PPPL

THEORY REVIEW

CRITICAL ISSUES IN THE MODELING OF MAJOR DISRUPTIONS IN TOKAMAKS

Report by the Study Group on "Plasma-wall boundary conditions for MHD simulations of disruption events"

Study group members:

A. Boozer (Chair), J. Breslau, E. Fredrickson, D. Stotler

PPPL Department of Theory and Computation

March 21, 2011

PPPL THEORY COMPUTATION

TABLE OF CONTENTS

Charge letter 2

Summary 3

Abstract 6

Introduction 7

Tokamak disruption reviews 10

Published disruption simulations 11

Electromagnetic boundary conditions in the halo 15

Boundary conditions on the halo plasma 17

Disruption issues other than boundary conditions 19

Conclusions 23

Appendix A: Magnetic islands in non-axisymmetric simulations... 24

Appendix B: Sonic flow conditions 29

Appendix C: Plasma-wall sheath 30

Appendix D: Halo model 33

References 35

The “BC” Study Group

TITLE: Plasma-wall boundary conditions (BC) for MHD simulations of disruption events

MEMBERS: A. Boozer (Chair), J. Breslau, D. Stotler, E. Fredrickson

BACKGROUND: Prior to the thermal quench, the plasma column is hot enough that it can be approximated as an ideal plasma within closed magnetic field lines. The last closed plasma surface is surrounded by an open field line region filled with a low-density and relatively-cold plasma (the halo). The MHD codes that are currently used to simulate disruptions are based on a fluid description for both regions. The resistivity and density vary by several orders of magnitude between the quasi-ideal plasma and the surrounding halo. The reduction in the toroidal magnetic flux enclosed by the plasma boundary during a disruption can induce wall currents producing dangerously large forces in the surrounding structures. Wall currents can also arise from sheath effects at the plasma-wall interface and from the currents associated with a non-axisymmetric kinking of the plasma. ***The issue to be addressed and resolved by the BC Study Group concerns the boundary conditions at the plasma-wall interface and how such boundary conditions affect the interaction of the main plasma with the wall and the resulting forces in the wall during a disruption.***

QUESTIONS TO BE ADDRESSED:

[1] Are the boundary conditions used in the present MHD codes appropriate for calculating the dominant interactions of the main plasma with the wall and the resulting forces in the resistive wall during the current quench phase of a disrupting plasma?

[2] Can these boundary conditions be improved to enhance the accuracy of the calculation of the forces? If so, how?

PROCESS, DELIVERABLES and SCHEDULE: The content and terms in this Charge Letter have been agreed upon by the Theory Department Head and the Study Group Chair. The Study Group will submit a report to the Science Committee (SC) by March 31st 2011. The Study Group will provide monthly briefings on the progress made at the meetings of the SC. The report will be edited by an editor appointed by the SC and published on the PPPL Theory Review. <u>A section of the report will summarize the findings and recommendations.</u>

R. Betti	Prof. Allen Boozer
Head, Department of Theory and Computation	BC Study Group Chair
Princeton Plasma Physics Laboratory	PPPL and Columbia University

Additional input on the background and relevant questions has been provided by S. Jardin and L. Zakharov, and is included as an attachment to this charge letter

SUMMARY

In September 2010, Riccardo Betti appointed this committee to answer two questions. The questions and their answers are:

(1) Are the boundary conditions used in the present MHD codes appropriate for calculating the dominant interactions of the main plasma with the wall and the resulting forces in the resistive wall during the current quench phase of a disrupting plasma?

The published simulations of disruptions, TSC[1] [2], DINA [3], and M3D [4] [5], make the vessel surrounding the plasma axisymmetric and prescribe a state for the halo plasma rather than using boundary conditions to find that state. The halo plasma is the part of the plasma that is in direct contact with the wall. In experiments, large forces on wall structures arise from halo currents, which are currents that flow for part of their path through the halo plasma and for part through the wall. The two most important parameters for determining the total halo current are the resistance and the width of the halo.

The boundary conditions used in the TSC, DINA, and M3D simulations are appropriate for obtaining an estimate of the maximum of the total force exerted on the wall by the halo current under certain approximations, such as axisymmetry in the TSC and DINA codes, by varying assumed values for the resistance and width of the halo.

The boundary conditions used in existing simulations are not appropriate for studying a number of important properties of the forces during disruptions. The localization and duration of the forces, which are critical for an assessment of the potential for damage, are dependent on a number of factors beyond just the halo resistance and width. The use of more physical boundary conditions would provide greater understanding and, consequently, increased confidence in extrapolations of these simulation models from present devices to ITER.

The total force that a disruption can exert on a wall has an obvious upper bound, $F = f_d \pi a_v I_p B$, where the poloidal circumference of vessel is $2\pi a_v$, the net plasma current is I_p, the magnetic field strength is B, and f_p is a coefficient, which empirically has a value of a few tenths but is certainly not a universal constant. The resistance and the width of the halo plasma can be varied in existing simulations to obtain information on f_p beyond that obtained from analytic estimates. Features not represented in existing simulations, such

as the complicated non-axisymmetric geometry of the structures surrounding the plasma, could change f_p by a factor of two or more.

The assumption in existing simulations that the plasma cannot flow into the wall, $\vec{v} \cdot \hat{n} = 0$, is unphysical. Since existing codes assume, rather than calculate, the properties of the halo, the impact of this boundary condition on current simulations is limited to essentially the inertia of the halo plasma, which is negligible in the overall simulation.

(2) Can these boundary conditions be improved to enhance the accuracy of the calculation of the forces? If so, how?

Yes, appropriate plasma boundary conditions could be imposed, which would clarify the physics issues that determine the forces, their concentration, and their duration. The electrostatic sheath at the plasma-wall interface is a major determinant of the plasma boundary conditions and could limit the current density to the ion saturation current enC_s, where C_s is the speed of sound. This would provide an important limit on the concentration of forces. The complicated geometry of actual walls affects both the plasma and the magnetic boundary conditions and should be represented to obtain an accurate simulation. A detailed discussion of physical boundary conditions is contained in a longer report.

The disruption simulations that were considered in the answers to the two questions are the TSC[1] [2], DINA [3], and M3D [4] [5] simulations. The NIMROD simulations of disruptions [6] [7] [8] neither include the plasma boundary conditions imposed by the sheath, nor do they have an appropriate boundary condition on the magnetic field. The NIMROD simulations assume a perfectly conducting wall, which effectively rules out the halo currents that are responsible for important forces.

Both the TSC and the DINA simulations assume axisymmetry, but non-axisymmetry is an important element in the forces exerted by disrupting plasmas as emphasized by Zakharov [9].

The empirical time scales for disruptions span the range form hundreds to hundreds-of-thousands of shear Alfvén times, R/V_A, so disrupting plasmas remain close to force balance, $\vec{\nabla} p = \vec{j} \times \vec{B}$, as is assumed in TSC and DINA. The published M3D simulations were fully non-axisymmetric but were limited by their numerical procedure to a modest number of shear Alfvén times. Future simulations using M3D-C^1 and the massively parallel version of M3D are not expected to have this limitation.

[1] R. O. Sayer, Y.-K. M. Peng, S. C. Jardin, A. G. Kellman, and J. C. Wesley, Nucl. Fusion **33**, 969 (1993).

[2] I. Bandyopadhyay, S. Gerhardt, S. C. Jardin, R. O. Sayer, Y. Nakamura, S. Miyamoto, G. Pautasso, and M. Sugihara, and the ASDEX Upgrade and NSTX Teams, *TSC modelling of major disruption and VDE events in NSTX and ASDEX-Upgrade and predictions for ITER,* poster ITR/P1-16 at the IAEA Fusion Energy Conference, Daejon, Korea, October 2010.

[3] M. Sugihara, M. Shimada, H. Fujieda, Y. Gribov, K. Ioki, Y. Kawano, R. Khayrutdinov, V. Lukash, and J. Ohmori, Nucl. Fusion **47**, 337 (2007).

[4] H. R. Strauss, R. Paccagnella, and J. Breslau, Phys. Plasmas **17**, 082505 (2010).

[5] R. Paccagnella, H. R. Strauss, and J. Breslau, Nucl. Fusion **49**, 035003 (2009).

[6] S. E. Kruger, D. D. Schnack, and C. R. Sovinec, Phys. Plasmas **12**, 056113 (2005).

[7] V.A. Izzo, Nucl. Fusion **46** 541 (2006).

[8] V.A. Izzo and P.B. Parks, Nucl. Fusion **50**, 058001(2010).

[9] L. E. Zakharov, Phys. Plasmas 15, 062507 (2008).

(Dated: March 21, 2011)

Abstract

Disruptions are considered a major obstacle to the use of tokamaks for fusion power; knowledge is required to avoid disruptions and to mitigate their damage. Phenomena associated with disruptions in large tokamaks take place on time scales between a hundreds and hundreds-of- thousands of shear Alfvén times, R/V_A. Inertial effects appear to play no large-scale role, and the plasma can be taken to be in equilibrium force balance, $\vec{j}\times\vec{B} \simeq \vec{\nabla}p$, though much effort has been spent on the inclusion of inertial effects in simulations. No codes exist that can accurately and efficiently solve force balance while addressing the critical disruption issues, nor is there an ongoing program focused on developing such a code. Critical simulation issues include: (1) appropriate boundary conditions between the plasma on open field lines and the walls, (2) the capability of calculating force balance when a non-axisymmetric current is flowing in the halo that contains tens of percent of the plasma current, and (3) a realistic evolution of the breaking and the healing of magnetic surfaces.

PACS numbers:

I. INTRODUCTION

The importance of avoiding disruptions in tokamaks and mitigating the damage of any that occur is accepted in the fusion community [1–3]. When disrupting plasmas strike the structures that surround the plasma, large forces are exerted by currents flowing along the open field lines of the plasma halo, as well as large heat loads. The importance of disruptions implies the need for simulations to develop the required base of knowledge, and a number of disruption simulations have been carried out [4–12]. Section III discusses the assumptions used in these simulations. The physical relevance of some of some of these disruption simulations has been brought into question, most notably by Leonid Zakharov's concerns about the boundary conditions [13].

The head of the theory department at the Princeton Plasma Physics Laboratory, Riccardo Betti, appointed a committee chaired by Prof. Allen Boozer to report on the appropriateness of the boundary conditions used in disruptions simulations. That was the stimulus for the description of existing simulations and the elements required in disruption simulations that is given here.

