DEPENDENCE OF CIT PF COIL CURRENTS ON PROFILE AND SHAPE PARAMETERS USING THE CONTROL MATRIX*

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ABSTRACT

The plasma shaping flexibility of the Compact Ignition Tokamak (CIT) poloidal field (PF) coil set is demonstrated through magnetohydrodynamic (MHD) equilibrium calculations of optimal PF coil current distributions and their variation with poloidal beta, internal inductance, plasma 95% elongation, and 95% triangularity. Calculations of the magnetic stored energy are used to compare solutions associated with various plasma parameters. The Control Matrix (CM) equilibrium code,[1] together with the nonlinear equation and numerical optimization software packages HYBRD[2] and VMCON[3] respectively, is used to find equilibrium coil current distributions for fixed divertor geometry, volt-seconds, and plasma profiles in order to isolate the dependence on individual parameters. A reference equilibrium and coil current distribution are chosen, and correction currents $d\mathbf{I}$ are determined using the CM equilibrium method to obtain other specified plasma shapes. The reference equilibrium is the κ = 2 divertor at beginning of flattop (BOFT) with a minimum stored energy solution for the coil current distribution. The pressure profile function is fixed.

THE CONTROL MATRIX EQUILIBRIUM CODE

Free-boundary magnetohydrodynamic (MHD) equilibria with prescribed constraints on plasma radii, magnetic X-point position, 95% shape parameters, and flux linkage are obtained efficiently using a Control Matrix (CM) method. Plasma shape control matrices $\mathbf{A}$ relating changes in the poloidal field (PF) coil currents $d\mathbf{I}$ to deviations from prescribed values in the plasma position, shape, and flux, i.e., $d\mathbf{I} = \mathbf{A}\epsilon$, are computed in the inner loop of an iterative solution to the nonlinear equilibrium equation. The desired external field is found as the error vector ϵ converges to zero.

A typical iterative algorithm for solving the separatrix-limited free-boundary equilibrium problem $\Delta^*\psi = r^2\nabla\bullet(r^{-2}\nabla\psi) = -\mu r J_\phi$, for the poloidal flux function ψ in a rectangular region Ω, consists of two major computational parts:

(1) For fixed plasma current density and coil current distributions, compute boundary values of ψ on $\partial\Omega$, solve for $\psi(r,z)$ in the interior of Ω, and locate the separatrix to evaluate ψ_x.

(2) Given ψ_x and $\psi(r,z)$ in Ω, compute the plasma current density distribution $\mathbf{J}_\phi(r,z)$ using the assumed pressure and toroidal flux profile functions $P(\psi)$ and $F(\psi)$, respectively.

That is, the first part computes the flux function from a knowledge of the currents, and the second computes the plasma current from the flux function and plasma profiles. The shape of the plasma boundary is controlled by the external flux $\psi_e(r,z)$, where $\psi = \psi_p + \psi_e$. Values of the plasma flux, ψ_p, and external flux, ψ_e, on $\partial\Omega$ are related through Green's functions to the plasma current density and external coil currents I_i, i = 1, ..., N, respectively.

The CM method (Fig. 1) consists of computing the shape control matrix $\mathbf{A}$ after step (1), using it to determine the vector of correction currents $d\mathbf{I}$ producing a desired change in the plasma shape, and recomputing step (1) with the coil current vector $\mathbf{I} = \mathbf{I} + d\mathbf{I}$. The calculation in step (2) is then carried out with the corrected coil current distribution.

For a fixed plasma current distribution J_ϕ, the shape control matrix $\mathbf{A}$ is computed by varying the PF coil current distribution about reference values in a set of M solutions to the linear elliptic problem $\Delta^*\psi = -\mu r J_\phi$. Using the error vectors associated with each solution, set $\sum a_{i,k}\epsilon_{kj} = dI_{i,j}$, i = 1, ..., N, j = 1, ..., M, and solve this system for the row vectors $\mathbf{a}_i$ of the $N \times K$ matrix $\mathbf{A}$. Typically, $M > K$ (where K is the length of the error vector), and a least-squares solution is computed.

With greater than a factor of 10 decrease in CPU time over previous methods,[5] new applications in the area of divertor equilibrium optimization are possible using the CM method.

