Massless Fluid-Electron Model in GEM

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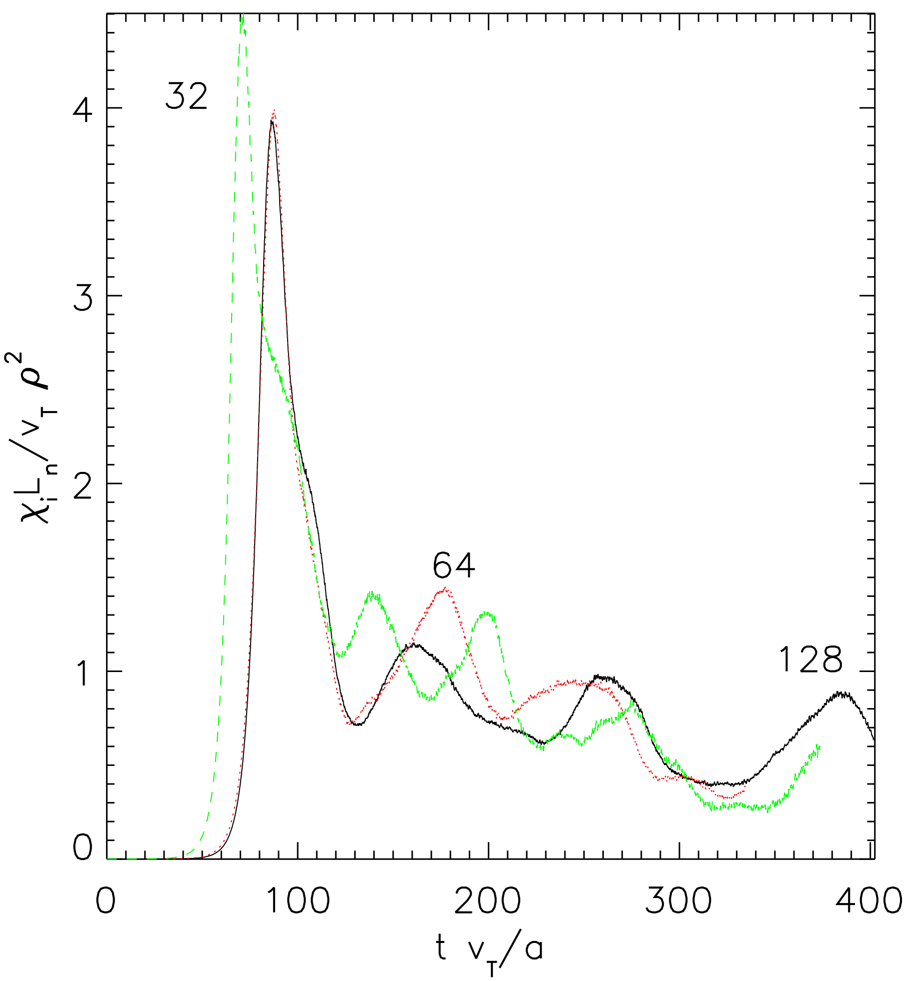
ABSTRACT