In experiments, large forces on wall structures arise from halo currents, which are currents that flow for part of their path through the halo plasma, the part of the plasma in direct contact with the wall, and for part through the wall. The two most important parameters for determining the total halo current are the resistance and the width of the halo. The total force that a disruption can exert on a wall has an obvious upper bound, $F = f_d \pi a_v I_p B$, where the poloidal circumference of vessel is $2\pi a_v$, the net plasma current is I_p, the magnetic field strength is B, and f_p is a coefficient, which empirically has a value of a few tenths but is certainly not a universal constant. See Figure (42) in [2].

The published simulations of disruptions, TSC [4, 5], DINA [6], and M3D [7, 8], make the vessel surrounding the plasma axisymmetric and prescribe a state for the halo plasma rather than using boundary conditions to find that state. The plasma boundary conditions imposed in the codes are not physical. Though not explicit in the published work, the resistance and the width of the halo plasma could be varied in TSC and DINA simulations to obtain information on f_p beyond that obtained from analytic estimates. Features not represented in existing simulations, such as the complicated non-axisymmetric geometry of the structures surrounding the plasma, could change f_p by a factor of two or more.

The boundary conditions used in existing simulations are not appropriate for studying a number of important properties of the forces during disruptions. The localization and duration of the forces, which are critical for an assessment of the potential for damage, are dependent on a number of factors beyond the halo resistance and width.

Physical boundary conditions could be imposed on the plasma, which would clarify the physics issues that determine the thermal loads and mechanical forces associated with disruptions. These boundary conditions are in large part determined by the electrostatic sheath that lies at the interface of the halo plasma with the surrounding structures. Section V discusses the physical boundary conditions that determine the halo density, temperature, current density and effective electrical resistivity.

The magnetic field boundary condition in existing simulations is that of a thin axisymmetric wall. A resistive wall is required to represent a plasma driven into a wall in a vertical displacement event, but in some disruption studies [9–11] the wall is taken to have zero resistivity. Actual walls have a complicated geometry, which should be represented to obtain accurate results. Section IV considers the boundary conditions on the magnetic field lines at conducting structures.

Major non-axisymmetric disruption simulations [7–11] have used codes designed to resolve effects on the shear Alfvén time scale, R/V_A. However, phenomena associated with disruptions in large tokamaks take place on time scales between hundreds and hundreds-of-thousands of shear Alfvén times, R/V_A, and inertial effects appear to play no large-scale role, cf. Section II.

An explanation for the slow evolution compared to the shear Alfvén time has been given by Zakharov [12]. Consider a path that is formed by a magnetic field line just outside the main plasma that is closed through the wall structure that the field line contacts. The evolution of the magnetic flux, $\sim 1 Tesla \cdot meters^2$, enclosed by the path implies a loop voltage, which is typically of order $10^6 V$ in a large tokamak if the time scale for the evolution is of order the shear Alfvén time scale, $R/V_A \sim 1\mu s$ as it would be, for example, in a strongly unstable kink. A far smaller loop voltage would cause a current to flow in the plasma and in the wall structures along that path. In analogy to a resistive wall mode, this current slows the evolution to a rate determined by the the resistance along the current path.

When the plasma evolution is slow compared to inertial times scales, as appears to be the case in disruptions, equilibrium force balance, $\vec{j} \times \vec{B} \simeq \vec{\nabla} p$, holds. Stellarator equilibrium codes, modified to include a halo current, cf. Appendix D, could be used in disruption simulations. Stated differently, any code that is able to accurately simulate strongly non-axisymmetric disruptions in tokamaks could accurately calculate stellarator equilibria. The theoretical and empirical knowledge of stellarator equilibria provide an important basis for verifying and validating codes for non-axisymmetric disruption simulations. Section VI discusses non-boundary conditions issues, such as the plasma force balance, that are important to disruption simulations.

An important issue in non-axisymmetric simulations, either for tokamaks or stellarators, is the breaking and healing of magnetic surfaces, which involves the shielding by internal plasma currents of the magnetic fields associated with surface breaking. The physics is discussed in Appendix A. The breaking of magnetic surfaces on a very short time scale compared to the global skin time of the plasma, $\tau_{skin} \equiv a^2\mu_0/\eta$, involves current sheets that have a complicated spatial structure and a width that is probably set by kinetic effects as shown in codes written for reconnection in space plasmas. The opening of islands is part of the plasma evolution and is difficult to follow in a realistic manner as an integral part of the force-balance calculation. Much more efficient and accurate disruption simulations should be possible if islands that contain a specified toroidal magnetic flux were included in the calculation of force balance, which would allow a separation of the equilibrium and the evolution.

The restoration of magnetic surfaces is a critical element in the formation of a dangerous number of runaway electrons [2]. The restoration of magnetic surfaces is also a transport problem, which can have a very different time scale from their destruction.

Disruptions frequently follow if the plasma rotation stops. The time interval between the stopping of the plasma rotation and the thermal collapse of the plasma may be due to the growth of islands by the Rutherford mechanism, which means without thin current sheets and with no effects from the plasma inertia. An estimate of this time is given in Equation (A8).

The body of the paper starts with Section II, which gives basic information about disruptions from well known reviews and ends with Section **??**, which is a summary and a discussion of possible simulations that would enhance the knowledge of tokamak disruptions. The appendices at the end of the paper derive basic physics results that are needed to understand the paper.

II. TOKAMAK DISRUPTION REVIEWS

The best known reviews of the physics of tokamak disruptions are two articles in *Nuclear Fusion* that are parts of publications on the physics basis of ITER: Section 4 of [1] and Section 3 of [2]. Another well know discussion of disruptions is in Sections 7.7 to 7.9 as well as Section 7.12 of the 2004 edition of *Tokamaks* by Wesson [3].

Four features of disrupting plasmas in these reviews are of particular importance for defining the requirements of disruption simulations:

1. The time scales for disruption effects are long compared to the inertial time, the shear Alfvén time $\tau_A \equiv R/V_A \sim 1\mu sec$, so force balance, $\vec{\nabla}p = \vec{j}\times\vec{B}$, should be an accurate approximation. The fastest time scale in ITER is expected to be the thermal quench of $\sim 700\mu sec$. The overall disruption time scale is expected to be $100's$ of ms.

2. The forces on wall structures are dominated by currents in a halo plasma, which is the plasma on magnetic field lines that intercept wall structures. The current in the halo plasma can reach several tenths of the original plasma current and is not axisymmetric.

3. Disruptions in diverted tokamaks generally involve both a thermal quench and a vertical movement of the plasma into the wall. The thermal quench and the vertical displacement event (VDE) can occur in either order.

4. When the bulk plasma temperature is at least a few tens of electron volts, the characteristic time scale for magnetic field decay in the bulk plasma is long compared to the time for the magnetic field to penetrate the surrounding conducting structures, which sets a time scale for the overall disruption.

The characteristic time scale for magnetic field decay in the bulk plasma is $\tau_{skin} \equiv a^2\mu_0/\eta \approx 45sec\ a^2T^{3/2}/Z_{eff}$, where T is the temperature in keV, a the radius in meters, and Z_{eff} the effective charge state of the plasma. J. Bialek in an unpublished calculation has found that the time for a horizontal field to penetrate the ITER wall is $\tau_w = 0.344s$. The plasma radius $a = 2m$, so $\tau_{skin} > \tau_w$ if $T \gtrsim Z_{eff}^{2/3}15eV$. It should be noted that the time scale for the rearrangement of the q profile is proportional to τ_{skin} but somewhat shorter [15].

The destructive properties of the forces produced by the halo currents are determined by their strength, their spatial concentration, which is in part determined by the width of the halo plasma, and their temporal duration. The impulse, which is the integral of the force over time, can be more important than the force itself if the inertia of the wall structures is large.

III. PUBLISHED DISRUPTION SIMULATIONS

Three types disruption simulations have been carried out: axisymmetric, non-axisymmetric, and Zakharov's calculations, which are dominated by plasma kinking.

A. Axisymmetric simulations

Axisymmetric simulations of disruptions have been carried out using the Tokamak Simulation Code (TSC) [4], [5] and the DINA code [6]. These simulations are carried out assuming force balance, $\vec{\nabla}p = \vec{j} \times \vec{B}$, which appears valid.

The need for boundary conditions on the halo plasma is largely circumvented in these simulations by assuming a priori values for the halo properties that dominate the simulation results. These properties are the halo resistance and the halo width. The halo width is set by assuming it contains a fixed fraction of the initial poloidal magnetic flux. The halo resistance and width are adjusted until the simulation results appear similar to experiments, but it is unclear how many features of disruptions can be consistently represented.

The boundary conditions on the magnetic field normal to conducting structures, Sec. IV B, are correctly calculated in the TSC and DINA simulations for a thin, smooth, axisymmetric wall. The plasma facing structures in actual tokamaks are neither axisymmetric, nor smooth, with many protrusions. The complexities of the wall structures is expected to have a strong effect on where the plasma strikes the wall.

B. Non-axisymmetric simulations

Actual disruptions have important effects that can only be represented in non-axisymmetric simulations. A simple example is that when both the wall and the plasma are axisymmetric, the part of the halo closest to the main plasma strikes the wall along a circle of fixed major radius. If the symmetry of either is broken, the closest part of the halo to the main plasma strikes at a point, and the width of the halo is critical in determining the closed current paths that encompass both the wall and the halo plasma.

1. M3D simulations

Published M3D simulations consider the wall forces produced by toroidal mode number $n = 1$ kinking during a disruption [7], and the evolution of halo currents during a vertical displacement event [8].

These studies assume a fixed low plasma temperature near the wall for magnetic field lines started inside the original plasma separartix and assume the plasma flow to the wall is zero. The resistivity on magnetic field lines that were outside the original separatrix is made sufficiently high that they carry negligible current. The effective resistivity of the halo is set by the low plasma temperature near the walls. The width of the halo is set by the region over which magnetic field lines started inside the original separatrix strike the wall due to the break up of the magnetic surfaces, which early in the simulations becomes essentially complete.

The surrounding conducting structures in the M3D simulations are assumed to form a thin axisymmetric wall.

The time step in M3D simulations is set by the shear Alfvén time, $\tau_A \equiv R/V_A$. For ITER, the time τ_w for the magnetic field to penetrate the surrounding conducting structures satisfies $\tau_w \sim 10^5\tau_A$. As noted in Ref. [8], realistic values of τ_w/τ_A using the existing M3D code are "completely out of the discussion, due to the prohibitive memory and computer time resources needed." The M3D simulations of the wall forces associated with $n = 1$ kinking [7], assume $\tau_w = 10\tau_A$, and the simulations of the vertical displacement events [8] assume $\tau_w = 100\tau_A$.