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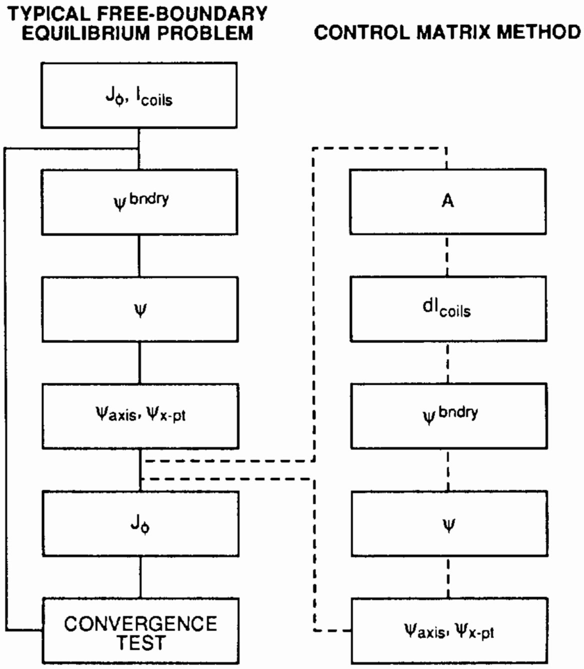

Fig. 1. For fixed X-point equilibria, the shape control matrix **A** is computed and used to determine a correction $d\mathbf{I}_{coils}$ to the coil current distribution, prior to updating the plasma current distribution $\mathbf{J}_\phi$.

PROFILE FUNCTIONS

Plasma pressure and toroidal magnetic flux profile functions used in the evaluation of the plasma current distribution,

$$J_\phi = r\, dP/d\psi + F(dF/d\psi)/(\mu r),$$

are given in terms of their derivatives:

$$dP/dx = P_0\,[\exp(-Ax) - \exp(-A)]/[\exp(-A) - 1],$$

$$dF^2/dx = 2\mu R_o^2\, P_0\,(1/\beta_J - 1)[\exp(-Bx) - \exp(-B)]/[\exp(-B) - 1],$$

where $x = (\psi - \psi_0)/(\psi_x - \psi_0)$. The parameter P_0 is scaled so that the total plasma current,

$$I_p = \iint J_\phi\, d\Omega,$$

is fixed at an input value. In general, β_J controls the plasma beta, and A and B control the width of the current profile.

Plasma equilibrium pressure and current density profiles are characterized by the poloidal beta,

$$\beta_p = 4 \int P\, dV/(\mu R_o I_p^2),$$

and the plasma internal inductance,

$$l_i/2 = \int B_p^2 dV/(\mu^2 R_o I_p^2),$$

respectively.

PROFILE CONSTRAINTS AND MINIMUM-ENERGY SOLUTIONS

The numerical software package HYBRD[2] is designed to solve n nonlinear equations in n variables. The CM equilibrium subroutine is called from HYBRD to compute values of free parameters β_J and B in the F-profile to constrain the profile parameters β_p and $l_i/2$.

In the CIT, seven coil sets control five plasma shape and flux parameters. It is possible to fix the values of two coil currents in the CM solution, i.e., to reduce the size of the control matrix from 7 × 5 to 5 × 5. In this case the nonlinear optimization package VMCON[3] is used to find values of the two additional currents to minimize the stored energy in the coil system $W_{PF} = 1/2\mathbf{I}^T M\mathbf{I}$. This is referred to as a minimum energy equilibrium solution.

REFERENCE EQUILIBRIUM AND THE CIT PF SYSTEM

The geometry considered in this study is based on a CIT design with R_o = 2.138 m, a = 0.661 m, field on axis B_t = 11 T, and plasma current I_p = -12.3 MA. The reference equilibrium is at BOFT with κ = 2.0, δ = 0.25, β_p = 0.16, $l_i/2$ = 0.37, and a flux-linkage ϕ = 38 V-s (Fig. 2). For the calculations in this study, the shape of the pressure profile is fixed (A = -0.49). The external PF coil system used is GEM-29[4] (Table I) and consists of seven coil groups, labeled PF1

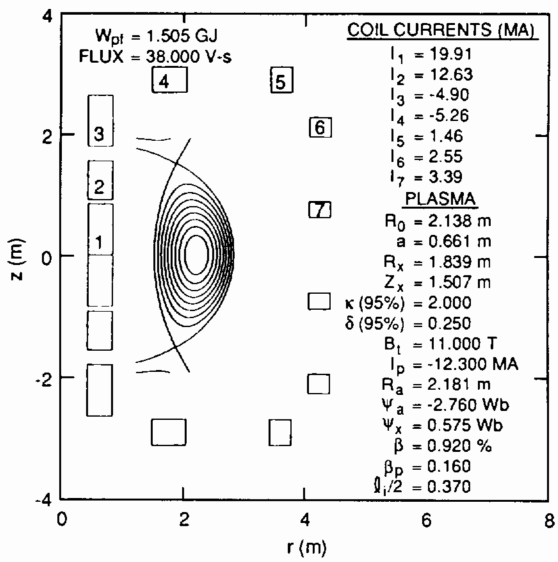

Fig. 2. The reference CIT equilibrium solution is the κ = 2, diverted plasma at beginning of flattop.

