current.

- As $n = 1$ mode grows in amplitude, region of contact between plasma and vacuum vessel becomes toroidally asymmetric. Eventually, contact region becomes toroidally localized. This implies toroidal localization of halo current, and halo current force. However, net force still has to be large enough to moderate $n = 0$ and $n = 1$ modes. Assuming that net force remains approximately constant, this implies significant increase in force density in contact region.

Critical Questions

- What is peak vertical force exerted on vacuum vessel during VDE?
- What is maximum toroidal peaking factor of this force?
- What is maximum sideways force exerted on vacuum vessel during VDE?

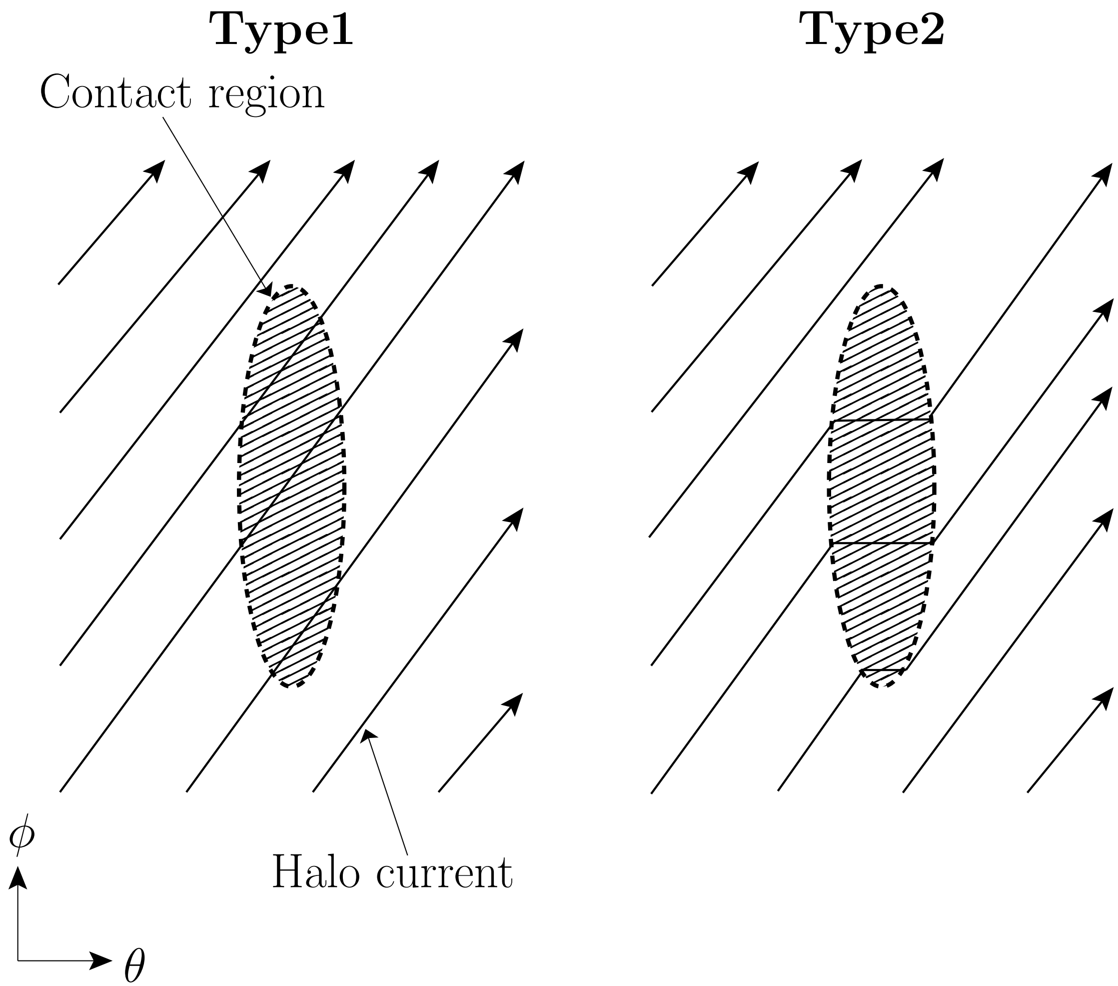
Sharp-Boundary Plasma Model

- Strongly elongated, large aspect-ratio, high beta (i.e., $\beta \sim \epsilon$) plasma with uniform internal pressure. All equilibrium currents flow on plasma boundary.

- Model allows fairly realistic treatment of $n = 0$ and $n = 1$ external modes.

- $n = 0$ and $n = 1$ stability calculations involve matching of vacuum-like solutions at plasma boundary. Can explicitly include effect of halo current force in matching process. Allows self-consistent calculation of effect of halo currents on $n = 0$ and $n = 1$ stability.

