Zonal flow dynamics and size-scaling of anomalous transport

Roscoe White

Liu Chen*, Fulvio Zonca+

Princeton Plasma Physics Lab
Princeton University
*University of California, Irvine
+Associazione Euratom-ENEA sulla Fusione, Frascati, Italy

Oct 2003

Supported by US DoE, NSF

• **Non-linear equations for the slow space-time evolution of the radial drift wave envelope and zonal flow amplitude have been self-consistently derived for a model nonuniform tokamak equilibrium within the coherent 4-wave drift wave-zonal flow modulation interaction model of Chen, Lin and White [Phys. Plasmas 7, 3129, (2000)].**

Solutions clearly demonstrate turbulence spreading due to non-linearly enhanced dispersiveness and, consequently, the device-size dependence of the saturated wave intensities and transport coefficients.

PACS numbers: 52.55.Dy, 52.55.Fa, 52.55.Tn

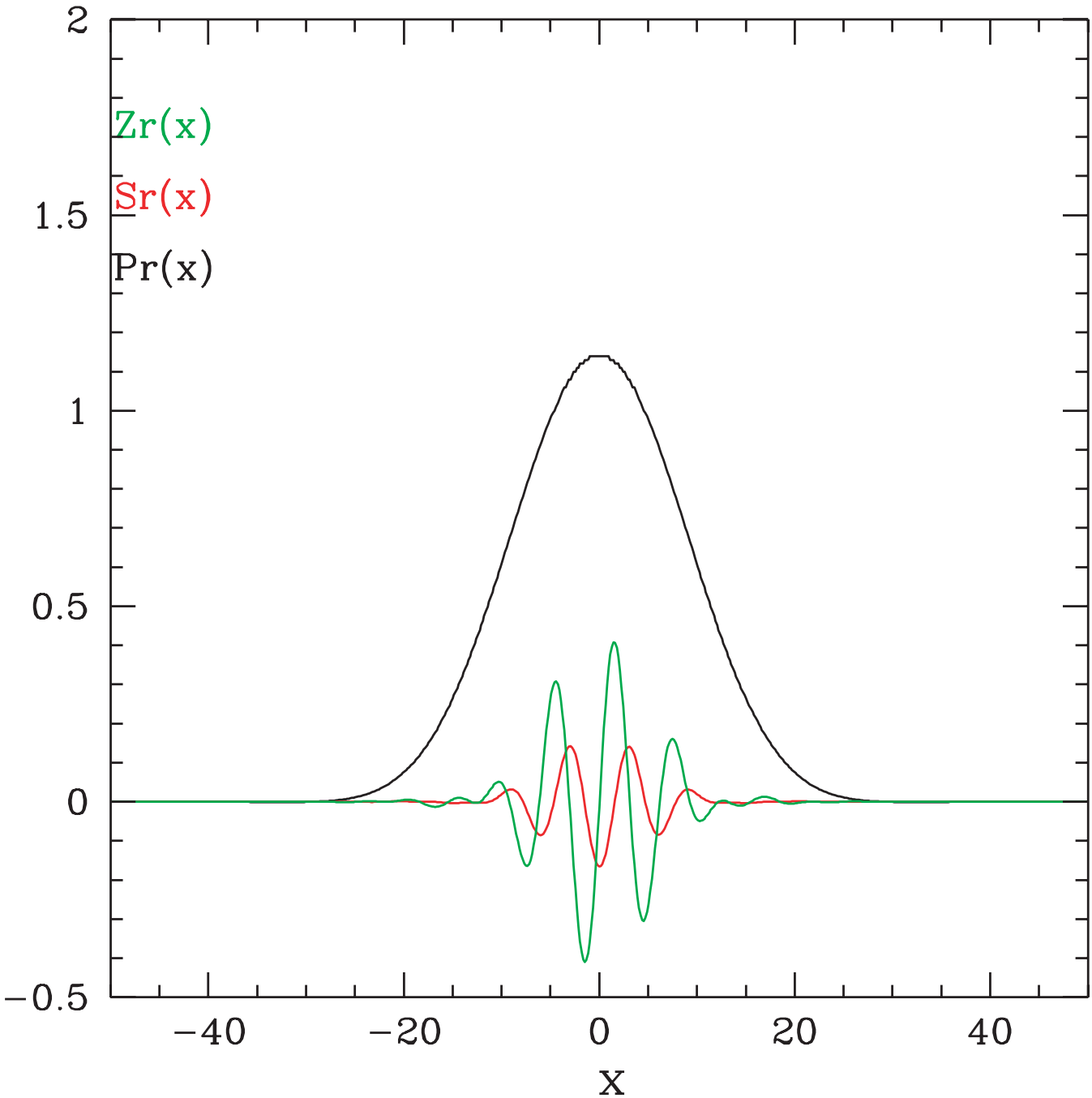
The dependence of plasma confinement on the device size is obviously a very crucial issue in fusion energy research. Assuming drift waves are responsible for the anomalous transport, size-scaling can be reduced, in the simplest model, to the dependence of drift-wave fluctuation intensity on ρ_*. Here $\rho_* = \rho_i/L_p$ with ρ_i and L_p being, respectively, the ion Larmor radius and the plasma inhomogeneity scale length. The coherent 4-wave drift wave-zonal flow modulation interaction model of Chen, Lin and White has captured the essential features observed in global gyrokinetic simulations in the $\rho_i/L_p \to 0$ limit.