The boundary condition $\vec{v} \cdot \hat{n} = 0$ that was used in the M3D simulations is unphysical [13]. In these simulations, not allowing an outflow of plasma to the wall, $\vec{v} \cdot \hat{n} = 0$, primarily affects the inertia of the halo plasma, which is a subdominant issue and probably has little effect on the results. However, the rate of plasma outflow would be very important if realistic plasma-wall boundary conditions were used to calculate quantities such as the temperature or the resistance of the halo plasma.

A major issue in the relevance of the published M3D simulations to the interpretation of experiments is having a time scale for wall penetration by the magnetic fields that is only an order of magnitude or two longer than the shear Alvén time. In addition the extremely rapid break up of magnetic surfaces, which is central to the calculation of the width of the halo region, is unlikely to be representative of an ITER-like plasma, Appendix A.

2. *Proposed M3D-C^1 simulations*

A new version of the M3D code called M3D-C^1 should be able to take arbitrarily long time steps compared to the shear Alfvén time [14]. Steven Jardin plans to use the M3D-C^1 code to study disruptions by imposing a fixed halo resistivity and width as in the TSC simulations. The imposition of a fixed halo width means a specification of the width of the wall region struck by magnetic field lines that pass through the halo. The exact meaning of the plasma halo is subtle when the magnetic field is stochastic at the plasma boundary; the lengths of the field lines that strike the walls has a complicated dependence on the field line considered as does the depth of penetration of the line into the plasma.

3. *NIMROD simulations*

Disruption simulations using NIMROD assume a perfectly conducting wall [9–11], which is an inappropriate boundary condition for a plasma being driven into a wall. The NIMROD disruption simulations focus on effects that arise from the breakup of the internal magnetic surfaces, though it appears unlikely these effects are realistically simulated, cf. Appendix A.

C. Zakharov's calculations of disruption forces

Leonid Zakharov has stressed the importance that plasma kinking can have to disruption simulations [12]. In the calculations reported in [12], the forces on conducting structures are dominated by currents flowing between the plasma and the structures due to plasma kinking. He calls these currents Hiro currents, though here all currents flowing between plasma and conducting structures are called halo currents.

His primary assumption involves the way kinks become unstable. This can be reduced to an assumption about the form of the function $\Delta_w(q_a)$, where q_a is the edge value of the safety factor and Δ_w is a measure of the maximum spatial separation between an axisymmetric plasma and a conducting wall for ideal MHD stability. For disruption simulations in a hot plasma, the function $\Delta_w(q_a)$ should be determined assuming that $q(r)$ is fixed for $r < a$, that the plasma radius a is a decreasing function of time, and that there is no plasma outside the magnetic surface at $r = a$. The assumption that Zakharov makes is that as q_a decreases the function $\Delta_w(q_a)$ goes from infinity to zero as q_a crosses a rational value, $q_a = m/n$. For q_a values just below m/n, the critical wall separation Δ_w is small, but non-zero. This assumption is correct for a circular cylindrical model of an infinite aspect ratio tokamak. The assumption can be checked for tokamak equilibria of practical interest using a standard ideal MHD stability code. Janardhan Manickam has recently undertaken such a study using PEST.

Taking Zakharov's assumption about the form $\Delta_w(q_a)$ as correct, only currents in the halo plasma can slow the growth of the kink below a rate determined by the Alvén time once a critical value of the edge safety factor is reached—induced currents in the actual wall would be too far away. Because the ideal growth rate is so rapid compared to the time scale for the vertical displacement, the voltage in the halo plasma can be be far larger due to kinking than due to the vertical displacement—by the ratio of the vertical displacement to the kink time scale, $\sim 10^5$. The current in the halo plasma due to the kinking must have a magnitude comparable to the plasma current I times the helical displacement ξ divided by the minor radius of the plasma to stabilize the ideal evolution. Consequently, the evolution is determined by the current-source case, cf. Section IV A 2, of the evolution of halo currents once kinking has begun.

If the reduction in the critical wall distance, $\Delta_w(q_a)$, is gradual rather than sudden as q_a becomes smaller then the development of non-axisymmtery could be qualitatively different with a closer resemblance to the growth of the vertical instability. The preliminary result of Manickam's study is that the kink switches over a very small range of $q(a)$ near $q(a) = 2$ from being stable with a wall at infinity to requiring an extremely closely fitting wall for stability.

Zakharov correctly notes that, when $q_a \approx m/n$, that the parallel current in a cylindrical model of a tokamak flows in the opposite direction to the equilibrium current in the spatial region in which the plasma moves towards the wall—where the radial displacement ξ is positive. This means the halo currents due to non-axisymmetric plasma displacements have a sign that is opposite to that of axisymmetric displacements.

IV. ELECTROMAGNETIC BOUNDARY CONDITIONS IN THE HALO

A. Drive for halo currents

Two types of currents flow in the conducting structures that surround a plasma: (1) direct-induction currents, sometimes called image currents, which do not have a current flowing between the plasma and the conducting structures, and (2) halo currents that flow between the halo plasma and the conducting structures. The drive for the halo currents has two limiting cases: (1) a voltage source and (2) a current source.

1. *Voltage Source*

A voltage-source drive for the halo current arises if the currents in the halo plasma are so small that they do not affect the overall plasma equilibrium. Voltage-source-like behavior was observed [16] on MAST.

The path taken by the halo currents is in part through plasma and in part through wall structures. The voltage that drives these currents is determined by the evolution of magnetic flux enclosed by this path. The halo current is determined by that voltage and the electrical resistance along the path. See page 396 of *Tokamaks* [3]. The resistance is given by the resistivity of the plasma and the structures as well as the sheath resistance at the interface between the plasma and the structures. Unless the mean free path of electrons in the halo is short, $< q_h R/\sqrt{m_i/m_e}$, where $2\pi q_h R$ is the typical length of a field line in the halo plasma, or there is strong electron emission by the conducting structures, cf. Appendix C 2, the sheath resistance determines the resistance along the path, cf. Appendix C 1 b.

For the voltage-source case, the evolution is determined by the currents directly induced in the conducting structures. These structures can be well separated from the objects that the halo plasma strikes.

An important question is whether a sensitive structure, such as an antenna, could be protected from the effects of halo currents by electrically insulating the structure from the wall. When the current drive in the halo is a voltage source, then the halo current could be blocked as long as the insulator can stand off the driving voltage.

In principle when the drive is a voltage source, the entire halo current can be eliminated. This occurs when the plasma facing structures are far more resistive than those somewhat deeper in the wall. The effect could be simulated by considering a wall made of two concentric shells with the shell closer to the plasma having a far higher resistivity.

2. Current Source

A current-source drive for the halo current arises if the halo currents are so large that they determine the overall plasma evolution. Many tokamaks have seen halo currents with a magnitude of a few tenths of the plasma current. See Figure (42) in [2].

Zakharov's theory of kinks, which was discussed in the Introduction and in Section III C, is an example of a current-source drive for the halo current. Zakharov's theory provides a natural explanation for the magnitude of the halo current reaching a certain fraction of the total plasma current. This fraction is given by the degree of helical distortion of the plasma.

For the current-source case, the halo currents are essentially determined by force balance The rate of evolution of the equilibrium is determined by the resistance felt by these currents, which flow both through the plasma halo and the surrounding structures, cf. Section V D. The resistance is determined by the resistivity and width of the halo plasma and the surrounding structures as well as by sheath resistance at the wall-plasma interface, cf. Appendix C 1 b.

If the resistance to the halo currents were variable, then the voltage- source case would be the high-resistance limit and the current-source case would be the low-resistance limit.

When the drive is a current source, the entire halo current cannot be eliminated by a resistive layer on the wall. However, it may still be possible to insulate a sensitive structure, so the halo current flows elsewhere.

B. Normal magnetic field evolution

The combination of Faraday's and Ohm's laws implies the normal magnetic field on a conducting surface evolves as $\partial\vec{B}\cdot\hat{n}/\partial t = \vec{\nabla}\cdot(\eta\hat{n}\times\vec{j})$, where η is the resistance of the conductor, $\hat{n}$ is the normal to the conductor, and $\vec{j}$ is the current density in the conductor.

The simplest model of the surrounding structure is a resistive shell, located on a surface

$\vec{x}_s(\theta, \varphi)$, in which $\vec{j} = \vec{\nabla}\kappa \times \hat{n}/\Delta$, where $\kappa(\theta, \varphi)$ is the current potential in the shell and $\Delta(\theta, \varphi)$ is its thickness. The field evolution is determined by η/Δ. By making the surface shape $\vec{x}_s(\theta, \varphi)$ sufficiently convoluted, complicated wall structures can be represented. In principle the only assumption of a shell model is that the thickness of the shell Δ be small compared to the wavenumber along the surface of the penetrating magnetic field. However for the published simulations discussed in Sections III A and III B, the ratio η/Δ was assumed constant and the shell axisymmetric.

For a realistic assessment of the forces to be expected from halo currents the complexities in shape of the structure contacted by the halo plasma should be represented. This is particularly true when the halo plasma is narrow, so a large current can flow through a small protrusion.

V. BOUNDARY CONDITIONS ON THE HALO PLASMA

The boundary conditions between the halo plasma and the conducting structures that surround a plasma have a central role in determining the halo density, temperature, current density, and resistance. These boundary conditions are discussed in this section.

The plasma boundary conditions are in large part determined by the sheath that forms at the interface between the plasma and the wall. Basic sheath physics, which is required to understand the plasma boundary conditions, is given in Appendix C. A far more extensive discussion of related issues can be found in [17].

A. Halo plasma density

The plasma density in the halo region is determined by transport from the main body of the plasma, ionization of neutral gas from the walls, as well as the outflow.

The outflow velocity is expected to be along the magnetic field with a speed comparable to

$$C_s \equiv \sqrt{(T_e + T_i)/m_i} \tag{1}$$

unless a cold dense plasma with a pressure equal to that of the halo plasma forms near the strike points of open magnetic field lines, cf. Appendix B. The formation of a such cold, dense plasma is called detachment in the divertor context.

B. Halo Plasma Temperature

The ion and electron temperatures in the halo region are determined by the radial heat transport from the bulk plasma and radiation—in particular impurity radiation—in addition to the parallel heat flux to the wall structures through the plasma-wall sheath, Appendix C.

