GLOBAL STABILITY STUDY OF THE ULTRALOW ASPECT RATIO TOKAMAK, ULART

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ABSTRACT. By introducing a slender current carrying conductor through the geometric centre axis of the TS-3 device at Tokyo Univesity, ultralow aspect ratio tokamak (ULART) configurations have been generated with aspect ratios as low as 1.1. In this extreme limit the transition of the spheromak ($q_{edge} = 0$, $I_{tf} = 0$) to an ULART plasma ($q_{edge} = 2$–20) is studied. The global MHD characteristics of ULART are investigated by comparing the theoretical results with the experimental data obtained. A small current (compared with the plasma current) in the centre conductor is found to improve significantly the global MHD stability characteristics of the plasmas formed by effectively stabilizing the global tilt/shift mode. Theoretical calculations of the threshold toroidal field current required for stability and the growth rates of the tilt/shift modes agree well with the TS-3 data.

1. INTRODUCTION

In recent years, low aspect ratio tokamak configurations have been the subject of intensive study in the search for a cost effective, high performance plasma regime that can lead to compact volumetric neutron sources as well as high beta advanced fuel reactors [1–4]. They can confine high beta plasmas with large natural elongation in the first stability regime. Recent results from START [2] have shown that the confinement properties of low aspect ratio tokamaks ($A = R/a < 2$) can be as good as conventional tokamaks with aspect ratios of 3 to 4. To extend the parameter range of the low aspect ratio tokamak regime, the construction of a class of 1 MA level devices is being considered [3]. The present work describes the exploration, both experimental and theoretical, of global MHD characteristics of tokamak plasmas in the ultralow aspect ratio tokamak (ULART) regime with aspect ratios $A \approx 1.1$–1.6. Tokamaks in this regime can manifest the highest advantages of low aspect ratio tokamaks for MHD stability and compact reactor design.

The ULART configuration [4] is similar to the spheromak [5] in its strong paramagnetism and magnetic helical pitch. As the aspect ratio and plasma configuration approach this extreme limit, the features of the magnetic well (average minimum B), the shear and their effects on the plasma's MHD stability should significantly deviate from those of standard tokamaks and the MHD characteristics are expected to change dramatically. By introducing a slender current carrying conductor through the geometric centre axis of the TS-3 device at Tokyo University, we have generated ULART plasmas with aspect ratios as low as 1.1. We study the global MHD stability characteristics of ULART by comparing theoretical results with data obtained in the TS-3 device.

The most significant contributions of this Letter are:

(a) It is found both theoretically and experimentally that a small current in the centre conductor ($I_{tf} < I_p$, where I_{tf} is the threshold toroidal field current and I_p the toroidal plasma current) significantly improves the global MHD stability characteristics of the ULART by effectively stabilizing the global tilt/shift mode.

(b) Theoretical estimates for I_{tf} and the growth rates of the $n = 1$ (n is the toroidal mode number) global modes agree well with the TS-3 data.

2. CHARACTERISTICS OF ULTRALOW ASPECT RATIO TOKAMAK (ULART) EQUILIBRIUM

MHD equilibrium calculations show that ULART plasmas with edge safety factors of $q(a) > 3$ (q is the inverse of the rotational transform of the field lines) are characterized by high maximum toroidal beta, low poloidal beta, high natural elongation, strong paramagnetism and a small ratio of toroidal field coil current to plasma current. Figure 1 shows flux plots and profiles of toroidal and poloidal magnetic fields for tokamak equilibria in the low aspect ratio regime. The paramagnetism is seen to increase as the aspect ratio decreases. In the ultralow aspect ratio limit, the field line pitch resembles that of a spheromak in the outer radial edge of the plasma, but the large toroidal field component in the inner radial edge makes the pitch resemble that of a tokamak. The global stability of an ULART is determined by the magnetic well depth (not always present) and the shear of the configuration.