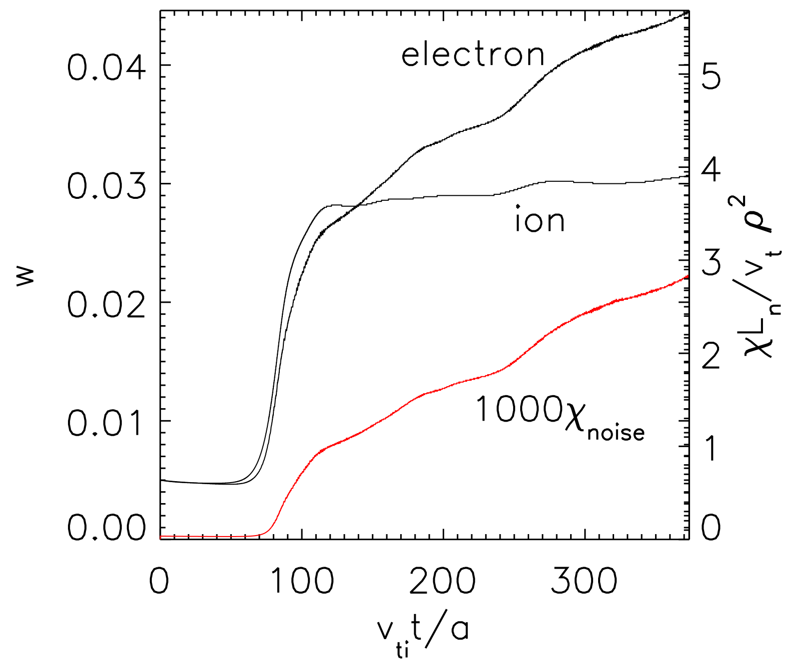
GEM is an explicit δf particle code that has kinetic electron dynamics and electromagnetic perturbations [1]. The code has been recently extended to handle general toroidal equilibrium magnetic field configuration, arbitrary equilibrium density and temperature profiles and arbitrary number of minority species. The perturbed magnetic field is given by $\delta \mathbf{B}_\perp = \nabla A_\| \times \mathbf{b}$, with $A_\|$ given by the parallel Ampere's law. The electric field is given by $\mathbf{E} = -\nabla\phi - (\partial A_\|/\partial t)\mathbf{b}$, with ϕ obtained from the quasi-neutrality condition. It has been shown [2] that this gyrokinetic model recovers the MHD equation for the shear Alfvén modes, and previous numerical studies have also shown that this model can accurately describe the slab shear Alfvén waves and the toridal Kinetic Ballooning Modes. A Maxwellian energetic particle species is included. The primary coupling of the hot particles to the bulk plasma comes from the hot particle term in the quasi-neutrality condition. While the electromagnetic algorithm of GEM has been extensively benchmarked, both with the dispersion relation in slab geometry and with other gyrokinetic codes in toroidal geometry, for ion-scale microinstabilities, for long-wavelength MHD modes the accuracy of the algorithm is uncertain. Recently the massless fluid-electron model has also been implemented in the general geometry. In this model the Ampere's equation is solved "backwards", i.e., to obtain the electron parallel flow velocity from the vector potential, which is evolved using the generalized Ohm's law.

[1] Y. Chen and S. E. Parker, J. Comp. Phys. 189 (2003) 463

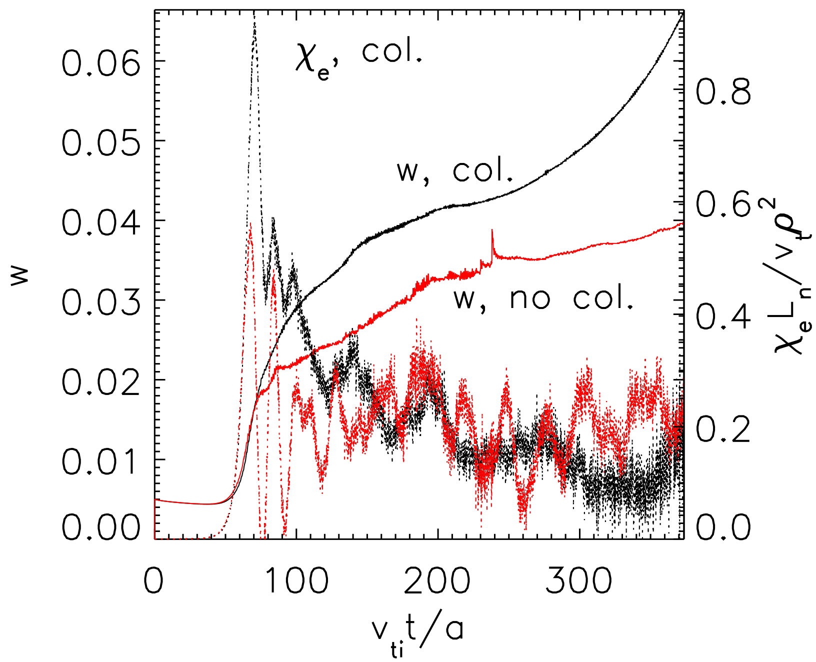
[2] H. Qin, W. M. Tang and G. Rewoldt, Phys. Plasmas 5 (1998) 1035

Noise in Finite-β ITG Simulation with Kinetic Electrons





Collisions Do Not Stop Weight Growing



- Collisions stop phase space granulation, limits δf
- However, $\delta f = \int wF(w)\ dw$. Weights continue to broaden
- "Thermostatted δf" (Krommes '99)
- Particle-Continuum Method (Vadlamani *et. al.* '04)