The energy flux through the plasma-wall sheath is generally written in the form $q_\epsilon = \gamma_s T_e n_0 C_s$, where $\gamma_s \simeq 8$ is called the sheath transmission factor, Appendix C 1 a, n_0 and T_e are the density and the electron temperature on the plasma side of sheath. However, the energy flux can be much larger, $q_\epsilon \sim T_e n_0 \sqrt{T_e/m_e}$ if the wall emits electrons, cf. Appendix C 2.

C. Halo current density

The current density in the halo can be determined by two effects: (1) the voltage along each field line of the halo in the evolving equilibrium, or (2) the maximum allowable current density, cf. Appendix C 1 b, which is the ion saturation current $j_s \equiv en_0 C_s$, when there is no electron emission from the walls.

The current density in the halo is an important determinant of the forces that can be exerted on components. The maximum force on a component is the current density times the presented area of the component times the strength of the magnetic field.

The current flowing in the halo I_h can be a few tenths of the total plasma current I, which gives a relation between the width of the halo Δ_h and the spatially averaged current density in the halo $\langle j\rangle = I_h/(2\pi a\Delta_h)$. Using identities, the ratio of the spatially averaged current density in the halo to the ion saturation current can be written as

$$\frac{\langle j\rangle}{en_0 C_s} = \frac{1}{\beta_p}\frac{\rho_p}{\Delta_h}\frac{I_h}{I}, \tag{2}$$

where β_p is the ratio of the halo plasma pressure to the local pressure of the poloidal magnetic field, and $\rho_p = (q_h R/a)\rho_s$ is the poloidal gyroradius in the halo. As will be discussed in Section VI A, the width of the halo is expected to lie in the range $\rho_p \lesssim \Delta_h \lesssim \sqrt{R\rho_p}$. Since β_p can be small in the halo, the ratio of the average current density to the ion saturation current is not immediately obvious.

D. Halo resistance

In the absence of electron emission by the walls, the resistance of the halo plasma tends to be dominated by the sheath resistance, cf. Appendix C 1 b, when the temperature in the halo is greater than roughly $50eV$. When the sheath resistance dominates, the relation between the current density and the voltage V across the plasma is $j = en_0C_s(eV/T_e)$.

The sheath resistance is equivalent to an effective resistivity, η_{sh}, along magnetic field lines of length q_hR,

$$\eta_{sh} \equiv \frac{T}{e^2nC_sq_hR} \approx \frac{0.9\times10^{-5}}{q_hR}\frac{10^{20}/m^2}{n}\sqrt{\frac{T}{eV}}. \tag{3}$$

The ratio of the sheath resistivity to the Spitzer resistivity, $\eta_{sp} \approx ne^2\tau_e/m_e$, where τ_e is the electron collision time, is

$$\frac{\eta_{sh}}{\eta_{sp}} \approx \sqrt{\frac{m_i}{m_e}}\frac{\lambda_e}{qR}. \tag{4}$$

Unless the electron mean free path, $\lambda_e \equiv v_e\tau_e$ is shorter than $q_hR/\sqrt{m_i/m_e}$ the sheath resistance dominates. The electron mean free path is $\lambda_e \approx 1m(T/100eV)^2\{(10^{20}/m^2)/n\}$.

The effective resistivity of the halo plasma tends to be large, either due to the sheath for $T > 50eV$ or due to the Spitzer resistivity for $T < 50eV$, when compared to the resistivity of metals; for copper $\eta = 1.7\times10^{-8}$.

The thickness of the halo plasma assumed in the simulations discussed in Section III A was broad compared to the thickness of the conducting walls. A thick halo compared in the wall thickness makes the wall contribution to the halo resistivity relatively larger and makes the resistance of the halo relatively smaller in comparison to the resistance for currents that are directly induced in the walls.

VI. DISRUPTION ISSUES OTHER THAN BOUNDARY CONDITIONS

A. Axisymmetry of halo plasma

The symmetry of the halo plasma is an important element in determining the forces and the heat loads on the wall. The axisymmetry of the halo plasma can be broken by two effects: (1) the breaking of axisymmetry in the magnetic field, and (2) the breaking of axisymmetry in the electric potential. In particular a strong electron emission in a narrow flux tube, cf. Appendix C 2, could give concentrated forces and heat loads.

Kinking of the overall plasma, such as that discussed by Zakaharov, Cf. Section III C, breaks the magnetic symmetry of the halo plasma and changes the force and heat patterns on the chamber walls.

Even if the magnetic field is perfectly axisymmetric, the electric potential in the halo need not be. An axisymmetric magnetic field can be represented as $2\pi\vec{B} = \mu_0 G(\psi_p)\vec{\nabla}\varphi + \vec{\nabla}\varphi \times \vec{\nabla}\psi_p$, where ψ_p is the poloidal flux.

The effect of even the axisymmetric electric potential, $\Phi(\psi_p, \theta)$, is non-trivail. A symmetric electric potential must vary along the magnetic field lines to preserve quasi-neutrality in the region in which the plasma accelerates to the speed of sound, $\vec{B}\cdot\vec{\nabla}(\Phi - (T_e/e)\ln n) = 0$, cf. Appendix B. The implied $E \times B$ drift moves the plasma radially a distance $\Delta_h \simeq (E_\theta/B)(q_h R/C_s)$ as the plasma flows along the magnetic field lines, where $2\pi q_h R$ is the length of the field lines in the halo. The poloidal electric field $E_\theta \simeq (T_e/e)/a$, where a is the plasma radius, so $\Delta_h \simeq (q_h R/a)\rho_s$, were ρ_s would be the gyroradius of ions moving with a velocity C_s. In other words, the halo plasma must have a radial width Δ_h at least as great as the poloidal gyroradius [19].

An asymmetric potential can produce stronger effects. The most important is the possibility that the heat flux can concentrate in a small tube of magnetic flux, which can heat the footpoints of the flux tube on the walls to such a high temperature that strong thermal emission can occur, cf. Appendix C 2. The poloidal width Δ_p of such a magnetic flux tube within the constant ψ_p surfaces cannot be arbitrarily narrow. The radial ambipolar electric field E_r across the halo plasma is roughly $T_e/e\Delta_h$, where Δ_h is the radial scale of the halo plasma. The width of the flux tube that is required in order to be consistent with the radial electric field is $\Delta_p \simeq (E_r/B)(q_h R/C_s) \simeq q_h R\rho_s/\Delta_h$. If Δ_h is narrow, $\Delta_h \simeq (q_h R/a)\rho_s$, as is possible in an axisymmetric halo, then the flux tube tends to be broad, $\Delta_p \simeq a$, which means little concentration of the heat flux. However, as will be seen in the next paragraph the halo can be much broader $\Delta_h \simeq \sqrt{q_h R\rho_s}$, which would give a similar width Δ_p for the flux tube in the constant ψ_p surface and a small hot spot on the wall. The existence of narrow flux tubes that carry a disproportionate fraction of the heat could be determined by infrared cameras, which could also estimate the area of the tubes, which is $\Delta_p\Delta_h$.

The variations in the sheath potential due to variations, cf. Appendix C 2, in the electron emission coefficient δ_e are an obvious cause of asymmetry. A variation in the potential $\delta\Phi$ within a constant ψ_p surface on a spatial scale Δ_p gives an $E \times B$ radial velocity $\delta\Phi/B\Delta_p$.

During the time it takes the plasma to flow along the magnetic field lines to the divertor $q_h R/C_s$, it can be transported radially a distance $\Delta_h \simeq (\delta\Phi/B\Delta_p)(qR/C_s)$. If the spatial scales for the variation in the potential radially and within the flux surfaces are comparable and if $\delta\Phi \simeq T/e$, then the natural width scale for the halo is then $\Delta_h \simeq \sqrt{q_h R \rho_s}$.

B. Evolution of the central plasma

Physics assumptions about the evolution of the central plasma affect the reliability of disruption simulations. The evolution of the central plasma is due to two effects: (1) the changing magnetic field produced by currents outside of the central plasma and (2) the evolution of the plasma profiles particularly the temperature and the current profiles.

The evolution of the central plasma appears sufficiently slow that inertia is negligible in the overall dynamics. What is required is a fast and accurate method of calculating force balance, $\vec{\nabla}p = \vec{j} \times \vec{B}$ that can include (1) a large force-free current flowing on the open field lines of the plasma halo, (2) a large departure from axisymmetry, and (3) magnetic islands enclosing a specified toroidal magnetic flux. No codes exist that can address these three requirements, nor is there an ongoing program to develop such a code.

1. *Evolution due to changes in the externally produced magnetic field*

The externally produced magnetic field is due to currents in the coils and in the surrounding conducting structures. Parts of the currents in the conducting structures are due to induction and parts are due to currents flowing into the structures from the halo plasma.

The halo plasma is observed to carry a current as large as a few tenths of the original plasma current. To represent this, the effect of the currents in the halo plasma on the overall equilibrium must be calculated. The halo presumably has a low pressure, so the currents in the halo are generally taken to be force free, $\vec{j} \times \vec{B} = 0$, which simplifies the calculation of their effects. Appendix D gives a method of augmenting a stellarator equilibrium code, such as the Variational Moments Equilibrium Code (VMEC) [18], with a thin halo that carries a force-free current.

2. Evolution due to changes in plasma and current profiles in the central plasma

The evolution of the overall plasma current profile is slow on the time scale expected for ITER disruptions when the plasma temperature is greater than a few tens of electron volts.

The relaxation of current sheets that prevent the opening of magnetic islands is also very slow in a hot plasma if that relaxation is by a Rutherford relaxation. However, a rapid opening of magnetic islands is possible, cf. Appendix A. For accuracy and for insight into the physics, the growth of magnetic islands in a non-axisymmetric simulation code should be treated as part of the plasma evolution and not as part of the equilibrium, which means the force-balance calculation has to be consistent with islands that enclose a specified toroidal magnetic flux. When a plasma resistance is included in the code used to find the equilibrium, island widths evolve at a rate that need have no relationship with the physically correct rate.

The time scale over which magnetic surfaces are lost and over which they can be regained may be very different. The speed with which they can be regained is a central element in calculating the runaway electron distribution.

The temperature of the central plasma is observed to undergo a very fast thermal collapse either near the beginning or the end of a disruption. The time scale for this thermal collapse in ITER is expected to be approximately $700\mu sec$, which is a few hundred times shorter than the overall disruption time.

