MHD CONSTRAINTS FOR ADVANCED TOKAMAK OPERATION*

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Abstract

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Advanced tokamaks are characterized by having a ratio of bootstrap driven to total current of near unity, or by having extreme shaping to maximize the plasma beta limits. The paper reports on the theoretical equilibrium and stability properties of such configurations.

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1. Properties of Plasma Equilibrium with High Bootstrap Fraction [1]

Detailed studies of the economic tradeoffs of fusion reactor designs[1] have shown us that economic viability depends on operating a tokamak reactor plasma in a regime where most of the current is produced by the bootstrap effect. We have carried out intensive theoretical studies of the stability properties of such plasmas, of the physics parameters needed to optimize them, and of the scaling relations which allow us to prototype certain advanced tokamak reactor features in smaller experiments such as PBX-M.

The bootstrap fraction, I_{bs}/I_p, and the shape of the bootstrap current density, J_{bs}, can depend sensitively on the shape of plasma profiles. We have expanded Hirshman's collisionless form[2] for J_{bs} in powers of $(a/R)^{1/2}$. We find that if the plasma is within the first regime stability limit (Troyon Limit), then I_{bs}/I_p can be maximized by peaking both the current and pressure profiles, but poor alignment prevents values of $I_{bs}/I_p > 0.5$ with moderately flat density profiles, or 0.7 for peaked profiles from being achieved. Second stability operation with concomitant high q_0 makes possible configurations with $I_{bs}/I_p > 0.9$.

We derive analytically a new scaling of the poloidal plasma beta, β_p, with the bootstrap fraction, f_b, for a steady-state plasma in the banana regime with a fixed seed current profile. This scaling, which assumes that $n(r)/n(0) = [T(r)/T(0)]^{\lambda}$, is of the form $\sqrt(\epsilon)\beta_p = A(\lambda, Z_i)[a_1 f_b - a_2 f_b^2]$, where ϵ is the aspect ratio, Z_i is the ionic charge, $A(\lambda, Z_i)$ is a monotonically decreasing function of λ, and a_1 and a_2 are numerical coefficients of order unity. For plasma operation in the first stability regime, we have obtained scaling relations by combining the bootstrap formula discussed above, the Troyon limit, and the L-mode scalings. The β limit for a high bootstrap fraction tokamak scales like $(a/R)^{1/2}$, while the total bootstrap current scales as $P^{1/2}a^{1/2}B^0$.

We have performed extensive parameter studies of the stability of equilibria with high bootstrap fraction using the ideal MHD codes PEST, BALLOON and CAMINO. For aspect ratio $A = 4.5$, we have focused attention on a *first stability regime* with central safety factor $q_0 = 1.3$, $I_{bs}/I_p = .70$, Troyon factor $C_t = 3.0$, and $\beta = 1.95\%$, which is stable to all ideal MHD modes with a wall at infinity, and a *second stability regime* with $q_0 = 2.4$, $I_{bs}/I_p = .98, C_t = 4.15$, and $\beta = 1.61\%$, requiring a wall at $b = 1.29$ for kink mode stabilization and a second stability regime with $q_0 = 4.0$, $I_{bs}/I_p = .98, C_t = 3.3$, and $\beta = 0.9\%$, which is stable to all ideal MHD modes without a conducting wall.

At very high β, asymptotic scaling relations and increased physical insight come from investigating the stability and bootstrap current properties of analytic very high β large aspect ratio equilibria.[3] It is found that there are regimes at arbitrarily large β that are stable to high-n ballooning modes. These equilibria

[1] Jardin, Bhattacharjee, Chance, Cowley, Greene, Hofmann, Hughes, Iacono, Ignat, Kessel, Manickam, Monticello, Perkins, Phillips, Pomphrey, Ramos, Reiman, Rutherford, Valeo, and Wang

have greatly modified neoclassical transport properties primarily due to a large reduction in the fraction of trapped particles on a surface at very high β. The presence of a magnetic well leads to the enhanced stability of some trapped particle modes at high β.

We have carried out TSC/LSC simulations to study the degree to which we can prototype advanced plasma configurations in PBX-M and/or in the proposed SSAT experiment. LSC (Lower-hybrid Simulation Code) does multiple ray tracing in arbitrary numerically specified equilibria, and a 1D quasilinear solution for the parallel velocity distribution function in each flux shell. Then, using the response functions[4] and the dc electric field specified with the equilibrium, it calculates the current driven in each flux shell.

There is a correlation between the appearance of MHD instabilities in TFTR 'supershot' plasmas and subsequent deterioration of their confinement. This phenomenon is variously referred to as 'beta-saturation' or 'beta-collapse' depending on the severity of the confinement degradation. A 3/2 instability is excited most frequently, though modes with n = 1, 3 and 4 are also observed. At the largest values of $C_t \sim 2.7$ some 90% of all plasmas are unstable. To elucidate the nature of these instabilities, a detailed study of the predictions of MHD theory based on equilibrium profiles obtained from TRANSP analysis of the experimental data has been performed.

