noble analysis of radial electric field formation, turbulence transport, and H-mode transition based on the gyrocenter shift (GCS)

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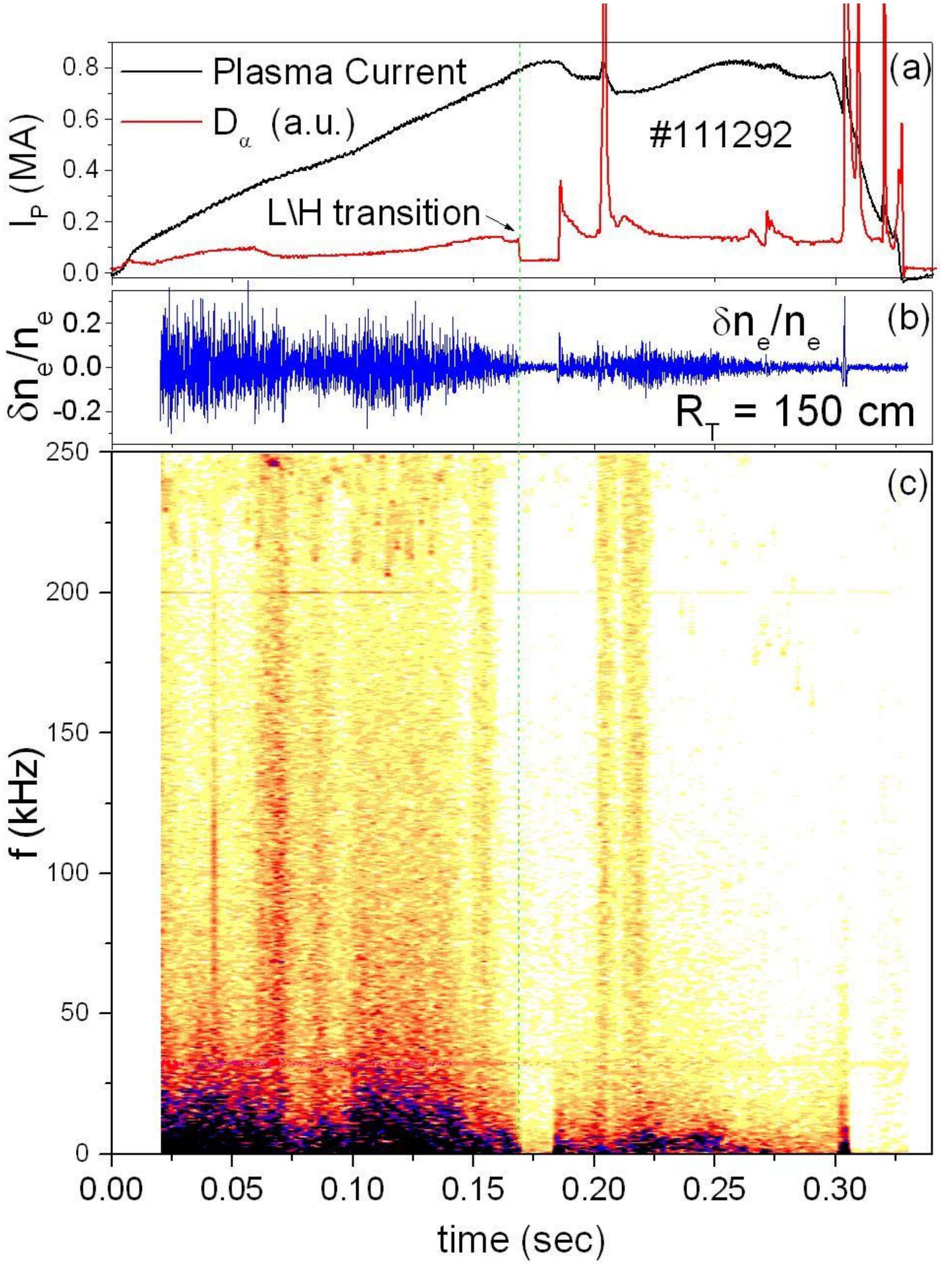
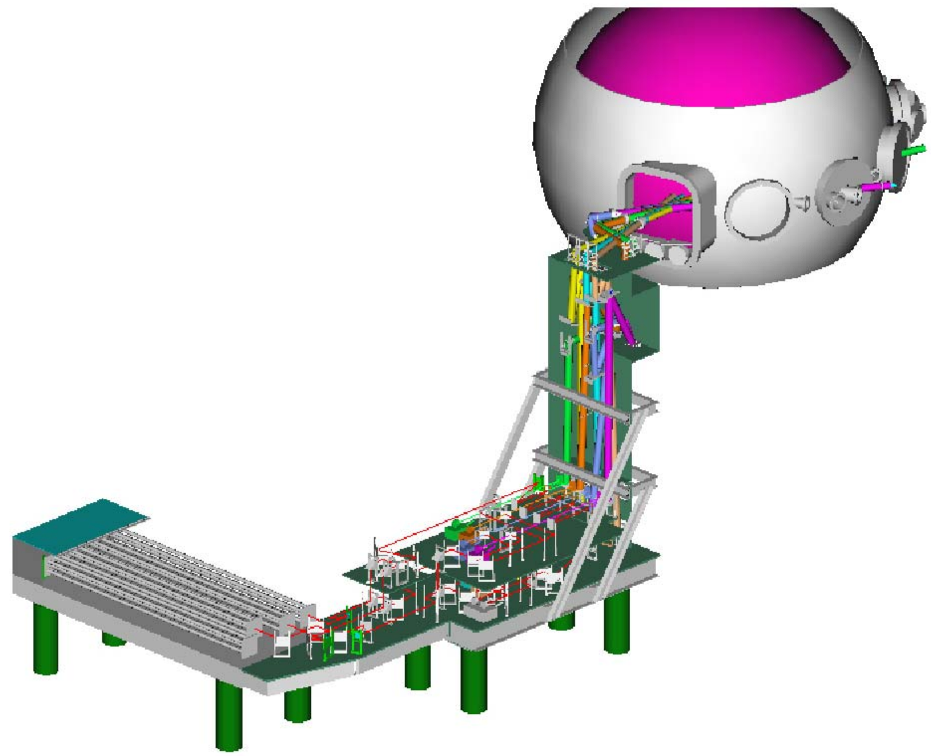
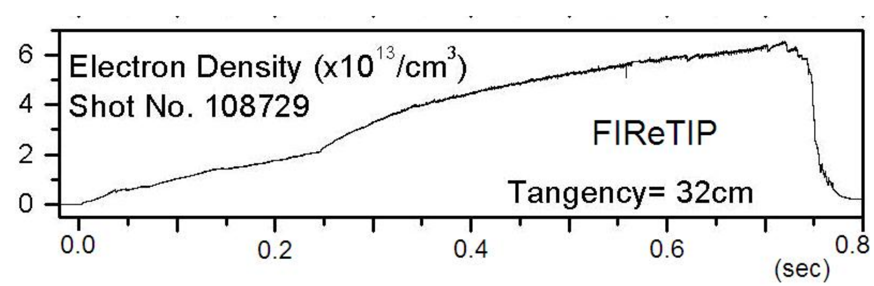


Plasma Current
D_α (a.u.)
(a)
#111292
L\H transition
I_P (MA)
$\delta n_e/n_e$
(b)
R_T = 150 cm
(c)
f (kHz)
time (sec)





Electron Density ($x10^{13}/cm^3$)
Shot No. 108729
FIReTIP
Tangency= 32cm
(sec)

Introduction

cross field plasma transport

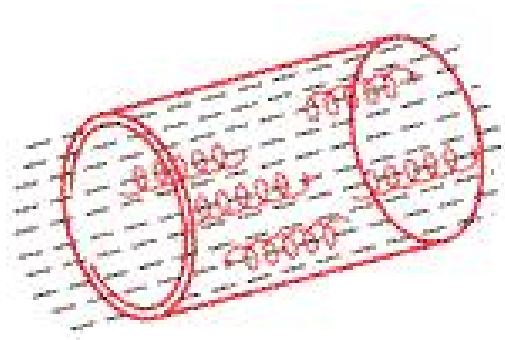
theories:
- ► classical diffusion
- ► neo-classical diffusion

experiments:
- ► Bohm diffusion(1946)
- ► anomalous transport

(100~1000 times higher than theory)

↓

turbulence transport (ITG, ETG…)



high confinement mode (H-mode)

experimental discovery:
- ► ASDEX (1982)

theories:
- ► turbulence suppression ($E_r x B_T$ shearing)

↓

E_r formation: Gyro-Center Shift (GCS) (Lee, PoP 2006)

poloidal momentum balance

$$0 = j_r B / n_i - m_i \mu_\theta \upsilon_{\theta i} - \underline{m_i \nu_n \upsilon_{\theta i}} + m_i \partial / \partial r (\langle \tilde{\upsilon}_{ri} \cdot \tilde{\upsilon}_{\theta i} \rangle)$$