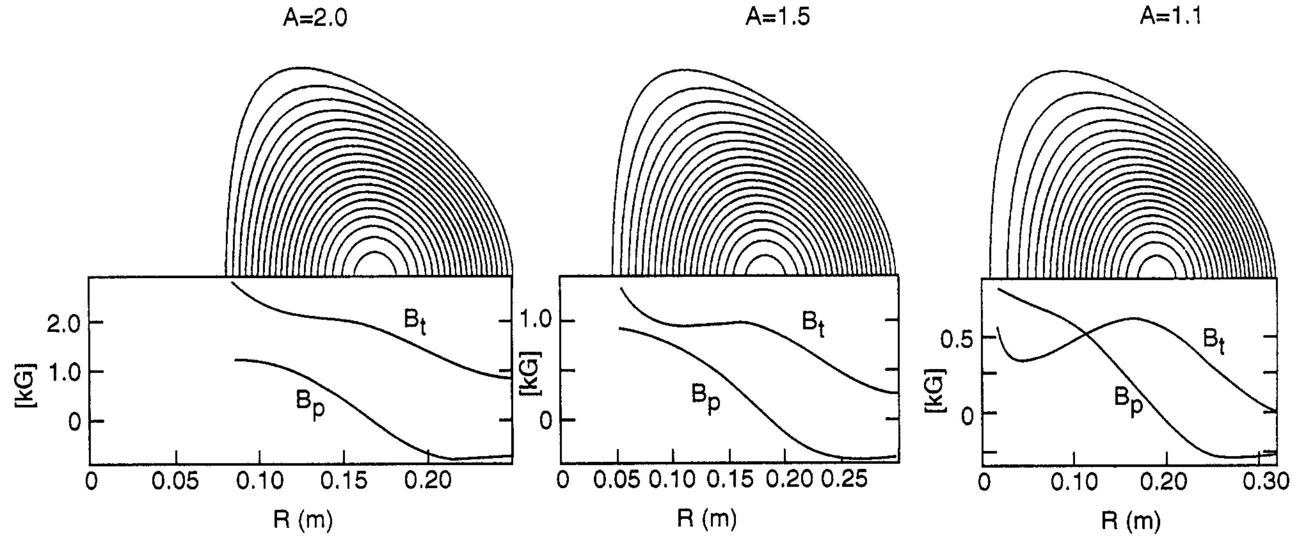

FIG. 1. Comparison of magnetic field profiles for equilibria of decreasing aspect ratio (PEST calculation) tokamaks. The plasma current, edge and central q *values are held constant at the values* $I_p = 35$ *kA,* $q(a) = 3.0$ *and* $q(0) = 0.6$*, respectively.*

The required toroidal field coil current, I_{tf}, for typical ULART equilibria has been estimated [6, 7] as

$$I_{tf}/I_p = 2\,q(a)\,(\pi a/2b)^2\,(A-1)^2$$

where I_p is the plasma current, $2b$ the vertical length of the plasma, a the minor radius, R_p the major radius and $A = R_p/a$ the aspect ratio. It should be noted from this simple scaling that an ULART requires substantially smaller toroidal field current than conventional tokamaks, and that the ratio (I_{tf}/I_p) can be as small as 0.1 for an ultralow aspect ratio tokamak with $A = 1.1$ and $q(a) = 3$.

A spheromak configuration which confines low aspect ratio toroidal plasmas without requiring a linked toroidal coil structure can offer many reactor advantages [5]. However, a serious drawback of the spheromak configuration is its susceptibility to global $n = 1$ tilt/shift instabilities in the absence of nearby conductors. The radial displacement eigenfunctions of these modes, $\xi = \Sigma_m \xi_m e^{i(m\theta - n\phi)}$, are formed primarily of a superposition of $n = 1$, $m = \pm 1$ modes (m is the poloidal mode number) of equal/opposite amplitude and opposite/equal phase, respectively. Insertion of a thin passive centre conductor is found to be ineffective in stabilizing the tilt/shift modes because the image currents generated are small. However, by driving a current through a thin centre conductor, thereby generating an external toroidal field, the spheromak can be converted into an ULART, as demonstrated by Bruhns et al. [8]. The stability of ULART configurations with $A \sim 1$ and small applied external toroidal field is expected to be related to the stability of spheromaks. In particular, for small enough I_{tf}, the remnants of the tilt and shift instabilities will exist. As I_{tf} is increased relative to the plasma current, the tokamak edge safety factor is increased and the eigenfunctions of the unstable modes change continuously into $n = 1$, $m = nq(a)$ free boundary modes. For sufficiently high $q(a)$, the increased shear near the plasma edge completely stabilizes the external kink mode.

