Energetic-Particle-Induced Geodesic Acoustic Mode

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A new energetic particle-induced geodesic acoustic mode (EGAM) is shown to exist. The mode frequency and mode structure are determined nonperturbatively by energetic particle kinetic effects. In particular the EGAM frequency is found to be substantially lower than the standard GAM frequency. The radial mode width is determined by the energetic particle drift orbit width and can be fairly large for high energetic particle pressure and large safety factor. These results are consistent with the recent experimental observation of the beam-driven $n = 0$ mode in DIII-D.

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Understanding of energetic particle physics in tokamaks is of fundamental importance for burning plasmas. Recent experimental results [1–3] indicated energetic particles can drive an $n = 0$ mode unstable with frequency comparable to that of geodesic acoustic mode (GAM) [4]. Here it is shown analytically and numerically that energetic particles can indeed excite a new GAM-like mode via free energy associated with velocity space gradient in energetic particle distribution. More importantly, it is shown that both the mode frequency and mode radial structure is determined nonperturbatively by energetic particle kinetic effects. Thus, the new mode, to be called EGAM (for energetic-particle-induced GAM), is intrinsically an energetic particle mode. As such, EGAM is qualitatively different from the global geodesic acoustic mode (GGAM) [1,2] which is a pure MHD mode. EGAM also is qualitatively different from the usual GAM observed in the edge region of tokamak plasmas [5]. While the usual GAM is nonlinearly driven by plasma micro-turbulence and is highly localized near the edge of plasmas [5], the EGAM is linearly driven by energetic particles and is located in the tokamak core with a much wider radial width. It should also be noted that the kinetic effects of *thermal ions* on stable GAM had been studied in previous work [6–11]. In contrast, this work considers the *energetic particle* excitation of the new EGAM. The new mode is important since it can degrade energetic particle confinement as shown recently in the DIII-D experiments [3]. The new mode may also affect the thermal plasma confinement. Finally, the new mode could be excited in burning plasmas since auxiliary heatings such as neutral beam heating can produce highly anisotropic energetic particle distribution.

The equation for radial electric field E_r of $n = 0$ GAM-like mode is given by

$$\frac{\partial}{\partial t}\left\langle\frac{\rho|\nabla r|^2}{B^2}\right\rangle E_r = -\langle G(r,\theta)(\delta P_\parallel + \delta P_\perp)\rangle, \qquad (1)$$

where the bracket $\langle\rangle$ denotes the flux surface average and $G(r,\theta) = -(B_\phi R/JB^3)(\partial B/\partial\theta)$, ρ is plasma mass density, B is the equilibrium magnetic field strength, $\delta P_\parallel$ and $\delta P_\perp$ is the perturbed parallel and perpendicular pressure, respectively, B_ϕ is the toroidal magnetic field, R is the major radius, J is the Jacobian of the flux coordinates (r, θ, ϕ) with r being the radial flux variable (or minor radius). Equation (1) is derived from the flux surface average of the vorticity equation. For simplicity, the electrostatic approximation is assumed. The ideal Ohm's law is also used along with the Chew-Goldberger-Low form of the perturbed stress tensor $\delta\mathbf{P} = \delta P_\perp \mathbf{I} + (\delta P_\parallel - \delta P_\perp)\mathbf{bb}$.

A hybrid model is used for the stress tensor response, namely, fluid model for thermal plasmas and kinetic model for energetic particles. Thus for thermal plasmas, $\delta P_{\perp\mathrm{th}} = \delta P_{\parallel\mathrm{th}} = \delta P_{\mathrm{th}}$ and

$$\frac{\partial}{\partial t}\delta P_{\mathrm{th}} = 2\gamma G(r,\theta) P_{\mathrm{th}} E_r, \qquad (2)$$

with the subscript th denotes the thermal species and γ is the coefficient of specific heat. On the other hand, the perturbed energetic pressures are calculated kinetically as

$$\delta P_{\parallel h} + \delta P_{\perp h} = \int d^3v\left(mv_\parallel^2 + \frac{1}{2}mv_\perp^2\right)\delta f \qquad (3)$$