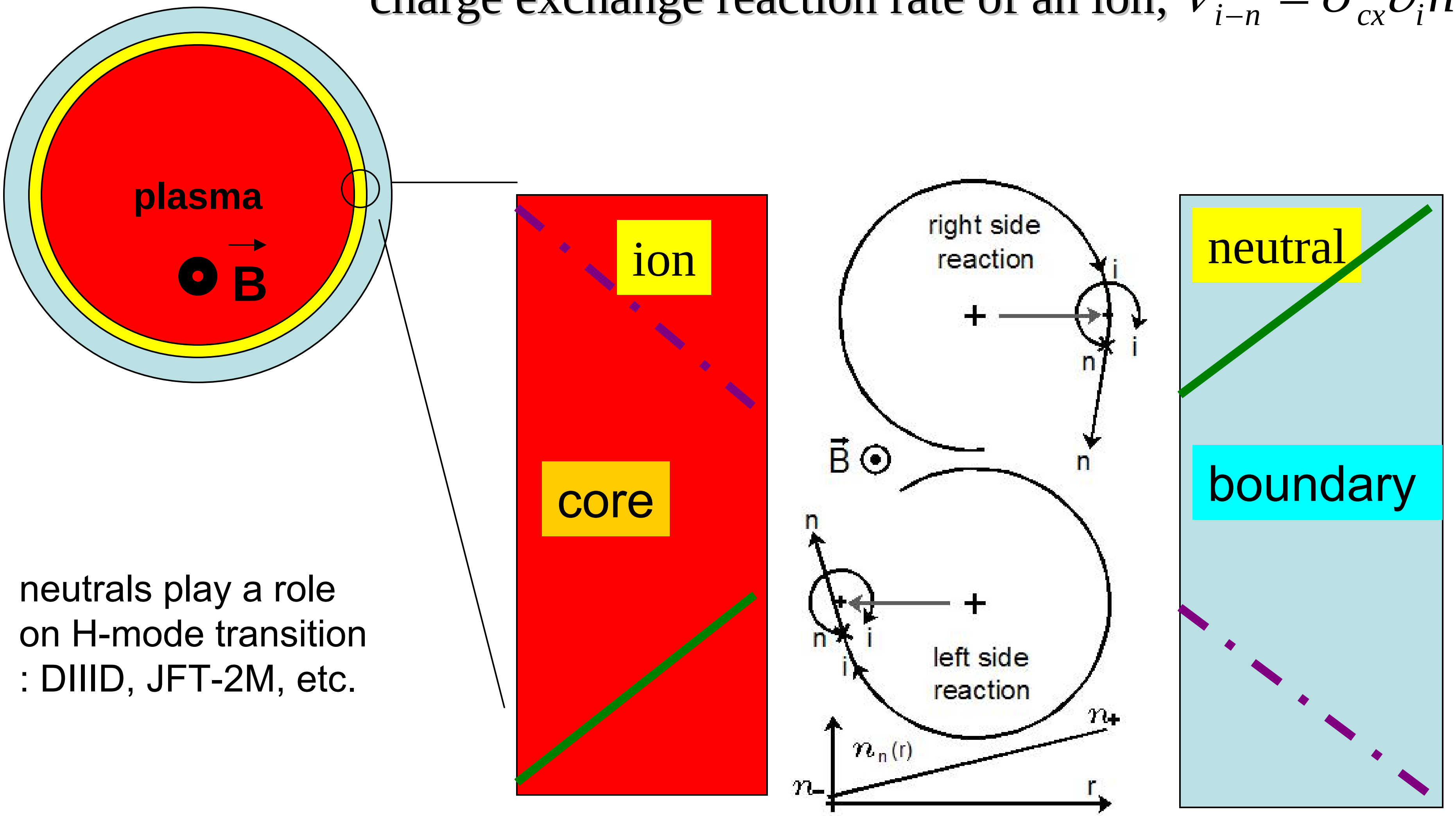
(Wagner, PPCF 2007)

turbulence diffusion by GCS

H-mode transition by GCS

Gyrocenter shift due to charge exchange

charge exchange reaction rate of an ion; $\nu_{i-n} = \sigma_{cx} \upsilon_i n_n$

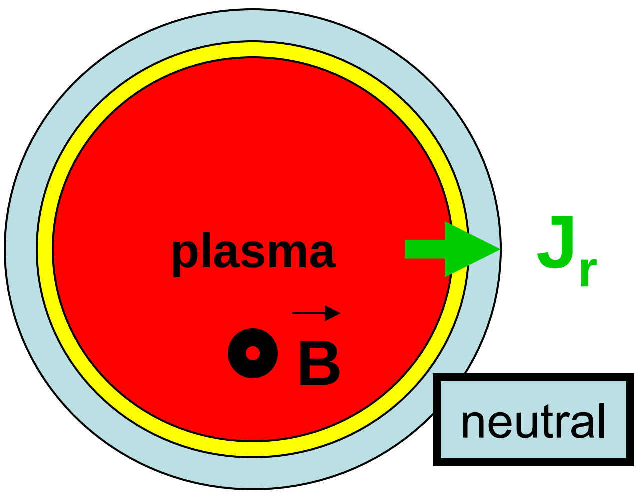


neutrals play a role on H-mode transition : DIIID, JFT-2M, etc.

Introduction to gyrocenter shift

momentum exchange of ion-neutral collisions $\rightarrow$ J_r
(charge exchange / elastic scattering)

$$J \times B = n_i \nu_{i-n} S_i^m$$

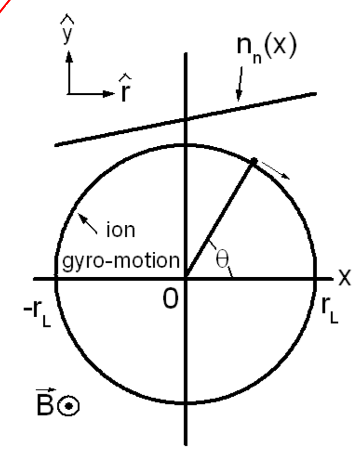


$$J_r^{GCS} = \frac{n_i \sigma_{cx} \upsilon_{i\perp} n_n}{B} m_i \Big(\underbrace{\frac{E}{B} - \frac{1}{eB}\frac{\nabla P_i}{n_i} + \frac{kT_i}{eB}\frac{\nabla n_n}{n_n}}_{v^*} \Big)$$

$$\upsilon_{E\times B} = \frac{E}{B}$$

$$\upsilon_D = -\frac{1}{eBn_i}\frac{\partial P_i}{\partial r}$$

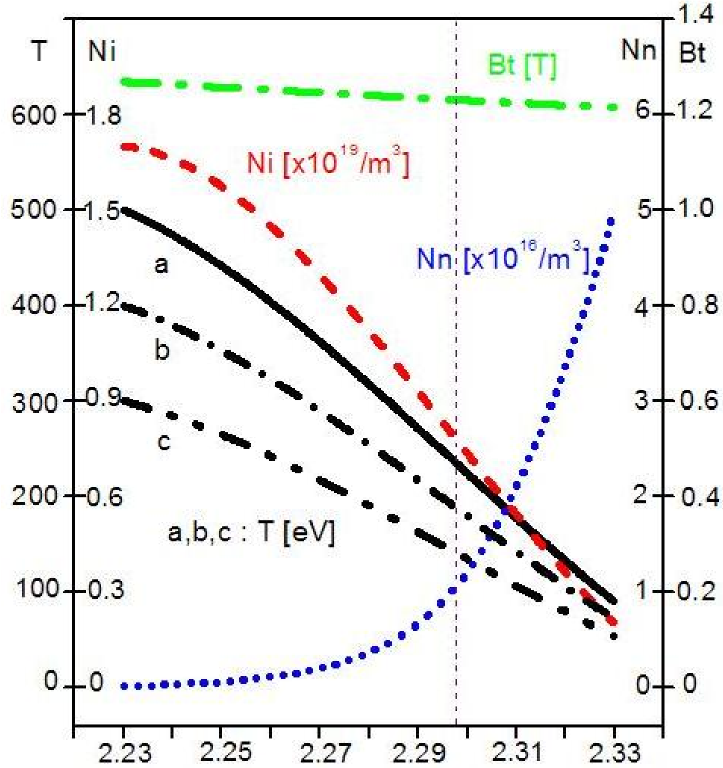
ExB drift is in opposite direction
=> return current (E_r saturation)



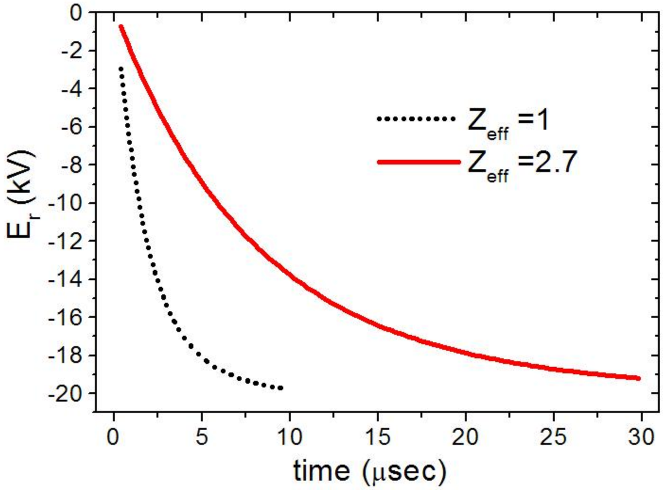
$$\upsilon_{av}^* = \frac{\sigma_{cx}\upsilon_{i\perp}\oint \vec{\upsilon}_{i\perp}(\theta) n_n(\theta) d\theta}{\sigma_{cx}\upsilon_{i\perp}\oint n_n(\theta) d\theta} = \boxed{\frac{1}{2} r_{Li}\upsilon_{i\perp}\frac{1}{n_n}\frac{\partial n_n}{\partial r}}$$