3. STABILITY ANALYSIS OF TILT/SHIFT MODES FOR ULART

As I_{tf} is increased relative to the plasma current, the toroidal field pressure which increases sharply near the geometric axis is expected to provide a strong stabilizing force against tilt or shift motion. The question then arises: what amount of external toroidal field is necessary to stabilize all the global $n = 1$ modes in ULART configurations?

To address this issue, we have carried out three different calculations, each of which assumes that the $n = 1$ modes are rigid. In the first method, presented below, the stability criteria are estimated by approximating the plasma as a rigid, low aspect ratio, toroidal

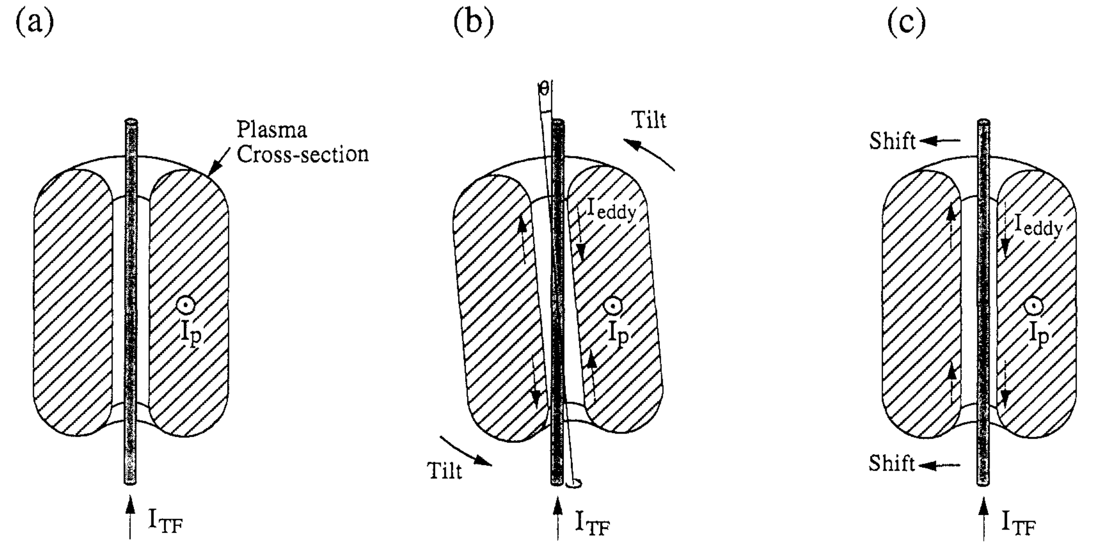

FIG. 2. Schematics of (a) induced ULART eddy currents for (b) tilt and (c) shift modes.

ring [5, 9] with rectangular cross-section and by calculating the tilt and shift moments due to surface currents induced on the inner edge of the displaced plasma, as shown in Fig. 2. This method, which provides simple analytical expressions for the stability criteria and the growth rates for the tilt/shift modes, has been verified by a second more elaborate calculation that takes into account the return passes of the induced surface current [10]. The third method, which will be reported in detail elsewhere [11], is an extension of Rebhan's work [12]. Using the energy principle, restricted to rigid plasma displacements, 'sufficient' conditions for stability are obtained for a Solovev class of analytic equilibria by demanding positivity of the plasma energy. The agreement between the three different calculations in required values of I_{tf}/I_p for tilt/shift stabilization is excellent (better than 30%). In this Letter, we present the simplest version of calculation to be compared with TS-3 experimental data.

