Slowing-down f_0 and collisions in GEM

- For alpha particles with distribution $f(\mathbf{x}, v, \lambda, t)$ ($\lambda = v_\parallel / v$) in the presence of a particle source, annihilation due to charge exchange, slowing down, and pitch-angle scattering:

$$\begin{aligned}\frac{D}{Dt}f &= \frac{\partial f}{\partial t} + \mathbf{V}_H \cdot \nabla f + \dot{v}_H \frac{\partial f}{\partial v} + \dot{\lambda}_H \frac{\partial f}{\partial \lambda} - \nu_d \frac{\partial}{\partial \lambda}(1-\lambda^2)\frac{\partial f}{\partial \lambda} - \frac{\nu}{v^2}\frac{\partial}{\partial v}[(v^3 + v_I^3) f] \\ &= S(\mathbf{x}, v, \lambda).\end{aligned}$$

- Alfvén waves are included in $\mathbf{V}_H$, $\dot{v}_H$ and $\dot{\lambda}_H$ by adding terms $\mathbf{V}_{H1}$, $\dot{v}_{H1}$ and $\dot{\lambda}_{H1}$—i.e.,

$$\begin{aligned}\mathbf{V}_H &= \mathbf{V}_{H0} + \mathbf{V}_{H1}, \\ \dot{v}_H &= \dot{v}_{H1}, \\ \dot{\lambda}_H &= \dot{\lambda}_{H0} + \dot{\lambda}_{H1}.\end{aligned}$$

  Since the zeroth-order Hamiltonian motion conserves energy, $\dot{v}_{H0} = 0$.

- Assume $f = f_0 + \delta f$,

$$\frac{D\delta f}{Dt} = -\mathbf{V}_{H1} \cdot \nabla f_0 - \dot{v}_{H1} \frac{\partial f_0}{\partial v} - \dot{\lambda}_{H1} \frac{\partial f_0}{\partial \lambda},$$

- Assuming particles are loaded according to the distribution g, the weight equation is

$$\dot{w} = \frac{1}{g}\left[-\mathbf{V}_{H1}\cdot\nabla f_0 - \dot{v}_{H1}\frac{\partial f_0}{\partial v} - \dot{\lambda}_{H1}\frac{\partial f_0}{\partial \lambda}\right].$$

- The distribution g satisfies

$$\frac{Dg}{Dt} = S(\mathbf{x}, v, \lambda)$$

Particles should be loaded such that $g(\mathbf{x}_j, \mathbf{v}_j, t)$ can be readily evaluated.

- Assume $\delta g/g \sim \delta f/f \sim \delta$ not too small, the approximation $g \approx g_0$ is equivalent to adding a $O(\delta^3)$ term in the gyrokinetic equation

$$\frac{\partial \delta f}{\partial t} + \cdots = 0$$

- If there are only pitch-angle scattering, no slowing down, $g \equiv \text{const}$ is an exact solution

$$\frac{\partial g}{\partial t} + \mathbf{V}_H \cdot \nabla g + \dot{v}_H \frac{\partial g}{\partial v} + \dot{\lambda}_H \frac{\partial g}{\partial \lambda} - \nu_d \frac{\partial}{\partial \lambda}(1-\lambda^2)\frac{\partial f}{\partial \lambda} = 0$$

Can $g \equiv \text{const} \equiv g_0$ be a rigorous solution by introducing an appropriate source term?

$$-\frac{\nu}{v^2}\frac{\partial}{\partial v}((v^3 + v_I^3)g_0) = -3\nu g_0$$

Particles need to be randomly selected and removed.

- There are other subtle difficulties near the phase space boundaries.

- If we choose not to change velocity and to evaluate the slowing-down term directly

$$\left(\frac{\partial \delta f}{\partial t}\right)_{\text{sl}} = \frac{\nu}{v^2}\frac{\partial}{\partial v}[(v^3 + v_I^3)\delta f]$$

then g can be a constant.

- With coarse-graining, δf is constructed on v-grids. How to reduce noise?