where the subscript h denotes the energetic particle species. The perturbed particle distribution δf satisfies the following drift kinetic equation:

$$\frac{d\delta f}{dt} = -\frac{dE}{dt}\frac{\partial f}{\partial E} = -\left(mv_\parallel^2 + \frac{1}{2}mv_\perp^2\right)\frac{\partial f}{\partial E} G(r,\theta)E_r, \qquad (4)$$

where $f = f(P_\phi, E, \mu)$ is the equilibrium distribution as function of constant of motion P_ϕ the toroidal angular momentum, E the particle energy, and μ the magnetic moment. Note that there is only dE/dt term in Eq. (4) because $dP_\phi/dt = 0$ for axisymmetric modes.

Equations (1)–(4) form a self-consistent kinetic-fluid hybrid model for EGAM including kinetic efforts of energetic particles.

The drift kinetic equation can be solved by integrating along the unperturbed orbit and the solution is given by

$$\delta f = -E\frac{\partial f}{\partial E}\sum_p \frac{H_p^\sigma(\bar{r})\exp[-i\omega t + ip\omega_b t^\sigma(\theta)]}{\omega - p\omega_b} \tag{5}$$

with

$$H_p^\sigma(\bar{r}) = \frac{1}{\tau_b}\int dt' K(\Lambda, r', \theta')E_r(r'), \tag{6}$$

$$K(\Lambda, r', \theta') = (2 - \Lambda B)G(r', \theta')\sin(p\omega_b t'), \tag{7}$$

where $\bar{r}$ is the particle orbit average of r, τ_b is the particle orbit period (transit period for passing particles and bounce period for trapped particles), $\omega_b = 2\pi/\tau_b$ is the orbit frequency, $\Lambda = \mu B_0/E$ is the particle pitch angle with B_0 being the magnetic field strength at the magnetic axis, σ is the sign of parallel velocity, p is an integer representing orbit harmonics, and t^σ is the orbit time integral given by $t^\sigma(\theta) = \sigma\int(J/v_\parallel)d\theta$ for passing particles.

By combining Eqs. (1)–(5), the following integral eigenequation is obtained for $n = 0$ electrostatic modes with effects of energetic particles:

$$\omega^2\left\langle\frac{\rho|\nabla r|^2}{B^2}\right\rangle E_r = \langle 4\gamma G^2(r,\theta)P_{\rm th}\rangle E_r - \left\langle\sum_{p,\sigma}\int d^3v E^2\frac{\partial f}{\partial E}\frac{2\omega^2 H_p^\sigma(\bar{r})}{\omega^2 - p^2\omega_b^2}K(\Lambda, r, \theta)\right\rangle, \tag{8}$$

where p sums over positive integer. Note that because of orbit integral in Eq. (6), Eq. (8) is an integral equation which couples different flux surfaces due to finite orbit width of energetic particles.

In the limit of zero orbit width for energetic particles, the integral mode equation becomes the local dispersion relation defined on each flux surface as

$$\omega^2 = \omega_{\rm GAM}^2 + \frac{\langle P_{\parallel h} + P_{\perp h}\rangle}{B_0^2R_0^2\langle\rho|\nabla r|^2/B^2\rangle}Q_h\left(\frac{\omega}{\omega_{b0}}\right), \tag{9}$$

where

$$\omega_{\rm GAM}^2 = \frac{\langle 4\gamma G^2(r,\theta)P_{\rm th}\rangle}{\langle\rho|\nabla r|^2/B^2\rangle}, \tag{10}$$

$$Q_h = -\sum_{p,\sigma}\frac{2\int\frac{\partial f}{\partial E}E^3 dEd\Lambda\tau_b(\bar{H}_p^\sigma)^2\frac{\omega^2}{\omega^2 - p^2\omega_b^2}}{\int fE^2dEd\Lambda\tau_b\langle 2 - \Lambda B\rangle}, \tag{11}$$

$$\bar{H}_p^\sigma = \frac{1}{\tau_b}\int dt'(2 - \Lambda B)B_0R_0G(r, \theta')\sin(p\omega_b t'), \tag{12}$$