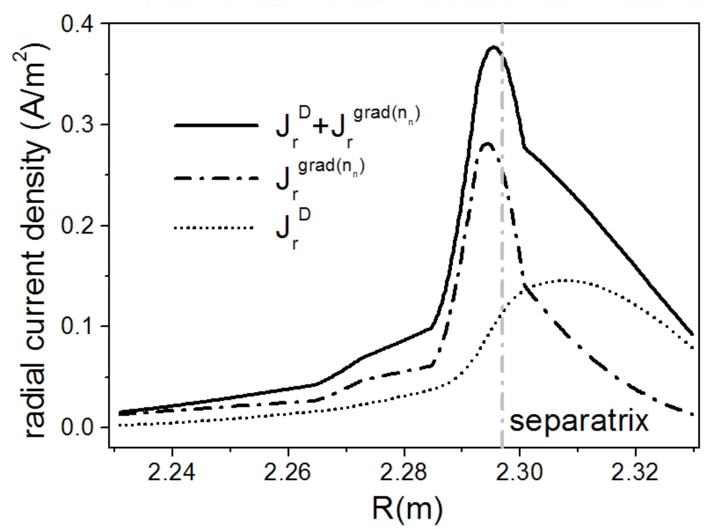
DIII-D example E_r calculation

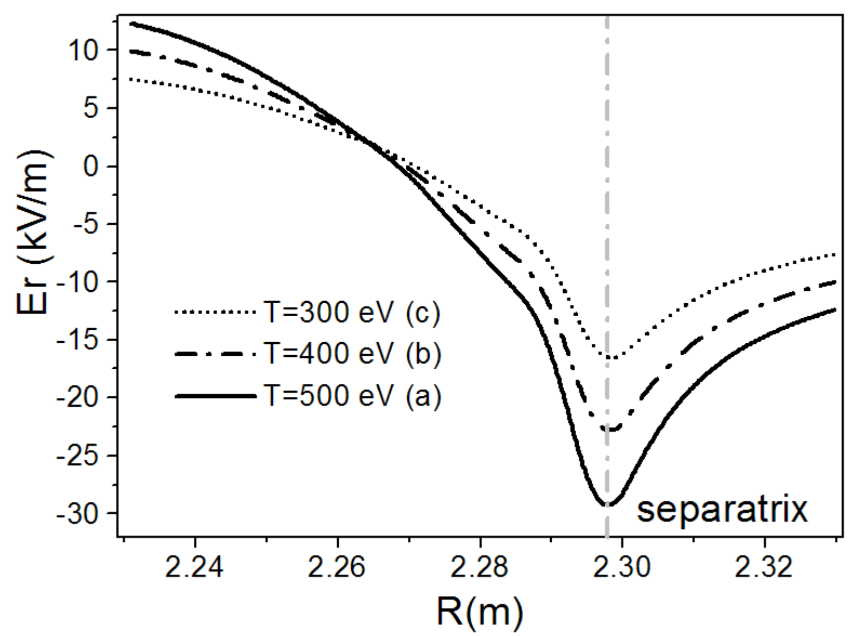
$$J_r^{GCS} = en_i \frac{r_{Li}}{\lambda_{i-n}} \left(\frac{E}{B} - \frac{1}{eB} \frac{\nabla P_i}{n_i} + \frac{kT_i}{eB} \frac{\nabla n_n}{n_n} \right)$$



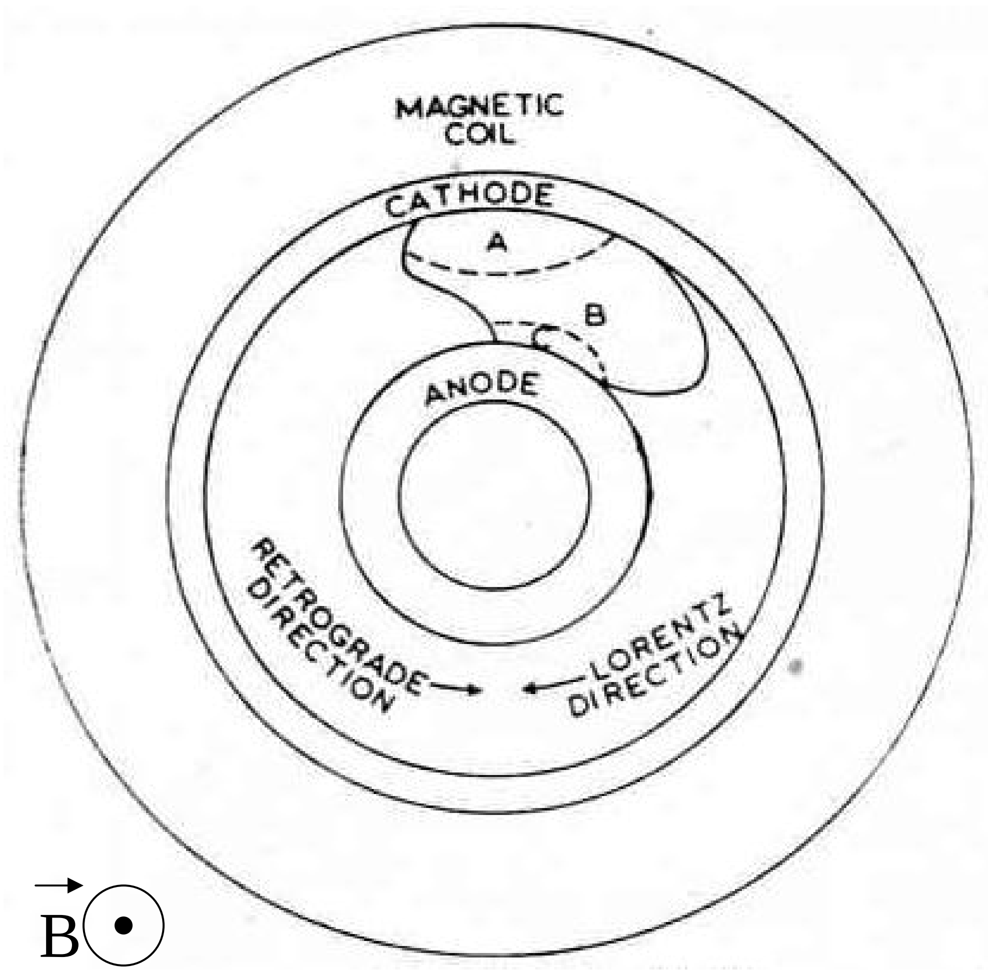
(Carreras, etc., PoP 98')







Retrograde motion of arc cathode



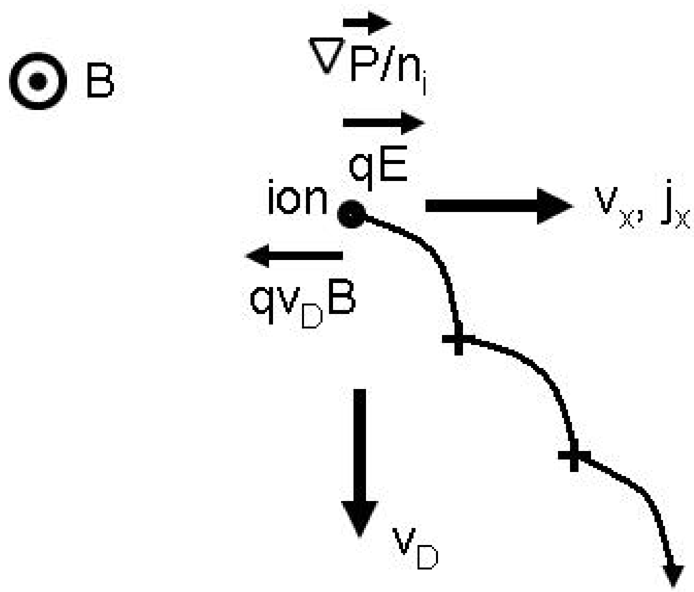
A. CATHODIC ARC REGION
B. POSITIVE COLUMN
C. ANODIC ARC REGION

► cathode spot moves opposite (retrograde) direction under B-field

► retrograde motion was noticed by Stark (1903)

► no satisfactory explanation despite numerous attempts.

Generalization for high collision case



$$v_d = \frac{1}{1 + r_L^2 / \lambda_{cx}^2} \left(\frac{E}{B} - \frac{\nabla p}{qBn_i} \right)$$

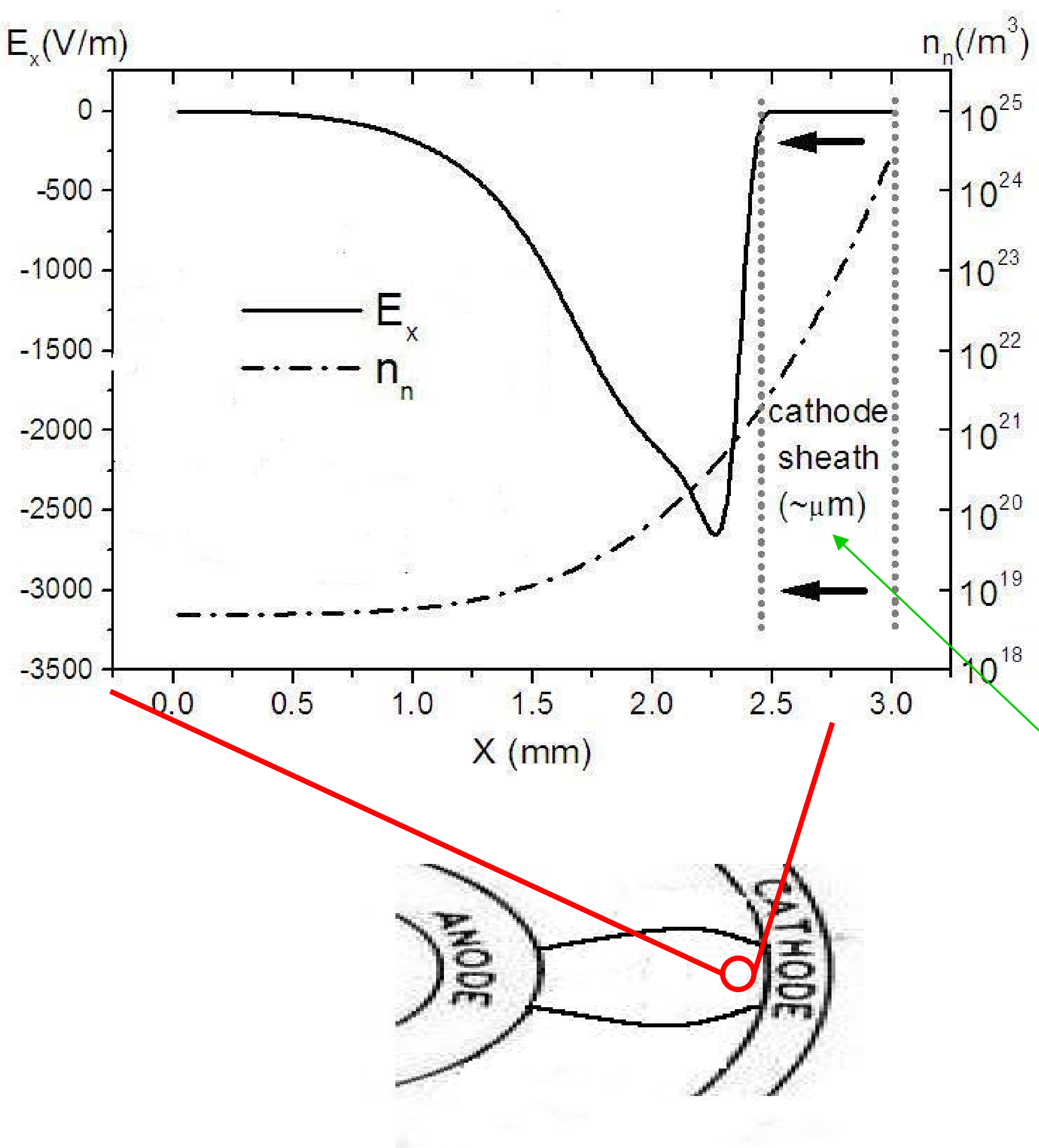
- only ions have longer λ_{cx} than r_L contribute to gyrocenter shift