Analysis of equilibria constructed from unstable shots identifies conditions that are near marginal to either low-n ideal ballooning modes or to the infernal modes, depending on the peakiness of the current profile. In each case, the limiting β follows a Troyon-like scaling, $\beta_{max} = C_t(I/aB)$, but with C_t sensitively dependent on the peakiness of the pressure profile; the more peaked pressure profiles are more severely limited. The infernal modes are excited only when the shear within the rational surface of interest is small, as illustrated in Fig. 1 and, therefore, q_0 is close to a rational value. On the other hand, the ideal ballooning mode does not rely on elevated values of q_0 and can be excited when q_0 is reduced to unity.

2. IDEAL-MHD BETA LIMITS AT HIGH ELONGATION [2]

The construction of strongly shaped and elongated tokamaks is to a large extent motivated by the theoretical prediction that the beta limit is proportional to the plasma current [5]. Experimental results confirm the advantageous effects of elongation; $\beta \equiv 2\mu_0\langle p\rangle/B_0^2 = 11\%$ was reached with an elongation $\kappa = 2.34$ at aspect ratio $A \approx 2.96$ in DIII-D [6]. The understanding of beta and other operational limits at very high elongation is, however, rather incomplete. It is not clear whether the beta limit can be further improved by increasing the elongation or by other types of shaping, and to what extent control of current and

[2] Ward, Eriksson, Villard, Bondeson, Hofmann, and Lütjens

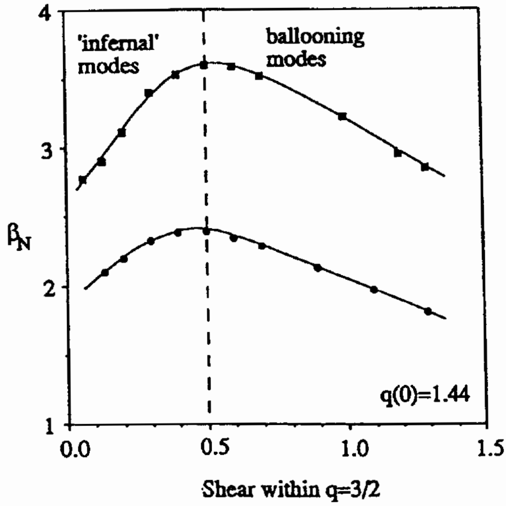

FIG. 1. Limiting β_N for ideal, pressure driven modes as a function of the shear. When the shear is small β is limited by the 3/2 infernal mode. At large values of the shear the β limit is determined by the $n = 2$ ballooning modes. The two curves correspond to pressure profiles with different peakedness, spanning the range of supershot data studied; the lower curve has the more peaked pressure profile.

pressure profiles is needed. Notably, at high elongation, the vertical instability ($n = 0$) gives an upper limit to the internal inductance, l_i, which may reduce the beta limit due to the kink ($n = 1$) [6] and ballooning ($n = \infty$) [7] modes.

We have studied the operational limits imposed by $n = 0, 1$ and ∞ stability for highly elongated tokamaks with shapes accessible to the TCV tokamak in Lausanne [8]. The plasma boundary is parametrized as

$$R/a = A + \cos(\theta + \delta \sin\theta + \lambda \sin 2\theta) \quad , \quad Z/a = \kappa \sin\theta \qquad (1)$$

Positive λ broadens the "tips" and the case $A = 3.7$, $\delta = 0.5$, $\lambda = 0.2$ is referred to as a TCV dee. Vertical stability has been calculated, assuming a resistive wall shaped as the TCV vacuum vessel, and that the active feedback stabilization is effective if the growth time in the absence of feedback is longer than 0.5 ms. No wall stabilization is assumed for the kink. To calculate beta limits, we choose the current profiles and optimize the pressure profiles for ballooning in the first region of stability. Subsequently, the pressure limits for vertical and kink stability are computed for pressure profiles that are scaled versions of those at the ballooning limit.