where Q_h is an order of unit kinetic integral depending only on ω/ω_{b0} and details of energetic particle distribution function. Here $\omega_{b0} \sim v/(qR)$ is the orbit frequency at $\Lambda = 0$. For large aspect ration low beta tokamak equilibria with circular flux surfaces, $J \sim rR$, $|\nabla r| \sim 1$, $G(r, \theta) \sim -\sin(\theta)/(B_0R_0)$. Then the GAM frequency given by Eq. (10) reduces to the well-known formula $\omega_{\rm GAM} = \sqrt{2\gamma P_{\rm th}/\rho}/R_0$ for $q \gg 1$. Also in this limit, the $p = 1$ term is dominant over other terms.

For energetic particles, ω_{b0} is typically comparable or larger than $\omega_{\rm GAE}$. This allows wave particle resonances. Depending on details of particle distribution function, the real part of Q_h can be either positive or negative for energetic particles. Thus, energetic particles can often contribute negatively to the mode frequency. Also, more than one mode can exist. Up to three modes have been found. Finally, for an anisotropic distribution, energetic particles can destabilize the GAM mode due to $\partial f/\partial E > 0$. This has been demonstrated by numerical solution of the dispersion relation as shown below.

The following anisotropic slowing-down distribution function is used for energetic particles in this study:

$$f = \frac{1}{v^3 + v_{\rm crit}^3}\exp\left[-\frac{P_\phi}{e\Delta\Psi} - \left(\frac{\Lambda - \Lambda_0}{\Delta\Lambda}\right)^2\right], \tag{13}$$

where f has peaked distribution in P_ϕ with $\Delta\Psi$ being the width, a slowing-down distribution in velocity and a peaked distribution in the pitch angle Λ with $\Delta\Lambda$ being the width. Figure 1 shows the real part (top) and imaginary part (bottom) of Q_h for an anisotropic pitch angle distribution function with $\Delta\Lambda = 0.2$ and $\Lambda_0 = 0.5$. The results are obtained at $\epsilon = r/R = 0.16$ of an tokamak equilibrium with circular flux surfaces at $R/a = 3$. The results show $\mathrm{Re}(Q_h) < 0$ for $0.23 < \omega/\omega_{b0} < 0.72$. Thus in this frequency region, the energetic particle effects actually reduce the mode frequency. Furthermore, $\mathrm{Im}(Q_h) > 0$ for

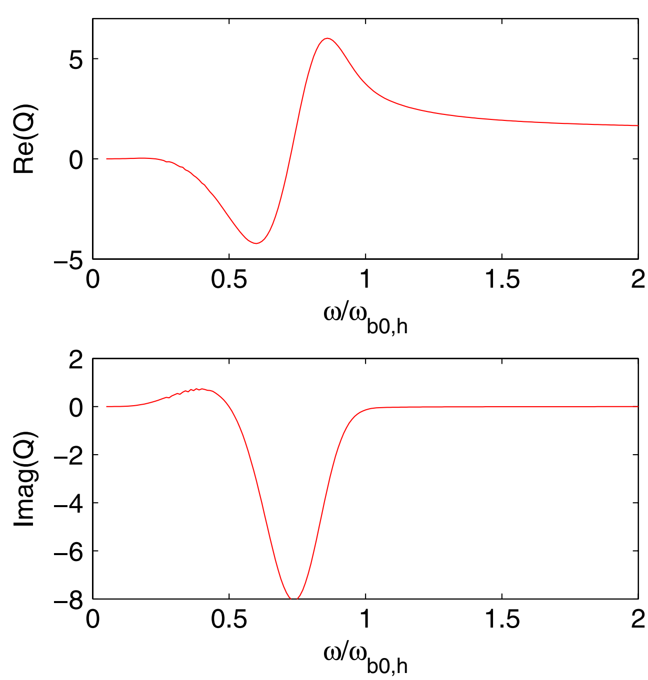

FIG. 1 (color online). Real (top) and imaginary (bottom) part of Q as function of mode frequency.