- $\dfrac{\text{number of ions that contribute}}{\text{total number of ions}}$ $= e^{-r_L / \lambda_{cx}}$

general formula for the gyrocenter shift

$$J_x^{GCS} = \frac{m_i n_i n_n}{B} < \sigma_{cx} v_i > \left[\frac{1}{1 + r_L^2 / \lambda_{cx}^2} \left(\frac{E}{B} - \frac{\nabla p}{qBn_i} \right) + \frac{T_i \nabla n_n}{qBn_n} e^{-r_L / \lambda_{cx}} \right]$$

Calculation of electric field in arc discharge

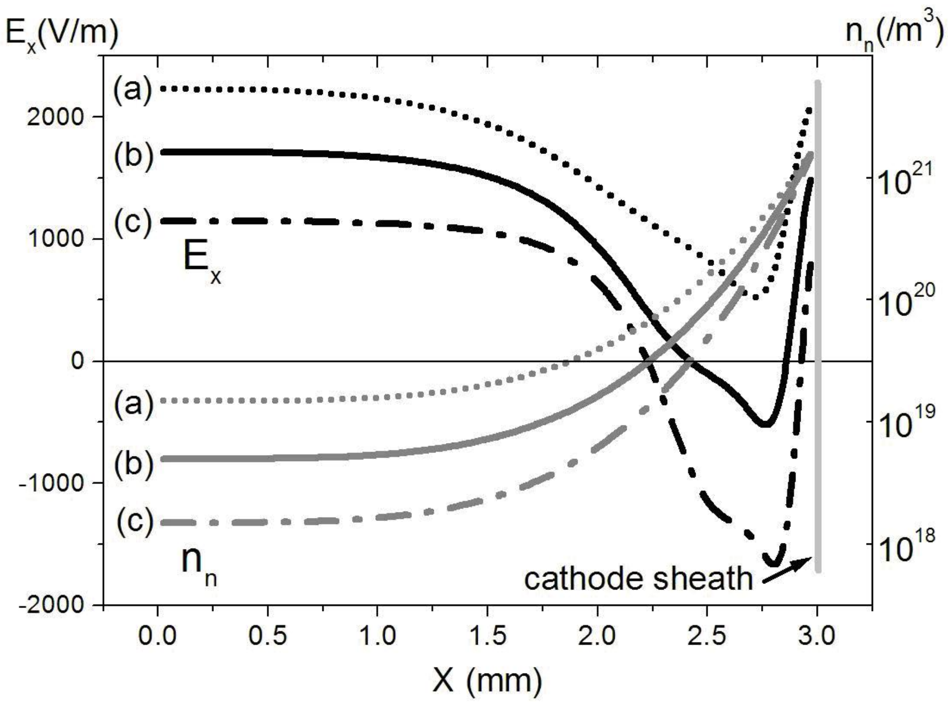


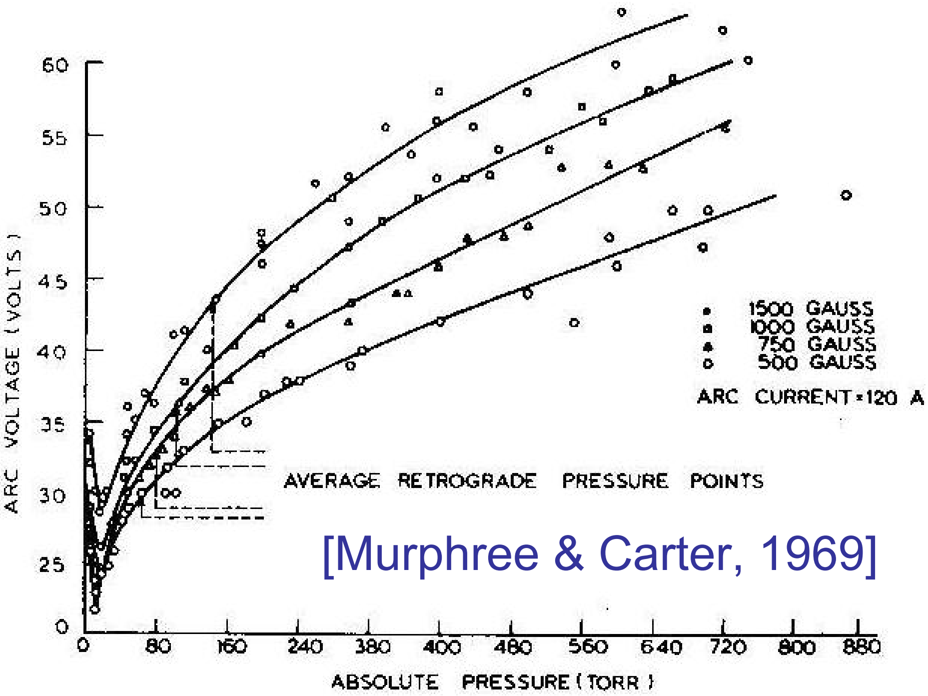
- no background E-field
- constant n_i ($5x10^{22}/m^3$)
- constant T_i (0.5 eV)
- B = 0.1 T
- gas pressure : ~100 Torr (Argon)
- gap :16.5 mm
- n_n is an exponential function with its gradient approach zero at middle of the discharge

- reversed electric field is formed
- E_x vanishes when
- $n_n > \sim 10^{21}/m^3$

► cathode sheath : massive ionizations take place (~µm) where rapid decrease of neutral and increase of ion

Calculation with arc column electric field



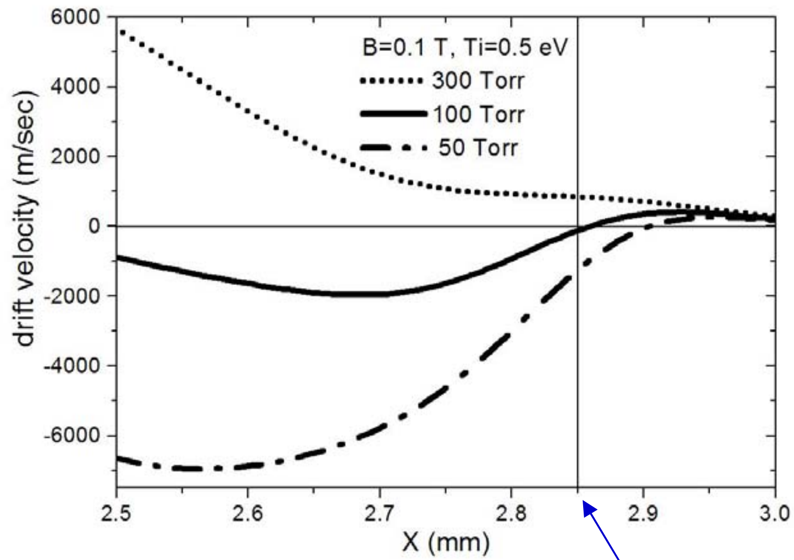


► higher neutral density → higher column electric field (constant current)

⇒ high gas pressure → positive electric field in front of cathode
low gas pressure → negative electric field in front of cathode