An explanation of the thermal collapse is a central element in the simulation of disruptions. The usual explanation is the loss of magnetic surfaces—and the shortness of the thermal quench ranks along with the voltage spike as the primary evidence for the loss of surfaces. The time required for magnetic islands to grow sufficiently to destroy the magnetic surfaces may be very long compared to the thermal quench time. When the loss of plasma rotation is a precursor to a disruption, the time between the stopping of rotation and the thermal quench may be the time required for islands to grow. A simple estimate of the required time for the growth of islands to destroy the magnetic surfaces is given in Appendix A 5.

The primary effect of the break up of magnetic surfaces is a greatly enhanced heat transport and a radial flattening of the net parallel current $j_{||}/B$. The $j_{||}/B$ flattening must conserve magnetic helicity when it is fast compared to the global resistive decay time, which gives a characteristic voltage spike. When unresolved islands dominate a helicityconserving evolution of the current profile [20], the parallel electric field has the form $\vec{E} \cdot \vec{B} = -\vec{\nabla} \cdot (\lambda_h \vec{\nabla}(j_{||}/B))$, where the coefficient λ_h is called the hyper-resistivity.

A presumably important but unstudied transport mechanism for heat, particles, and impurities in a stochastic magnetic field is the strong but complicated $E \times B$ flow that results from the slow spatial variation of the electric potential along the magnetic field to enforce quasi-neutrality. In a stochastic magnetic field, neighboring magnetic field lines separate exponentially, so the electric potential on neighboring magnetic field lines is determined by ambipolarity requirements in widely separated spatial regions.

If the current is flattened in a region of space, then it is not necessary for an equilibrium code to resolve the details of the trajectories of the magnetic field lines in that region because the actual direction of the magnetic field lines, and hence the current, is changed only slightly between stochastic and non-stochastic magnetic fields.

VII. CONCLUSIONS

Although existing simulations of disruptions do not impose physical boundary conditions between the plasma and walls, there is no fundamental reason such boundary conditions could not be imposed. The largest practical impediment is probably the existence of an appropriate code for calculating plasma force balance, $\vec{\nabla} p = \vec{j} \times \vec{B}$.

What is needed is a fast and accurate force-balance code that can include (1) a large force-free current flowing on the open field lines of the plasma halo, (2) a large departure from axisymmetry, and (3) magnetic islands that enclose a specified toroidal magnetic flux.

The lack of an appropriate force-balance code is not due to a lack of physics understanding but the absence of a program focused on this programmatic need.

Much effort has been expended on the inclusion of Alfvénic time scales in the non-axisymmetric codes that are used for disruption studies. However, there is no evidence that such times scales are directly relevant to the simulation of disruptions, and their inclusion may greatly slow and complicate simulation efforts.

Any code that could reliably calculate non-axisymmetric disruption physics could reliably calculate stellarator equilibria. The extensive data base of stellarator experiments and of computations of stellarator equilibria provide an obvious basis for validation and verification of non-axisymmetric codes for disruption studies. The major missing element in the stellarator equilibrium codes for direct relevance to disruption studies is the omission of the halo current. This omission can be addressed in a simple way under the approximation of a thin halo compared to the plasma radius.

The major physics uncertainties in the halo plasma involve the plasma-wall interaction and the effects of toroidal asymmetries in the electric potential. A particular area of uncertainty is secondary electron emission from the walls. Even axisymmetric simulations of disruptions could be useful as a platform for building understanding of plasma-wall interactions.

The major physics uncertainties in the core plasma during a disruption involve the thermal quench and the destruction of magnetic surfaces. An important issue is how rapidly can strongly-driven islands open. In the language of the reconnection community, this is rapid reconnection in the presence of a strong guide field with periodicity in two directions. The rapid opening of an island involves thin boundary layers and kinetic effects and probably should be studied as a stand-alone problem rather than as an element in a calculation of plasma force balance. When the magnetic field lines near the plasma edge become stochastic, the distinction between the stochastic region and the halo plasma is subtle. For runaway electrons to reach dangerous relativistic energies about 10^5 toroidal circuits are required, which implies a certain quality of magnetic surfaces. The time scale required for magnetic surfaces to reform after the thermal quench is distinct from the time scale for their destruction an may be the greatest uncertainty in runaway-electron calculations.

Acknowledgments

This work was supported by U. S. Department of Energy through the grant ER54696 to Columbia University.

APPENDIX A: MAGNETIC ISLANDS IN NON-AXISYMMETRIC SIMULATIONS

Disruptions studies can neither assume with assurance that magnetic islands do not have time to open nor that the shielding currents for magnetic islands will be fully relaxed.

Let δ_I be the instantaneous half width of the most important island chain and δ_d be the island half width if the shielding current near the resonant rational surface of radius r_r is eliminated keeping all other currents fixed. Then island opening occurs at a relatively slow Rutherford rate, Eq. (A6) when $\delta_I \gtrsim 2k\delta_d^2$, where $k = m/r_r$ is the wavenumber of the island. When $\delta_I \gtrsim k\delta_d^2/4$, the island X- point can stretch into a current sheet and secondary islands can open, which can greatly enhance the rate of island opening, cf. Section A 3. The island growth rate can also be greatly enhanced when $\delta_I \lesssim \rho_s$ [21].

Work remains to be done to determine the rate at which islands open when the conditions $\delta_I \gtrsim 2k\delta_d^2$ and $\delta_I > \rho_s$ are not satisfied. For $\delta_I \lesssim k\delta_d^2/4$ the rate will be determined by a thin boundary layer with complicated spatial structure, which makes the break up of magnetic surfaces in fusion grade plasmas too subtle to be convincingly calculated using large scale non-axisymmetric codes such as M3D, M3D-C^1, and NIMROD. Specialized studies of island opening should be carried out instead, as in the space science community [22], [23].

1. Slab model

Central features of magnetic islands can be understood using a slab model of the magnetic field, $\vec{B} = B_0\hat{z} - \hat{z}\times\vec{\nabla}A(x,y)$, where B_0 is a constant. Since $\vec{B}\cdot\vec{\nabla}A = 0$, the field lines lie in constant A surfaces.

The $\hat{z}$ component of the vector potential has the form $A(x,y) = A_0(x) + \mathcal{A}(x)\cos(ky)$. Ampere's law in the slab is $\nabla^2 A = -\mu_0 j$, where j is the $\hat{z}$ component of the current. When the slab model is valid, the x variation of A is rapid compared to the y variation, so Ampere's law can be approximated as $\partial^2 A/\partial x^2 = -\mu_0 j(x,y)$. Near a resonance $A_0 = -\mu_0 J_0 x^2/2$, where $j(x,y) = J_0(x) + \delta j\cos(ky)$. If the perturbed vector potential has the form $\mathcal{A} = B_I/k$ at $x = 0$, then the constant A-surfaces split to form an island around the $x = 0$ surface. The half width of this island is $\delta_I = \sqrt{4B_I/k\mu_0 J_0}$, and the constant B_I is the $\hat{x}$ component of the magnetic field at $x = 0$.

2. Slab approximation to torus

The slab model is closely related to the general representation of a magnetic field with surfaces in a torus. When the magnetic surfaces are nested, the magnetic field has the representation $2\pi\vec{B} = \vec{\nabla}\psi_t\times\vec{\nabla}\theta + \iota(\psi_t)\vec{\nabla}\varphi\times\vec{\nabla}\psi_t$. Even when a rational surface $\iota(\psi_r) = n/m$

is split by a magnetic island, non-nested magnetic surfaces can remain. That is a function $A(\vec{x})$ exists such that $\vec{B}\cdot\vec{\nabla}A=0$, but $A=A_0(\psi_t)+\delta A(\psi_t,\theta-n\varphi/m)$, where the δA gives the splitting of the magnetic surfaces,

$$2\pi\vec{B}=\vec{\nabla}\psi_t\times\vec{\nabla}(\theta-n\varphi/m)+\vec{\nabla}\times(A\vec{\nabla}\varphi). \tag{A1}$$

The function $A_0(\psi_t)=-\left(d\iota/d\psi_t\right)_r(\psi_t-\psi_r)^2/2$, gives the shear in the rotational transform, $\iota(\psi_t)=n/m+(d\iota/d\psi_t)_r(\psi_t-\psi_r)$, near the rational surface $\iota(\psi_r)=n/m$ in the absence of splitting, $\delta A=0$.

Equation (A1) can be placed in the form of the slab model by letting $z=R\varphi$, where R is the major radius. The coordinate x is the distance from the rational surface, $d\psi_t/dx=2\pi B_0 r_r$, where r_r is the minor radius of the resonant surface, and the coordinate y is defined by $ky=m\theta-n\varphi$, where $k=m/r_r$ is a wavenumber, so $k\hat{y}=(m/r_r)\hat{\theta}+(n/R)\hat{\varphi}$, which implies $\hat{y}\approx\hat{\theta}+\epsilon\iota\hat{\varphi}$, where $\epsilon=r_r/R$ is the inverse aspect ratio of the resonant surface. Since $\vec{B}\cdot\vec{\nabla}A=0$, the magnetic field lines lie in surfaces of constant $A(x,y)=A_0(x)+\delta A(x,y)$, where $A_0=-\epsilon\iota' B_0x^2/2$ and $\iota'\equiv d\iota/dx$. The quantity $J_0=\epsilon\iota' B_0/\mu_0$.

3. Critical island width for fast reconnection

The rate an island opens was found by Rutherford [24] assuming the perturbed vector potential $\mathcal{A}$, or equivalently the normal magnetic field $\delta B_x=k\mathcal{A}\sin ky$ changes little across the island. When $\mathcal{A}(x)$ changes little across the island of half width δ_I, the current that is induced by the opening of the island [24], [25] flows in a channel of width $\sim\delta_I$, and the vector potential can be determined using the equations for a surface current,

$$\begin{aligned}\mathcal{A}(x) &= \frac{B_d}{k}\left(e^{kx}-e^{-kx}\right)+\frac{B_I}{k}e^{-kx} \text{ for } x>\delta_I\\ &= \frac{B_I}{k}e^{kx} \qquad\qquad\qquad\quad \text{for } x<-\delta_I.\end{aligned} \tag{A2}$$

In the absence of the surface current, the normal magnetic field on the rational surface would be B_d, which is called the driving field for the island, and in the presence of the surface current, the normal field on the rational surface is B_I.

