Workshop on Long Time Simulations of Kinetic Plasmas

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One man's noise is other man's signal. How do we proceed to settle this dispute?

Chairman’s charge:

1.) What is the metric of good performance against noise, i.e. - How many particles per wavelength are needed to resolve e-phi/T of X% on scale y for time T?

2.) What sets noise and Limits Performance in Gyrokinetic-PIC simulations?

3.) How does resolution limit performance in Gyrokinetic-Continuum Codes?

4.) What aspects of drift wave physics are most likely to be obscured in noisy PIC simulations?

5.) How might we mitigate noise in PIC codes and still extract the relevant physics/

6.) What might constitute a sensible PIC GPSC research program for the coming 1-2 years?

Governing Equations for Turbulence Simulation in Fusion Plasmas

- Gyrokinetic Vlasov Equation

$$\frac{\partial F_{\alpha gc}}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial F_{\alpha gc}}{\partial \mathbf{R}} + \frac{dv_\parallel}{dt} \frac{\partial F_{\alpha gc}}{\partial v_\parallel} = 0,$$

$$\frac{d\mathbf{R}}{dt} = v_\parallel \mathbf{b}^* + \frac{v_\perp^2}{2\Omega_{\alpha 0}} \hat{\mathbf{b}}_0 \times \nabla ln B_0 - \frac{c}{B_0} \nabla \bar{\phi} \times \hat{\mathbf{b}}_0$$

$$\frac{dv_\parallel}{dt} = -\frac{v_\perp^2}{2} \mathbf{b}^* \cdot \nabla ln B_0 - \frac{q_\alpha}{m_\alpha} \left(\mathbf{b}^* \cdot \nabla \bar{\phi} + \frac{1}{c} \frac{\partial \bar{A}_\parallel}{\partial t} \right)$$

-- Velocity Nonlinearity

$$\mu_B \equiv \frac{v_\perp^2}{2B_0} \left(1 - \frac{mc}{e} \frac{v_\parallel}{B_0} \hat{\mathbf{b}}_0 \cdot \nabla \times \hat{\mathbf{b}}_0 \right) \approx cons.$$

$$\mathbf{b}^* \equiv \mathbf{b} + \frac{v_\parallel}{\Omega_{\alpha 0}} \hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0 \cdot \nabla) \hat{\mathbf{b}}_0, \quad \mathbf{b} = \hat{\mathbf{b}}_0 + \frac{\nabla \times \bar{\mathbf{A}}}{B_0}$$

$$\begin{pmatrix} \bar{\phi} \\ \bar{\mathbf{A}} \end{pmatrix} (\mathbf{R}) = \langle \int \begin{pmatrix} \phi \\ \mathbf{A} \end{pmatrix} (\mathbf{x}) \delta(\mathbf{x} - \mathbf{R} - \rho) d\mathbf{x} \rangle_\varphi,$$

-- Coordinate Transformation

$$F_{\alpha gc} = \sum_{j=1}^{N_\alpha} \delta(\mathbf{R} - \mathbf{R}_{\alpha j}) \delta(\mu - \mu_{\alpha j}) \delta(v_\parallel - v_{\parallel \alpha j})$$

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Governing Equations for Turbulence Simulation in Fusion Plasmas (cont.)

- Gyrokinetic Poisson's Equation

$$\nabla^2\phi + \frac{\tau}{\lambda_D^2}[\phi(\mathbf{x}) - \tilde{\phi}(\mathbf{x})] = -4\pi\rho_{gc}(\mathbf{x}) \quad \overset{(k_\perp\rho_i)^2 \ll 1}{=========>} \quad \boxed{\frac{\rho_s^2}{\lambda_D^2}\nabla_\perp^2\phi(\mathbf{x}) = -4\pi\rho_{gc}(\mathbf{x})}$$

$$\tilde{\phi}(\mathbf{x}) \equiv < \int \bar{\phi}(\mathbf{R})F_i(\mathbf{R},\mu,v_\parallel)\delta(\mathbf{R}-\mathbf{x}+\rho)d\mathbf{R}d\mu dv_\parallel >_\varphi$$

$$\rho_{gc}(\mathbf{x}) = \sum_\alpha q_\alpha \langle \int F_{\alpha gc}(\mathbf{R})\delta(\mathbf{R}-\mathbf{x}+\rho)d\mathbf{R}dv_\parallel d\mu\rangle_\varphi$$

- Gyrokinetic Ampere's Law

$$\boxed{\nabla^2\mathbf{A} - \frac{1}{v_A^2}\frac{\partial^2\mathbf{A}_\perp}{\partial t^2} = -\frac{4\pi}{c}\mathbf{J}_{gc}} \qquad \omega^2/k^2v_A^2 \ll 1$$