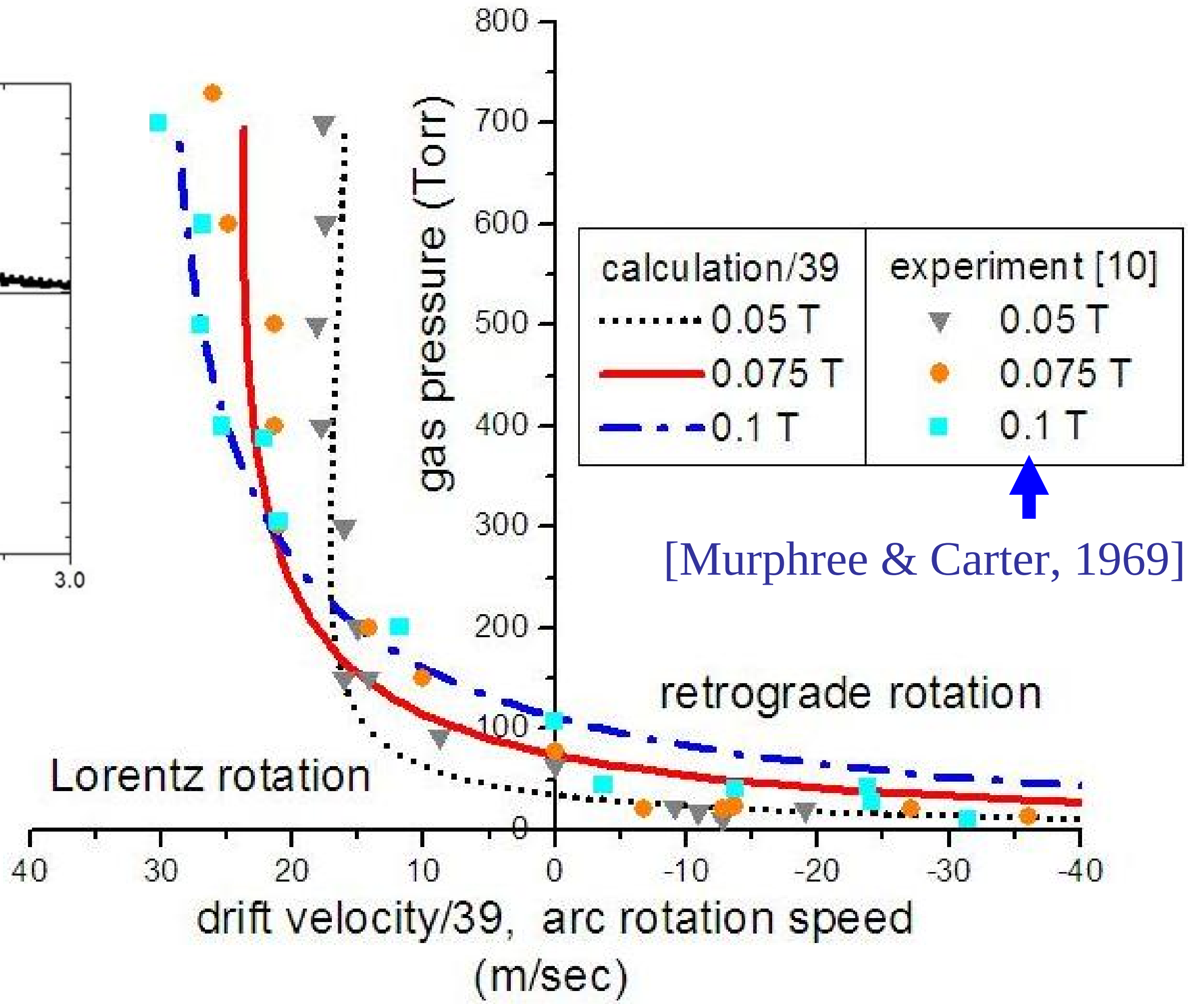
► negative electric field (seems unnatural) : gyrocenter shift is a process of putting ions in a direction which is independent of electric force

Comparison of calculation with experiment



1/20 of neutral decay length

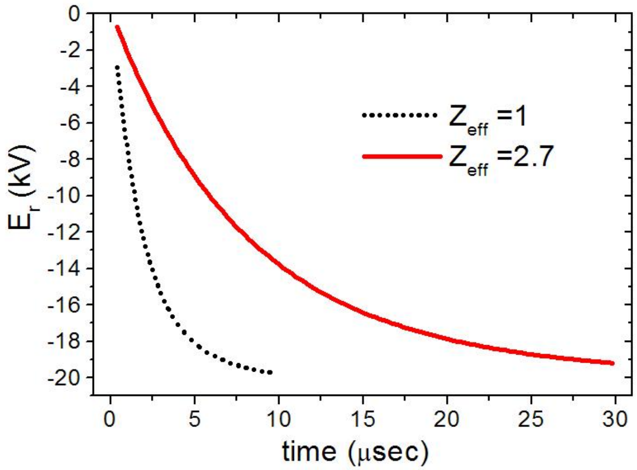
$$v_D = \frac{1}{1 + r_L^2 / \lambda_{cx}^2} \left(\frac{E}{B}\right)$$



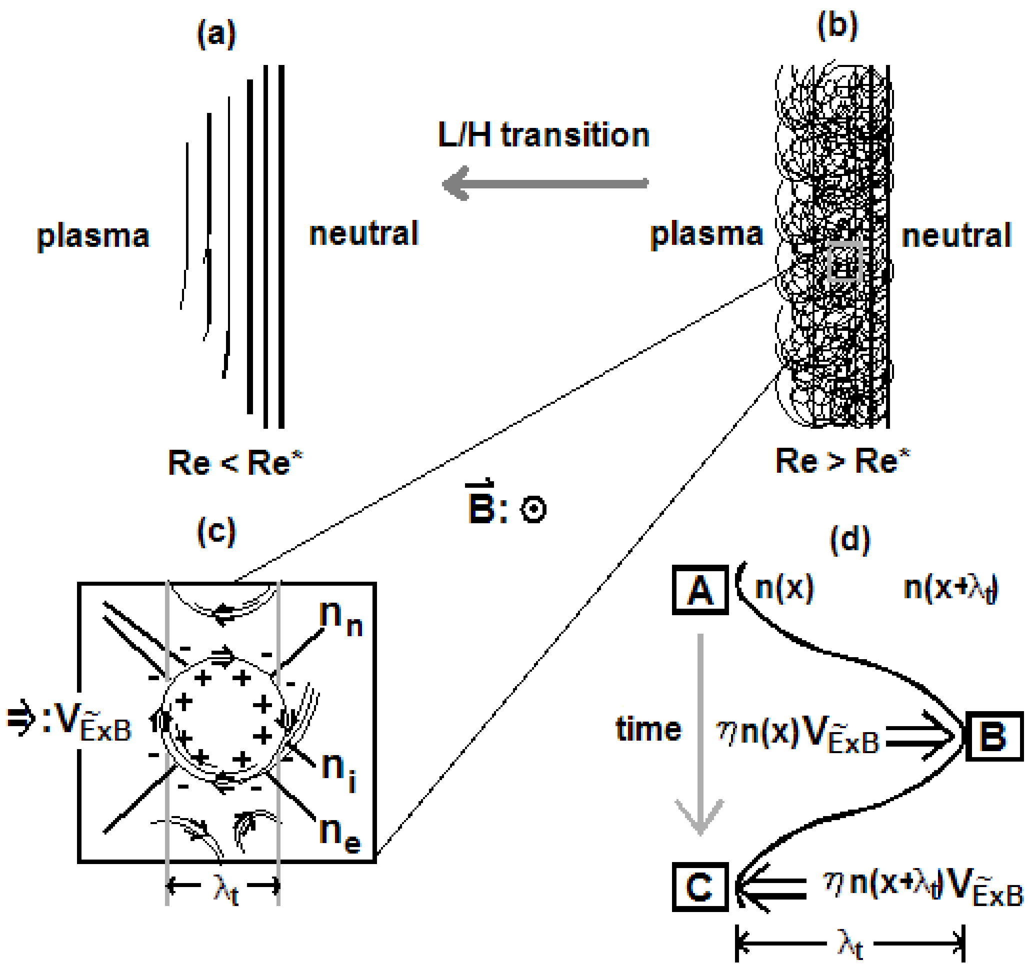
[Murphree & Carter, 1969]

[K.C. Lee, PRL, Vol 99, 065003 (2007)]

$$J_r^{GCS} = en_i \frac{r_{Li}}{\lambda_{i-n}} \left(\frac{E}{B} - \frac{1}{eB} \frac{\nabla P_i}{n_i} + \frac{kT_i}{eB} \frac{\nabla n_n}{n_n} \right)$$



Turbulence induced diffusion and E_r saturation condition of GCS

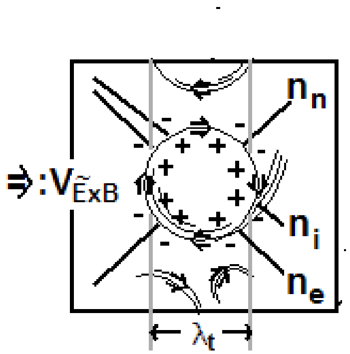


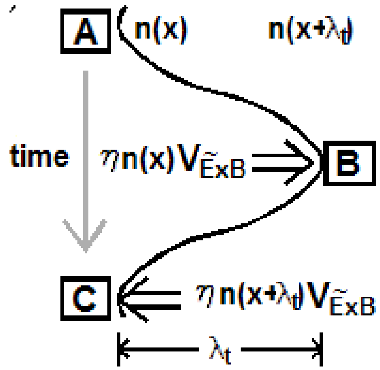
► J_r and E_r saturate before J_r=0

► E_r saturates when ion movement is same as electron movement (ambipolar electric field => classical diffusion)

► only for ideal case of no density fluctuation

► turbulence induces real condition of E_r saturation





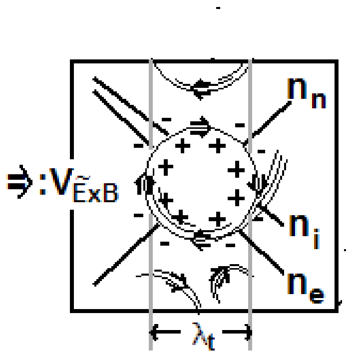
Turbulence induced diffusion of particles

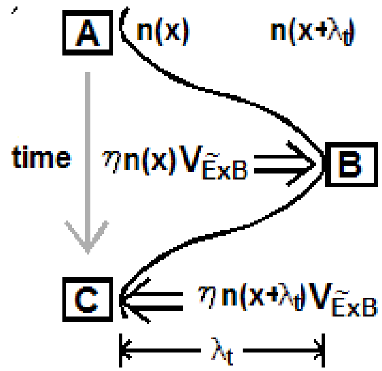
$$\eta \equiv \frac{\tilde{n}}{n}, \quad n' \equiv \frac{\partial n}{\partial x} < 0$$

	x	$x+\lambda_t$
[A]	$n_{i,e}(x) \equiv n_{i,e}$	$n_{i,e}(x+\lambda_t) = n_{i,e} + \lambda_t n'_{i,e}$
[B]	$n_{i,e} - \eta n_{i,e}$	$n_{i,e} + \lambda_t n'_{i,e} + \eta n_{i,e}$
[C]	$n_{i,e} - \cancel{\eta n_{i,e}} + \cancel{\eta n_{i,e}} + \eta\lambda_t n'_{i,e} + \cancel{\eta^2 n_{i,e}}$ $\approx n_{i,e} + \eta\lambda_t n'_{i,e} = n_{i,e}(x) + \eta\lambda_t n'_{i,e}$	$n_{i,e} + \lambda_t n'_{i,e} + \cancel{\eta n_{i,e}} - \cancel{\eta n_{i,e}} - \eta\lambda_t n'_{i,e} - \cancel{\eta^2 n_{i,e}}$ $\approx n_{i,e} + \lambda_t n'_{i,e} - \eta\lambda_t n'_{i,e} = n_{i,e}(x+\lambda_t) - \eta\lambda_t n'_{i,e}$

