Force balance equation and neoclassical transport in gyrokinetic simulations

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Introduction

- Full-f gyrokinetic codes: require an accurate calculation of the mean electric potential.
- Requires a calculation of the neoclassical equilibrium, including force balance equation. Controversial for full-f codes.
- Regularisation of small scales in velocity space is also needed.
- Full collision operator quite complicated : interest for model collision operators. Must verify that force balance and neoclassical results are recovered.

Outline

- Entropy variational principle: a fast way to check a model collision operator.
- Model collision operators: advantages and drawback.
- Implementation in the GYSELA code and verification.

Basics

- Fokker-Planck equation

$$\partial_t F_s - [H_s, F_s] = \sum \mathcal{C}_{ss'}(F_s, F_{s'})$$

$$\mathcal{C}_{ss'}(F_s, F_{s'}) = \partial_{\mathbf{p}} \cdot \int d^3\mathbf{p}' \boldsymbol{\gamma}_{ss'} \cdot [F_{s'} \partial_{\mathbf{p}} F_s - F_s \partial_{\mathbf{p}'} F_{s'}]$$

$$\boldsymbol{\gamma}_{ss'} = 2\pi \frac{e_s^2 e_{s'}^2}{(4\pi\epsilon_0)^2} \ln \Lambda_s \frac{w^2\mathbf{I} - \mathbf{w}\mathbf{w}}{w^3}$$

$$\mathbf{w} = \partial_{\mathbf{p}} H_s(\mathbf{x}, \mathbf{p}, t) - \partial_{\mathbf{p}'} H_{s'}(\mathbf{x}, \mathbf{p}', t)$$

Basics (cont.)

- Introduce departure U_s from thermodynamical equilibrium

$$F_s = F_{s0} \exp\left(-\frac{H_s - U_s}{T}\right)$$

- Reformulation of the collision operator

$$\mathcal{C}_{ss'}(U) = \partial_{\mathbf{p}} \cdot \int d^3\mathbf{p}' F_s F_{s'} \gamma_{ss'} \cdot [\partial_{\mathbf{p}} U_s - \partial_{\mathbf{p}'} U_{s'}]$$

Near integrable system

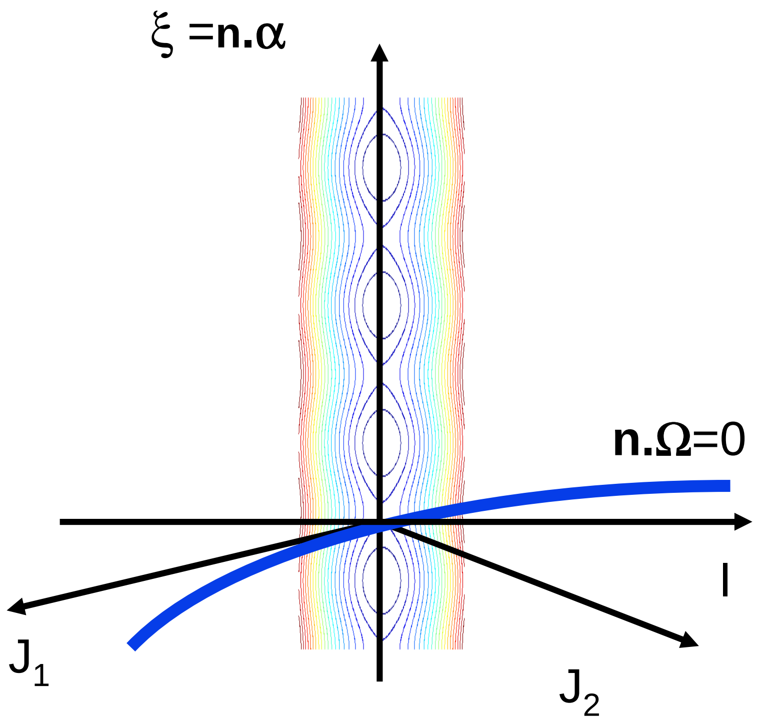
- Integrable system – angle/action variables

$$d_t \boldsymbol{\alpha} = \partial_{\mathbf{J}} H_s$$
$$d_t \mathbf{J} = -\partial_{\boldsymbol{\alpha}} H_s$$

$$\mathrm{H_s} = \mathrm{H_{s,eq}}(\mathbf{J}) + \delta \mathrm{H_s}(\boldsymbol{\alpha}, \mathbf{J})$$