The tilt moment is calculated from the poloidal force balance and from the toroidal pressure on the inner surface of an elongated low aspect ratio toroidal plasma [9]. For a tilt angle θ with respect to the horizontal plane (Fig. 2(b)), the tilt moment due to the force on plasma from the equilibrium field is

$$\begin{aligned} M_p &= 2I_p(\theta B_z - B_R)R_p^2 \int_{-\pi/2}^{\pi/2} \cos^2\varphi \, d\varphi \\ &= \pi R_p^2 I_p(\theta B_z - B_R) \qquad (1) \end{aligned}$$

where I_p, B_z, B_R and R_p are the toroidal plasma current, axial and radial equilibrium fields and the major radius of the plasma, respectively; toroidal angle is denoted by φ.

Using the relationships $B_z = (\mu_0 I_p/4\pi R_p)\, C_L$ and $B_R = n^*\theta B_z$, we obtain

$$M_p = \frac{\mu_0 I_p^2}{4}(1 - n^*)\theta R_p C_L \qquad (2)$$

where n^* denotes the field index at the magnetic axis defined by $n^* = -(R/B_z)(\partial B_z/\partial R)$ and C_L is a coefficient (~ 1) that determines the self-inductance for plasma rings of a rectangular cross-section [13] with half height b, minor radius a and major radius R_p. Here we note that

$$C_L \Rightarrow \ln\frac{8R_p}{a} - 1$$

in the large aspect ratio limit for circular cross-section rings.

The external toroidal field, which increases sharply at the inner edge (near the geometric axis) of an ULART, provides a strong stabilizing (pressure) force against tilt motion. The stabilizing moment due to the inner toroidal field B_{tf} is

$$M_{tf} = -2\frac{B_{tf}^2}{\mu_0}\theta\pi b\int_0^b y^2 \, dy = -\frac{2\pi b^3}{3\mu_0}B_{tf}^2\theta \qquad (3)$$

where B_{tf} is expressed in terms of the centre toroidal field coil (radius $\sim R_{tf}$) current I_{tf} as $B_{tf} = \mu_0 I_{tf}/2\pi R_{tf}$.

From Eqs (2) and (3) the total tilt moment M_{tilt} is derived as

$$M_{tilt} = \frac{\mu_0 I_p^2}{4}\,\theta R_p \times \left[(1-n^*)C_L - \frac{2}{3\pi}\left(\frac{I_{tf}}{I_p}\right)^2\left(\frac{\kappa}{A-1}\right)^2\left(\frac{\kappa}{A}\right)\right] \qquad (4)$$

where A and κ $(= b/a)$ are the aspect ratio and elongation of the rectangular cross-section toroidal ring. The first term in the large brackets is the contribution from the poloidal field and the second stabilizing term is due to the toroidal field. The external toroidal field, which increases sharply at the inner edge, provides a strong stabilizing (pressure) force against tilt or shift motion. This is clearly seen in the second term of Eq. (4) which depends on $1/(A-1)^2$. It follows from Eq. (4) that the stability condition for the tilt mode is

$$\frac{I_{tf}}{I_p} \geq \left(\frac{3}{2}\pi(1-n^*)C_L A\right)^{1/2}\frac{A-1}{\kappa^{3/2}} \qquad (5)$$

The growth rate of the tilt mode can be straightforwardly derived by writing an equation of motion for the tilt [12]. It is found that

$$\gamma_{tilt} = \frac{1}{2}\frac{V_A^p}{R_p}A\left(\frac{2\pi(1-n^*)C_L}{\kappa}\right)^{1/2} \times \left(1 - \frac{I_{tf}^2}{I_{crit}^2}\right)^{1/2} \qquad (6)$$

where $I_{crit} = I_p \times$ (RHS of Eq. (5)) and $V_A^p = \sqrt{B_p^2/\mu_0\rho}$ is the Alfvén velocity for the average poloidal field.

