Simulation of Hasegawa-Wakatani Equation

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Bifurcation Analysis of Low Dim. Model Predicts Transition

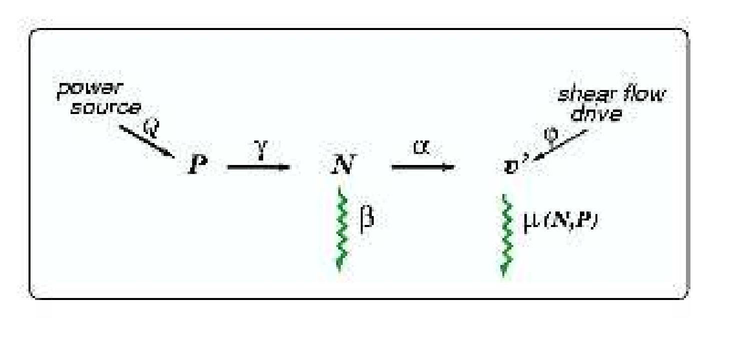
A low dimensional dynamical model is a reduction of a fluid system (infinite degree of freedom) to a system of rate equations of some macroscopic variables described by coupled ODEs (few degree of freedom).

Recently, Ball *et al*. (2002) derived a low dim. model for confinement transitions by integrating the reduced MHD equations. The model consists of three macroscopic state variables: P is the potential energy production, N is the turbulent kinetic energy, F is the shear flow kinetic energy,

$$\varepsilon \frac{\mathrm{d}P}{\mathrm{d}t} = q - \gamma P N \quad (1)$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \gamma P N - \alpha F N - \beta N^2 \quad (2)$$

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \alpha F N - \mu(P,N)F + \varphi F^{1/2} \quad (3)$$



- Model provides economical tool to predict transitions over parameter space
- Requires validation against numerical simulation and/or real experimental data

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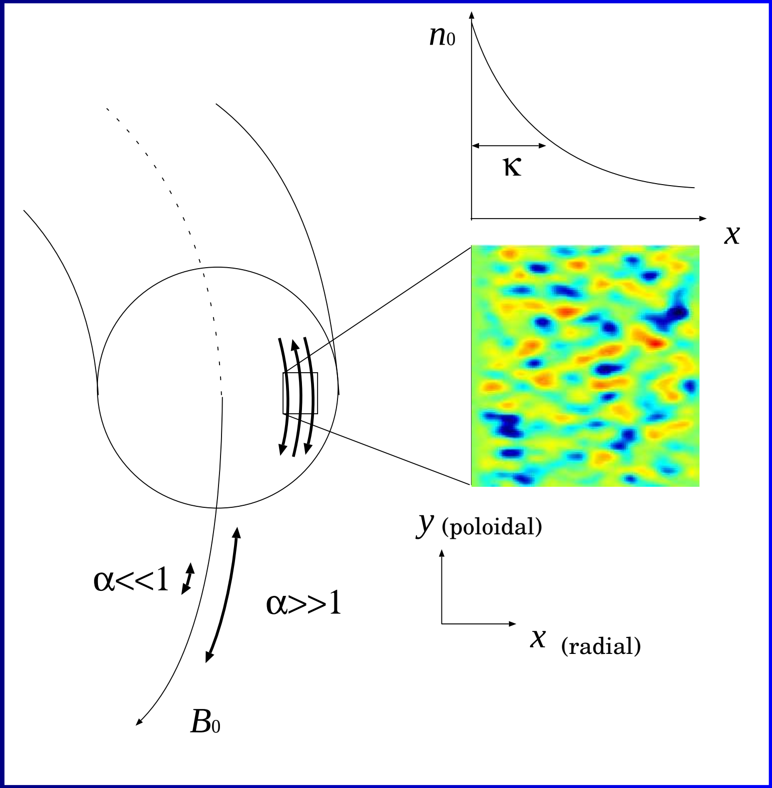
Hasegawa-Wakatani Model