- Helical perturbation $\rightarrow$ trapping

$$\delta H = h(\mathbf{J}) cos\left(\mathbf{n} \cdot \boldsymbol{\alpha}\right)$$



Entropy production rate

Rosenbluth '72

- FP equation can be recast into an entropy variational principle

$$\delta \dot{S} = \sum_s \dot{S}_s^t + \sum_s \dot{S}_s^{res} + \sum_{ss'} \dot{S}_{ss'}^{bulk}$$

Time evolution $\longrightarrow$

$$\dot{S}_s^t = \frac{2}{T^2} \int d\Gamma F_{\mathcal{H}s,eq} \mathcal{U}_{s,eq}^\dagger \partial_t \mathcal{U}_{s,eq}$$

Neoclassical transport $\longrightarrow$

$$\dot{S}_s^{res} = -\frac{2}{T^2} \int d\Gamma F_{\mathcal{H}s,eq} \delta\mathcal{U}_{s,eq}^\dagger \left[\delta\mathcal{H}_s, \delta\mathcal{U}_s\right]$$

// dynamics - force balance equation $\longrightarrow$

$$\dot{S}_{ss'}^{coll}(U^\dagger, U^\dagger) = \frac{1}{2T^2} \frac{\gamma_{ss'}}{2} \int d^3\mathbf{x} d^3\mathbf{p} d^3\mathbf{p}' F_{Hs,eq} F_{Hs',eq} \left\{\partial_{\mathbf{p}} U_s^\dagger - \partial_{\mathbf{p}'} U_{s'}^\dagger\right\} \cdot \mathfrak{L} \cdot \left\{\partial_{\mathbf{p}} U_s^\dagger - \partial_{\mathbf{p}'} U_{s'}^\dagger\right\}$$

Single hamiltonian perturbation

- Allows a compact reformulation of the resonant entropy production rate Samain ’77, Berk&Breizman ’90s

$$\dot{S}_{res} = \frac{1}{T_0^2} \int d\Gamma F_{\mathcal{H},eq} \left\{ \mathbf{n} \cdot \partial_{\mathbf{J}} \mathcal{U}^{\dagger}_{eq} \right\}^2 \Lambda(\mathbf{J}) h^2 \delta(\mathbf{n} \cdot \mathbf{\Omega})$$

- Plateau (“Landau”): $\Lambda(\mathbf{J}) = \frac{\pi}{2}$

- Trapping regime Zakharov&Karpman ‘63:

$$\Lambda(\mathbf{J}) = 4\mathfrak{I}\nu^* \qquad \mathfrak{I} = 1.38$$

Neoclassical theory

- Hamiltonian

$$H_s = \frac{1}{2} m_s v_\parallel^2 + \mu B_0 (1 - \epsilon \cos\theta) + e_s \phi(r)$$

δH

- Distribution function

$$F_{Ms}(\mathbf{J}, t) = \frac{N_s}{[2\pi m_s T_s]^{3/2}} \exp\left[-H_s/T_s\right] \left[1 + \frac{m_s W_s v_\parallel}{T_s}\right]$$

- Departure from thermodynamical equilibrium

$$U_s\left(H_s, J_3, t\right) = T_0 \left\{ \ln N_s - \frac{3}{2} \ln T_s - H_s \left\{ \frac{1}{T_s} - \frac{1}{T_0} \right\} + \frac{m_s W_s v_\parallel}{T_s} \right\}$$

Perturbative theory

- Obviously, if ε=0, $[H_{eq},F]=0$.
- J_3 is the canonical momentum $J_3 = e\psi + mRv_\varphi$. At first order $o(\rho_*)$

$$U_s = U_{s,\psi} + U_{s,v_\parallel}$$

Local Maxwellian

Shifted Maxwellian

$$U_{s,v_\parallel} = T\left(\frac{R_0}{e_s}\partial_\psi \Xi_s + \frac{W_s}{T_s}\right) m_s v_\parallel$$

$$\partial_\psi \Xi_s = \partial_\psi \ln n_s + \frac{e_s}{T_s}\partial_\psi \phi + \left(\frac{E}{T_s} - \frac{3}{2}\right)\partial_\psi \ln T_s$$