Fluid Electron Model

$$\frac{\partial \delta n_e}{\partial t} + n_0 \mathbf{B} \cdot \nabla \frac{u_{\parallel \mathrm{e}}}{B} + \mathbf{v}_E \cdot \nabla n_e + \frac{1}{m_e \Omega_e B^2} \mathbf{B} \times \nabla B \cdot \nabla (\delta P_\perp + \delta P_\parallel) + \frac{2 n_0}{B^3} \mathbf{B} \times \nabla B \cdot \nabla \phi = 0$$

$$e n_0 E_\parallel = -\tilde{\mathbf{b}} \cdot \nabla \delta P_\parallel - \frac{\delta \mathbf{B}_\perp}{B} \cdot \nabla (P_{\parallel 0\mathrm{e}} - e n_0 \phi)$$

$$\tilde{\mathbf{b}} \cdot \nabla (T_{0\mathrm{e}} + \delta T_e) = 0$$

$$\frac{\partial A_\parallel}{\partial t} = -\nabla_\parallel \phi - E_\parallel$$

- Easy to take MHD limit (e.g., set $E_\parallel = 0$)
- No “Cancellation Problem”
- Tested in flux-tube. Implemented in general geometry, global code.

General Equilibrium

- General equilibrium magnetic field

$$\mathbf{B} = \frac{f(\Psi)}{R}\hat{\zeta} + \nabla\zeta \times \nabla\Psi$$

- Specified to the Miller equilibrium model

$$\begin{aligned} R &= R_0(r) + r\cos[\theta + (\sin^{-1}\delta(r))\sin\theta] \\ Z &= \kappa(r) r \sin\theta \end{aligned}$$

- q is related to f through

$$\oint \frac{f}{Rq \mid \nabla\Psi \times \nabla\theta \mid} d\theta = 2\pi$$

- Given $f(r)$, $q(r)$, flux surface shape $R(r,\theta)$, $Z(r,\theta)$,

$$\begin{aligned} &\longrightarrow \nabla r, \nabla\theta, \\ &\longrightarrow \Psi'(r), \\ &\longrightarrow \hat{q} = \frac{\mathbf{B}\cdot\nabla\zeta}{\mathbf{B}\cdot\nabla\theta} \end{aligned}$$

- Field-line-following coordinates, r is the distance to the magnetic axis on the outer mid-plane,

$$
\begin{aligned}
x &= r - r_0 \\
y &= \frac{r_0}{q_0}\left(\int_0^\theta \hat{q}\,d\theta - \zeta\right) \\
z &= q_0 R_{\Psi 0}\theta
\end{aligned}
$$

- Four points for doing gyro-averaging

$$
\begin{aligned}
&\mathbf{r}(x+\triangle x_i, y+\triangle y_i) - \mathbf{r}(x,y) = \frac{\partial \mathbf{r}}{\partial x}\triangle x_i + \frac{\partial \mathbf{r}}{\partial y}\triangle y_i = \boldsymbol{\rho}_i \\
&\triangle x_i = \boldsymbol{\rho}_i \cdot \nabla x \\
&\triangle y_i = \boldsymbol{\rho}_i \cdot \nabla y \\
&\boldsymbol{\rho}_1 = \rho\hat{r}, \quad \boldsymbol{\rho}_2 = -\rho\hat{r} \\
&\boldsymbol{\rho}_3 = \rho\hat{b}\times\hat{r}, \quad \boldsymbol{\rho}_4 = -\rho\hat{b}\times\hat{r} \\
&\hat{r} = \nabla r / \mid \nabla r \mid
\end{aligned}
$$

- For field equations,

$$
\nabla_\perp^2 \phi = \frac{\partial^2 \phi}{\partial x^2} \mid \nabla x \mid^2 + 2\frac{\partial^2 \phi}{\partial x \partial y}\nabla x \cdot \nabla y + \frac{\partial^2 \phi}{\partial y^2} \mid \nabla y \mid^2
$$