HW model describes evolution of density fluctuation n and vorticity $\zeta = \nabla^2\varphi$ (φ: electrostatic potential)



$$\frac{\partial}{\partial t}\zeta + \{\varphi, \zeta\} = \alpha(\varphi - n) - D_\zeta \nabla^4 \zeta$$

$$\frac{\partial}{\partial t}n + \{\varphi, n\} = \alpha(\varphi - n) - \kappa\frac{\partial \varphi}{\partial y} - D_n \nabla^4 n$$

$\{a, b\} = \partial a/\partial x \partial b/\partial y - \partial a/\partial y \partial b/\partial x$
$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$
D_ζ and D_n are dissipation coefficients
$\kappa \equiv -\partial/\partial x \ln n_0$
$\alpha \equiv \frac{T_e k_z^2}{\eta n_0 \omega_{ci} e^2}$: adiabaticity parameter

α

0 Hydrodynamic ... Adiabatic ∞ (Hasegawa-Mima)

Modified Hasegawa-Wakatani Model

- Resistive coupling term comes from parallel electron response $\partial j_z/\partial z = 1/\eta\partial^2(\varphi - n)/\partial z^2$ (Ohm's Law)
- Zonal components subtracted from resistive coupling term since the zonal components ($k_y = k_z = 0$) do not contribute to this term [Smolyakov *et al* (2000)]

$$\alpha(\varphi - n) \longrightarrow \alpha(\tilde{\varphi} - \tilde{n})$$

Non-zonal $\tilde{\cdot}$ and zonal components $\langle\cdot\rangle$

$$\tilde{\varphi} = \varphi - \langle\varphi\rangle, \quad \tilde{n} = n - \langle n\rangle$$
$$\langle f\rangle = \frac{1}{L_y}\int f\mathrm{d}y \quad (f = \varphi \text{ or } n)$$

Modified HW model

$$\begin{aligned}\frac{\partial}{\partial t}\zeta + \{\varphi,\zeta\} &= \alpha(\tilde{\varphi} - \tilde{n}) - D_\zeta\nabla^4\zeta \\ \frac{\partial}{\partial t}n + \{\varphi,n\} &= \alpha(\tilde{\varphi} - \tilde{n}) - \kappa\frac{\partial\varphi}{\partial y} - D_n\nabla^4 n\end{aligned}$$