Force balance equation

- Extremalisation of bulk entropy production $\rightarrow$ generalised force balance equation

$$\frac{R_0 T_s}{e_s}\partial_\psi \Xi_s + W_s = V_{Ts}$$

$$\partial_\psi \Xi_s = \partial_\psi \ln n_s + \frac{e_s}{T_s}\partial_\psi \phi + \left(\frac{E}{T_s} - \frac{3}{2}\right)\partial_\psi \ln T_s$$

- contains usual force balance equation

$$-\partial_r \phi + V_{ps} B_0 - V_{Ts} B_p = \frac{\partial_r p_s}{n_s e_s}$$

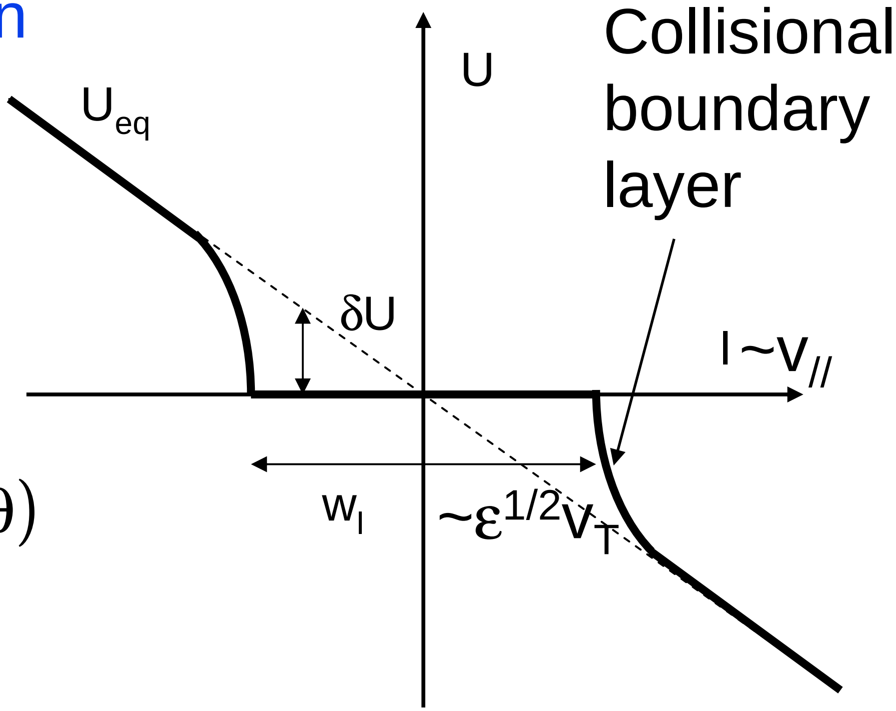
- Correct up to o(ρ_*)

Next order: friction+radial transport

- Calculation of the perturbed distribution function

- Island in the phase space

$$K = \frac{1}{2} m v_{//}^2 - \mu B \varepsilon \cos(\theta)$$



Single ion species

- Plateau regime

  - poloidal velocity $$V_{pi} = -\frac{1}{2}\frac{\partial_r T_i}{e_i B_0}$$

  - diffusivity $$\chi_i = 3\sqrt{\frac{\pi}{2}}\, q \frac{v_{Ti}}{R_0}\rho_{Ti}^2$$

- Banana regime

  - poloidal velocity $$V_{pi} = k_{neo}\frac{\partial_r T_i}{e_i B_0} \qquad k_{neo} \simeq 1.17$$

  - diffusivity $$\chi_i = \chi_B \frac{q^2}{\epsilon^{3/2}}\nu_{Ti}\rho_{Ti}^2 \qquad \chi_B \simeq 1.35$$