The condition that the vector potential, Eq. (A2), change little across the island is that $2B_d\delta_I\lesssim B_I/k$, when $k\delta_I$ is assumed small. The ratio $B_d/B_I=\delta_d^2/\delta_I^2$, where δ_d is the width the island would have in the absence of the surface current. The condition for the validity of the Rutherford rate of island opening is then

$$\delta_I \gtrsim 2k\delta_d^2. \tag{A3}$$

The criterion for the validity of the Rutherford theory, Eq. (A3) is often given in the literature using the jump in the derivative of the perturbed vector potential, $\Delta' \equiv [d\mathcal{A}/dx]/\mathcal{A}$, evaluated at $x = 0$, so

$$\Delta' = 2k\frac{\delta_d^2 - \delta_I^2}{\delta_I^2}. \tag{A4}$$

Island formation can be seen from two perspectives: (1) as a tearing mode with Δ' slowly varying [26], or (2) as a perturbed equilibrium with the driven-island width $2\delta_d$ slowly varying. Although the physics in the vicinity of the island can be same, the two perspectives can give apparent contradictions. For example, assume $\delta_I << \delta_d$, so $\Delta'\delta_I = 2k\delta_d^2/\delta_I$. When Δ' is fixed, the island X-point is known to stretch into a current sheet in resistive MHD calculations [27, 28] when island grows sufficiently that $\Delta'\delta_I \gtrsim 8$. The reconnection rate of magnetic field lines is enhanced by the current sheet and can be enhanced further by the break up of the sheet by secondary islands. If driven island width is assumed constant, then the same effect occurs when the island is sufficiently narrow that $\delta_I \lesssim k\delta_d^2/4$. In other words, in the tearing mode perspective, rapid reconnection occurs when the island is too wide, $\delta_I \gtrsim 8/\Delta'$. In the driven-island perspective, rapid reconnection occurs when the island is too narrow

$$\delta_I \lesssim k\delta_d^2/4. \tag{A5}$$

The criterion of $\delta_I \lesssim k\delta_d^2/4$ is a necessary condition for rapid growth of an island, at least when $\delta_I \gtrsim \rho_s$ so the fluid picture of reconnection is valid.

Linear resistive MHD, which gives a time scale $\tau_A^{2/5}\tau_{skin}^{3/5}$, has a current channel far thinner than the ion gyroradius in large tokamaks, cf. p. 323 of [3]. Non-linear ideal MHD implies the width of the overall current layer near the rational surface cannot become narrower [29] than $k\delta_d^2$ although part of that current flows in a delta function distribution.

A signature of rapid reconnection is the generation of resonant harmonics, which might be detected in an experiment by external magnetic measurements. In Rutherford reconnection, the current at the rational surface $\iota = n_0/m_0$ produces an almost pure Fourier (m_0, n_0) magnetic field in response [25]. However, the elongation of the X-point into a singular surface when $\delta_I \gtrsim k\delta_d^2/4$ implies the shielding current produces resonant harmonics $(2m_0, 2n_0)$, $(3m_0, 3n_0)$, *etc.*

4. Rutherford growth rate

The rate at which an island grows when $\delta_I << 2k\delta_d^2$ was found approximately by Rutherford [24]. An exact treatment [25] yields

$$\frac{d\delta_I^3}{dt} = c_R k \frac{\eta}{\mu_0}\left(\delta_d^2 - \delta_I^2\right), \text{ where } c_R = 1.22\cdots \tag{A6}$$

The form of Rutherford's equation is easily found under the assumption the current near the resonant surface flows in a channel of width $2\delta_I$. Ampere's law, $d^2\mathcal{A}/dx^2 = -\mu_0\delta j(x)$, so $\mu_0\langle\delta j\rangle \simeq -(B_I/k)\Delta'/2\delta_I = -B_I(\delta_d^2 - \delta_I^2)/\delta_I^2$. Ohm's law is $-\partial\mathcal{A}/\partial t = \eta\delta j$, so $d\delta_I^2/dt = (\eta/\mu_0)k(\delta_d^2 - \delta_I^2)/\delta_I$, which yields Equation (A6) but with a coefficient $c_R = 3/2$ instead of $1.22\cdots$.

5. Transition to a stochastic magnetic field

Magnetic field lines become stochastic roughly when islands from neighboring rational surfaces overlap, which is called the Chirikov criterion [30]. The critical island half width δ_c for stochasticity is then $n/m + 2\iota'\delta_c = n/(m-1)$, which implies $2s_\iota\delta_c = r_r/(m-1)$, where the shear $s_\iota \equiv |d\ln\iota/d\ln r|$. The Chirikov overlap parameter is

$$S_I \equiv 2ks_\iota\delta_I; \tag{A7}$$

the field lines are stochastic if $S_I \gtrsim 1$.

If the islands grow at the Rutherford rate, Eq. (A6), then $dS_I^3/dt = (S_d^2 - S_I^2)/\tau_R$, where S_d is the Chirikov overlap parameter calculated with the driven island half-width δ_d rather than the actual half-width δ_I. The characteristic time required for the islands to grow at the Rutherford rate is

$$\tau_R = \frac{\tau_{skin}}{2m^2 s_\iota c_R}, \tag{A8}$$

the resistive skin time is $\tau_{skin} \equiv r_r^2\mu_0/\eta$, and the Chirikov criterion for the driven islands of half width δ_d is S_d. The time required to reach field line stochasticity is approximately

τ_R/S_d^2 for S_d significantly larger than unity. More precisely the time $t(S_I)$ required to reach a specific Chirikov criterion with fixed S_d is

$$t(S_I) = 3\tau_R S_d \left\{ \ln\left(\sqrt{\frac{1+S_I/S_d}{1-S_I/S_d}} \right) - \frac{S_I}{S_d} \right\}. \tag{A9}$$

The condition that island growth be consistent with the Rutherford rate, $\delta_I \simeq 2k\delta_d^2$, is equivalent to $S_I \simeq S_d^2/s_\iota$. When the islands are driven to opening all the way to stochasticity, the validity of the Rutherford theory is only marginally satisfied.

The condition for rapid reconnection Equation (A5) all the way to a stochastic field $S_I \simeq 1$ is $S_d \gtrsim 2\sqrt{s_\iota}$. This condition is of interest in the theory of the control of edge localized modes (ELM's) by non-axisymmetric magnetic fields. The condition says the drive for islands, as calculated by a code such as the Ideal Perturbed Equilibrium Code (IPEC) [31], should satisfy $S_d \gtrsim 2\sqrt{s_\iota}$ and not just $S_d \gtrsim 1$ for a rapid reconnection, reconnection that is too rapid to be easily impeded by effects such as plasma rotation.

APPENDIX B: SONIC FLOW CONDITION

The characteristic speed for a plasma to flow into a wall is the sound speed, which is $C_s = \sqrt{(T_i + T_e)/m_i}$ for isothermal ions and electrons. The derivation of this speed is simple for a narrow halo plasma.

Force balance in the halo is $\rho\vec{v}\cdot\vec{\nabla}\vec{v} + \vec{\nabla}p = \vec{j}\times\vec{B}$ with $\vec{\nabla}\cdot(\rho\vec{v}) = 0$ and $\rho = mn$ the mass density of the plasma. The plasma flow is rapid along the magnetic field. Let $\rho\vec{v} = \Gamma\hat{b} + \rho\vec{v}_\perp$, then $\vec{\nabla}\cdot(\rho\vec{v}) = 0$ implies

$$\vec{B}\cdot\vec{\nabla}\left(\frac{\Gamma}{B}\right) = S, \tag{B1}$$

where the source $S \equiv -\vec{\nabla}\cdot(\rho\vec{v}_\perp)$. Equation (B1) gives the flux of plasma Γ along a magnetic field line.

Since the flow is essentially parallel to the magnetic field, the parallel component of force balance is $\Gamma\hat{b}\cdot\vec{\nabla}(\Gamma/\rho) + \hat{b}\cdot\vec{\nabla}p = 0$, which can be written as

$$\Gamma(\ell)\frac{d(\Gamma/\rho)}{d\ell} + \frac{dp}{d\ell} = 0, \tag{B2}$$

where $d/d\ell \equiv \hat{b}\cdot\vec{\nabla}$ and ℓ is the distance along a magnetic field line.

If the density $\rho(\ell)$ is used as the independent variable,

$$\frac{d\Gamma^2}{d\rho} = -2\rho\left(C_s^2 - \frac{\Gamma^2}{\rho^2}\right), \tag{B3}$$

where $C_s^2 \equiv dp/d\rho$. That is, the effective sound speed is $C_s^2 \equiv (dp/d\ell)/(d\rho/d\ell)$. Equation (B3) is well known from the theory of nozzles in fluid mechanics and says the density drops as the flux Γ increases when the flow starts with zero speed, $\Gamma/\rho = 0$.

The maximum flux Γ that can be obtained is $\Gamma = \rho C_s$, which implies a flow at the speed of sound C_s. If the wall exerts a sufficiently small back pressure, then the plasma flow will reach the sonic rate as it flows down the field lines. In the other limit in which the back pressure p_b keeps the flow slow compared to the sound speed, the flux reaches $\Gamma^2 = 2\rho_0(p_0 - p_b)$ with p_0 the pressure where $\Gamma = 0$.

APPENDIX C: PLASMA-WALL SHEATH

To preserve the quasineutrality of the overall plasma an electrostatic structure, called a sheath, must form at the interface between a plasma and a wall in which there is a jump Φ_s in the electrostatic potential. If the magnetic field lines have a normal incidence to the wall, the spatial scale of the sheath is the Debye length, but the sheath is thicker when the field lines have a grazing incidence [32].

1. Classical sheath theory

The classical theory of the interface between a plasma and a wall assumes no emission of electrons by the wall. The effects of electron emission are considered in Appendix C 2.