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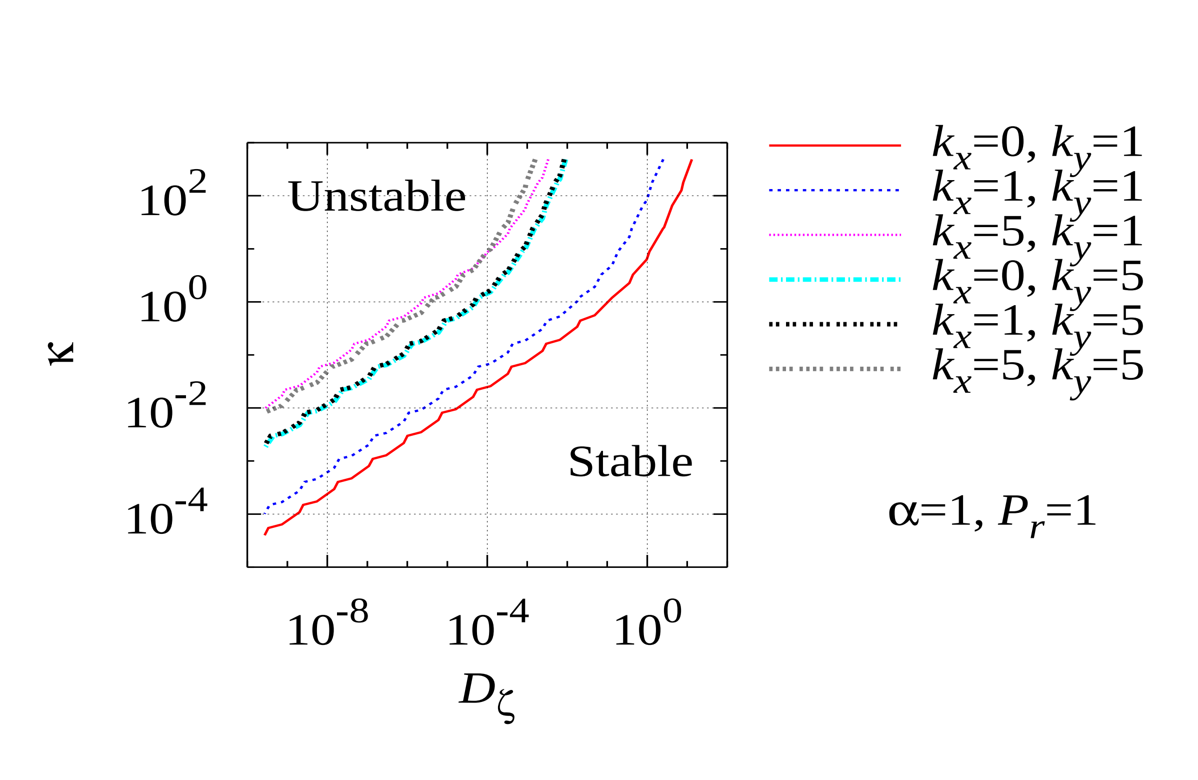
Stability Diagram Provides Indication of Transition Points

Stability threshold in D_ζ (dissipation) – κ (drive) space



- Strong drive (large κ) causes strong instability, which can be stabilized by strong dissipation (large D_ζ)

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Algorithm to Solve MHW Model

- Numerical simulation solves MHW model in the slab geometry
- Box size L, determined by smallest wavenumber $\Delta k = 0.15$ $[(2L)^2 = (2\pi/\Delta k)^2]$
- Periodic boundary in y direction; periodic or Dirichlet boundary in x direction eg. Dirichlet condition in x

$$\varphi(x = \pm L, y) = 0, \;\; n(x = \pm L, y) = 0, \;\; \zeta(x = \pm L, y) = 0$$