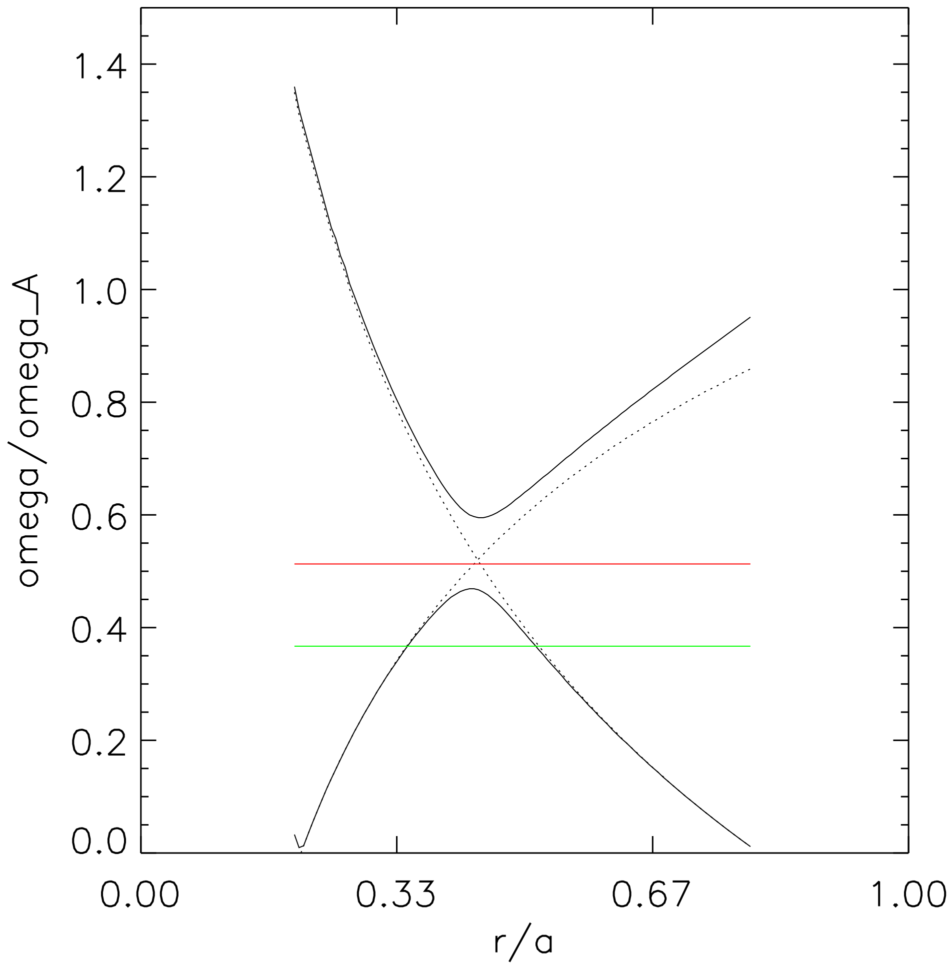
$$\mathbf{B}\cdot(\nabla r\times\nabla\theta)=\frac{f}{R}\mid\nabla r\times\nabla\theta\mid$$
$$\mathbf{B}\cdot(\nabla x\times\nabla y)=\frac{\partial y}{\partial\theta}\mathbf{B}\cdot(\nabla r\times\nabla\theta)$$
$$\mathbf{B}\cdot(\nabla r\times\nabla\zeta)=-\frac{1}{R^2}\nabla r\cdot\nabla\Psi$$
$$\mathbf{B}\cdot(\nabla\theta\times\nabla\zeta)=-\frac{1}{R^2}\nabla\theta\cdot\nabla\Psi$$
$$\nabla x\cdot\nabla y=\frac{\partial y}{\partial r}\mid\nabla r\mid^2+\frac{\partial y}{\partial\theta}\nabla r\cdot\nabla\theta$$
$$\frac{\partial y}{\partial r}=\frac{r_0}{q_0}\int_0^\theta\hat{q}d\theta$$
$$\frac{1}{J_s}=\nabla z\cdot\nabla x\times\nabla y=\frac{r_0R_{\psi0}}{R}\mid\nabla r\times\nabla\theta\mid$$

drift motion

$$V_{\rm dx}=-\frac{m(v_\parallel^2+v_\perp^2/2}{qB^3}\frac{f}{R}\frac{\partial B}{\partial\theta}\mid\nabla r\times\nabla\theta\mid$$
$$V_{\rm dy}=\frac{m(v_\parallel^2+v_\perp^2/2}{qB^3}\frac{f}{R}\left(-\frac{\partial y}{\partial r}\frac{\partial B}{\partial\theta}+\frac{r_0}{q_0}\hat{q}\frac{\partial B}{\partial r}\right)\mid\nabla r\times\nabla\theta\mid$$