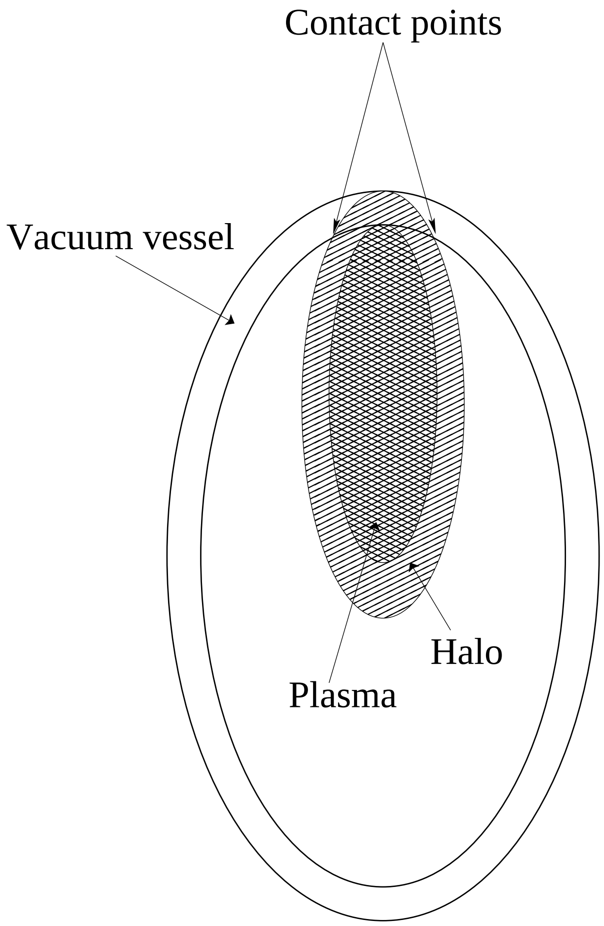
Halo Current Patterns



Circuit Equations

- Model allows for two types of halo current pattern. Type 1 gives rise to no halo current force. Type 2 maximizes force.
- Can calculate mean resistance of halo current circuit associated with each pattern (for specified SOL and v.v. resistivities), as well as mean emf generated by plasma shrinkage. (Assuming that circuit path covers LCFS ergodically.) Associated circuit equations determine relative mix of Type 1 and Type 2 patterns.
- Net halo current force adjusted such that $n = 0$ (and $n = 1$, when it is unstable) mode marginally stable. Circuit equations then give $n = 0$ (and $n = 1$) growth-rate.

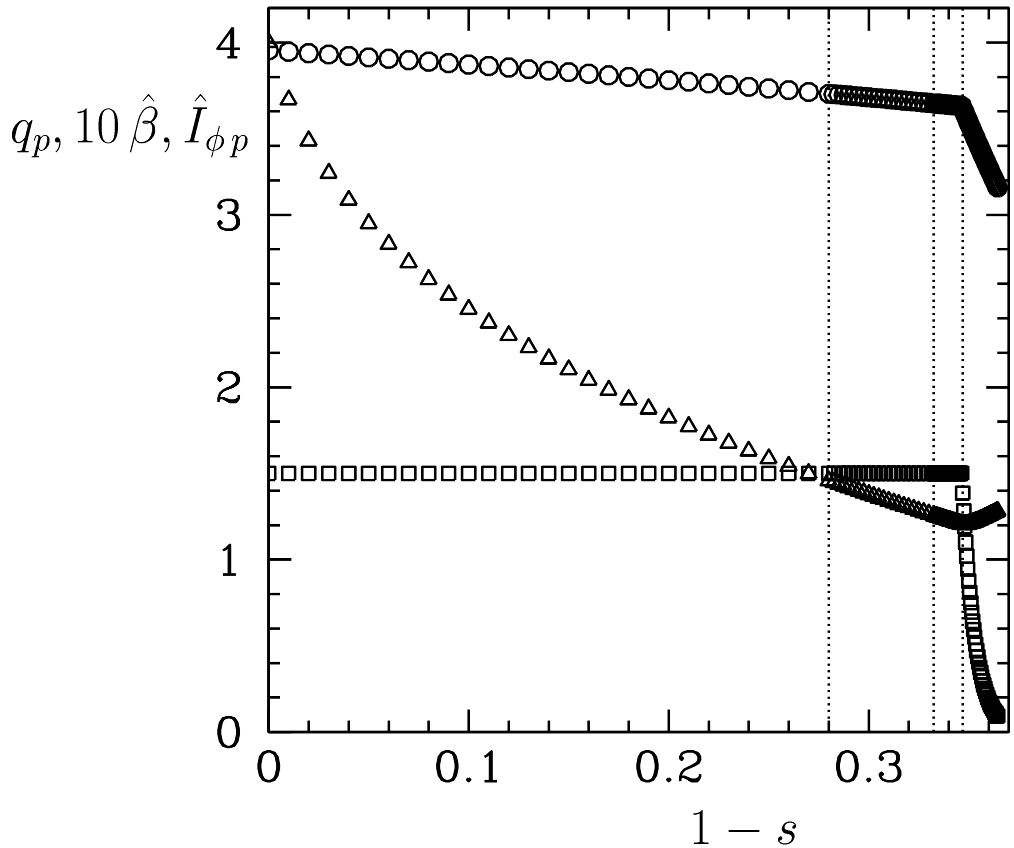
Determination of Poloidal Extent of Contact Region