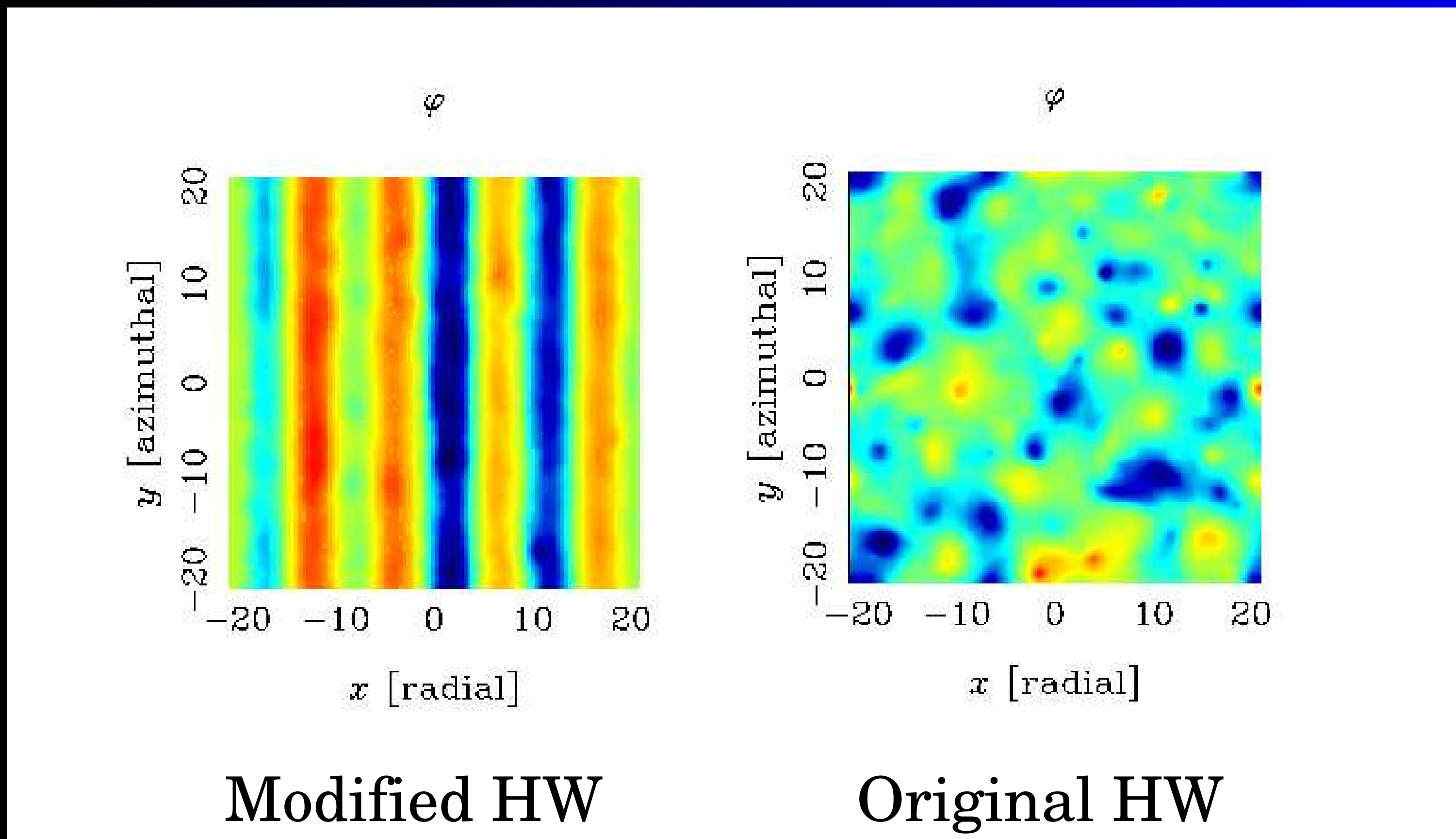
- Time stepping algorithm is a 3rd order explicit linear multistep method. The method for $\mathrm{d}\boldsymbol{x}/\mathrm{d}t = \boldsymbol{f}(t, \boldsymbol{x})$ is expressed by

$$\frac{11}{6}\boldsymbol{x}_n - 3\boldsymbol{x}_{n-1} + \frac{3}{2}\boldsymbol{x}_{n-2} - \frac{1}{3}\boldsymbol{x}_{n-3} = 3\boldsymbol{f}(t_{n-1}, \boldsymbol{x}_{n-1}) - 3\boldsymbol{f}(t_{n-2}, \boldsymbol{x}_{n-2}) + \boldsymbol{f}(t_{n-3}, \boldsymbol{x}_{n-3})$$

- Finite difference method is used for spatial discretization
- Poisson bracket term evaluated by the Arakawa's method (Arakawa (1966))
- Implemented on the APAC SGI Altix 3700 Bx2 cluser in ANU