$0.0 < \omega/\omega_{b0} < 0.49$ for this case. Thus, the energetic particles can destabilize the EGAM due to anisotropic distribution (i.e., driven by $\partial f/\partial E > 0$). The dispersion relation can be solved perturbatively when the fast ion pressure is much smaller than the thermal pressure. When the energetic particle pressure is comparable to the thermal pressure, the dispersion relation has to be solved nonperturbatively. The eigenfrequencies can be expressed in terms of two dimensionless parameters: $Z = \omega_{b0}/\omega_{\mathrm{GAM}}$ being the ratio of particle orbit frequency and the GAM frequency, and $Y = \langle P_{\parallel h} + P_{\perp h}\rangle/2\gamma P_{\mathrm{th}}$ being the ratio of energetic particle pressure and thermal plasma pressure. Figure 2 plots the real part (top) and imaginary part (bottom) of eigenmode frequencies as a function Y at $Z = 1.0$ for the parameters of Fig. 1. The corresponding results for $Z = 1.8$ are plotted in Fig. 3. It is observed that, for small energetic particle pressure (i.e., small Y), only one mode (red squares) can exist and this mode transits smoothly to the conventional GAM as Y approaches zero. Furthermore, these results show that there is a critical energetic pressure fraction above which two new modes emerge (black diamonds and blue circles). The mode frequencies and the stability of the new modes depend critically on the value of Z. In fact, it is found analytically and numerically that there exists a critical threshold $Z = Z_{\mathrm{crit}}$ ($Z_{\mathrm{crit}} = 1.4$ for parameters of Fig. 1) such that for $Z < Z_{\mathrm{crit}}$, the new modes start at the mode frequency well below the GAM frequency ω_{GAM}(see black diamonds and blue circles in Fig. 2). On the other hand, for $Z > Z_{\mathrm{crit}}$, the new modes start at the mode frequency well above the GAM frequency (see Fig. 3). Interestingly, the value of Z is also critically important for the mode which starts from the conventional GAM frequency (red squares in Figs. 2 and 3). The mode frequency of this mode increases with increasing Y for $Z < Z_{\mathrm{crit}}$ (see Fig. 2). For $Z > Z_{\mathrm{crit}}$, the mode frequency decreases with increasing Y (see Fig. 3). Finally, it is found that there exists only one unstable mode for the parameters of Fig. 1. For $Z < Z_{\mathrm{crit}}$, one of the new modes is unstable (blue circles) when energetic particle pressure exceeds a threshold (i.e., $\omega_i > 0$ for $Y > 0.26$ in Fig. 2). For $Z > Z_{\mathrm{crit}}$, the GAM mode can be destabilized by the energetic particles (see red squares in Fig. 3).

It is important to relate these local results to the experimental situation. First, the importance of the parameters Z has been shown above. It can be estimated as $Z^2 \sim E_h/(2.75q^2T)$ where E_h is the energetic particle energy, q is the safety factor, and T is the plasma temperature. For typical parameters of neutral beam-heated reversed shear plasmas, Z is comparable to Z_{crit}. Second, the unstable mode's frequency is always below the GAM frequency regardless of the value of Z. When the energetic particle pressure is comparable to the thermal pressure (i.e., $Y \sim 1$), the unstable mode frequency is substantially below GAM frequency and the growth rate can be very large. These results are consistent with the recent experimental observations in DIII-D [3].

Now consider the case where effects of finite orbit width of energetic particles are retained. In general, the integral equation given by Eq. (8) can only be solved numerically. To make analytic progress, the particle orbit width is assumed to be much smaller than the mode radial width. In this limit, a Taylor series expansion can be made for