TAE Frequency Eigenmode Observed at Low β

- $B_0 = 1.91T$, $T_i = T_e = 2KeV$, $1/L_T = 1/L_n = 0$, $R_0 = 1.67m$, $a = 0.36R_0$
- $q(r) = 1.3(\frac{r}{r_0})^{0.3}$, $r_0 = a/2$, $rq'/q = 0.3$
- $m_p/m_e = 500$, $m_i/m_p = 2$
- Scan over β is equivalent to changing n_0, with above parameters fixed
- Simulation domain $[0.2a, 0.8a]$.
- Add external $n = 2$ current for 200 steps, then observe the subsequent oscillation and mode structure
- Fluid electron case $E_\parallel = 0$



Continuum Gap Forms due to Toroidal Coupling of $m = 2$ and $m = 3$

- In ideal MHD limit and neglecting kinetic effects, the two Alfven continuum branches for $m = 2$ and $m = 3$ are given by

$$\omega_1^2 = k_{\parallel,2}^2 v_A^2, \quad \omega_2^2 = k_{\parallel,3}^2 v_A^2$$

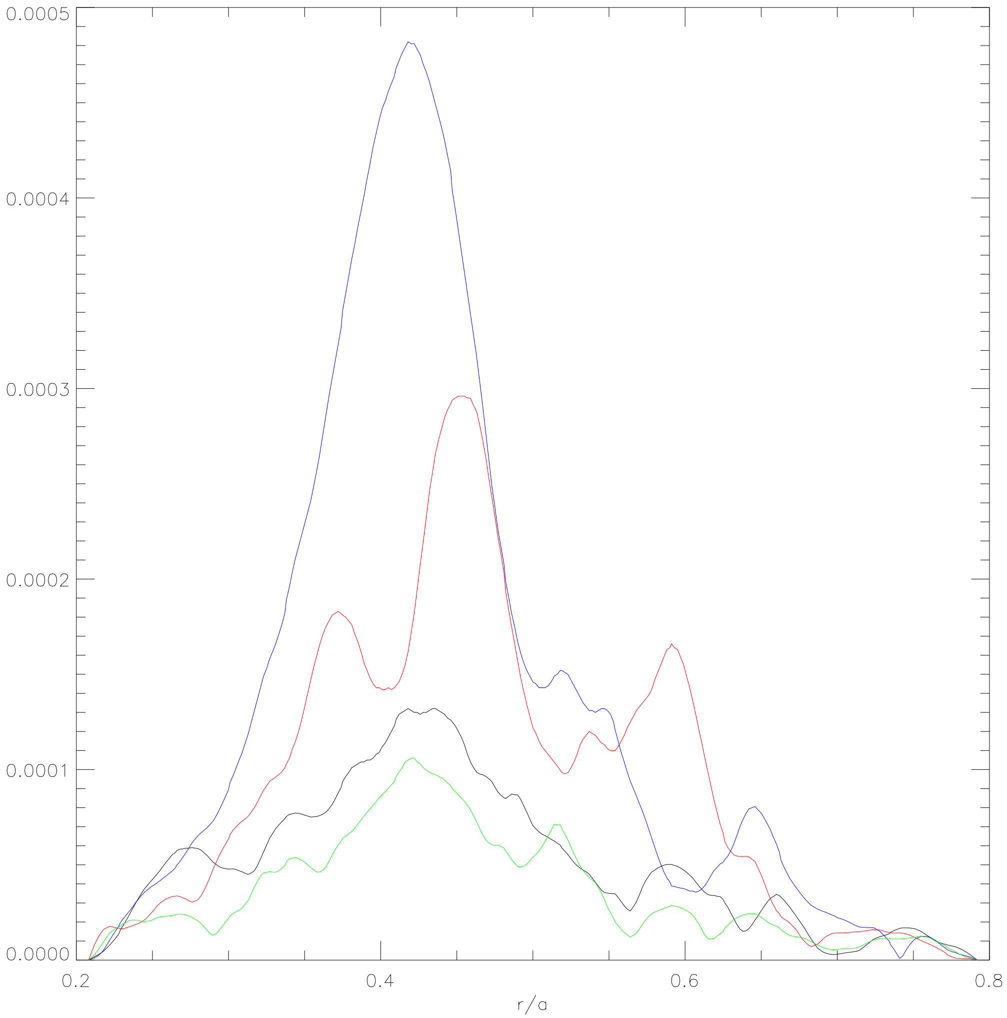
$$k_{\parallel,\mathrm{m}} = (n - m/q)/R$$