Zonal Structure Generated in MHW Model

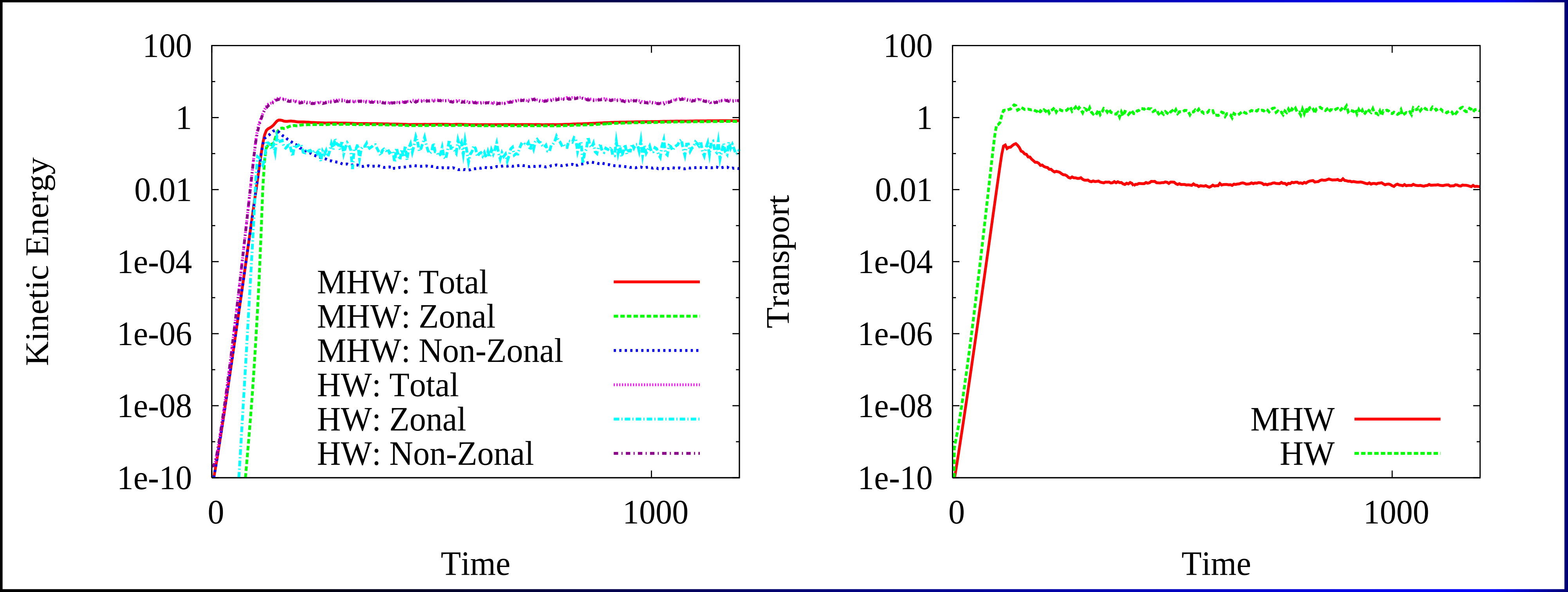
Saturated state of electrostatic potential φ