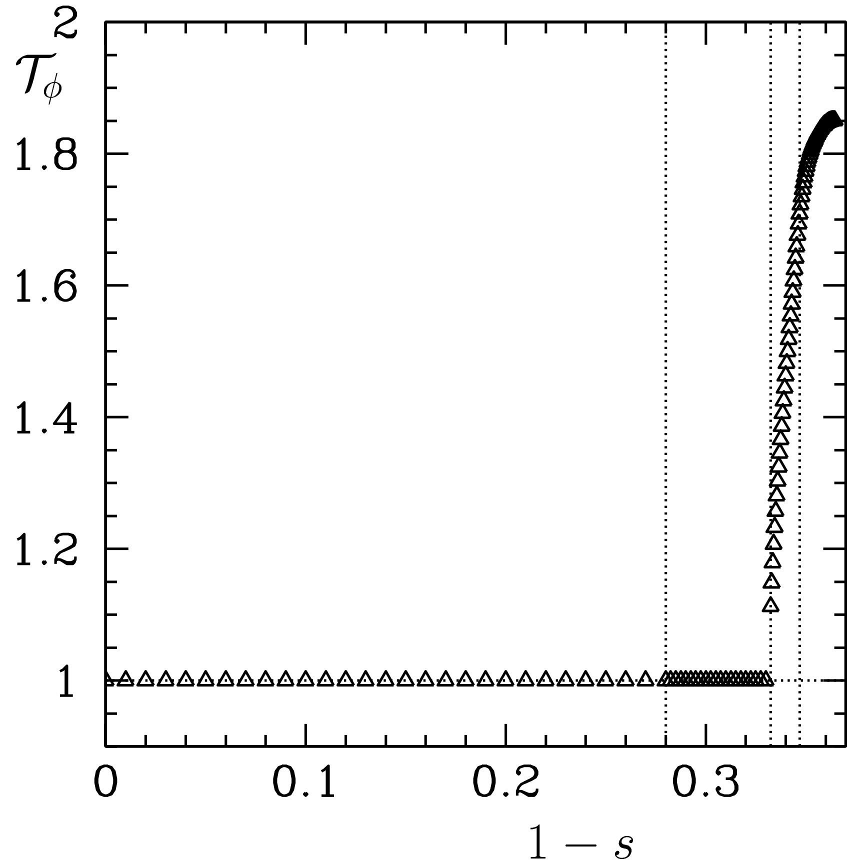
Inputs to Model

- Vacuum vessel shape and thickness.
- Initial edge-q.
- Plasma shape, current, and beta as plasma shrinks.
- SOL and vacuum vessel resistivities.
- Critical halo current fraction that triggers current quench.