► net movement of one cycle is $\eta\lambda_t \nabla n$: same result from L-R-L and R-L-R cycles

► diffusion takes place from high density region to low density region





Turbulence induced diffusion of charge

$$\eta \equiv \frac{\tilde{n}}{n}, \quad n' \equiv \frac{\partial n}{\partial x} < 0$$

	x	$x+\lambda_t$
[A]	$\rho(x) = e(n_i - n_e) \equiv \rho$	$\rho(x+\lambda_t) = \rho + \lambda_t e(n_i' - n_e')$
[B]	$\rho - \eta\rho$	$\rho + \lambda_t e(n_i' - n_e') + \eta\rho$
[C]	$\rho(x) + \eta\lambda_t e(n_i' - n_e')$	$\rho(x+\lambda_t) - \eta\lambda_t e(n_i' - n_e')$

► turbulence induced ion and electron diffusion : $\eta\lambda_t \nabla n$

► turbulence induced charge diffusion : $-\eta\lambda_t \nabla \rho$

- ► ion and electron move toward boundary => ***diffusion***
- ► charge (ρ) moves toward core => ***dilution current =>*** ***Saturation by J^{GCS}***

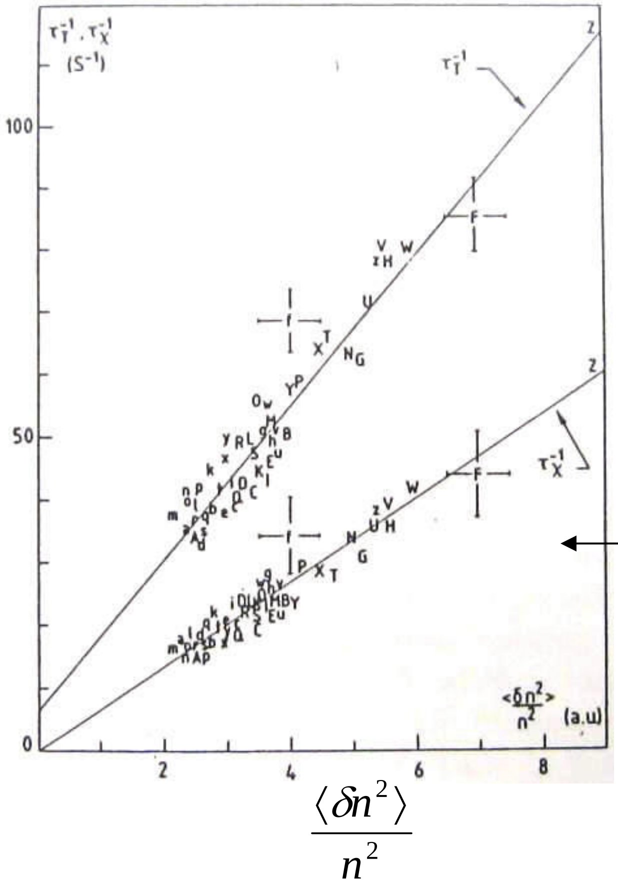
Turbulence induced diffusion coefficient

$$\Gamma = \partial n \cdot \tilde{\upsilon} \quad \Rightarrow \quad D = \frac{\eta}{\pi}\frac{\tilde{E}\lambda_t}{B} \quad \Rightarrow \quad D = \frac{2}{\pi}\eta^2\frac{kT_e}{eB}$$

$\partial n \approx \eta\lambda_t \nabla n$, $\tilde{\upsilon} \approx \frac{1}{\pi}\frac{\tilde{E}}{B}$

$$\tilde{E}\lambda_t \approx 2\eta\frac{kT_e}{e} \leftarrow \left(\frac{e\tilde{\phi}_t}{kT_e} \approx \frac{\tilde{n}_e}{n_e}\right.$$: Boltzmann relation, $\left.\tilde{\phi}_t \approx \tilde{E}\frac{\lambda_t}{2}\right)$

$$D = \frac{2}{\pi}\eta^2\frac{kT_e}{eB}$$



- ► $\propto \frac{1}{B_T}$ and similar to Bohm diffusion : $\propto \frac{kT_e}{eB}$
- ► proportional to η^2: agreed by experiments

[TFR group, Nuclear Fusion (1986)]
NSTX density fluctuation, APS poster: (2008)

- ► characteristics close to “anomalous” diffusion

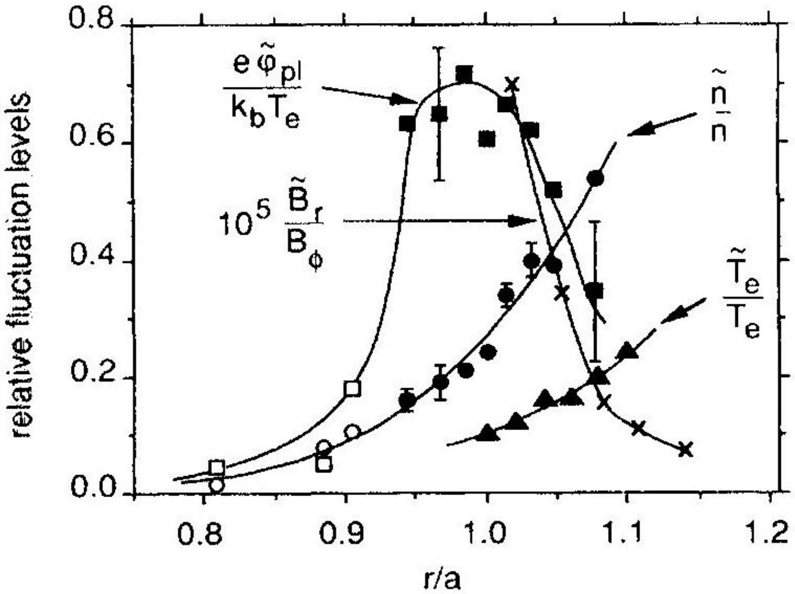
Modified Boltzmann relation

$$F = J_i^{GCS} \times B = m_i n_i \nu_{i-n} \left(\frac{\tilde{E}}{B} - \frac{1}{eB} \frac{\nabla P_i}{n_i} + \frac{kT_i}{eB} \frac{\nabla n_n}{n_n} \right) \approx 0$$

$$\frac{e\tilde{\phi}_t}{kT_e} \approx \frac{\tilde{n}_e}{n_e}$$

$$e\tilde{E} - kT_i \frac{\nabla n_i}{n_i} + kT_i \frac{\nabla n_n}{n_n} \approx 0$$

$$\left(-\frac{\nabla n_n}{n_n} \approx \frac{1}{L_{\tilde{n}}} \right)$$



[Ritz, TEXT, 1989]

$$\frac{e\tilde{E}}{kT_i} - \frac{1}{L_{\tilde{n}}} = \frac{\nabla n_i}{n_i}$$

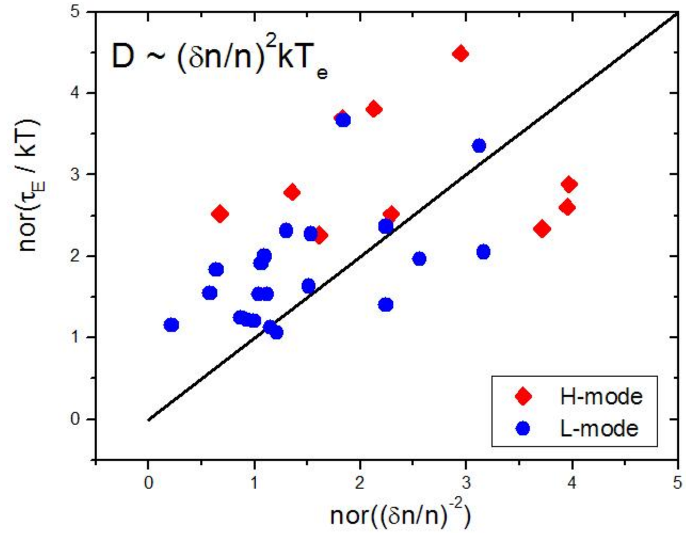
$$\frac{\tilde{n}}{n} = \frac{e\tilde{E}\lambda_t}{2kT_i} - \frac{\lambda_t}{2L_{\tilde{n}}}$$

$$D = \frac{2}{\pi} \eta \left(\eta + \frac{\lambda_t}{2L_{\tilde{n}}} \right) \frac{kT_i}{eB}$$