Mean radial electric field

- Plasma is quasi-neutral and E_r is given by force balance equation

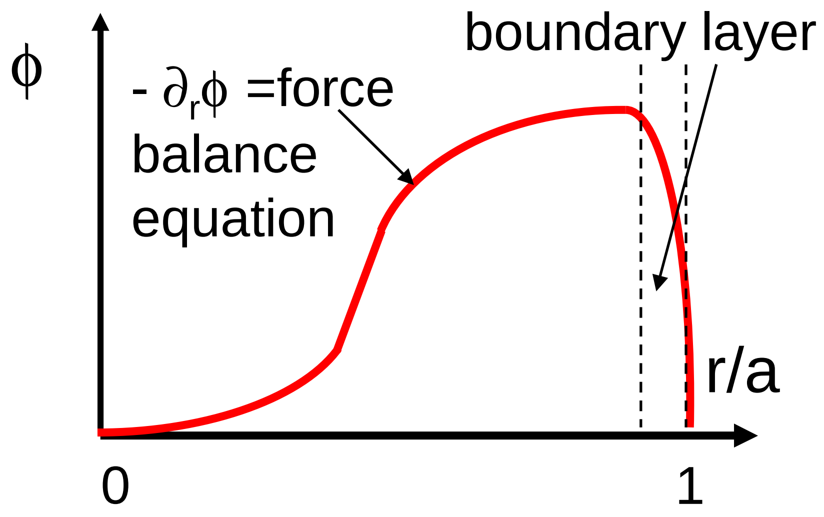
$$-\partial_r\phi + V_{ps}B_0 - V_{Ts}B_p = \frac{\partial_r p_s}{n_s e_s}$$

- Consistent with Poisson equation

$$-\rho_i^2\nabla_\perp\cdot n_i\nabla_\perp\frac{e\phi}{T} = \langle J_0\cdot \bar{F}_i\rangle - n_e$$

up to $o(\rho_*^2)$ (Parra & Catto 08)

- Except in the edge: shear layer.



Collision operator in $w=v_\perp^2$, $v_{//}$ coordinates

- General form of the linearised operator (identical to Xu 91)

$$
\begin{aligned}
C_{lin}(\delta F) = & -\partial_w\left(A_\perp \delta F\right) - \partial_{v_\parallel}\left(A_\parallel \delta F\right) \\
& + \partial_{ww}\left(D_\perp \delta F\right) + 2\partial_{wv_\parallel}\left(D_\Lambda \delta F\right) + \partial_{v_\parallel v_\parallel}\left(D_\parallel \delta F\right) + D_\gamma \partial_{\gamma\gamma}\delta F \\
& - \frac{v_\parallel}{v_T^2}\frac{F_M}{n}\int d^3\mathbf{v} A_\parallel \delta F - \frac{2}{3}\left(\frac{w}{v_T^2} + \frac{v_\parallel^2}{2v_T^2} - \frac{3}{2}\right)\frac{F_M}{nv_T^2}\int d^3\mathbf{v}\left(D_\parallel + A_\perp + v_\parallel A_\parallel\right)\delta F
\end{aligned}
$$

\+ terms coming from gyroaverage Xu 91, Brizard 04, Barnes 08.

- still not optimal for parallelization … and linearized.

Simplified collision operator

- Lorentz type operator (Lenard-Bernstein 58, Watanabe 06)

$$\mathcal{C}_i\left(F_i\right) = \partial_w \left\{ D_\perp F_{Mi} \partial_w \left(\frac{F_i}{F_{Mi}} \right) \right\} + \partial_{v_\parallel} \left\{ D_\parallel F_{Mi} \partial_{v_\parallel} \left(\frac{F_i}{F_{Mi}} \right) \right\}$$

$$F_{Mi} = \frac{n_i}{\left(2\pi m_i T_i\right)^{3/2}} \exp\left\{ -\frac{m_i \left(\mathbf{v} - V_{\parallel i}\mathbf{b}\right)^2}{2T_i} \right\}$$

- Temperature and velocity calculated to ensure momentum and energy conservation (Connor '73, Xu '91).

