TOOLS FOR THREE-DIMENSIONAL MHD EQUILIBRIUM CONFIGURATION STUDIES*

J. L. Johnson, D. Monticello, N. Pomphrey and A. H. Reiman

Plasma Physics Laboratory, Princeton University,
Princeton, New Jersey 08543

Computer hardware and numerical algorithms have improved so that it is possible to carry through elaborate MHD calculations for fully three-dimensional systems. The PIES code[1] has been developed for this purpose. It is a fully three-dimensional MHD equilibrium code capable of handling islands and stochastic regions. Three-dimensional MHD computations are still being complemented with results from two-dimensional averaged techniques like the STEP code[2,3] which are not as time-consuming and require less careful monitoring in their execution. Here we describe a major improvement that has been made to the PIES code, discuss some comparisons with analytic models for code validation, and introduce some techniques to couple the PIES and STEP codes in order to take advantage of the positive features of both approaches.

The PIES code uses an iterative algorithm that updates the magnetic field at each step by first solving for the pressure driven currents in the current magnetic field, and then solving Ampere's law to determine a new magnetic field. A numerical diagnostic determines the location of the islands and stochastic regions. Pressure and current profiles are flattened there. The pressure-driven currents are calculated on the good flux surfaces by a transformation to magnetic coordinates.[4] The computed current is then input to a three-dimensional Ampere's law solver, which employs a discretization in the radial direction and a Fourier representation in the angles.[5]

The major improvement has been the implementation of a coordinate transformation that improves the efficiency and robustness of the code, and allows us to handle islands at finite β. Our Ampere's law solver is capable of working in an arbitrary, numerically specified coordinate system. The initial implementation of the PIES code used the magnetic coordinate system in which the current is computed. When stochastic regions and islands appeared, the coordinates were calculated in those regions by interpolating between flux surfaces. The resulting coordinates behave poorly in the neighborhood of a separatrix. The resonant ripple of the flux surfaces increases rapidly as a separatrix is approached. The lines of constant magnetic angle converge at the x-point. Calculations in such a coordinate system encounter numerical problems. To cure those problems, we have implemented a transformation to a well-behaved "background" coordinate system, which is invoked before the Ampere's law solver. An earlier modification implemented this transformation at zero-β, allowing the calculation of zero pressure magnetic islands.[6] It was found that the transformation improved the robustness of the code even in the absence of islands, eliminating the need for the radial filtering that had often been required to stabilize short wavelength modes. The transformation has now been implemented for finite pressure, allowing us to calculate finite-β islands. A similar improvement in robustness has been seen for finite β calculations with good flux surfaces.

One application of the PIES code is to calculate the nonlinear saturation of tearing modes. We have benchmarked our zero-β tearing mode calculations against analytic calculations for narrow islands in the large aspect ratio limit. Comparisons have been done for a range of different current profiles. We find good agreement, not only for the predicted island saturation widths, but also for the detailed profiles of the perturbed current and magnetic field inside and outside the island. The comparison with analysis provides strong verification of the efficacy and accuracy of the code.[7]

In the STEP code,[2,3] the averaged contributions of the helical field to the magnetic flux and curvature are used to convert the problem of determining the MHD equilibrium and stability properties of a three-dimensional configuration to a two-dimensional one. With these contributions, $\Psi^{(0)}_{\mathrm{vac}}(R,Z)$ and $\Omega(R,Z)$, and the pressure $p(\Psi)$ and total current $I(\Psi)$ inside a magnetic surface prescribed, the code produces an equilibrium solution with the magnetic field given by

$$\boldsymbol{B}(R,\phi,Z) = R_0B_0(1-p(\Psi)/B_0)\nabla\phi + \nabla\phi\times\nabla A^{\sigma}(R,Z) + \boldsymbol{B}^{\delta}(R,\phi,Z). \tag{1}$$

The stream function A^{σ}, associated with axisymmetric currents in the plasma, is given on an X,Z array and the nonaxisymmetric vacuum field $\boldsymbol{B}^{\delta}$ has been prescribed (usually as a Fourier decomposition in R,Z,ϕ). One can convert the output of the STEP code into an input form for the PIES code by providing a subroutine which is capable of giving the magnetic field at any point, together with a specification of the pressure and current profiles as functions of the toroidal flux. A simple spline fit can provide the values of $p(\Psi)$ and $I(\Psi)$ at the desired surfaces. It is useful to compute $\boldsymbol{B}^{\delta}$ in a divergence free form; for example the vacuum field

$$\boldsymbol{B} = R_0B_0\nabla\{[(R/b)^N/2N]\cos N\phi\}, \tag{2}$$

which was used in a study of the effects of plasma currents on the toroidal ripple field in a tokamak,[8] should be written as

$$\boldsymbol{B} = \nabla Z\times R_0B_0\nabla\{[(R/b)^N/2N]\sin N\phi\}. \tag{3}$$

A two-dimensional cubic spline provides the necessary interpolation for A^{σ} and $\boldsymbol{B}^{\delta}$. The PIES code then follows field lines so as to establish magnetic coordinates, determine the Fourier decomposition of the different field components, and obtain the shape of the plasma boundary. This conversion enables the STEP code output to be studied with powerful line-following techniques.[4] It allows the PIES code to start with field and position values which are close to the desired equilibrium so that fewer iterations are necessary and the calculations are simplified. It also introduces a technique to incorporate a free plasma-vacuum interface instead of prescribing a fixed interface.

It has been proposed previously[9] that the output of a three-dimensional code could be used as input for the STEP stability studies. Since the physics restrictions on the unstable modes that are introduced by the stellarator expansion are quite reasonable even for fully three-dimensional configurations, the averaged results from the PIES code should give a good idea of the low-n MHD stability properties of a fully three-dimensional stellarator or tokamak configuration. The PIES code provides a description of the magnetic surfaces and fields in terms of a Fourier decomposition in the poloidal and toroidal angle variables θ and ϕ,

$$B_R(\Psi,\theta,\phi)=\sum_n\sum_m B_{R,m,n}(\Psi)\,\sin(m\theta)\,\cos(n\phi), \tag{4}$$

etc. We can easily average over the toroidal angle ϕ to obtain the shapes of the averaged magnetic surfaces,

$$\bar{R}(\Psi,\theta)=\sum_m R_{m,0}(\Psi)\,\cos(m\theta),\quad \bar{Z}(\Psi,\theta)=\sum_m Z_{m,0}(\Psi)\,\sin(m\theta), \tag{5}$$

and the associated Jacobian and metric elements. Averaging over ϕ gives expressions for the contributions of the nonaxisymmetric fields to the poloidal flux,

$$\Psi^{(0)}_{\mathrm{vac}}(\bar{R},\bar{Z})=\frac{1}{R_0B_0}\int_0^{2\pi}\left[R^3\tilde{B}_Z(\Psi,\theta,\phi)\int_0^{\phi}\tilde{B}_R(\Psi,\theta,\phi)d\phi\right]d\phi, \tag{6}$$

and to the average field line curvature,

$$\Omega(\bar{R},\bar{Z})=\left\langle\frac{\bar{R}^2(\Psi,\theta)}{R_0^2}+\frac{\tilde{B}^2(\Psi,\theta,\phi)}{B_0^2}\right\rangle, \tag{7}$$

which are the input functions required for the STEP stability code.

These techniques for improving and validating the PIES code and for coupling it to the STEP code should strengthen our ability to study three-dimensional MHD equilibrium and stability problems. The ideas could be incorporated equally well into other two- and three-dimensional approaches.

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