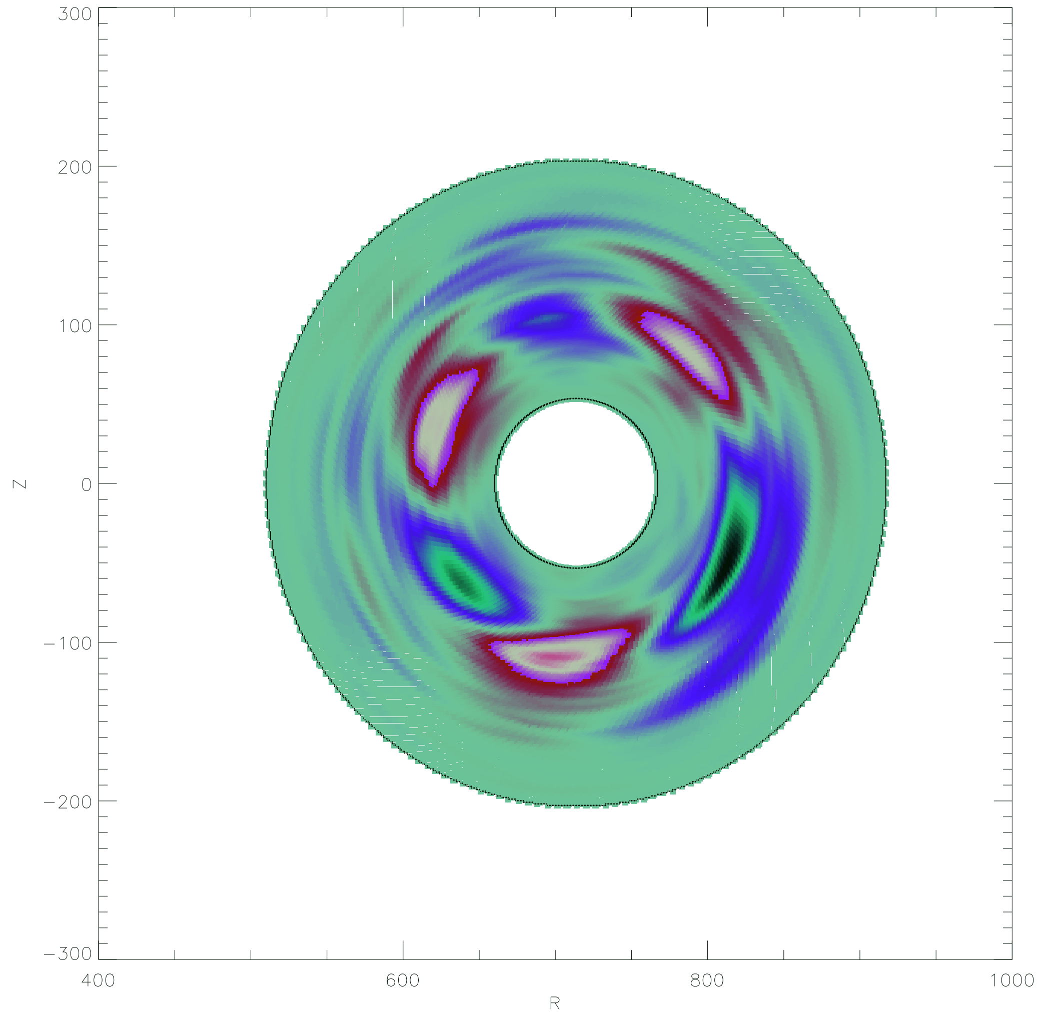
- Due to toroidal coupling, the upper and lower continuum are given by

$$\omega_\pm^2 = \frac{k_{\parallel\mathrm{m}}^2 v_A^2 + k_{\mathrm{pam+1}}^2 v_A^2 \pm \sqrt{(k_{\parallel\mathrm{m}}^2 v_A^2 - k_{\parallel\mathrm{m+1}}^2 v_A^2)^2 + 4\epsilon^2 x^2 k_{\parallel\mathrm{m}}^2 v_A^2 k_{\parallel\mathrm{m+1}}^2 v_A^2}}{2(1 - \epsilon^2 x^2)}$$

