PRESSURE DRIVEN MODES IN LOW-SHEAR REGIONS*.

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Ideal MHD instabilites are expected to play a role in determining the maximum achievable ß in tokamaks. Recent studies [1] have shown that a new form of pressure driven internal instability is possible which cannot be described by standard ballooning theories, and requires a full δW analysis. These have been termed as ' infernal modes' . They are shown to exist when there is a large enough pressure gradient in a region of low shear ($dq/d\psi$). In this study we shall briefly review the nature of this form of instability and its parametric dependence on plasma profiles. We shall then address its possible role in experiment.

Ballooning theory predicts that the largest-n modes are the most unstable, and that as n is decreased the growth-rate decreases monotonically. In certain circumstances this picture is modified to include an oscillatory dependence of the growth-rate on the mode number. This has been described in reference 2, along with conditions for its occurrence. Briefly; the standard ballooning theory is valid when, $n \gg (q')^{-2} \gg 1$, where $q' \equiv \psi\, dq/d\psi$. When the shear is reduced so that $(q')^{-2} \gg n \gg 1$, then a new theory is required, which indicates that the growth-rate will be an oscillatory function of n ($\Delta n \approx 1/q$), with a decreasing amplitude ($A \approx 1/n^2$). This implies a transition back to standard ballooning theory as n approaches infinity. However as shown in Ref. [1], if the shear is reduced even further, then even this Hastie-Taylor theory breaks down, and the connection with standard ballooning theory is severed. What remains, are alternating bands of stable and unstable toroidal mode numbers. This is well into the infernal mode regime. These results are summarized in figure 1 which shows the dependence of the growth-rate on n for a series of equilibria with circular cross-section, the same aspect-ratio (4), and the same functional form for the pressure profile. $p = p_o\,(1 - \psi^{\alpha_2})^{\alpha_1}$, α_1=4 and α_2=1.5, p_o is adjusted so that $ß$ = 1.5% in all cases. The pressure profile has been chosen so that the maximum of p' is in the region of reduced shear. The q-profile is specified through the function $q(\psi) = q_o + q_1\,\psi^{\alpha_q}$, with q_o=1.05 and q_1=2.05, this makes q_a=3.1. The different curves in figure 1 are obtained for values of the q-profile parameter α_q=1.5, 2 , 3 , and 4. The lowest value , 1.5, represents the case with the greatest shear near the axis, while α_q=4 has the lowest shear. Figure 1

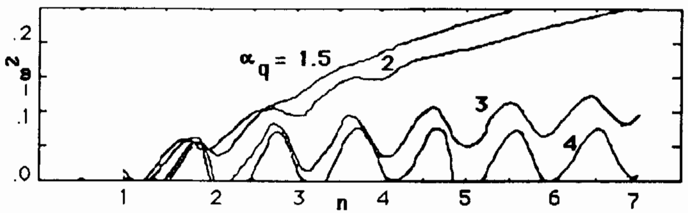

Figure 1. Plot of $-\omega^2$ vs. n for equilibria with varying shear . α_q controls the shear, the case with α_q =1.5 has the greatest shear near the axis and the one with α_q = 4 the least shear. As the shear is reduced infernal modes appear.

shows the corresponding growth-rate dependence and clearly indicates a transition from the mostly monotonic dependence expected in standard ballooning theory for α_q=1.5, through the regime where the Hastie-Taylor theory is valid for α_q=2, to the infernal mode regime, for α_q=4 where the banded structure is clearly visible. It should be noted that the role of the pressure profile is critical, as shifting the maximum of the gradient away from the low-shear regions to regions of larger shear, can restore the validity of the standard ballooning theory. Details of these and other observations are available in reference 1.