In a similar way, the threshold toroidal field current for stabilizing shift modes and the growth rate of the shift instability can be calculated as

$$\frac{I_{tf}}{I_p} \geq \left(\frac{\pi}{2}n^*C_L\right)^{1/2}\frac{A-1}{\sqrt{A\kappa}} \qquad (7)$$

$$\gamma_{shift} = \frac{1}{2}\frac{V_A^p}{R_p}A\left(\frac{2\pi n^*C_L}{\kappa}\right)^{1/2}\left(1 - \frac{I_{tf}^2}{I_{crit}^2}\right)^{1/2} \qquad (8)$$

A significant observation from these expressions is the effectiveness of the central toroidal field current for stabilizing tilt and shift modes: a small axial current ($I_{tf} \ll I_p$) in the centre conductor can stabilize both modes when A approaches unity. An important finding is that the condition of $q(a) > 1$ is not a deciding factor for suppressing tilt/shift modes. The stability criteria expressed in terms of I_{tf}/I_p in Eqs (5) and (7) are translated to $q(a) = 2$–4 for an aspect ratio of $A = 1.6$–1.1.

4. EXPERIMENTAL RESULTS

Experimental investigations of merging spheromaks and low aspect ratio tokamaks have been extensively carried out in the TS-3 device [10, 14, 15] in the past few years. By introducing a slender current-carrying conductor assembly ($R_{tf} \geq 1$ cm) through the geometric centre axis of this device, ULART configurations have been generated with an extremely low aspect ratio of 1.05 to 1.6. In this extreme limit the transition of the spheromak ($q(a) = 0$, $I_{tf} = 0$) to an ULART plasma ($q(a) = 2$–20) has been investigated experimentally. The plasma current is in the range of 25 to 35 kA with a variable toroidal field of 0 to 1 kG for $R_p = 18$ cm, $a \leq 16$ cm. The evolution of the current and magnetic field profiles [$q(R)$, $B(R)$, $\Psi(R)$] is monitored by an array of magnetic probes inserted into the plasma. Figure 3 presents the basic geometry of the present experiment [9, 10].

Experiments were performed to examine the effectiveness of the central toroidal field current in suppressing tilt, shift and other $n = 1$ kink modes. Decreasing the aspect ratio while keeping $I_p \sim 30$ kA $\sim$ const, it was observed that a decreasingly small I_{tf} is required to stabilize the $n = 1$ modes. The threshold value of I_{tf} for $n = 1$ global stability decreases substantially from about 30 to 6 kA as A is reduced from 1.6 to 1.1. Magnetic field measurements show that the threshold conditions correspond to $q = 2$–3 at the plasma edge. In Fig. 4, the threshold currents expressed as a ratio of the plasma current are plotted as a function of aspect ratio A and compared with our calculations based on rigid instability. The plasma shape, determined by flux measurements in the TS-3 experiments [10], corresponds to $\kappa = 1.6$. Our calculations (both analytical and numerical), which predict $I_{tf} \geq 0.2I_p$, are in good agreement with the experimental results.

A conventional large aspect ratio theory applied to the ULART would yield a threshold value for tilt mode