When vertical stability is disregarded and only kink and ballooning modes are considered, the highest beta limits result for current profiles with a high internal inductance [6]. We have studied the dependence of the kink-ballooning limit on geometry, Eq. (1), using current profiles with high l_i [8]. The beta

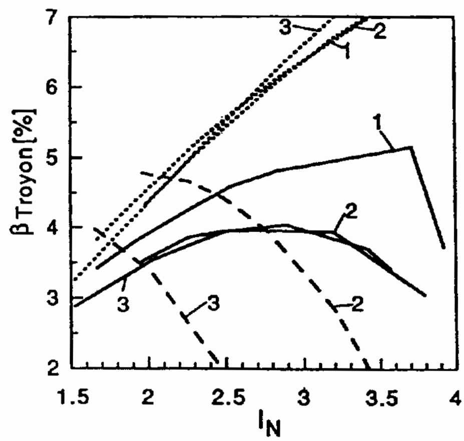

FIG. 2. Beta limits for ballooning (dotted), n = 0 (dashed) and n = 1 (solid) versus normalized plasma current for a TCV dee (κ = 3, A = 3.7) and the equilibrium sequences described in the text.

limit increases when the elongation increases from 2 to 2.5, if the boundary is sufficiently triangular. However, the limit decreases again when κ is further increased to 3. Concerning the current limit for the $n = 1$ kink, we find clearly different behavior at $\kappa = 3$ than for near circular cross sections; at $\kappa = 3$ the current limit is no longer correlated with integer q_ψ but is strongly dependent on the current profile. Typical values are $q_{95} \geq 3.2$ at $\kappa = 3$ while $q_\psi = 2$ can be reached at $\kappa = 2.5$ [8]. The current limit is almost identical for $\kappa = 2.5$ and $\kappa = 3$.

While kink stability is favored by high internal inductance, vertical stability is favored by low l_i. With increasing elongation, the set of current profiles that are stable to both modes becomes more and more restricted. At $\kappa = 2.5$, the requirements of $n = 1$ and $n = 0$ stability are quite easily reconciled and we find completely stable equilibria with TCV dee shape and $\beta \approx 7.5\%$. Increased triangularity is beneficial; we find $\beta \approx 9.2\%$ for $\delta = 0.8$ and $\lambda = 0$.

At $\kappa = 3$, vertical stability requires sufficiently low l_i, and this clearly reduces the beta limit for the kink mode. We have found the most advantageous results for $\kappa = 3$ by using current profiles with $q_0 \approx 1$ and "shoulders" near the edge of the plasma, around the $q = 2$ surface [8]. In TCV dee configurations with $\kappa = 3$, such profiles remain completely stable for beta up to about 4.5%.

Figure 2 shows the beta limits as $\beta_{\text{Troyon}} \equiv 2\mu_0\langle p\rangle/\langle B^2\rangle$ vs. normalized current $I_N = \mu_0 I_p/aB_0$ for ballooning (dotted), kink (solid) and vertical stability (dashed) curves for a TCV dee. Three sequences of current profiles have been tested. In order of decreasing l_i at fixed I_N these are: 1 – standard profile without shoulders, and two profiles with 2 – weak and 3 – large shoulders in the current density. Figure 3 shows l_i vs. I_N at the $n = 1$ beta limit for these equilibrium sequences.

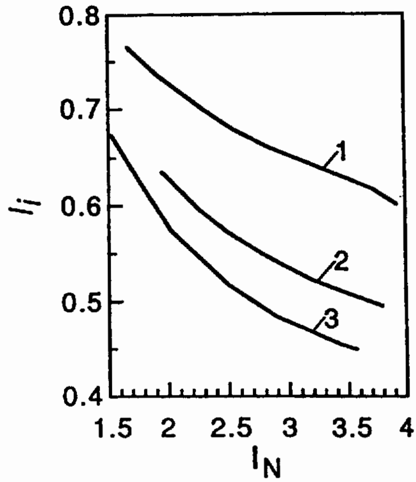

FIG. 3. Internal inductance versus normalized plasma current for the equilibrium sequences in Fig. 2.

As shown by Fig. 2, vertical stability is favored not only by low inductance but also, in a significant way, by high pressure (the $n = 0$ mode is stable above and to the right of the marginal curves in Fig. 2). The pressure stabilization appears to be due to an outward shift of the maximum in the current density which redistributes the eddy currents in the wall towards the outboard side. In fact, at high elongation, the window in l_i for stability to both $n = 0$ and $n = 1$ modes is wider for moderate pressure than for zero pressure.

The beta limit for kink modes is highest for the equilibrium sequence with high inductance (curve 1). However, for these equilibria, vertical stability requires $\beta > 10\%$, clearly above the kink limit, so the high-l_i sequence is always unstable. The sequences with medium (curve 2) and low (curve 3) l_i give a maximum β_{Troyon} close to 4%. The vertical stability is better for the low inductance profile (curve 3). These equilibria are stable to all modes over a fairly large range of plasma current. The sequence of intermediate inductance (curve 2) gives a smaller operational window because of poorer vertical stability.

Figure 2 shows that at $\kappa = 3$, the ballooning limit is only weakly dependent on the inductance, as opposed to the near-circular case, where the ballooning limit increases with inductance [7]. We conclude from Fig. 2 that, at elongation 3, the beta limit is set entirely by the $n = 0$ and $n = 1$ modes, the ballooning limit being less restrictive.

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