Entropy production rate

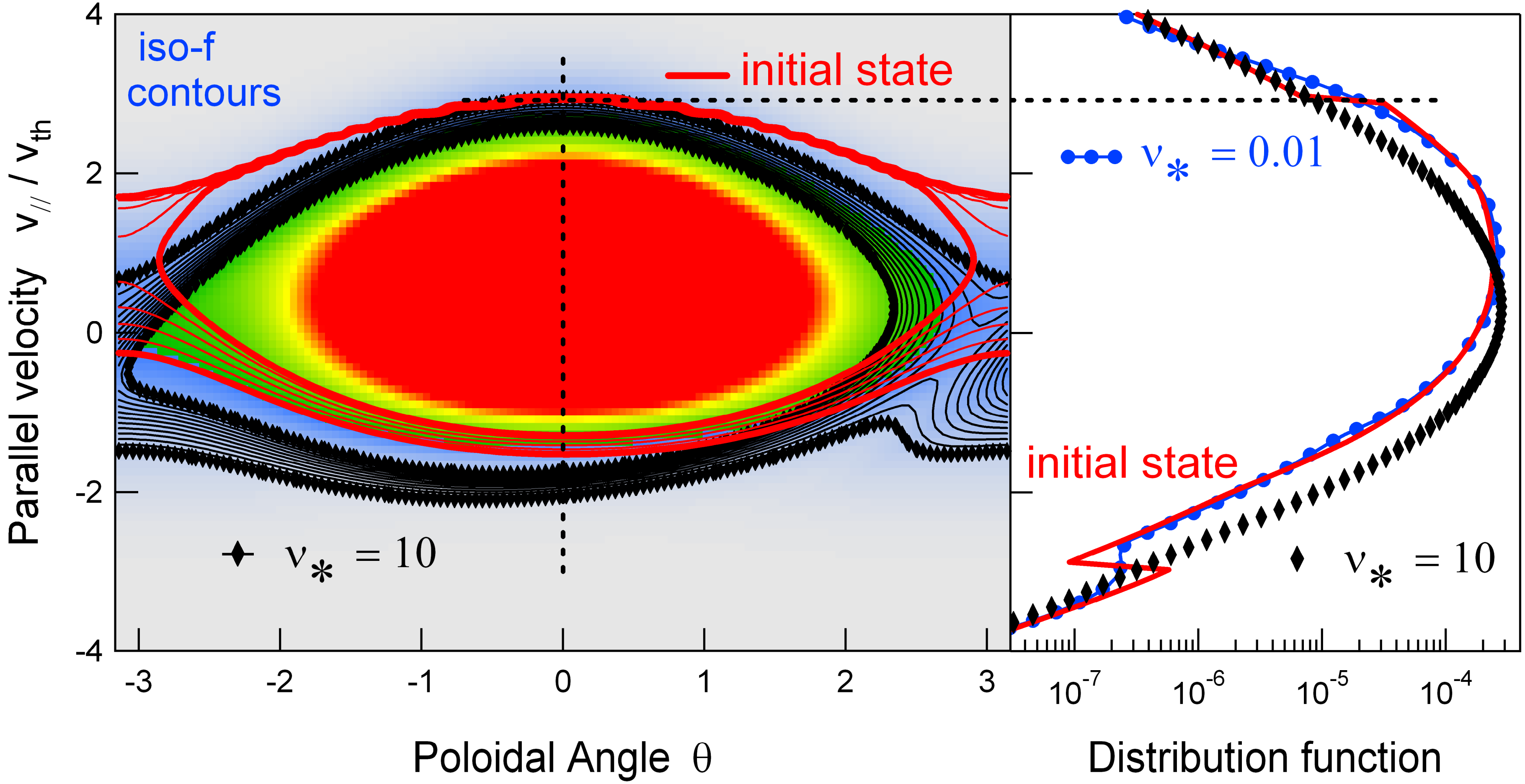
- Production rate with a // Lorentz operator

$$\dot{S}_i^{coll}(U,U) = \frac{1}{2}\int d^3\mathbf{x}d^3\mathbf{p}F_{Mi}D_{\parallel}\frac{m_i^2}{T_i^2}\left(\frac{RT_i}{e_i}\partial_\psi\Xi_i + W_i\right)^2$$