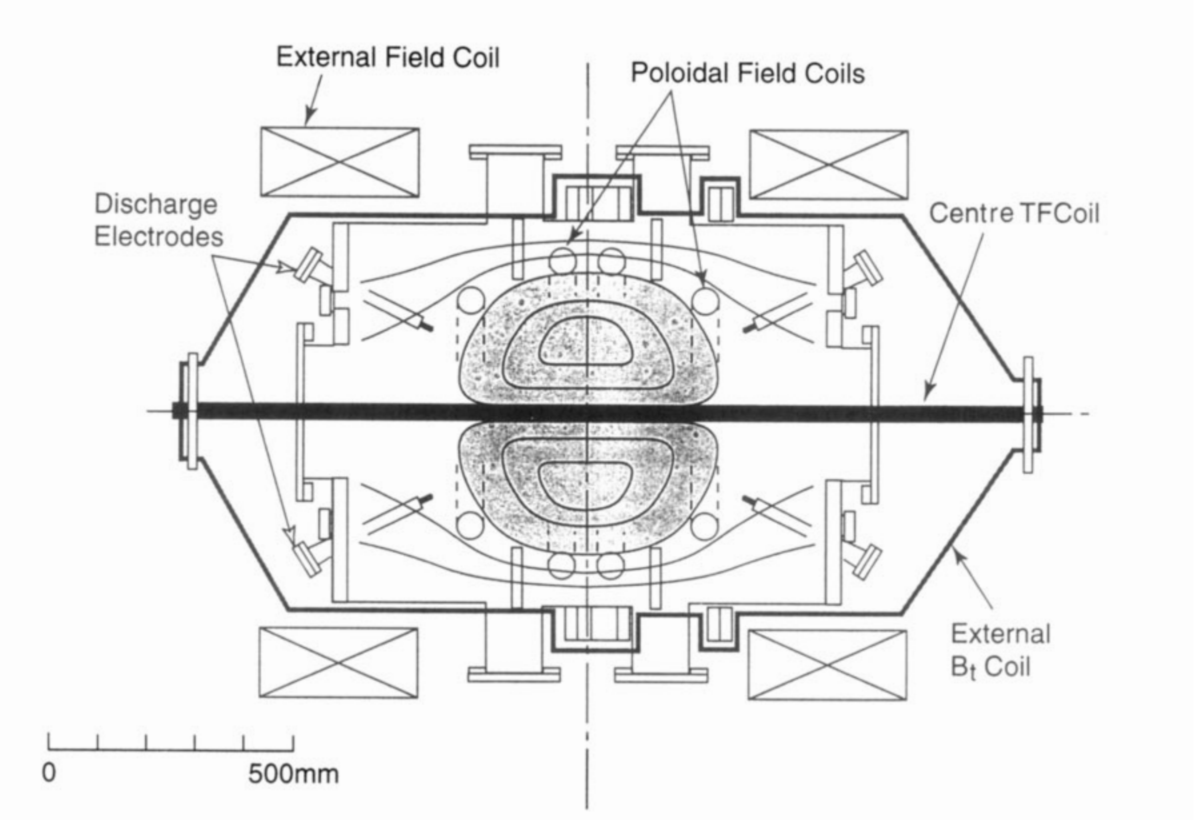

FIG. 3. TS-3 apparatus for ULART experiments.

stability [7, 10] of

$$q(a) \sim q_{cyl}(a) = aB_t/R_pB_p \sim 1$$

or $I_{tf} \geq I_p$ (where $q_{cyl}(a)$ denotes the cylindrical q value; $q(a) \gg q_{cyl}(a)$ for $A < 1.5$). However, our calculation and experiment have demonstrated that a notably smaller I_{tf} is required for $n = 1$ tilt/shift global stability. In Fig. 4, the stable regime is shown to be $I_{tf} \geq 0.2\ I_p$ for $A = 1.1$.

Figure 5 depicts the growth rate of $n = 1$ modes due to tilt/shift or kink modes versus toroidal field coil currents at the centre conductor for three different aspect ratios. It is observed that the growth rates of $n = 1$ modes decrease dramatically as the toroidal field coil current is raised or q increases, as predicted by the above theory. The excellent agreement between the data and the rigid conductor model suggests that the ULART is robust against global MHD instabilities.

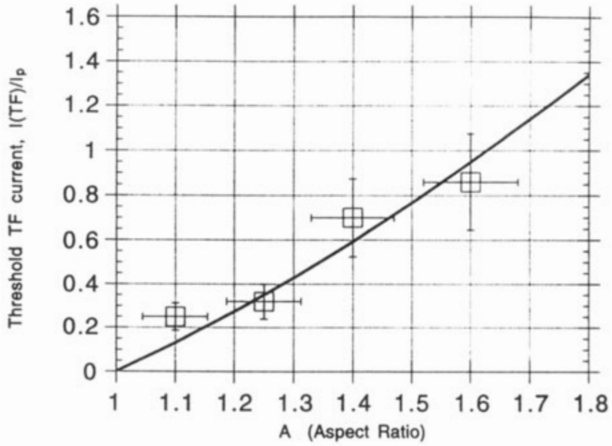

FIG. 4. Threshold toroidal field current expressed by I_{tf}/I_p versus aspect ratio for stabilizing n = 1 tilt modes: comparison of TS-3 data with theory, Eq. (5). For n = 0.2, tilt modes have a higher threshold value. (Data are for all n = 1 modes, error bars denote the range of the data.)*