Table I
Poloidal field coil set based on GEM-29

Coil	R_c (m)	Z_c (m)	ΔR (m)	ΔZ (m)
PF1	0.592	0.410	0.406	0.821
PF2	0.592	1.307	0.406	0.770
PF3	0.592	2.179	0.406	0.770
PF4	1.630	2.891	0.563	0.423
PF5	3.190	2.870	0.291	0.271
PF6	4.187	2.000	0.275	0.324
PF7	4.225	0.749	0.354	0.448

through PF7, that provide the equilibrium vertical field, shaping field, and inductive flux for a divertor plasma.

The central solenoid stack is divided into three sections (PF1, PF2, and PF3) for added flexibility in providing a field null at startup and shaping the plasma through a discharge. Solenoid sections are modeled by multiple filaments (3×7), while ring coils are modeled with a single filament. In general, all external PF coils contribute to the equilibrium, shaping, and control of the CIT plasma, as well as the ohmic heating function.

VARIATION WITH PLASMA PROFILE

The variation in the CM equilibrium coil current distribution with poloidal beta is almost linear (Table II), with PF7 increasing in current with β_p to account for the increase in the required vertical field,[6] while the remaining currents are adjusted to maintain the fixed volt-second condition.

Table II
PF coil current variation with poloidal beta

	β_p			
	0.08	0.16	0.24	0.32
Currrent (MA)				
PF1	20.406	19.900	19.414	18.915
PF2	12.651	12.660	12.624	12.638
PF3	-5.095	-4.914	-4.708	-4.530
PF4	-5.237	-5.273	-5.294	-5.333
PF5	1.633	1.468	1.285	1.121
PF6	2.553	2.550	2.544	2.541
PF7	3.229	3.383	3.543	3.695
W_{PF} (GJ)	1.512	1.506	1.501	1.499

CM coil current distributions for various values of the plasma internal inductance are presented in Table III and Fig. 3. Figure 4 shows that stored energy rises sharply for broad plasma current profiles (small $l_i/2$) and shows a minimum energy point in the interval $0.4 < l_i/2 < 0.45$.

Table III
PF coil current variation with internal inductance

	$l_i/2$			
	0.27	0.37	0.47	0.52
Current (MA)				
PF1	15.121	19.900	23.347	24.607
PF2	25.410	12.660	3.185	-0.181
PF3	-12.144	-4.914	0.705	2.308
PF4	-9.895	-5.273	-2.200	-1.014
PF5	5.858	1.468	-1.899	-3.269
PF6	3.538	2.550	1.827	1.592
PF7	1.367	3.383	5.000	5.644
W_{PF} (GJ)	2.593	1.506	1.448	1.600

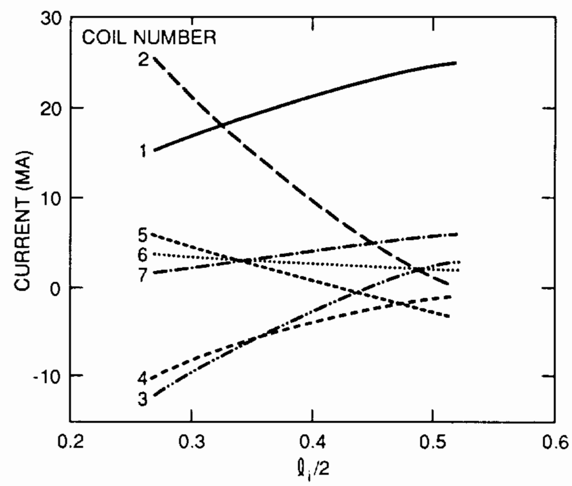

Fig. 3. The dependence of coil currents PF1 through PF7 on internal inductance for fixed plasma 95% shape parameters, poloidal beta, and flux linkage (volt-seconds).

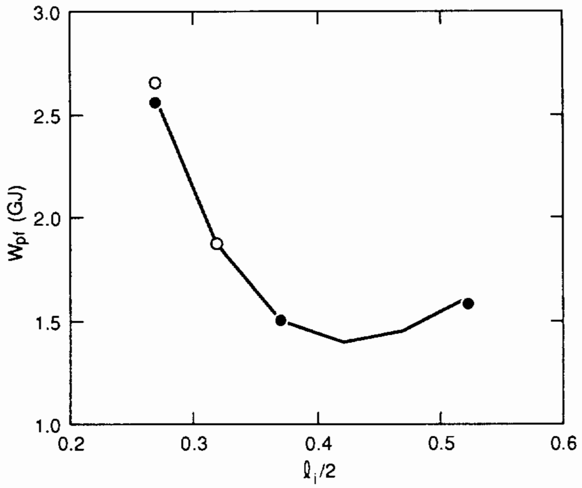

Fig. 4. The dependence of PF coil stored energy on internal inductance (solid line). The dots are minimum-energy solutions. The open circles are CM solutions with I_3 = -7.5 MA (fixed).