- Zonally elongated structure is clearly seen in MHW mode

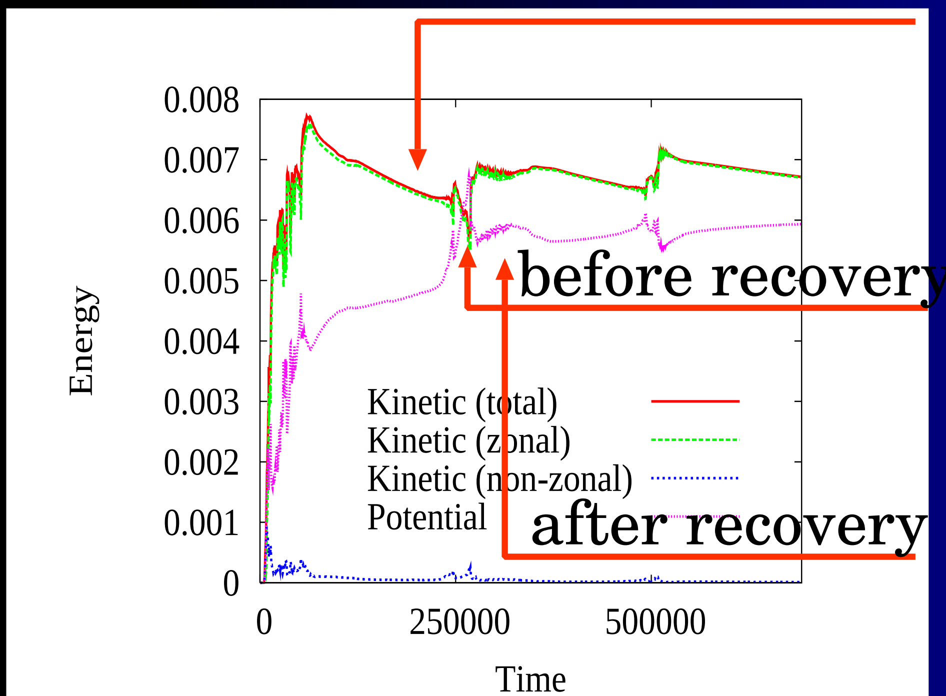
Zonal Flow Suppresses Transport

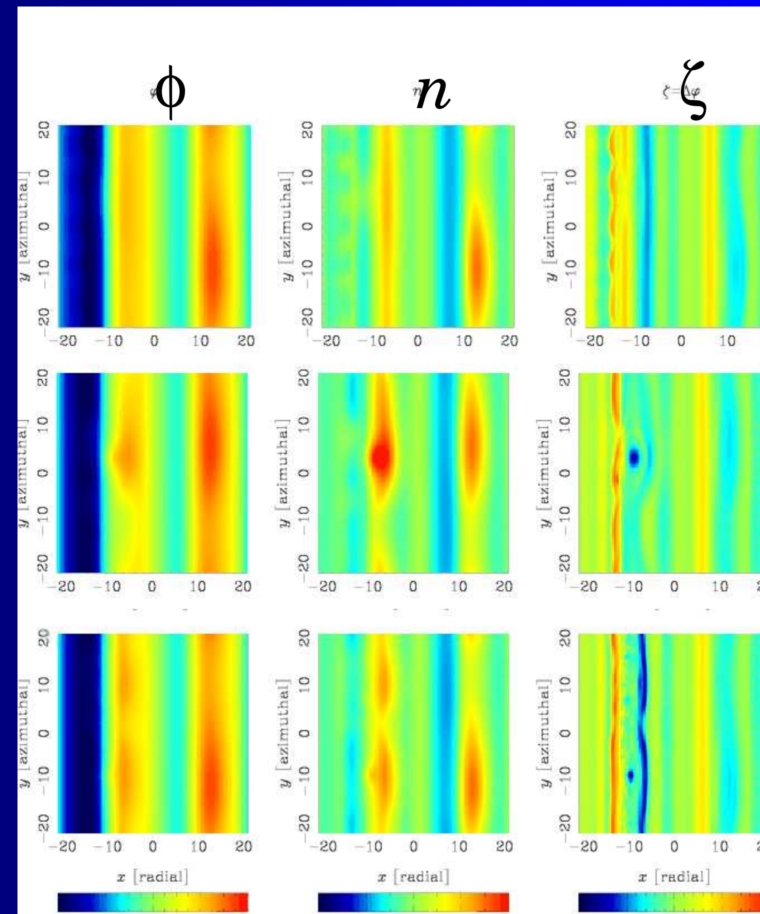
Time evolution of kinetic energy and transport for modified and unmodified HW model



- Zonal flow components dominates kinetic energy in MHW model
- Once zonal flow is generated, transport level is significantly suppressed

Sawteeth Like Behavior Observed in Weaker Drive Case ($\kappa = 0.1$)





- Instability $\longrightarrow$ Energy exchange between kinetic and potential energy $\longrightarrow$ Resisitve dissipation (parallel motion) [relatively large adiabaticity]
- Kinetic energy and potential energy are comparable