Load a Slowing-down Distribution

$$g = \frac{1}{C(v^3 + v_I^3)}, \qquad C = \frac{4\pi}{3} \ln \frac{v_B^3 + v_I^3}{v_{\min}^3 + v_I^3}$$

so that $\int g \, d\mathbf{x} \, d\mathbf{v} = V$

Collisions are implemented as ($\Gamma(t)$ Gaussian white noise)

$$\frac{dv}{dt} = \dot{v}_H - \nu \left(v + \frac{v_I^3}{v^2} \right)$$

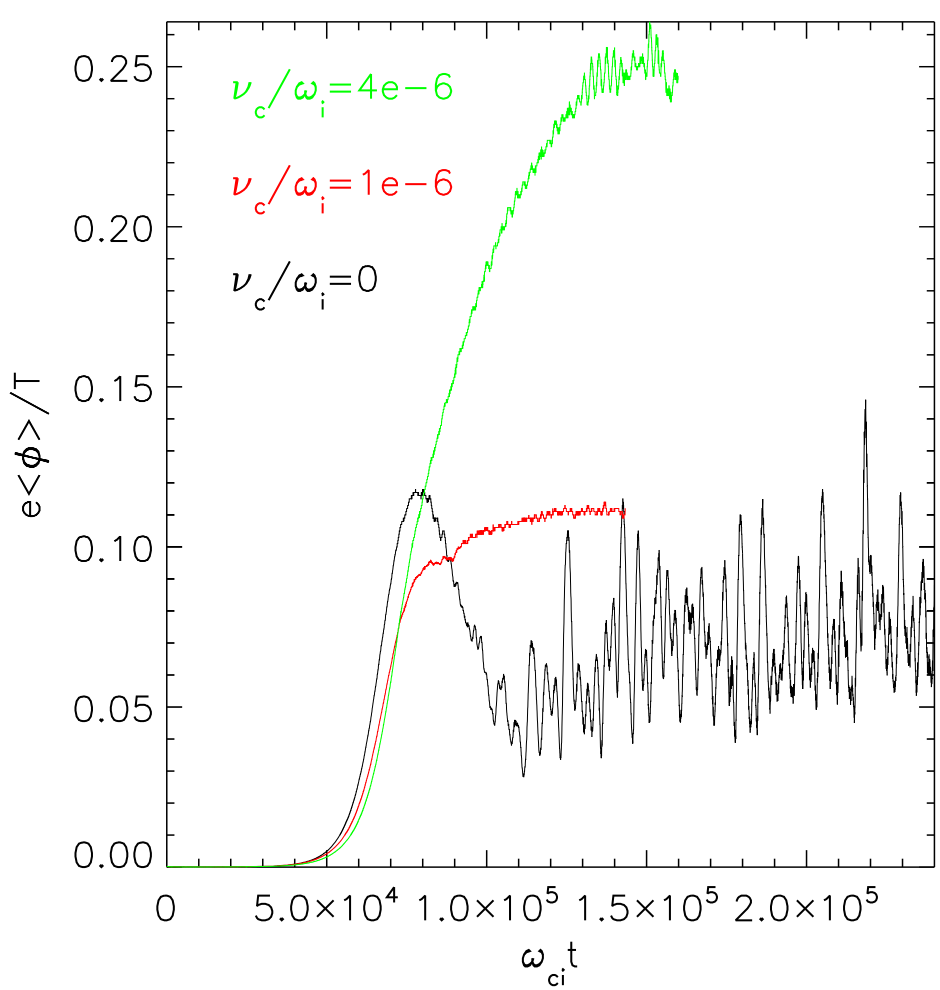
$$\frac{d\lambda}{dt} = \dot{\lambda}_H - 2\nu_d \lambda + \left[2\lambda_d (1 - \lambda^2) \right]^{1/2} \Gamma(t)$$

Equilibrium distribution

$$f_0 = \frac{n_h H(v_B - v)}{v^3 + v_I^3} \exp(- \langle \Psi_n \rangle / L_h)$$

for passing particles, $\langle \Psi_n \rangle \approx \Psi_n$

for trapped particles, $\langle \Psi_n \rangle \approx -R_0 P_\zeta / q$



$n_h = 0.015n_0$, $L_h = 0.5$, $v_B/v_T = 20$, $v_I/v_T = 5.8$.