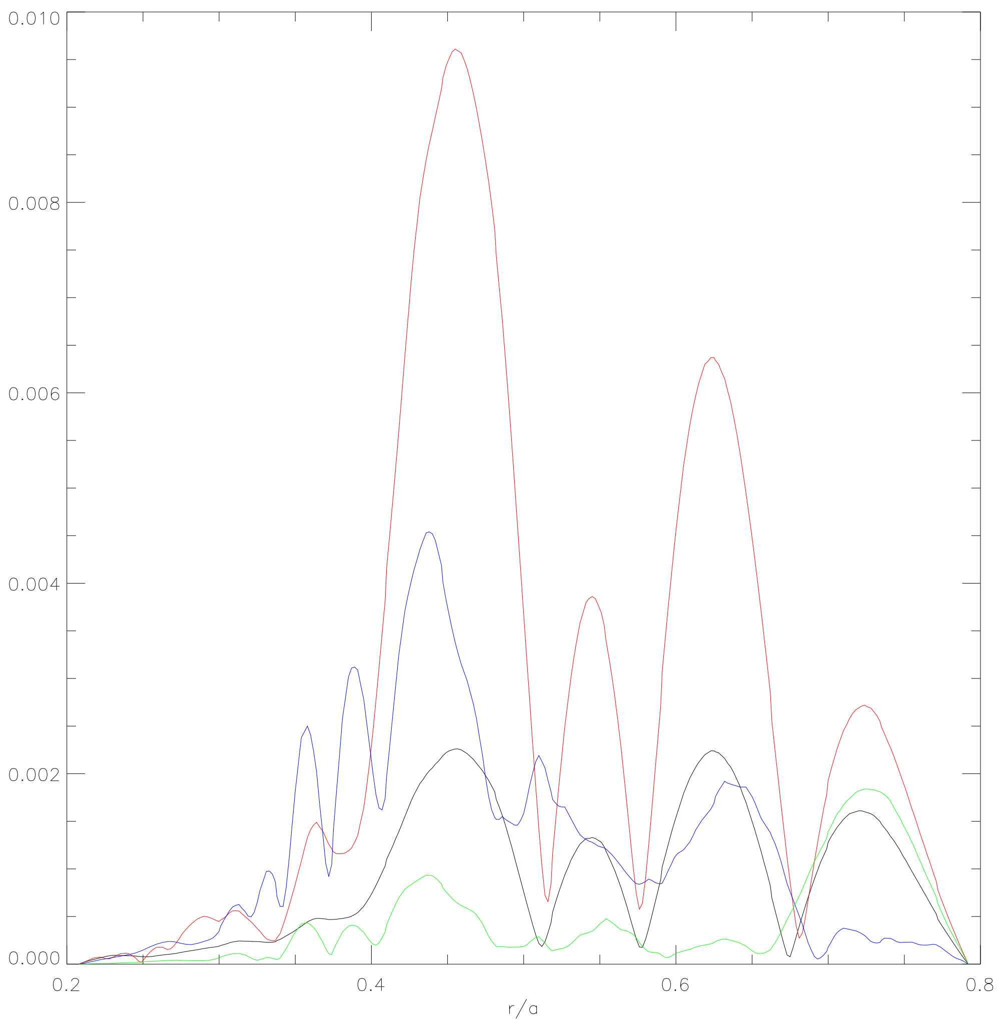
$x = r/a$. $\epsilon = 3a/2R$.