For a minimum-energy solution at β_p = 0.16 used as an initial coil current distribution for the CM method, Fig. 4 shows that CM solutions with a 7 × 5 control matrix are near minimum stored energy for variations in plasma profiles. If the current in PF3 is constrained (I_3 = -7.5 MA), as may be necessary to reduce the vertical separating forces on the solenoid, the size of the control matrix is reduced to 6 × 5, and there is an associated increase in W_{PF} (Fig. 4).

VARIATIONS WITH PLASMA SHAPE

CM solutions for different values of the 95% elongation are given in Table IV. For fixed plasma position (major and minor radius), shaping coil currents (e.g., PF3 and PF4) and stored energy decrease in magnitude with increasing elongation (Fig. 5). If elongation is increased by reducing the plasma minor radius, W_{PF} tends to increase, as shown for fixed X-point solutions in Fig. 5. Also shown in Fig. 5 is the stored energy associated with solutions with varying elongation but fixed safety factor at the 95% flux surface (q = 3.2).

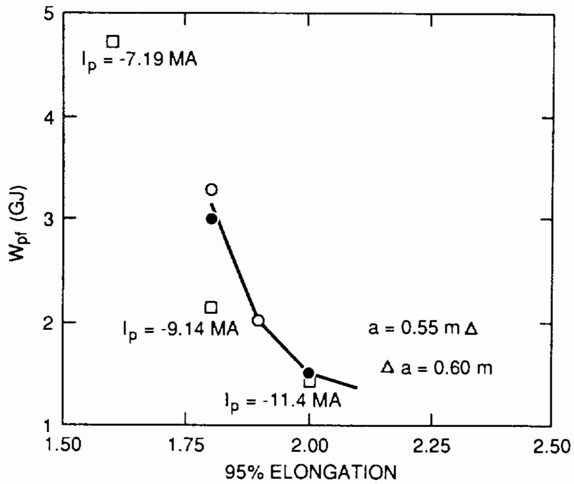

Fig. 5. The dependence of PF coil stored energy on 95% elongation (solid line). The dots are minimum-energy solutions. The open circles are CM solutions with I_3 = -7.5 MA (fixed). The squares are CM solutions with fixed 95% safety factor (q = 3.2, variable I_p). The diamonds correspond to solutions where κ is increased by reducing the plasma minor radius.

PF coil current distributions for variations in 95% plasma triangularity are presented in Table V. There is only a modest decrease in W_{PF} with increasing triangularity in the interval 0.2 < δ < 0.4 for κ = 2 and fixed profile parameters.

Table IV
PF coil current variation with plasma elongation

	κ_{95}			
	1.80	1.90	2.00	2.10
Current (MA)				
PF1	16.939	18.694	19.900	20.938
PF2	22.597	16.623	12.660	9.415
PF3	-17.146	-9.832	-4.914	-0.907
PF4	-12.299	-8.104	-5.273	-3.316
PF5	6.924	3.681	1.468	-0.134
PF6	3.915	3.109	2.550	2.088
PF7	1.369	2.563	3.383	4.025
W_{PF} (GJ)	3.155	1.965	1.506	1.351

Table V
PF coil current variation with plasma triangularity

	δ_{95}				
	0.20	0.25	0.30	0.35	0.40
Current (MA)					
PF1	18.997	19.900	20.774	21.873	22.904
PF2	14.728	12.660	10.588	7.842	5.333
PF3	-4.311	-4.914	-5.335	-5.360	-6.422
PF4	-5.656	-5.273	-4.774	-3.956	-2.609
PF5	1.005	1.468	1.780	1.844	1.852
PF6	2.519	2.550	2.546	2.483	2.353
PF7	3.610	3.383	3.223	3.171	3.116
W_{PF} (GJ)	1.544	1.506	1.470	1.416	1.364

SUMMARY

The CM equilibrium code, together with numerical software for nonlinear equations and constrained optimization, is used to find equilibrium coil current distributions for fixed divertor geometry, volt-seconds, and plasma profiles in order to determine the variation with plasma profile and shape parameters. Two input toroidal flux function profile parameters in the equilibrium code are varied by HYBRD to constrain poloidal beta and internal inductance. The variation in coil currents with internal inductance is much stronger than the variation with poloidal beta and, for $\beta_p = 0.16$, shows a distinct minimum in stored energy in the interval $0.40 < l_i/2 < 0.45$. With a minimum-energy reference equilibrium, CM solutions for variations in profile parameters are near minimum energy. For constant minor radius, shaping coil currents and stored energy rise in magnitude with decreasing elongation. For variable minor radius, stored energy rises as elongation increases.

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