The jump in the electric potential across the sheath, Φ_s, retards the flow of the electrons by preventing electrons from leaving the plasma that enter the sheath with a velocity along the magnetic field that satisfies $\frac{1}{2}m_e v_{||}^2 < e\Phi_s$. Assuming the electron distribution function is Maxwellian and $e\Phi_s/T_e$ is significantly greater than unity, the electron flux to the wall is given by

$$\begin{aligned}\Gamma_e &= \frac{\int_0^\infty v_{||} e^{-m_e v_{||}^2/2T_e} dv_{||}}{\int_{-\infty}^\infty e^{-m_e v_{||}^2/2T_e} dv_{||}} n_0 e^{-e\Phi_s/T_e} \\ &= \sqrt{\frac{T_e}{2\pi m_e}} n_0 e^{-e\Phi_s/T_e},\end{aligned} \tag{C1}$$

where n_0 is the density on the plasma side of the sheath. The ion flux is $\Gamma_i = n_0 C_s$, where $C_s = \sqrt{(T_e + T_i)/m_i}$ for isothermal ions and electrons, so ambipolarity, $\Gamma_e = \Gamma_i$, implies

$$\Phi_s = \frac{T_e}{e} \ln\left(\sqrt{\frac{m_i}{2\pi m_e}\frac{T_e}{T_e + T_i}}\right) \simeq 3\frac{T_e}{e}. \tag{C2}$$

a. *Sheath energy transmission*

The energy flux parallel to the magnetic field through the plasma- wall sheath has the form $q_\epsilon = \gamma_s T_e n_0 C_s$ where the sheath transmission factor $\gamma_s \approx 8$. The energy flux to the wall can be much larger if there is a strong emission of electrons from the wall structures, cf. Appendix C 2.

The energy flux across the sheath from the electrons is given by

$$\begin{aligned} q_{\epsilon_e} &= \frac{\int_0^\infty \frac{m_e v_{||}^2}{2} v_{||} e^{-m_e v_{||}^2/2T_e} dv_{||}}{\int_{-\infty}^\infty e^{-m_e v_{||}^2/2T_e} dv_{||}} n_0 e^{-e\Phi_s/T_e} + \\ &\quad +(T_e + e\Phi_s)\Gamma_e \\ &= (2T_e + e\Phi_s)\Gamma_e. \end{aligned} \tag{C3}$$

Each electron that leaves must have at least a parallel energy of $e\Phi_s$ as well as its transverse energy, which is T_e. The integral in the first line of Equation (C3) is the parallel energy flux carried out by electrons with a parallel energy above $e\Phi_s$. The energy flux coming from the ions is $q_{\epsilon_i} = (\frac{5}{2}T_i + m_i C_s^2/2)\Gamma_i$, which gives $q_{\epsilon_i} = (3T_i + \frac{1}{2}T_e)\Gamma_i$.

Although the energy $e\Phi_s$ comes out of the electrons, the ions passing through the sheath are accelerated by this potential and strike the wall with an enhanced energy, which affects their interaction with the wall, in particular the sputtering.

b. *Sheath resistance and limit on the current density*

The current density in the halo region is limited by the ion saturation current,

$$enC_s \approx \frac{0.2MA}{m^2}\frac{n}{10^{20}/m^3}\sqrt{\frac{T}{eV}}, \tag{C4}$$

unless there is a strong emission of electrons from the wall structures, cf. Appendix C 2.

The current density between the plasma and the wall, $j = e(\Gamma_i - \Gamma_e)$, is

$$j = en_0 \left(C_s - \sqrt{\frac{T_e}{2\pi m_e}} e^{-e\Phi_s/T_e} \right), \tag{C5}$$

when there is no emission of electrons by the wall. The ions flow to the wall at the mass flow speed, C_s, and the electrons flow to the wall at the speed of a half-Maxwellian but with their density at the wall reduced by the electric potential Φ_s in the plasma relative to the wall. The maximum density for a current flowing into the wall is en_0C_s, for that is the current density when $e\Phi_s/T_e \to \infty$. If the potential Φ_s deviates by a small amount V from its ambipolar, $j = 0$, value then

$$j = en_0 C_s \frac{eV}{T_e}, \tag{C6}$$

so the total current between the plasma and the wall has the form $V = IR_s$ where the sheath resistance

$$R_s = \frac{T_e}{A_h n_0 e^2 C_s} \tag{C7}$$

and A_h is the presented area of the halo plasma on the walls.

A halo current can also be driven without a voltage across the plasma if the sheath potential differs at the two ends of the field line. Though of possible importance, this is not discussed further in this paper.

2. Electron emission by walls

When wall structures can emit electrons, the effective electron flux is enhanced by a factor $1/(1-\delta_e)$, where δ_e is the electron emission coefficient, the ratio of the emitted to the incoming electrons. When this emission is weak, $\delta_e << 1$, the sheath potential is changed by $\delta\Phi_s = -\delta_e\Phi_s$. Strong electron emission, $\delta_e \geq 1$, can produce extremely large localized heating and forces on wall structures.

Two potentially important causes of electron emission are secondary emission and thermal emission. In secondary emission an incoming electron knocks an electron out of the structure. Secondary emission coefficients, cf. Section 3.2 of Reference [17], are usually less than unity if the energy of the exiting electrons is significantly lower than $500ev$. Thermal emission of electrons by an object at a temperature T is given by the Richardson equation

$$j = \frac{em}{2\pi^2\hbar^3} T^2 e^{-w/T}, \tag{C8}$$

where w is the work function of the wall material, which is typically about $3eV$. At $1500K$, this formula gives $j = 2.7 \times 10^{12} \exp(-w/T)$ in Amperes per meter squared. Thermal emission is enormous unless $\exp(w/T) \gtrsim 10^6$, or $w \gtrsim 14T$, which is satisfied for a work function of $3eV$ if $T \lesssim 2.5K$.

With strong emission, $\delta_e > 1$, the current density is limited by space charge. Emitted electrons move from the wall only if $\vec{E} \cdot \hat{n} \leq 0$ and leave the wall as fast as they are emitted if $\vec{E} \cdot \hat{n} > 0$. Consequently, the strong emission limit is characterized by $\vec{E} \cdot \hat{n} = 0$ on the wall. The Child- Langmuir theory assumes the emitted electrons are collisionless, so they conserve their flux nv and their energy $m_e v^2/2 - e\Phi$. The boundary conditions at the wall are $v \to 0$ and $d\Phi/dx = 0$. With these assumptions plus Poisson's equation $d^2\Phi/dx^2 = en/\epsilon_0$, the Child-Langmuir law can be derived as $j_{CL} = 4\epsilon_0 \sqrt{2e/m_e} V^{3/2}/9L^2$, where L is the distance over which the electrons are accelerated by the voltage V. The electron density drops with increasing L as $n = (4/9L^2)(\epsilon_0 V/e)$, so at the distance L at which the electron density equals the plasma density n_0, the current density predicted by the Child-Langmuir law is

$$j_{CL} = \sqrt{\frac{eV}{T}} e n_0 v_e. \tag{C9}$$

When strong electron emission from the wall structures occurs, the heat flux can be limited by the electron thermal speed rather than C_s, which means it is a factor of $\sqrt{m_i/m_e}$ larger. The current density can be as large as $en_0 v_e/\sqrt{2\pi}$, which is about 24 times larger than the ion saturation current.

APPENDIX D: HALO MODEL

A halo can be fitted to a non-axisymmetric, fixed-boundary equilibrium code, such as VMEC [18], if the halo region is assumed to be far thinner than the plasma radius.

The boundary of the plasma in the fixed boundary equilibrium code, $\vec{x}_p(\theta, \varphi)$, is also the inner boundary of the halo. The coordinate system that will be used is $\vec{x}(r, \theta, \varphi) = \vec{x}_p(\theta, \varphi) + (r - a)\hat{n}$ where the unit vector $\hat{n} \propto (\partial \vec{x}_p/\partial\theta) \times (\partial \vec{x}_p/\partial\varphi)$. The Jacobian of these coordinates satisfies $\mathcal{J} = |(\partial \vec{x}_p/\partial\theta) \times (\partial \vec{x}_p/\partial\varphi)|$.

The magnetic field lies in constant r surfaces, so

$$\vec{B} = \vec{\nabla} \times (A\vec{\nabla}r) = -\frac{1}{\mathcal{J}} \frac{\partial A}{\partial\theta} \frac{\partial \vec{x}}{\partial\varphi} + \frac{1}{\mathcal{J}} \frac{\partial A}{\partial\varphi} \frac{\partial \vec{x}}{\partial\theta}. \tag{D1}$$

On the boundary of halo with the main plasma, the vector potential has the form

$$A_p(\theta,\varphi) = \left(\frac{d\psi_p}{dr}\right)_a \frac{\theta}{2\pi} - \left(\frac{d\psi_t}{dr}\right)_a \left(\frac{\varphi}{2\pi} + \frac{\lambda}{2\pi}\right), \tag{D2}$$

where λ is a single valued function of θ and φ, $(d\psi_p/dr)_a$ is the radial derivative of the poloidal flux, and $(d\psi_t/dr)_a$ is the radial derivative of the toroidal magnetic flux at the plasma boundary. Using this expression for the vector potential, the magnetic field of Equation (D1) can be written as $2\pi\vec{B} = \vec{\nabla}\psi_t \times \vec{\nabla}(\theta+\lambda) + \vec{\nabla}\varphi \times \vec{\nabla}\psi_p$ on the surface of the main plasma.

To calculate the curl of the magnetic field, the covariant representation of $\vec{B}$ is required. Since $\vec{B}\cdot(\partial\vec{x}_s/\partial r) = 0$, $\vec{B} = B_\theta\vec{\nabla}\theta + B_\varphi\vec{\nabla}\varphi$, where

$$\begin{aligned} B_\theta \equiv \vec{B}\cdot\frac{\partial\vec{x}_s}{\partial\theta} &= \frac{g_{\varphi\varphi}}{\mathcal{J}}\frac{\partial A}{\partial\varphi} - \frac{g_{\theta\varphi}}{\mathcal{J}}\frac{\partial A}{\partial\theta} \\ B_\varphi \equiv \vec{B}\cdot\frac{\partial\vec{x}_s}{\partial\theta} &= \frac{g_{\theta\varphi}}{\mathcal{J}}\frac{\partial A}{\partial\varphi} - \frac{g_{\theta\theta}}{\mathcal{J}}\frac{\partial A}{\partial\theta} \end{aligned} \tag{D3}$$

The metric tensor $g_{\theta\varphi} \equiv (\partial\vec{x}/\partial\theta)\cdot(\partial\vec{x}/\partial\theta)$ *etc.* obeys the identity $g_{\theta\theta}g_{\varphi\varphi} - g_{\theta\varphi}^2 = \mathcal{J}^2$.