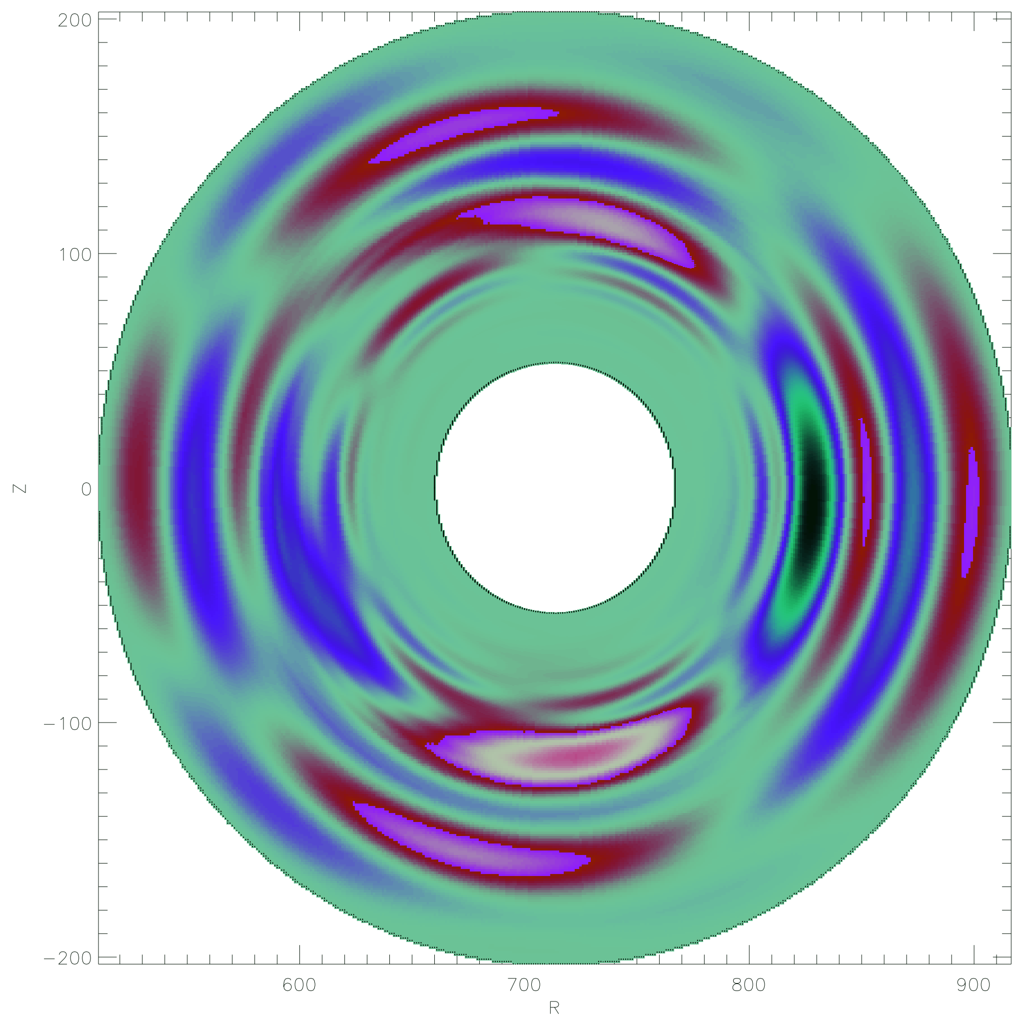
Mode structure–kinetic electrons





Mode structure–fluid electron





Numerical instability

$$\frac{\partial \delta n_e}{\partial t} + n_0 \mathbf{B} \cdot \nabla \frac{u_{\|\mathrm{e}}}{B} + \mathbf{v}_E \cdot \nabla n_e + \frac{1}{m_e \Omega_e B^2} \mathbf{B} \times \nabla B \cdot \nabla (\delta P_\perp + \delta P_\|) + \frac{2n_0}{B^3} \mathbf{B} \times \nabla B \cdot \nabla \phi = 0$$

- This term is the divergence of the electron $E \times B$ motion in nonuniform B
- $E_\| = 0$, $\nabla n_0 = \nabla T_0 = 0$, then $\delta T = 0$, $\delta P = T_0 \delta n$
- No ions
- Numerical instability for some q-profile.

Implementation of Energetic Particles

- Energetic particles have $n_h \ll n_b$, $\beta_h \sim \beta_b$
- Coupled to the bulk plasmas mainly through the quasi-neutrality condition

$$\phi - \tilde{\phi} = \int f_i \delta(\mathbf{R} + \boldsymbol{\rho} - \mathbf{x})\, d\mathbf{R}\, d\mathbf{v} - \int f_e \, d\mathbf{v} + \int f_h \delta(\mathbf{R} + \boldsymbol{\rho} - \mathbf{x})\, d\mathbf{R}\, d\mathbf{v}.$$

- Hot particle polarization density and current are also implemented in the quasi-neutrality condition and Ampere's equation, but not important
- Maxwellian distribution thus far. Slowing-down distribution to be implemented.

SUMMARY

- GEM is extended to
  - Kinetic electrons and $\delta \mathbf{B}_\perp$
  - General geometry
  - General equilibrium profiles
  - Arbitrary number of minority species
- The massless fluid electron model has been implemented in GEM
  - Avoid the "Cancellation Problem" for low-n MHD modes

Future Work

- Try "Thermostatted δf method" or the Particle-Continuum method for long-term simulation
  - Is fine structure in δf important?
- Benchmark the hybrid model with MHD eigenmode codes (NOVA-K)
- Implement slowing-down energetic particle distribution
- Include $\delta \mathbf{B}_\parallel$ through perpendicular force ballance (or perpendicular Ampere's equation)