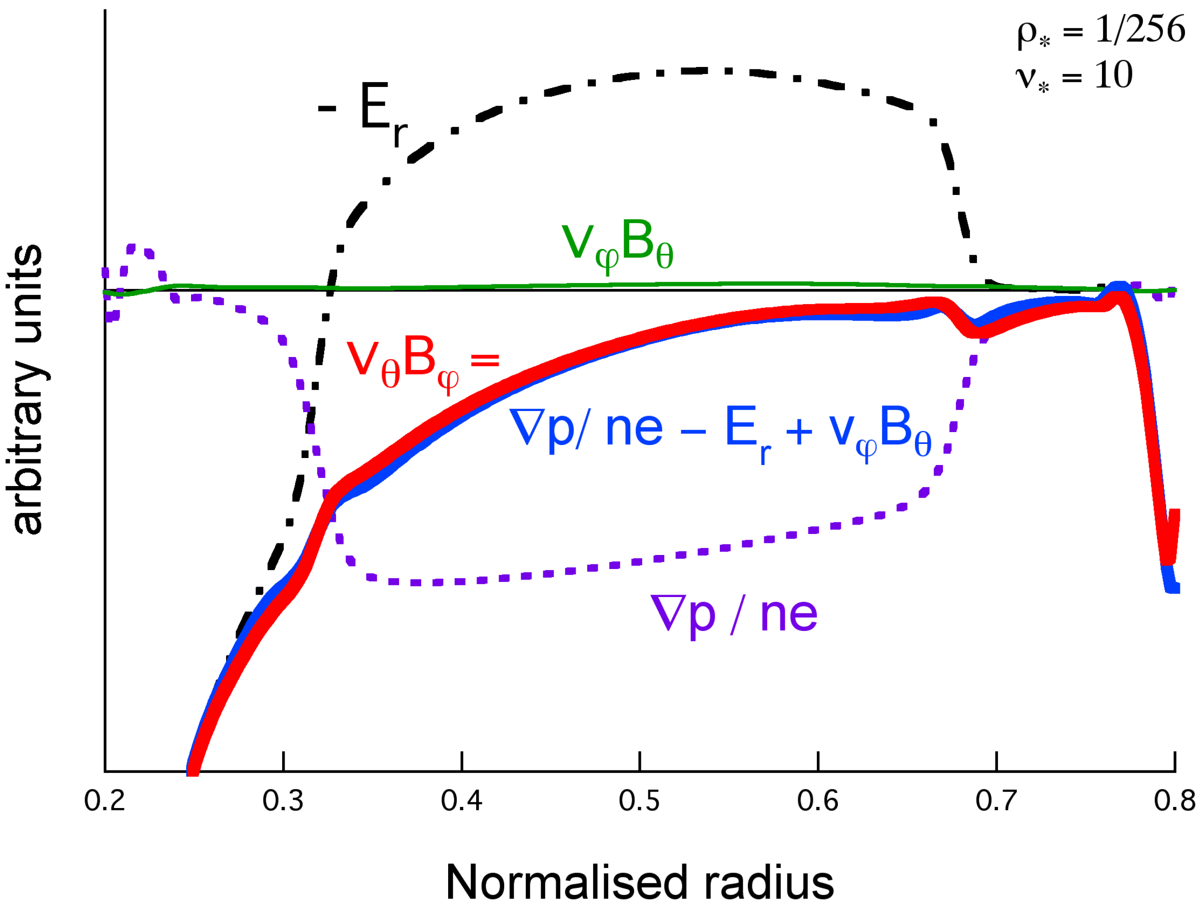
- All results from neoclassical theory are recovered, even if $D_{\perp}$=0.

- Does not work likely in the Pfirsch-Schlüter regime.