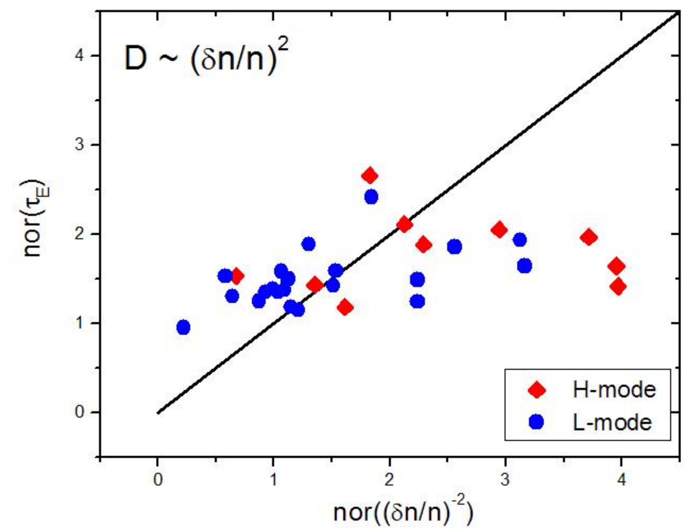
2008 Data NSTX

$$D = \frac{2}{\pi}\eta^2 \frac{kT_e}{eB}$$

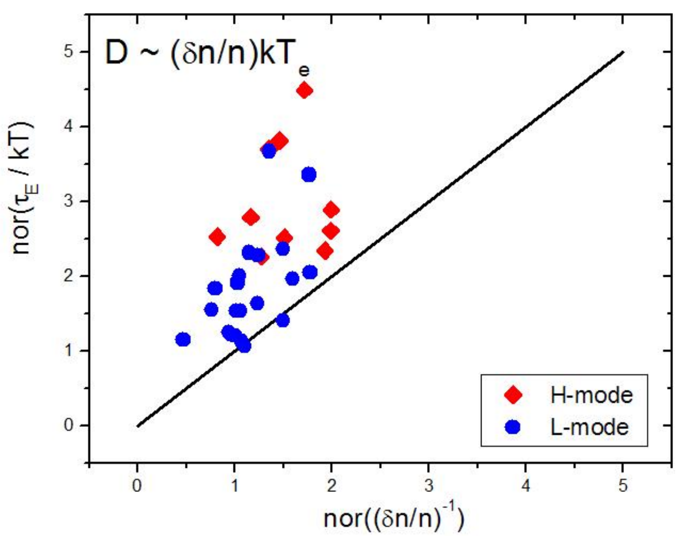
t_E with η^2 : including T_e effect



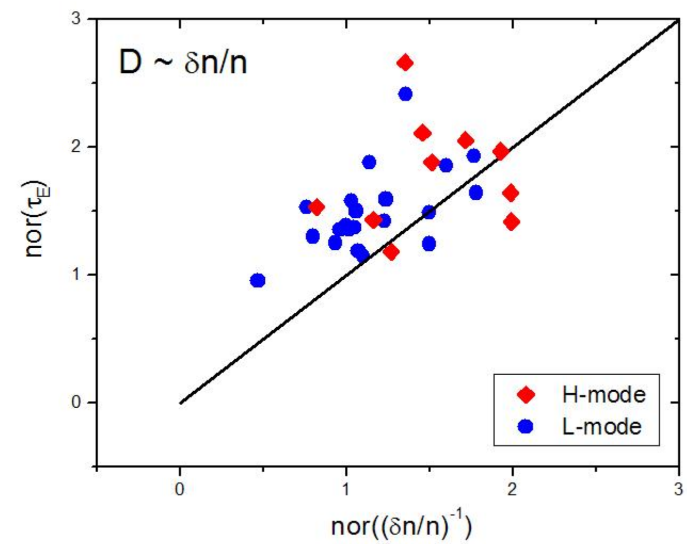
t_E with η^2 : no T_e effect



t_E with η : including T_e effect



t_E with η : no T_e effect

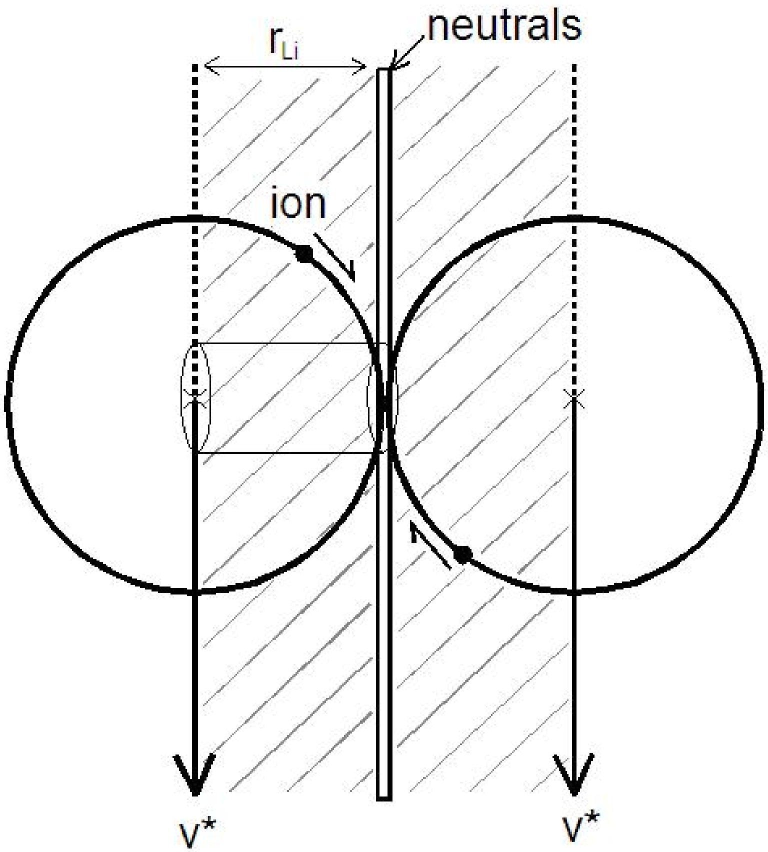


Turbulence

- ion and electron move toward boundary => ***diffusion***
- charge (ρ) moves toward core => ***dilution current =>*** ***saturation condition***

$$J_r^{GCS} = en_i \frac{r_{Li}}{\lambda_{i-n}} \left(\frac{E}{B} - \frac{1}{eB}\frac{\nabla P_i}{n_i} + \frac{kT_i}{eB}\frac{\nabla n_n}{n_n}\right)$$

v*



Inertia force

$$\mathrm{Re} \equiv \frac{n_i m_i \upsilon^{*2} / r_{Li}}{n_i m_i \nu_{i-n} \upsilon^*} = \frac{eB}{kT_i}\lambda_{i-n}\upsilon^*$$

viscosity force

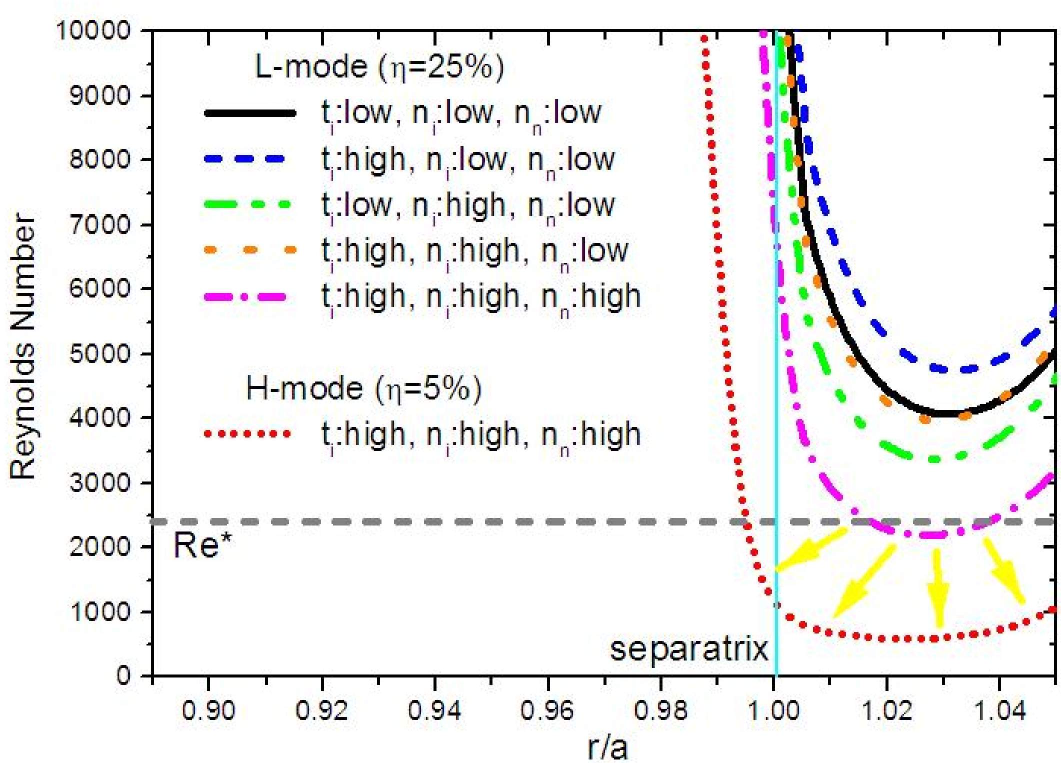
$$\mathrm{Re} = \frac{eB}{kT_i}\lambda_{i-n}\left(\frac{E}{B} - \frac{1}{eB}\frac{\nabla P_i}{n_i} + \frac{kT_i}{eB}\frac{\nabla n_n}{n_n}\right)$$

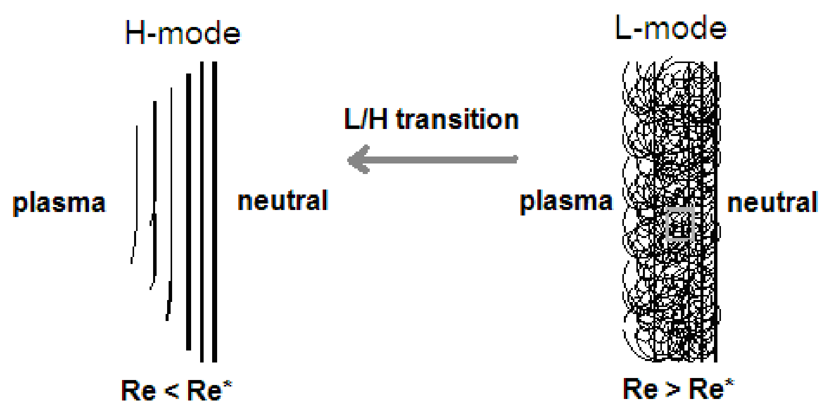
(saturation condition : $J_r^{GCS} = D\nabla\rho$)