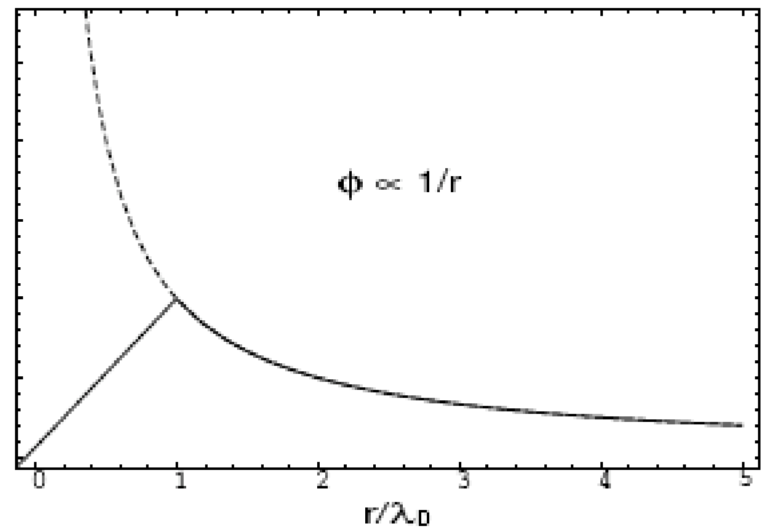
$$\mathbf{J}_{gc}(\mathbf{x}) = \mathbf{J}_{\parallel gc}(\mathbf{x}) + \mathbf{J}^M_{\perp gc}(\mathbf{x}) + \mathbf{J}^d_{\perp gc}(\mathbf{x})$$

$$= \sum_\alpha q_\alpha \langle \int (\mathbf{v}_\parallel + \mathbf{v}_\perp + \mathbf{v}_d)F_{\alpha gc}(\mathbf{R})\delta(\mathbf{R}-\mathbf{x}+\rho)d\mathbf{R}dv_\parallel d\mu\rangle_\varphi$$

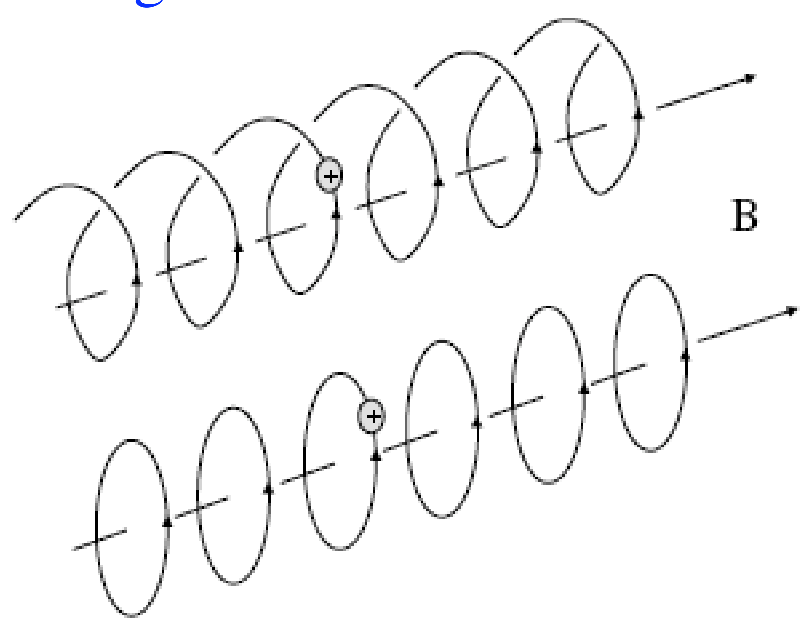
$$\mathbf{v}_d \equiv \frac{v_\parallel^2}{\Omega_{\alpha 0}}\hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0\cdot\nabla)\hat{\mathbf{b}}_0 + \frac{v_\perp^2}{2\Omega_{\alpha 0}}\hat{\mathbf{b}}_0 \times \nabla ln B_0$$

Deviations from the original Vlasov-Maxwell system

- Debye shielding



- Gyromotion becomes a charged ring



- Billions of particle have been used to study plasma turbulence on MPP platforms using the standard explicit PIC method.

- How many more do we need and why? Can this workshop help us to answer this question?

What have we done?

Billions of particle have been used to study plasma turbulence on MPP platforms using the standard explicit PIC method for thousands of time steps.

What do we also want to know?

How many more particles do we need and why?

- If a simulation satisfies all the conservation properties, should we believe it's signal?
- Is noise energy conserving?
- Can noise enhance or reduce diffusion?
- We know particle codes produce noise, how about continuum codes?
- How many orders of magnitude increase in resolution do we need to achieve in order to do the convergence test?

• How long should we run?

- GTC: 5000 - 10,000 time steps with predictor-corrector method.
- BEST: 1,000,000 time steps with 2nd order Runge-Kutta
- Scott Tremaine (Princeton University) : 9 planets plus sun interact with 1/r^3 using leap-frog to study stability of the solar system for millions of years.