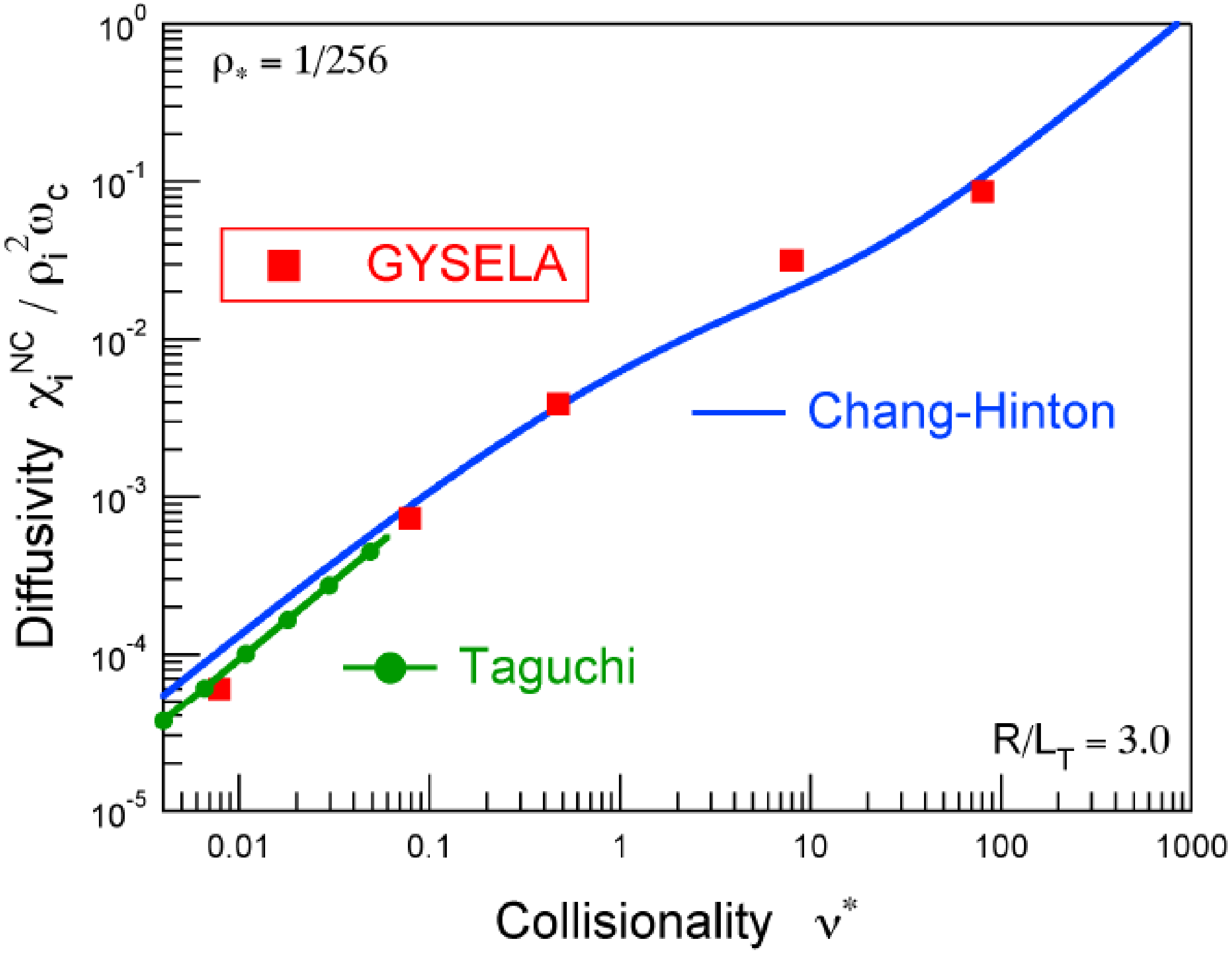
Trapping domain (Gysela code)