Reynolds number of ion-neutral collision

$$\mathrm{Re} = \frac{2}{\pi}\eta^2 \frac{B}{m_i n_i (\sigma_{i-n} n_n)^2 \upsilon_\perp}\nabla\rho$$

L/H transition by critical Reynolds number





$$\mathrm{Re} = \frac{2}{\pi}\eta^2 \frac{B}{m_i n_i (\sigma_{i-n} n_n)^2 \upsilon_\perp} \nabla\rho$$

► Re > Re* : turbulent flow
Re < Re* : laminar flow

(Re*~2400)

► turbulent flow (L-mode): high η
laminar flow (H-mode): low η

► plasma heating & neutrals
=> Reynolds number
=> L/H power threshold

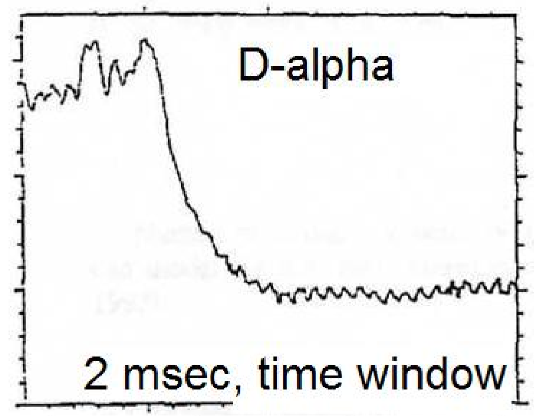
► P_{th} dependence on
neutral density , **isotopes**
=> agrees to experiments

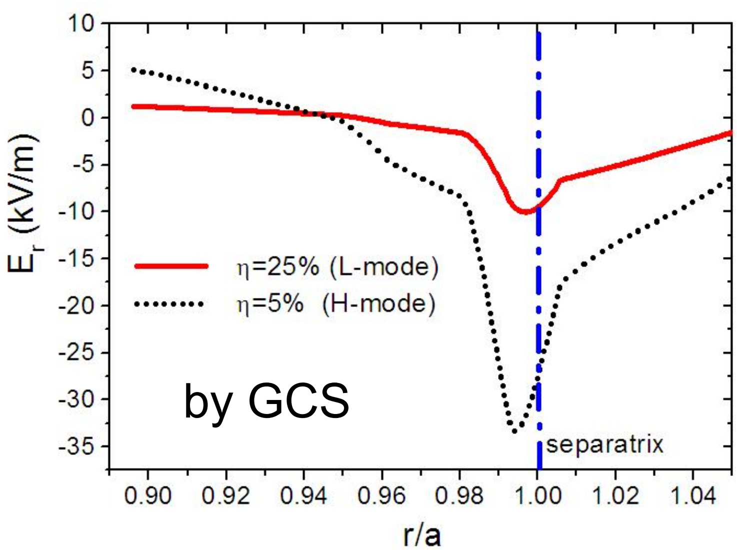
fast and slow changes of H-mode transition

$$\mathrm{Re} = \frac{2}{\pi}\eta^2 \frac{B}{m_i n_i (\sigma_{i-n} n_n)^2 \upsilon_\perp} \nabla \rho$$

		L-mode	H-mode
fast change (~50 μsec)	η v*(η) E_r	high high low	low => Re ↓ low => Re ↓ high => Re ↑ (deeper saturation)
slow change (~ msec)	$\nabla P, \nabla n_n$ E_r	low low	high => Re ↑ High back transition

clear H-mode





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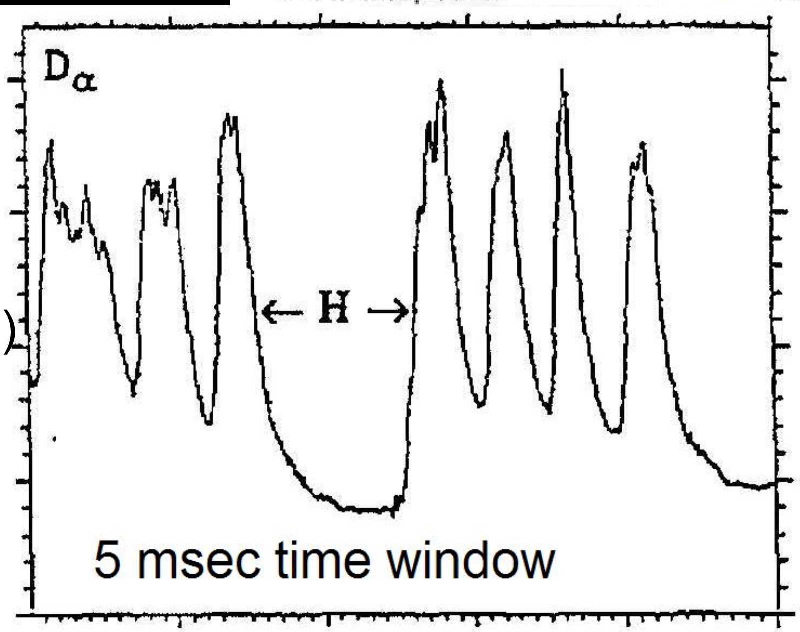
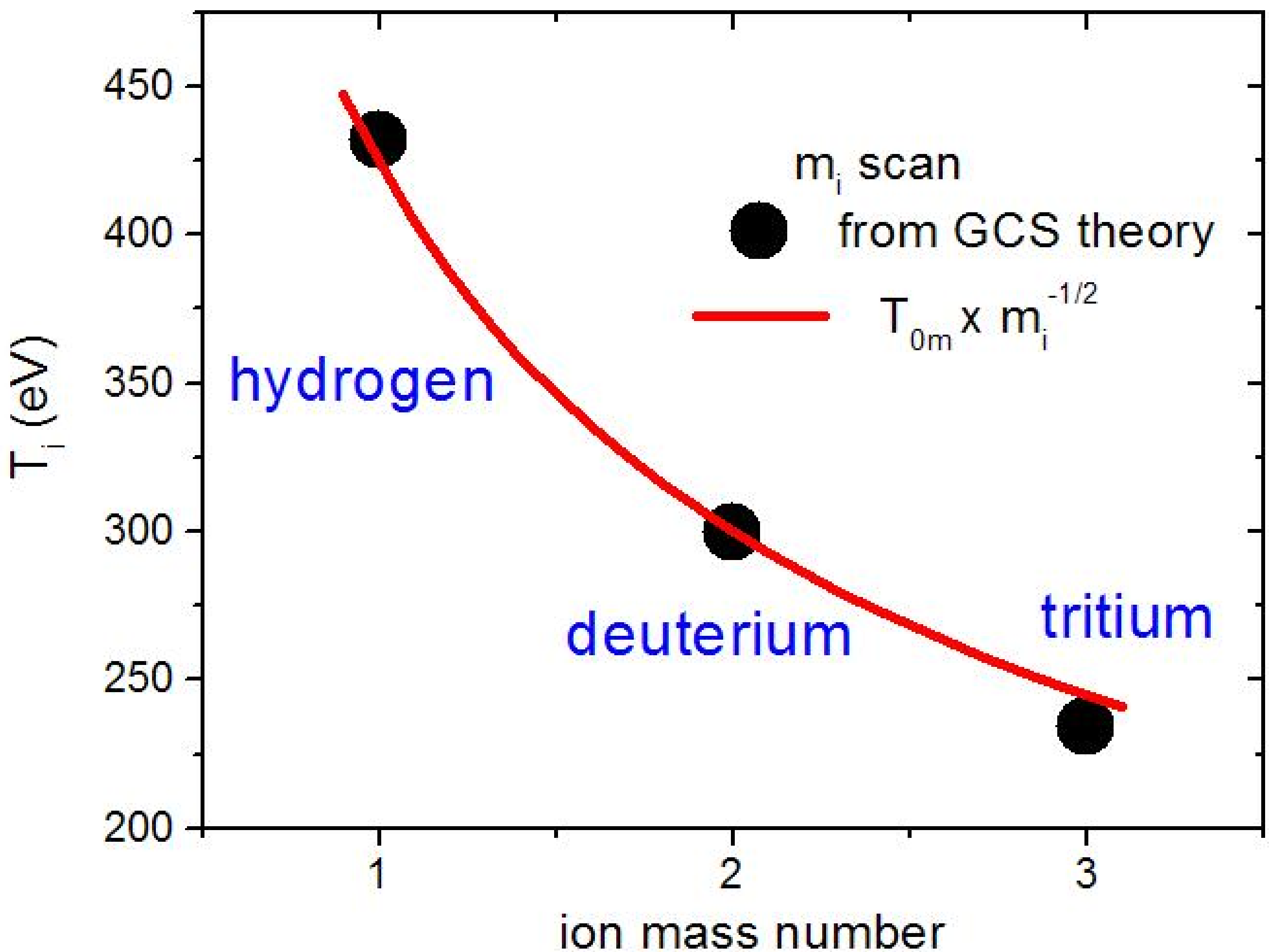
dithering H-mode, (Holzhauer, etc,PPCF,94') ASDEX



$$\mathrm{Re} = \frac{2}{\pi}\eta^2 \frac{B}{m_i n_i (\sigma_{i-n} n_n)^2 \upsilon_\perp} \nabla\rho$$

(Re -> Re*~2400)



gyrocenter shift

$$J_r^{GCS} = en_i \frac{r_{Li}}{\lambda_{i-n}} \left(\frac{E}{B} - \frac{1}{eB} \frac{\nabla P_i}{n_i} + \frac{kT_i}{eB} \frac{\nabla n_n}{n_n} \right)$$

turbulence diffusion

$$D = \frac{2}{\pi} \eta^2 \frac{kT_e}{eB}$$

Reynolds number of ion-neutral collision

$$\mathrm{Re} = \frac{2}{\pi} \eta^2 \frac{B}{m_i n_i (\sigma_{i-n} n_n)^2 \upsilon_\perp} \nabla \rho$$

anomalous transport

L/H transition

tokamak research