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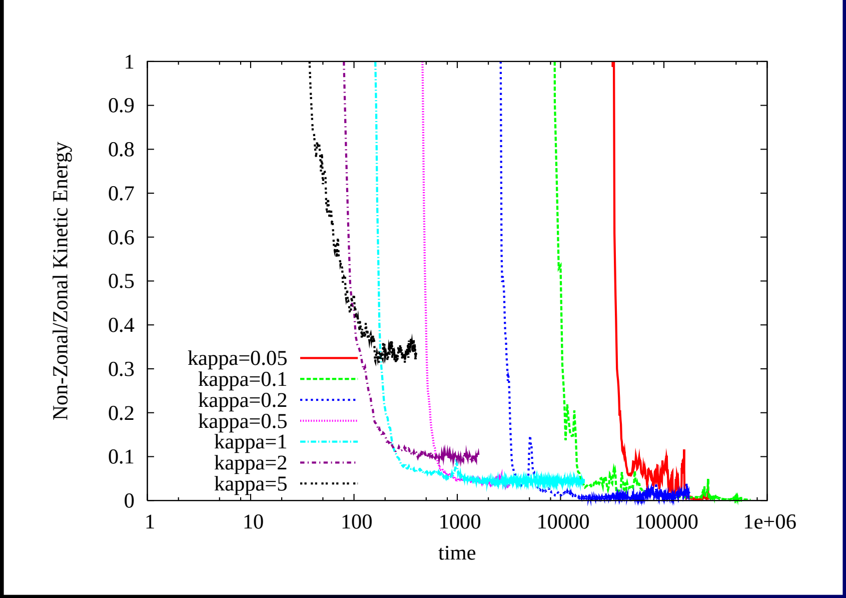
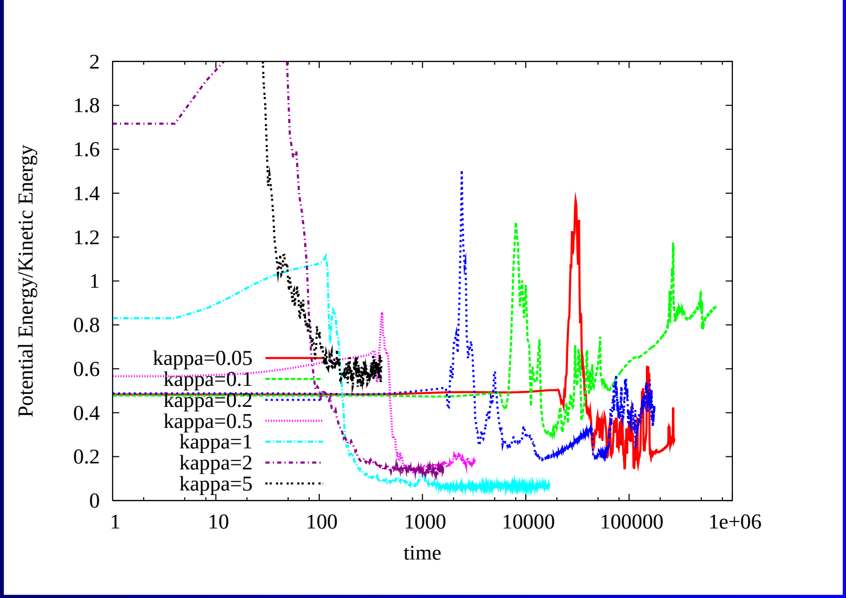
Energy Partition in Bursting Events





- Non zonal kinetic energy $\ll$ Zonal kinetic energy
- Competition between zonal kinetic energy and potential energy

Summary and Conclusion

- We have performed simulations of the modified Hasegawa-Wakatani model.
- Modification of the Hasegawa-Wakatani model is essential to generate zonal flows.
- Long time simulation shows sawteeth like behavior in low κ (weak drive) case.
- In bursting events, potential energy is comparable to zonal kinetic energy.