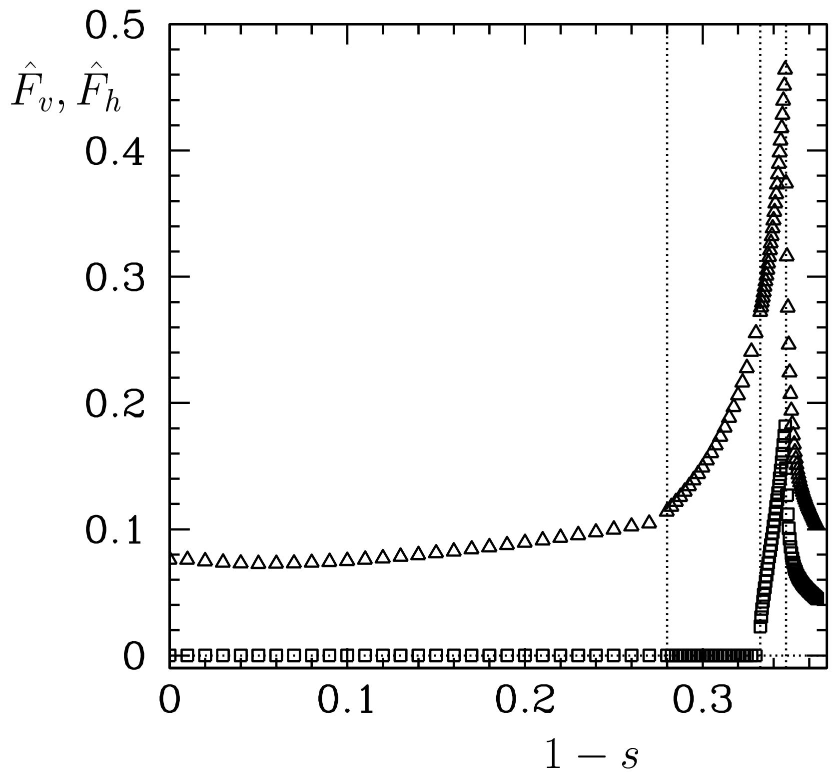
Example Calculation: Edge-q, Current



Example Calculation: Toroidal Peaking Factor

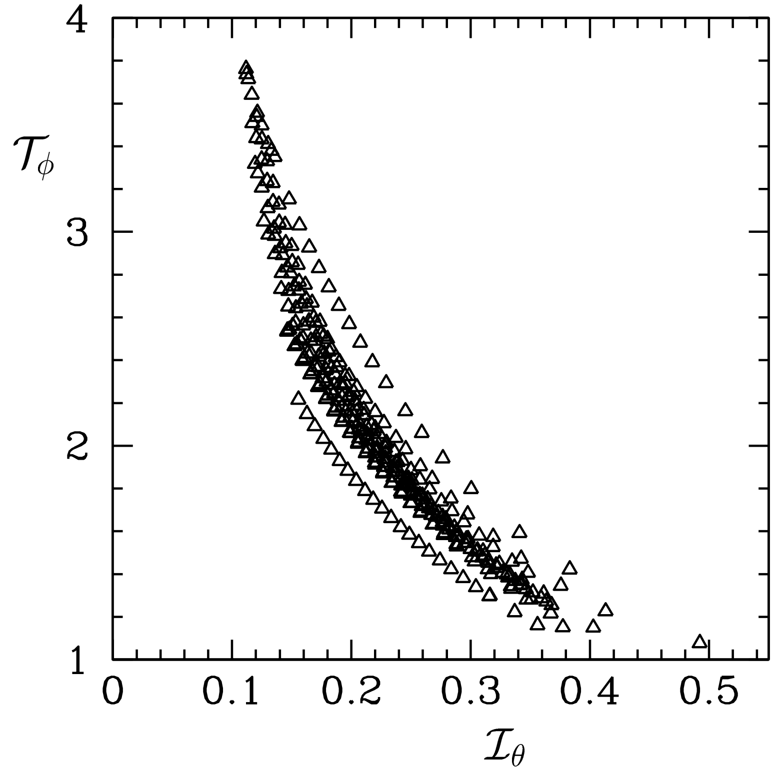


Example Calculation: Vertical and Sideways Forces



For typical ITER parameters, peak vertical and horizontal forces are 31 MN and 12 MN, respectively.

TPF vs Poloidal Halo Current Fraction



Ansatz that current quench triggered when halo current faction exceeds critical value reproduces inverse relation between TPF and poloidal halo current fraction seen experimentally.

Simulations of asymmetric VDEs with M3D: model validation and comparison with experimental cases

R. Paccagnella
Consorzio RFX and IGI-CNR, Italy

In collaboration with:
H. Strauss(HRS Fusion),
J. Breslau, S. Jardin (PPPL), L. Sugiyama (MIT)
P. Bettini (RFX &Padua Univ.), M. Furno Palumbo (RFX&Padua Univ.)
R. Specogna (Udine Univ.)
G. Pautasso and M. Maraschek (AUG)
M. Baruzzo (RFX), S. Gerasimov (Jet)

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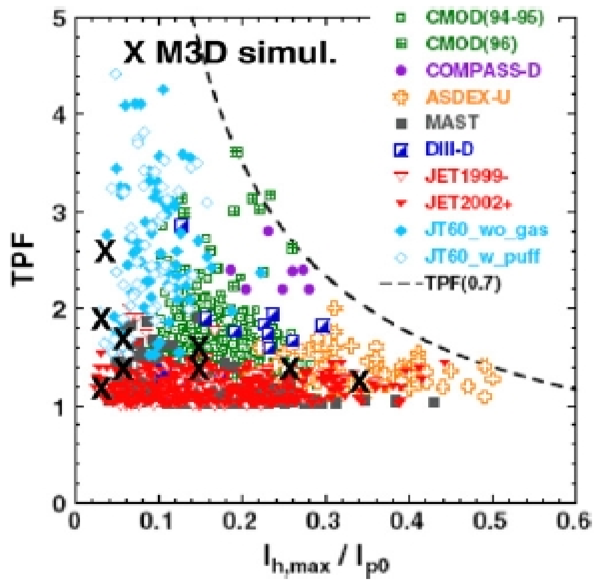
PPPL Workshop on Theory and Simulation of disruptions , July 17-19 2013

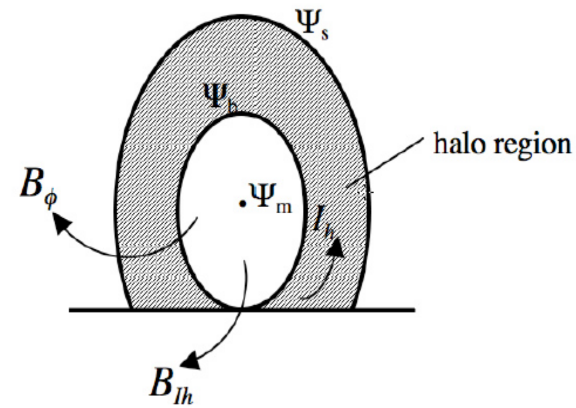
Some of the previous results

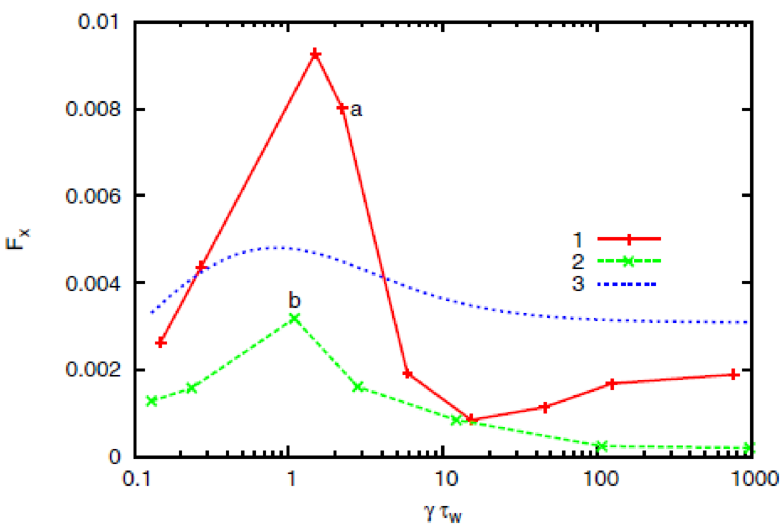


from NF (2009)