The radial, or $\vec{j}\cdot\vec{\nabla}r = 0$, component of Ampere's law implies $\partial B_\theta/\partial\varphi = \partial B_\varphi/\partial\theta$. Therefore, the vector potential must satisfy a Laplacian-like equation, $\mathcal{L}[A] = 0$, within the surface

$$\begin{aligned} \mathcal{L}[A] \equiv\ & \frac{\partial}{\partial\theta}\left(\frac{g_{\theta\theta}}{\mathcal{J}}\frac{\partial A}{\partial\theta} - \frac{g_{\theta\varphi}}{\mathcal{J}}\frac{\partial A}{\partial\varphi}\right) \\ &+ \frac{\partial}{\partial\varphi}\left(\frac{g_{\varphi\varphi}}{\mathcal{J}}\frac{\partial A}{\partial\varphi} - \frac{g_{\varphi\theta}}{\mathcal{J}}\frac{\partial A}{\partial\theta}\right) \\ =\ & 0. \end{aligned} \tag{D4}$$

The current is force free, so $\vec{\nabla}\times\vec{B} = k\vec{B}$, where $k \equiv \mu_0 j_{||}/B$. Dotting both sides with $\vec{\nabla}\theta$, and using $k\vec{B}\cdot\vec{\nabla}\theta = k(\partial A/\partial\varphi)/\mathcal{J}$ and $(\vec{\nabla}\times\vec{B})\cdot\vec{\nabla}\theta = -(\partial B^\varphi/\partial r)/\mathcal{J}$, one obtains the first equation of the set

$$\begin{aligned} \frac{\partial B_\varphi}{\partial r} &= -k\frac{\partial A}{\partial\varphi}; \\ \frac{\partial B_\theta}{\partial r} &= -k\frac{\partial A}{\partial\theta}. \end{aligned} \tag{D5}$$

The second equation is obtained by dotting $\vec{\nabla}\times\vec{B} = k\vec{B}$ with $\vec{\nabla}\varphi$. Assuming the metric tensor is independent of radius and using Equation (D3),

$$\begin{aligned} \frac{g_{\theta\varphi}}{\mathcal{J}}\frac{\partial A'}{\partial\varphi} - \frac{g_{\theta\theta}}{\mathcal{J}}\frac{\partial A'}{\partial\theta} &= -k\frac{\partial A}{\partial\varphi} \\ \frac{g_{\varphi\varphi}}{\mathcal{J}}\frac{\partial A'}{\partial\varphi} - \frac{g_{\theta\varphi}}{\mathcal{J}}\frac{\partial A'}{\partial\theta} &= -k\frac{\partial A}{\partial\theta}, \end{aligned} \tag{D6}$$

where the prime is a radial derivative. These two equations can be rewritten as

$$\begin{aligned}\frac{\partial A'}{\partial \theta} &= k\left(\frac{g_{\varphi\varphi}}{\mathcal{J}}\frac{\partial A}{\partial \varphi} - \frac{g_{\varphi\theta}}{\mathcal{J}}\frac{\partial A}{\partial \theta}\right) \\ \frac{\partial A'}{\partial \varphi} &= -k\left(\frac{g_{\theta\theta}}{\mathcal{J}}\frac{\partial A}{\partial \theta} - \frac{g_{\theta\varphi}}{\mathcal{J}}\frac{\partial A}{\partial \varphi}\right)\end{aligned} \tag{D7}$$

and have a consistency constraint: the θ derivative of the second must equal the φ derivative of the first. This constraint is equivalent to Equation (D4).

Assuming k is independent of radius, an algebraic elimination of the $\partial A/\partial\theta$ terms in Equation (D6) implies $\partial A''/\partial\varphi = -k^2\partial A/\partial\varphi$ using the identity $(g_{\theta\theta}g_{\varphi\varphi} - g_{\theta\varphi}^2)/\mathcal{J}^2 = 1$. Therefore Ampere's law is

$$\frac{\partial^2 A}{\partial r^2} = -k^2 A. \tag{D8}$$

Letting $x = r - a$, the general solution is

$$\begin{aligned}A &= A_p\cos(kx) + A_d\sin(kx) \\ A' &= -kA_p\sin(kx) + kA_d\cos(kx),\end{aligned} \tag{D9}$$

where $A_p(\theta,\varphi)$ is the vector potential on the plasma surface and $kA_d(\theta,\varphi)$ is the radial derivative of the vector potential as the main plasma is approached within the halo. Equation (D7) implies

$$\begin{aligned}\frac{\partial A_d}{\partial \theta} &= \frac{g_{\varphi\varphi}}{\mathcal{J}}\frac{\partial A_p}{\partial \varphi} - \frac{g_{\varphi\theta}}{\mathcal{J}}\frac{\partial A_p}{\partial \theta} \\ \frac{\partial A_d}{\partial \varphi} &= -\frac{g_{\theta\theta}}{\mathcal{J}}\frac{\partial A_p}{\partial \theta} + \frac{g_{\theta\varphi}}{\mathcal{J}}\frac{\partial A_p}{\partial \varphi}.\end{aligned} \tag{D10}$$

---

[1] ITER Physics Expert Group on Disruptions, Plasma Control, and MHD and ITER Physics Basis Editors, Nucl. Fusion **39**, 2251 (1999).

[2] T.C. Hender, J.C Wesley, J. Bialek, A. Bondeson, A.H. Boozer, R.J. Buttery, A. Garofalo, T.P Goodman, R.S. Granetz, Y. Gribov, O. Gruber, M. Gryaznevich, G. Giruzzi, S. Gunter, N. Hayashi, P. Helander, C.C. Hegna, D.F. Howell, D.A. Humphreys, G.T.A. Huysmans, A.W. Hyatt, A. Isayama, S.C. Jardin, Y. Kawano, A. Kellman, C. Kessel, H.R. Koslowski, R.J. La Haye, E. Lazzaro, Y.Q. Liu, V. Lukash, J. Manickam, S. Medvedev, V. Mertens, S.V. Mirnov, Y. Nakamura, G. Navratil, M. Okabayashi, T. Ozeki, R. Paccagnella, G. Pautasso, F. Porcelli,

V.D. Pustovitov, V. Riccardo, M. Sato, O. Sauter, M.J. Schaffer, M. Shimada, P. Sonato, E.J. Strait, M. Sugihara, M. Takechi, A.D. Turnbull, E. Westerhof, D.G. Whyte, R. Yoshino, H. Zohm and the ITPA MHD, Disruption and Magnetic Control Topical Group, Nucl. Fusion **47**, S128-S202 (2007). S161 to S197 article end)

[3] J. Wesson, *Tokamaks, Third Edition* (Oxford University Press, New York, 2004).

[4] R. O. Sayer, Y.-K. M. Peng, S. C. Jardin, A. G. Kellman, and J. C. Wesley, Nucl. Fusion **33**, 969 (1993).

[5] I. Bandyopadhyay, S. Gerhardt, S. C. Jardin, R. O. Sayer, Y. Nakamura, S. Miyamoto, G. Pautasso, and M. Sugihara, and the ASDEX Upgrade and NSTX Teams, *TSC modelling of major disruption and VDE events in NSTX and ASDEX-Upgrade and predictions for ITER,* poster ITR/P1-16 at the IAEA Fusion Energy Conference, Daejon, Korea, October 2010.

[6] M. Sugihara, M. Shimada, H. Fujieda, Y. Gribov, K. Ioki, Y. Kawano, R. Khayrutdinov, V. Lukash, and J. Ohmori, Nucl. Fusion **47**, 337 (2007).

[7] H. R. Strauss, R. Paccagnella, and J. Breslau, Phys. Plasmas **17**, 082505 (2010).

[8] R. Paccagnella, H. R. Strauss, and J. Breslau, Nucl. Fusion **49**, 035003 (2009).

[9] S. E. Kruger, D. D. Schnack, and C. R. Sovinec, Phys. Plasmas **12**, 056113 (2005).

[10] V.A. Izzo, Nucl. Fusion **46** 541 (2006). depth on disruption mitigation by massive high-pressure gas jet

[11] V.A. Izzo and P.B. Parks, Nucl. Fusion **50**, 058001(2010). tokamak disruptions

[12] L. E. Zakharov, Phys. Plasmas **15**, 062507 (2008). events in tokamaks

[13] L. E. Zakharov, Phys. Plasmas **17**, 124703 (2010).

[14] S.C. Jardin, N. Ferraro, X. Luo, J. Chen, J. Breslau, K.E. Jansen, and M.S. Shepard, J. Phys.: Conf. Series 125, 012044 (2008).

[15] D. Mikkelsen, Phys. Fluids B **1**, 333 (1989).

[16] G. F. Counsell, R. Martin, T. Pinfold, and D. Taylor, Plasma Physics Controlled Fusion **49**, 435 (2007). on MAST

[17] P. C. Stangeby, *The Plasma Boundary of Magnetic Fusion Devices*, (Institute of Physics, Philadelphia, 2000).

[18] S. P. Hirshman, W. I. van Rij and P. Merkel, Comput. Phys. Commun. **43**, 143 (1986).

[19] A. H. Boozer, Phys. Fluids **19**, 1210 (1976).

[20] A. H. Boozer, J. Plasma Phys. **35** 133 (1986).

[21] A. N. Simakov, L. Chacoń, and A. Zocco, Phys. Plasma **17**, 060701 (2010).

[22] P. Ricci, J. U. Brackbill, W. Daughton, and G. Lapenta, Phys. Plasmas **11**, 4102 (2004).

[23] J. F. Drake, M. Swisdak, K. M. Schoeffler, B. N. Rogers, and S. Kobayashi, Geophysical Research Letters **33**, L13105 (2006).

[24] P. H. Rutherford, Phys. Fluids **16**, 1903 (1973).

[25] A. H. Boozer, Phys. Fluids **27**, 2055 (1984).

[26] H. P. Furth, P. H. Rutherford, and H. Selberg, Phys. Fluids **16**, 1054 (1973).

[27] F. L. Waelbroeck, Phys. Rev. Lett. **70**, 3259 (1993).

[28] N. F. Loureiro, S. C. Cowley, W. D. Dorland, M. G. Haines, and A. A. Schekochihin, Phys. Rev. Lett. textbf 95, 235003 (2005). Reconnection

[29] A. H. Boozer and N. Pomphrey, Phys. Plasmas **17**, 110707 (2010).

[30] B.V.Chirikov, At. Energ. **6**, 630 (1959) (in Russian); J. Nucl. Energy Part C: Plasma Phys. **1** 253 (1960) (in English).

[31] J. K. Park, A. H. Boozer, and A. H. Glasser, Phys. Plasmas **14**, 052110 (2007).

[32] R. Chadura, Phys. Fluids **25**, 1828 (1982).