We now restrict ourselves to integer n and consider a mode with n=1. Figure 1 shows that n=1 does not coincide with a peak in the growth-rate but if we extend the observed periodicity to n less than unity, we would expect a peak at $n \approx 0.94$. An examination of δW shows that within the plasma n appears in the form nq, this implies that when q_o is changed from 1.05 to 1.01 the peak would shift to n = 0.98, and overlap with n=1. This would lower the ß limit for the n=1 mode, possibly below the ballooning threshold. We demonstrate this by obtaining the ß limits for these modes at these two values of q_o. The procedure is to fix the q-profile and the pressure profile functional and vary the peak pressure to obtain several equilibria with different values of ß, these are then examined for their stability properties and the critical ß is determined for each instability. This exercise is repeated for each value of α_q. The results are shown in figure 2, for q_o=1.05 (heavy curves), and 1.01 (light curves). When q_o = 1.05, we note that the ballooning mode ß limit lies below the n=1 mode

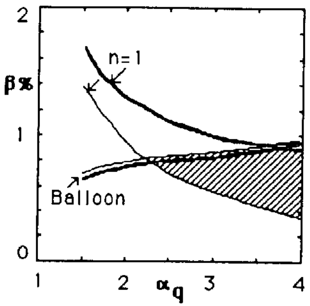

Figure 2. ß thresholds for the internal n=1 and ballooning modes for q_o = 1.05 (heavy curves) and q_o = 1.01 (light curves). The shaded region shows the infernal mode regime for q_o = 1.01 .

limit for most of the range, with a slight overlap near α_q=4 . This implies that for this q_o the ballooning mode sets the lower limit, and the n=1 infernal mode is not significant. When q_o is equal to 1.01, the ballooning mode limit remains virtually unchanged, however the n=1 mode threshold drops considerably, and for α_q greater than about 2.5, its threshold lies well below that of the ballooning mode. This region is clearly in the infernal mode regime.

We note here that the observed instability is not an internal kink mode as q_o is greater than unity, and the q=1 surface is not in the plasma. The mode structure of this instability is shown in figure 3, and clearly has the ballooning mode characteristic, rather than that of an internal kink mode. However, even though q_o is greater than unity, the low-shear near the axis as well as toroidicity, couple the poloidal harmonics and a large m=1 component is observed. This is typical of such low shear equilibria when q_o is close to unity.

This naturally leads to the speculation that the infernal mode may play a significant role in experiment. Reviewing the principal ingredients of the infernal mode: a low shear region and a large pressure gradient conspire to produce oscillations in the growth-rate as a function of the toroidal mode number. Slight modification of the q-profile, especially near the axis, can shift the peaks towards integer n values. The threshold for the low-n modes will drop rapidly in the process, reducing them well below that of the ballooning mode. These ingredients are all naturally present in experiment. It has long been assumed that the sawtoothing

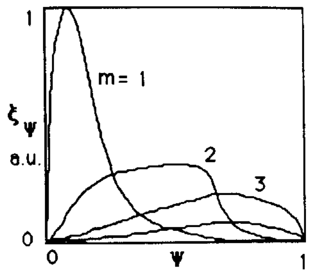

Figure 3. Poloidal harmonic content of the n=1 mode for the case with q_o= 1.01. Note that the m = 1 component is dominant even though the q=1 surface is not present in the plasma.

process is associated with a lowering of the shear in the central regions, especially when the q=1 surface is close to the center, and having q_o less than unity is conducive to driving an n=1 infernal mode, which leaves the peakedness of the pressure profile as the only remaining factor. In the model profiles of this study the pressure profile was strongly peaked, in experiment this may or may not be true. However even with a broader profile, we expect these effects to be induced at higher values of ß. Clearly such a mode could be responsible for some of the unusual MHD activitiy observed in high ß tokamaks, such as giant saw-teeth and ß collapse phenomena.

It is important to note that this study has concentrated on internal modes, by invoking a perfectly conducting shell at the plasma edge. It would be more appropriate to allow for a vacuum region surrounding the plasma. This does not affect the results at high α_q, as the enhanced edge shear stabilizes the external mode, but has a profound effect on the low α_q results, effectively lowering the thresholds even further for these profiles.

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References

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[2] Hastie, R. J., and Taylor, J. B., Nucl. Fusion 21 (1981) 187.