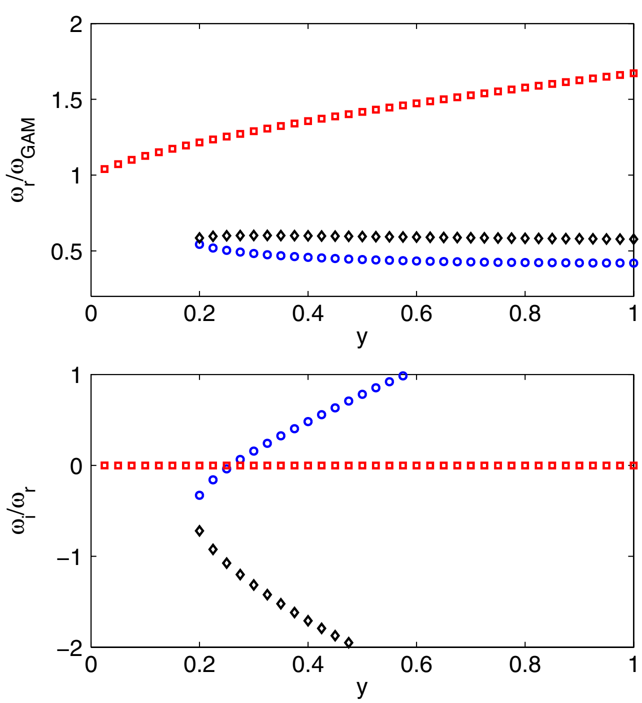

FIG. 2 (color online). Real and imaginary part of the eigenfrequencies as function of $Y = \langle P_{\parallel h} + P_{\perp h}\rangle/2\gamma P_{\mathrm{th}}$ for $Z = 1.0$.

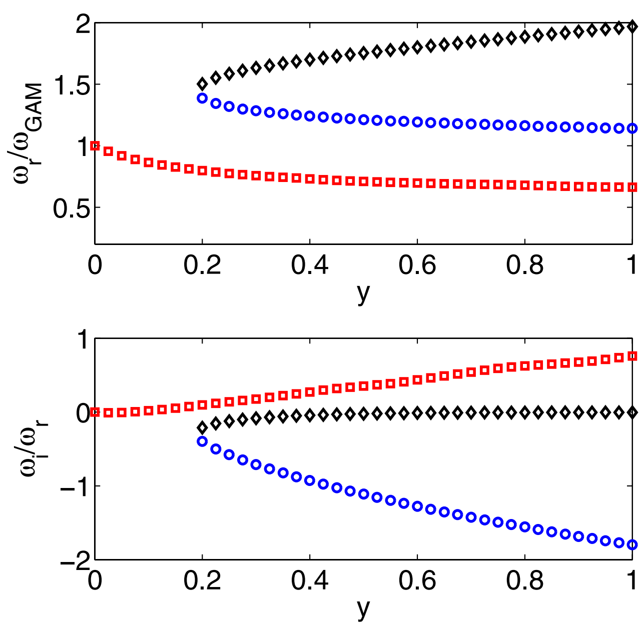

FIG. 3 (color online). Real and imaginary part of the eigenfrequencies as function of $Y = \langle P_{\parallel h} + P_{\perp h}\rangle/2\gamma P_{\mathrm{th}}$ for $Z = 1.8$.

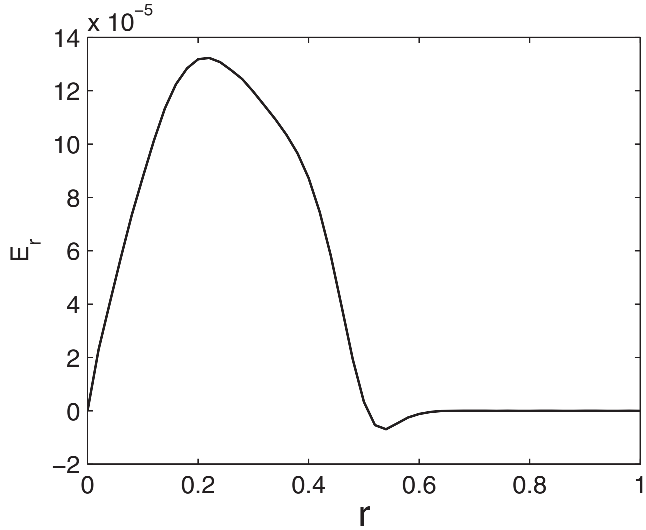

FIG. 4. The perturbed radial electric field versus radius of a global EGAM.