$$I_{\text{halo}}(\phi) = \frac{1}{2}\int |\hat{n}\cdot \mathbf{J}| R dl,$$

$$\text{TPF} = \frac{2\pi I_{\text{halo(max)}}}{\int I_{\text{halo}} d\phi}.$$





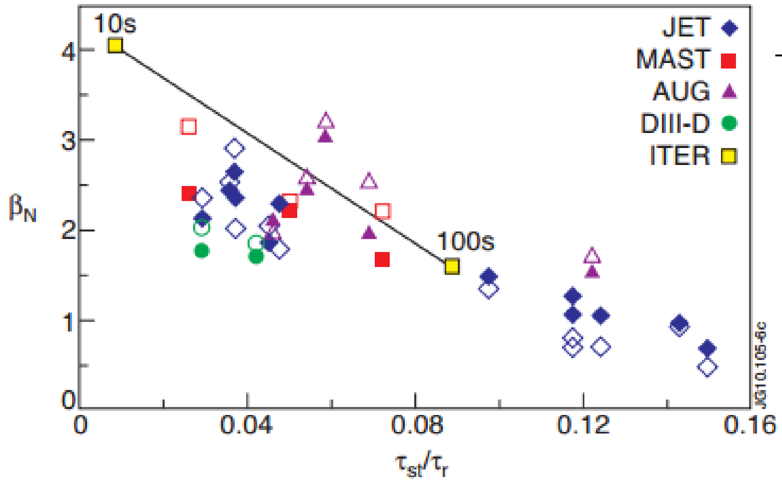
from PoP (2010)

Horizontal force is maximum for
$\gamma\, \tau_W = 1$

..what does this means really in ITER?

ITER vs Simulations: (1)