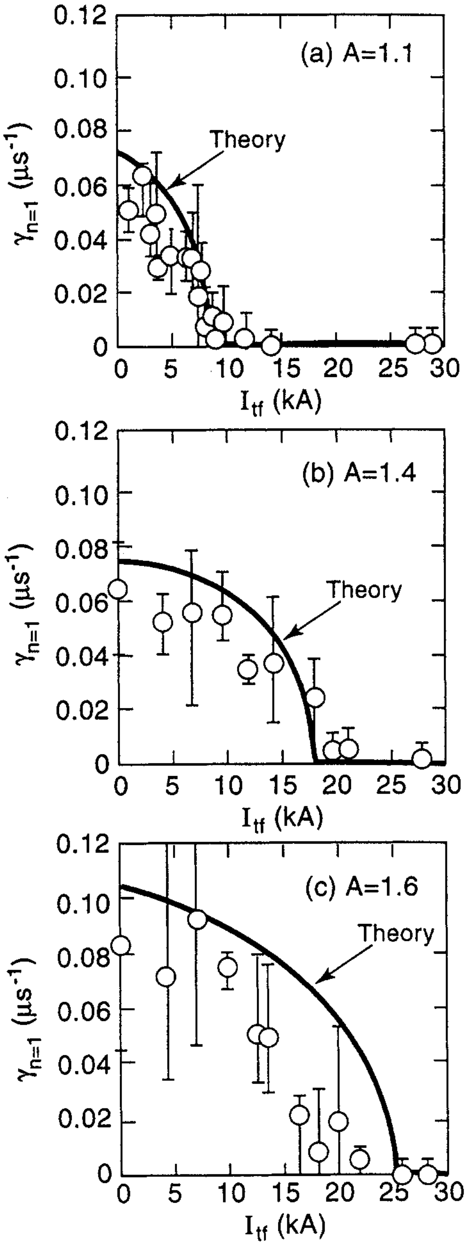

FIG. 5. Growth rate of the toroidal $n = 1$ *mode versus* I_{tf} *in the TS-3 ULART regime with* $A = 1.1$*, 1.4 and 1.6; comparison of experimental data with theory (Eqs (6) and/or (8)). For* $n^* = 0.2$*, tilt modes have higher growth rates.*

5. SUMMARY AND CONCLUSIONS

By introducing a slender current carrying conductor through the geometric centre axis of the Tokyo University TS-3 device, ULART configurations have been generated with aspect ratios as low as 1.1. The ULART configuration has been found to be similar to the spheromak in its strong paramagnetism and magnetic helical pitches. As the aspect ratio and plasma configuration approach this extreme limit, the features of the magnetic configuration are seen to deviate significantly from those of standard tokamaks. We study the global MHD stability characteristics of ULART by comparing theoretical results with data obtained in the Tokyo University TS-3 device.

The most significant findings are:

(a) It is found both theoretically and experimentally that a small current in the centre conductor ($I_{tf} < I_p$) significantly improves the global MHD stability characteristics of the ULART by effectively stabilizing the global tilt/shift mode.

(b) Theoretical estimates for the threshold toroidal field current (I_{tf}) and the growth rates of the tilt and shift modes agree well with the TS-3 data.

Another important finding is that the condition of $q(a) > 1$ is not a deciding factor for suppressing $n = 1/m = 1$ tilt/shift modes. Our simple analytical result expressed in terms of I_{tf}/I_p is translated to $q(a) = 2$–4 for the range of aspect ratios studied ($A = 1.6$–1.1). This result is in good agreement with the experimental data in which the measured threshold conditions correspond to $q(a) = 2$–3. Numerical calculations [11] based on the PEST II stability code show that the growth rate of $n = 1$ modes for an ULART with $A = 1.1$ decreases notably as $q_\psi(a)$ increases beyond 3. The present result from the ULART experiment on TS-3 may be related to the CDX result in which a significant enhancement of $m = 1$–$2/n = 1$ MHD modes is observed as the $q(a)$ is decreased [7]. We expect a coupling of low-n internal modes with external modes. A further extensive investigation with the aid of MHD numerical stability codes will clarify this important issue and reveal more unique MHD characteristics of the ULART configuration [11].

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