$E_r(r')$ in Eq. (6) as $E_r(r') = E_r(\bar{r}) + (\partial E_r/\partial \bar{r})\delta r + 0.5(\partial^2 E_r/\partial \bar{r}^2)(\delta r)^2$ where $r' = \bar{r} + \delta r$ with $\bar{r}$ being the orbit-averaged radius and δr being the drift orbit deviation from $\bar{r}$. Then, another Taylor expansion can be made in Eq. (8) using $\bar{r} = r - \delta r$. Note that the flux average in Eq. (8) is to be performed at fixed r. After some algebra, the following eigenmode equation is obtained:

$$\frac{d}{dr}\left[\frac{\langle P_{\parallel h} + P_{\perp h}\rangle}{\rho R^2}(q\rho_h)^2 W\left(\frac{\omega}{\omega_{b0}}\right)\right]\frac{d}{dr}E_r = -(\omega^2 - \omega_{\mathrm{EGAM}}^2)E_r \tag{14}$$

where ω_{EGAM}^2 is given by the right-hand side of Eq. (9), ρ_h is the Larmor radius of the energetic particles, W is an order of unity kinetic integral and is a function of ω/ω_{b0}. Note that $q\rho_h$ is approximately the drift orbit width of the energetic particles. Depending on details of particle distribution, W can be either positive or negative. For $W > 0$, the mode propagates in the region of $\omega > \omega_{\mathrm{EGAM}}$ and for $W < 0$, the mode propagates in the region of $\omega < \omega_{\mathrm{EGAM}}$. Near $\omega^2 - \omega_{\mathrm{EGAM}}^2 = 0$, the solution of Eq. (14) is given by Airy function. Thus, the radial width of EGAM scales as

$$\Delta r \sim (\beta_h/\beta_{\mathrm{th}})^{1/3} L_\omega^{1/3}(q\rho_h)^{2/3}, \tag{15}$$

where L_ω is the radial scale length of ω_{EGAM}. Note that the radial orbit width is mainly determined by the drift orbit width $\sim q\rho_h$ of the energetic particles and can be fairly large for DIII-D reversed shear plasmas.

Analysis of Eq. (14) indicates the existence of a global EGAM with radial scale length given by Eq. (15). Hybrid simulations have been carried out using Eqs. (1)–(4). In the simulations, the perturbed thermal pressure is evolved using Eq. (2) and the perturbed energetic particle pressures are evolved according to Eqs. (3) and (4) using PIC simulation method. The parameters of Fig. 1 are used along with central safety factor $q(0) = 5.0$, the minimum $q = q_{\mathrm{min}} = 4.0$ at $r/a = 0.4$, plasma pressure profile $P_{\mathrm{th}} = P_{\mathrm{th}}(0)(1 - \Psi)^2$ with Ψ being the normalized poloidal flux. The plasma density profile is uniform. For parameters of energetic particles, $\rho_h/a = 0.016$, $Z = \omega_{b0}/\omega_{\mathrm{GAE}}(0) = 1.42$, $Y \sim 1$, and $\Delta\Psi = 0.3$. Only counter passing particles are included. Figure 4 shows the eigenmode structure of the perturbed radial electric field of an unstable global EGAM obtained from the linear simulation. The calculated mode frequency and growth rate are $\omega_r/\omega_{\mathrm{GAE}}(0) = 0.63$ and $\omega_i/\omega_r = 0.5$. The calculated mode is clearly global. This result confirms the existence of global EGAMs. Details of simulation results will be presented elsewhere.

In conclusion, a new energetic particle-induced geodesic acoustic mode (EGAM) is shown to exist. The mode is driven by velocity space anisotropy in the energetic particle distribution function. The mode frequency and mode structure are determined nonperturbatively by energetic particle kinetic effects. In particular it is found that the EGAM frequency is substantially lower than the standard GAM frequency. The radial mode width is mainly determined by the energetic particle drift orbit width and can be fairly large for high energetic particle pressure and large safety factor. These results are consistent with the recent experimental observation of the beam-driven $n = 0$ mode in DIII-D [3].

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