Chapman et.al., NF (2010)
Neocl.tearing threshold



$\tau_{wall}^{iter} \approx$ 200 ms

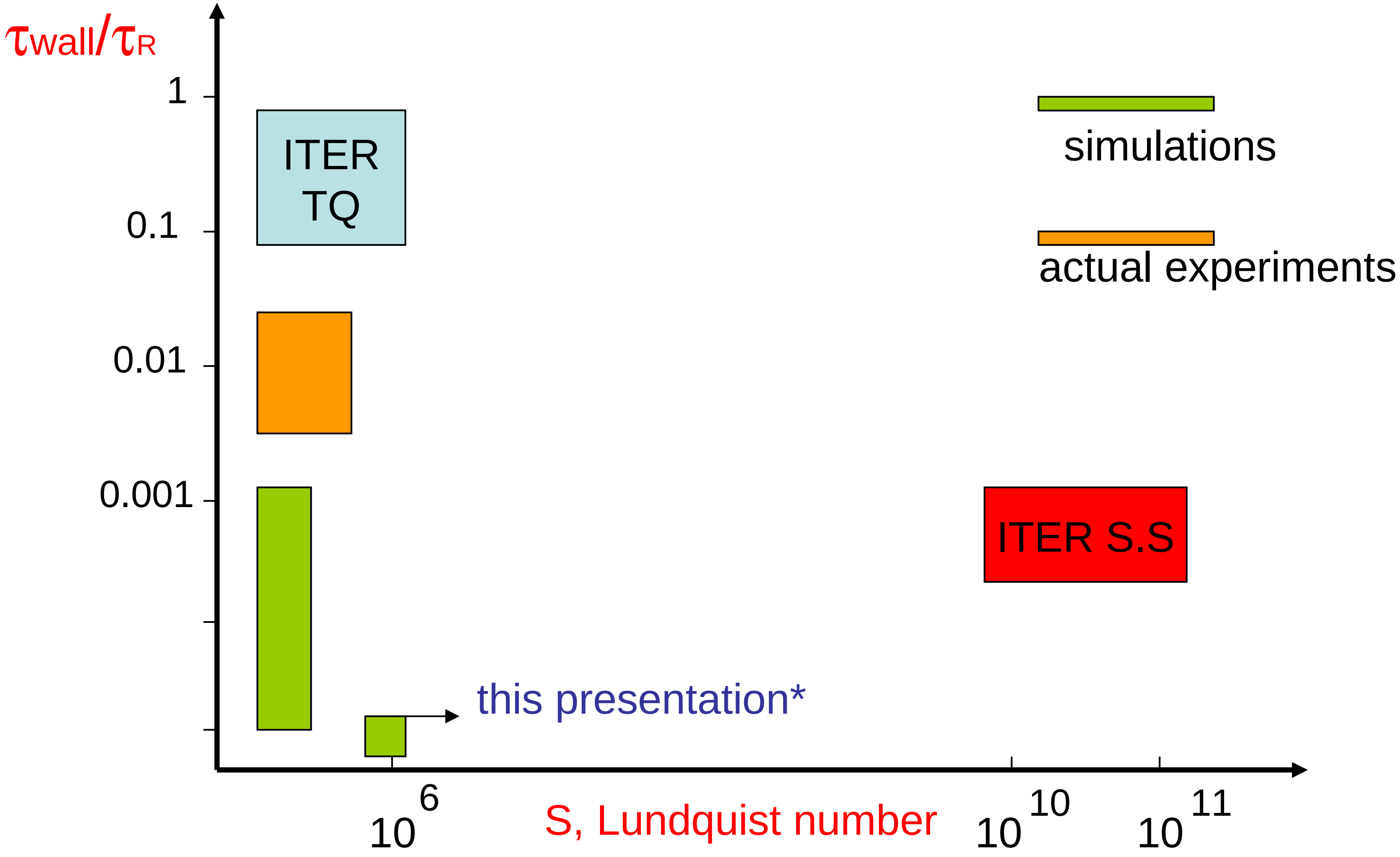
τ_R = 1000 sec

$\tau_{Wall} / \tau_R = 10^{-4} - 10^{-3}$

However after thermal quench with T → 100-10 eV range :

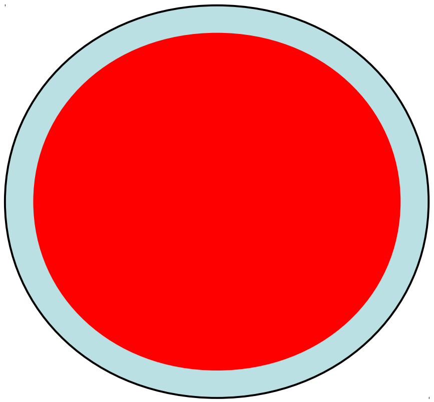
$\tau_{Wall} / \tau_R = 0.1 - 1$

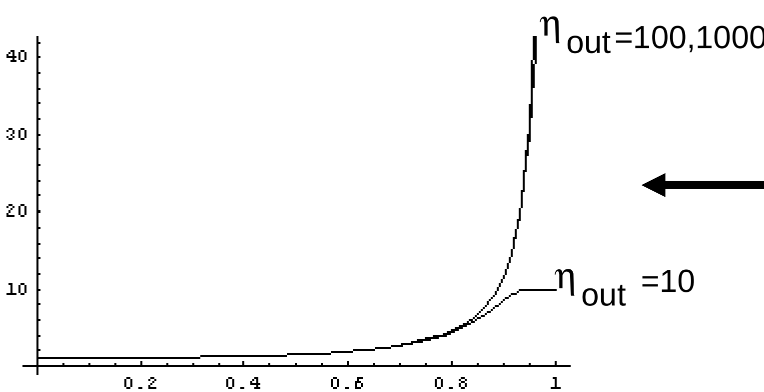
ITER vs Simulations: (2)

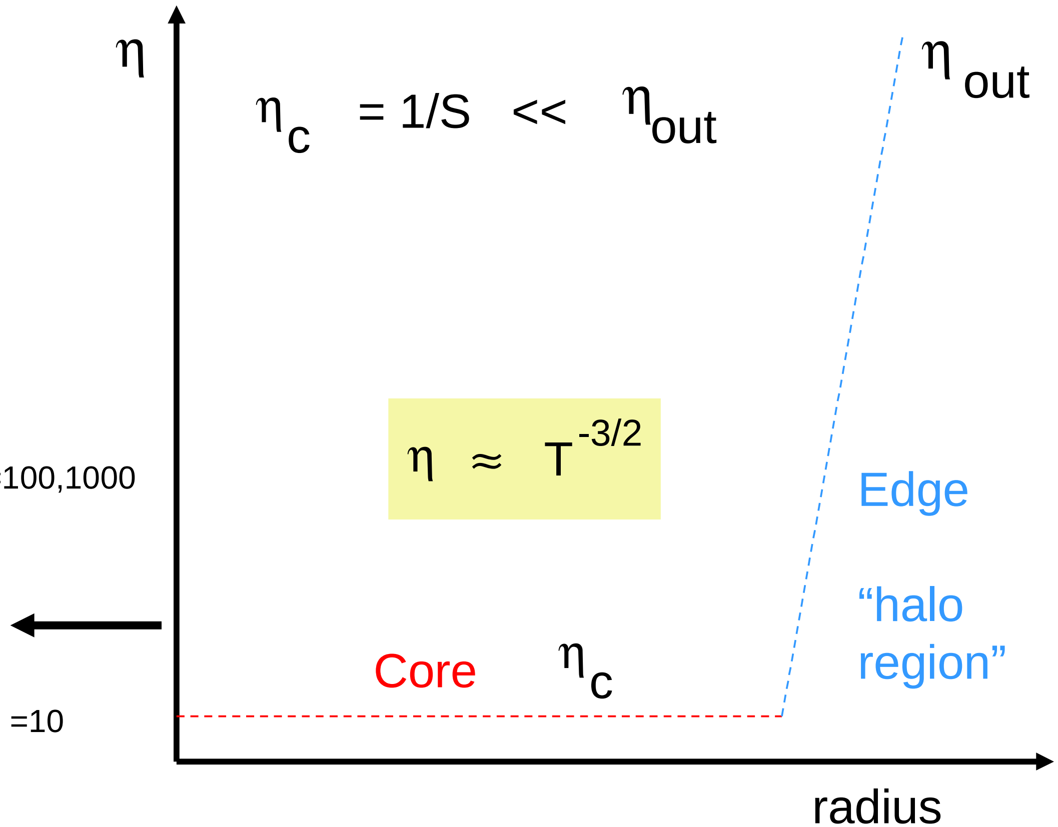


* At high S it is very time consuming having long wall time constants

M3D 2 resistivity regions:







Apart the arbitrariness of η_{out} the halo region is self-consistently determined by the time evolution of temperature

SOME GENERAL OBSERVATONS

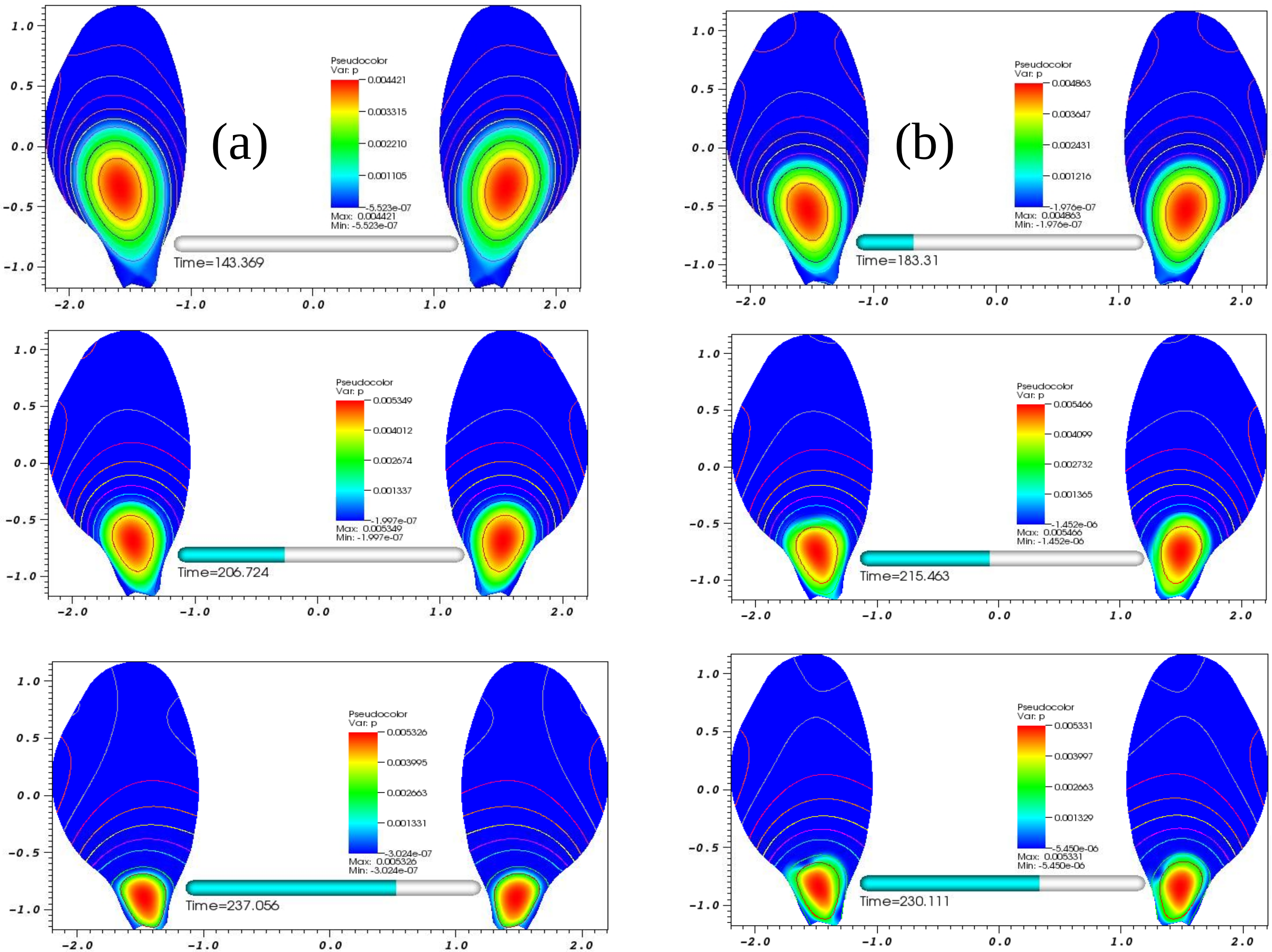
- Other AUG cases have confirmed the observation done for the FED case i.e. higher resolution and higher S, contribute to produce more symmetric VDE's.

(2D VDE overtakes 3D effects -> due to unrealistically low τ_{wall} and S mode scaling)

- non axi-symmetry can be obtained by enhancing the amplitude of an arbitrary initial perturbation to the plasma (at time t=0) .

- Enhancing the plasma viscosity for a given resistivity (or S) (i.e. enhancing the Prandtl number) has also the effect to smooth out non axi-symmetric modes, and to produce more symmetric VDE's.

- there is a clear competition in the system between the VDE time scale (mainly determined by the wall time constant), the current and temperature evolution in the plasma (determined by the transport and by the Lundquist number) and the evolution of the resistive modes, which determine the final TPF and halo fraction.