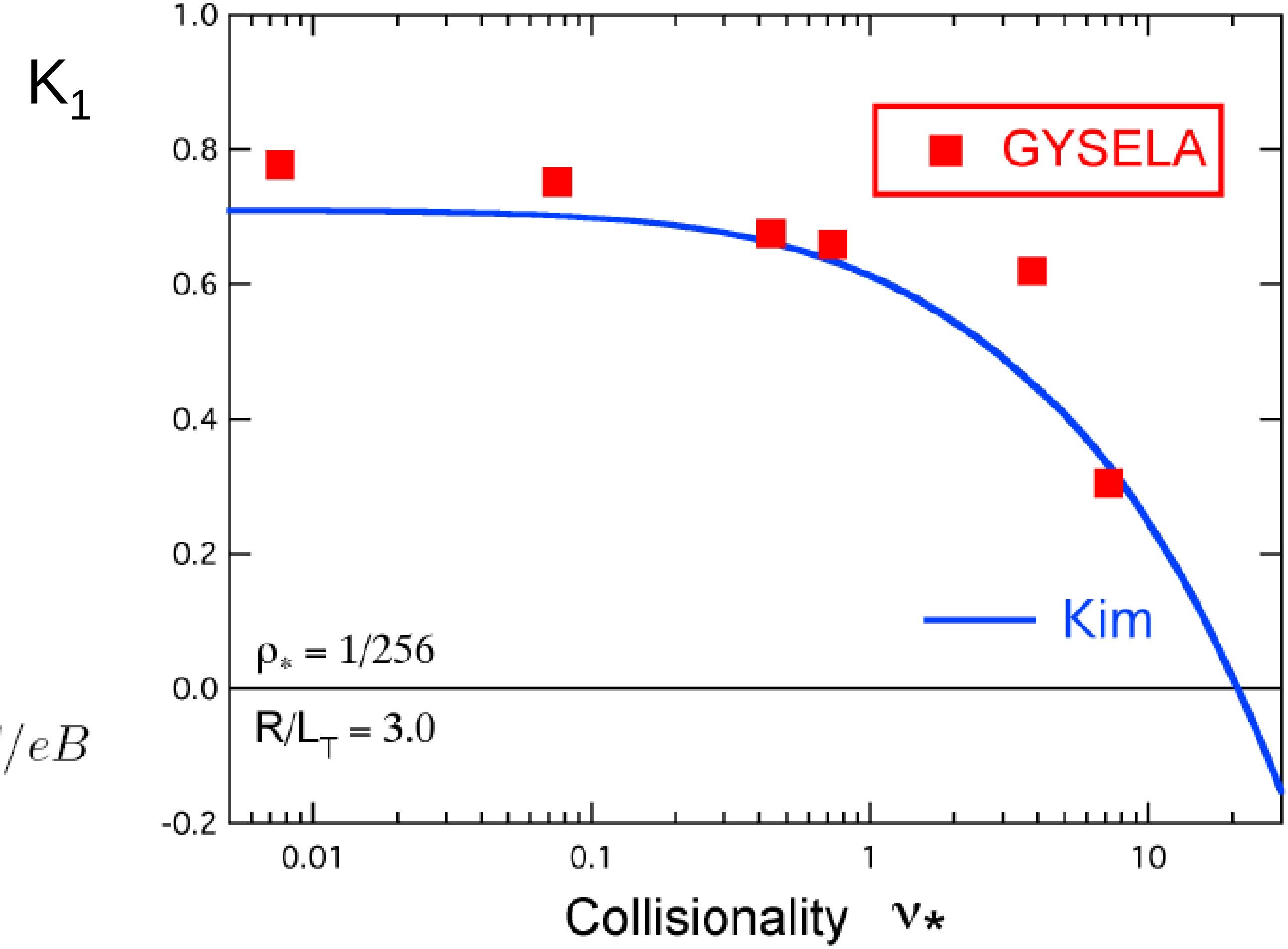
Example of force balance (GYSELA)



Test of heat diffusivity in the GYSELA code



and of neoclassical velocity



$$v_\theta = K_1 \nabla T / eB$$

Conclusions

- Possible to build a simple Lorentz operator that is consistent with neoclassical transport.
- Some advantages:

  - can be easily parallelized as in involves derivatives with respect to $v_{//}$ only.

  - ensures force balance equation

  - converges to a Maxwellian in absence of external forcing: guaranteed by an entropy extremum principle.
- GYSELA is not the only GK code that implements neoclassical transport (Lin 97, GTC-Neo, ELMFIRE, Stefi, GT5D, XGC, Tempest,…), but full-f and flux driven. Interest: interplay between turbulent and neoclassical transport (poloidal flow, effect of ν^* on χ_i, …)