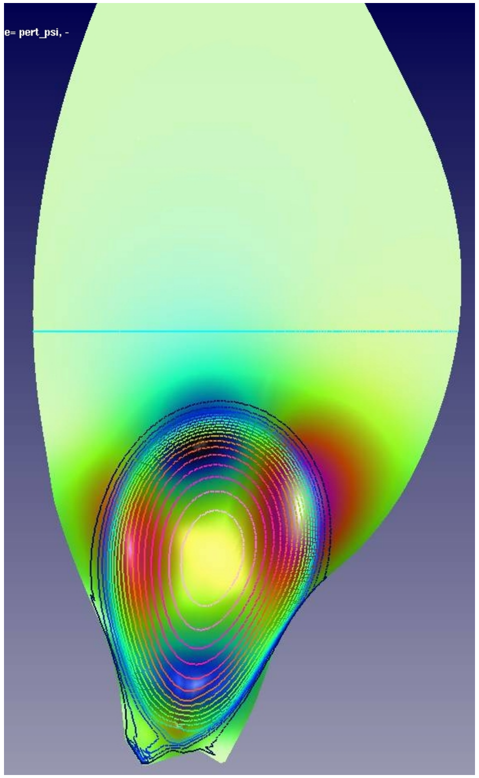
TPF = 1

(no applied initial perturbation)

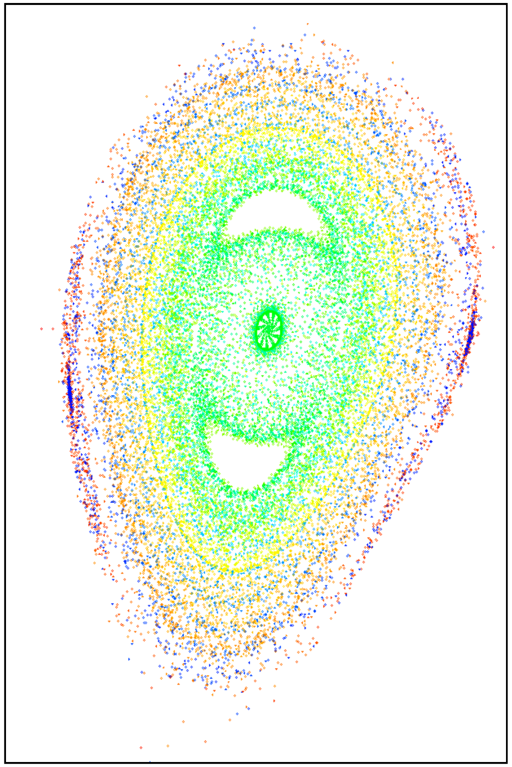
S=5e6

TPF = 1.3

(with initial perturbation)

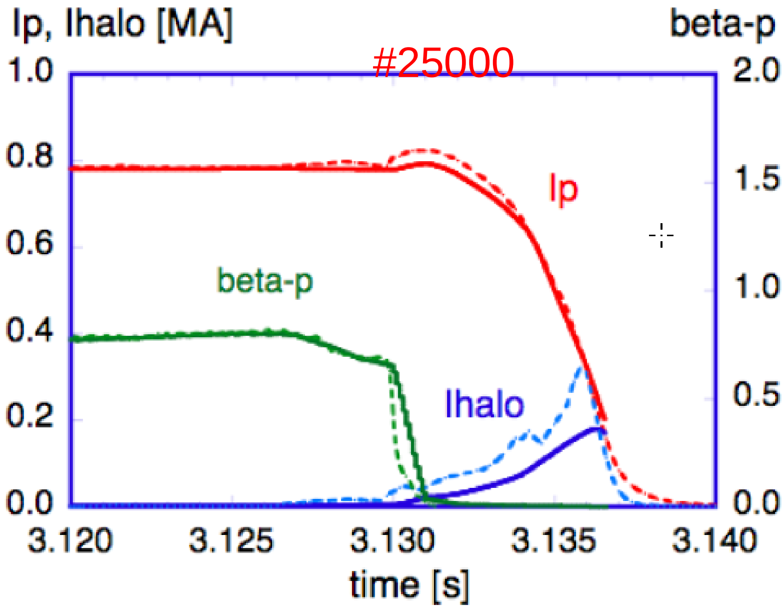


Perturbed poloidal flux and pressure contours



Poincarè puncture plot

a 2/1 resistive mode is dominant in these simulations



Several discrepancies with the experiment

- **the thermal quench is well before the current quench**
(instead similar rate in simulation)

- **high perpendicular transport, can reproduce a faster pressure decay:**
in this case however TPF and hcf can become unrealistic

- **a 2/1 resistive kink responsible for asymmetry in simulations**
Experimentally unclear (role of pure ideal modes?)

- **high resolution simulations resilient to asymmetry (init. pert. needed)**
What happens in experiments?

Conclusions (1)

- Progress in 3D disruptions simulations/validation have been acchieved
- Thermal quench (TQ) remains a big issue (simulation possible?)
- plasma conditions after TQ crucial in determining evolution
- role of transport and transport scaling after TQ also crucial
- resistive instabilities seem to play the main role in simulations

(2/1 mode and in some case 1/1 also)

- more data on relevant modes probably needed from experiments
- ITER simulations need to be completed at high resolution
- force calculations can be refined by using 3D electromag. wall codes