We adopt the same model, including finite L_p plasma inhomogeneities. In this finite-ρ_* coherent 4-wave model, not only the drift wave (pump) radial envelope will be localized, leading to reduction in the modulational instability growth rate due to the finite interaction region; but more interestingly the damped pump and sidebands will disperse outward leading to radial spreading of the drift wave turbulence, qualitatively similar to that observed in recent simulations.

We assume that fluctuating fields are given by a single $n \neq 0$ drift wave, $\delta\phi_d$, and a zonal ($n = m = 0$) scalar field perturbation $\delta\phi_z$:

$$
\begin{aligned}
&\delta\phi_d = \delta\phi_0 + \delta\phi_+ + \delta\phi_- \quad , \\
&\delta\phi_0 = e^{in\varphi} \sum_m A_{0,m} e^{-im\vartheta} \phi_0(nq - m, r) + c.c. \quad , \\
&\delta\phi_\pm = e^{\pm in\varphi} \sum_m A_{\pm,m} e^{\mp im\vartheta} \phi_\pm(nq - m, r) + c.c. \quad , \\
&\delta\phi_z = A_z(r) + c.c. \quad , \qquad (1)
\end{aligned}
$$

Equations (1) explicitly indicate the existence of two characteristic spatial scales for high-n drift waves. The long scale reflects the characteristic radial variation of $A_0, A_\pm, A_z$, i.e. of mode envelopes and zonal flow, and is typically shorter than the equilibrium scale L_p. The short radial scale is associated with the parallel (to the ambient magnetic field) mode structure. It is $\approx n^{-1} dr/dq$

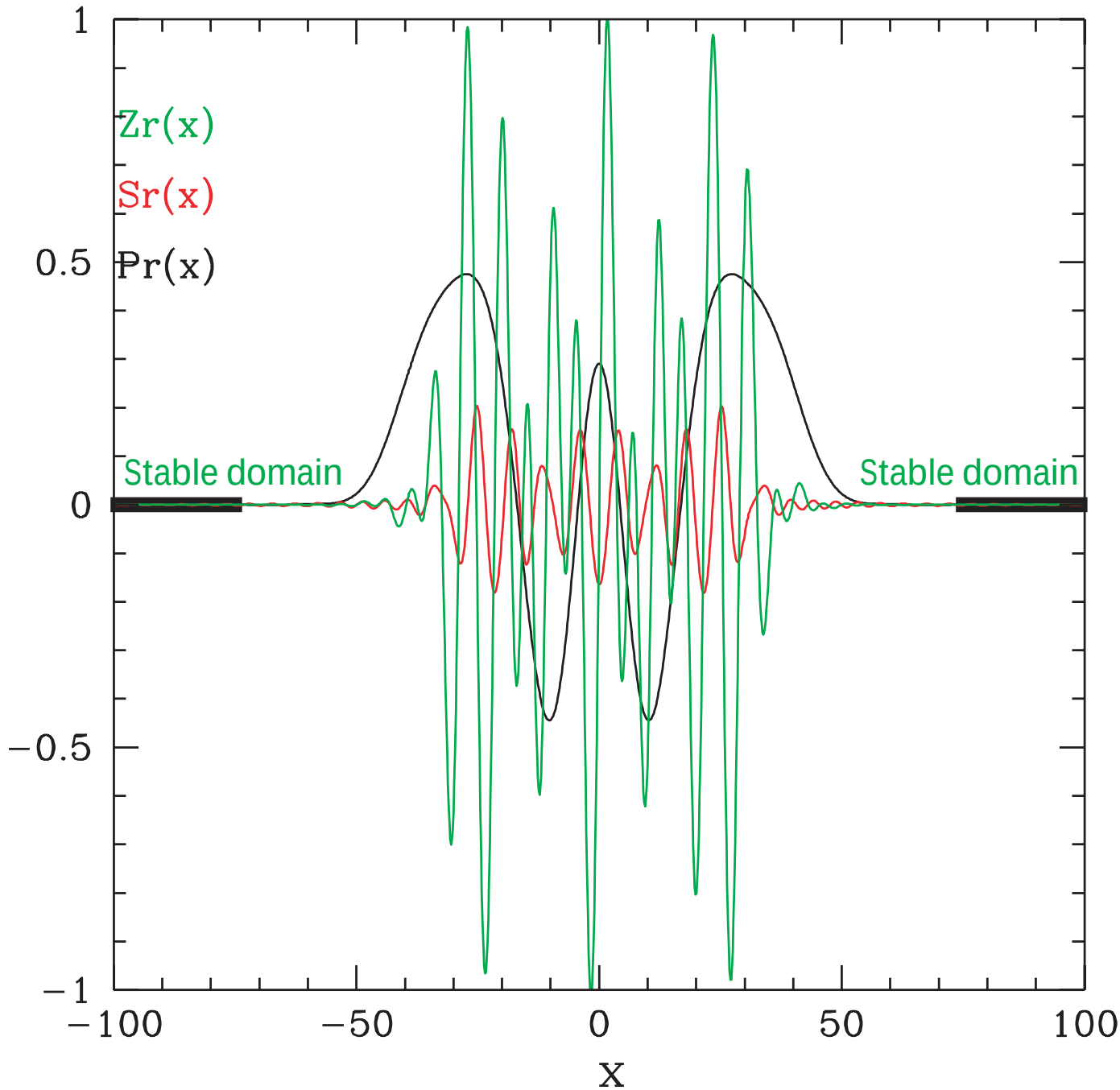


Radial structure of the real part of the wave fields, $\mathbb{R}\mathrm{e}P$ (black), $\mathbb{R}\mathrm{e}S$ (red) and $\mathbb{R}\mathrm{e}Z$ (green), $\tau = 20$, for $A = 1.15$ and $L = 200$.

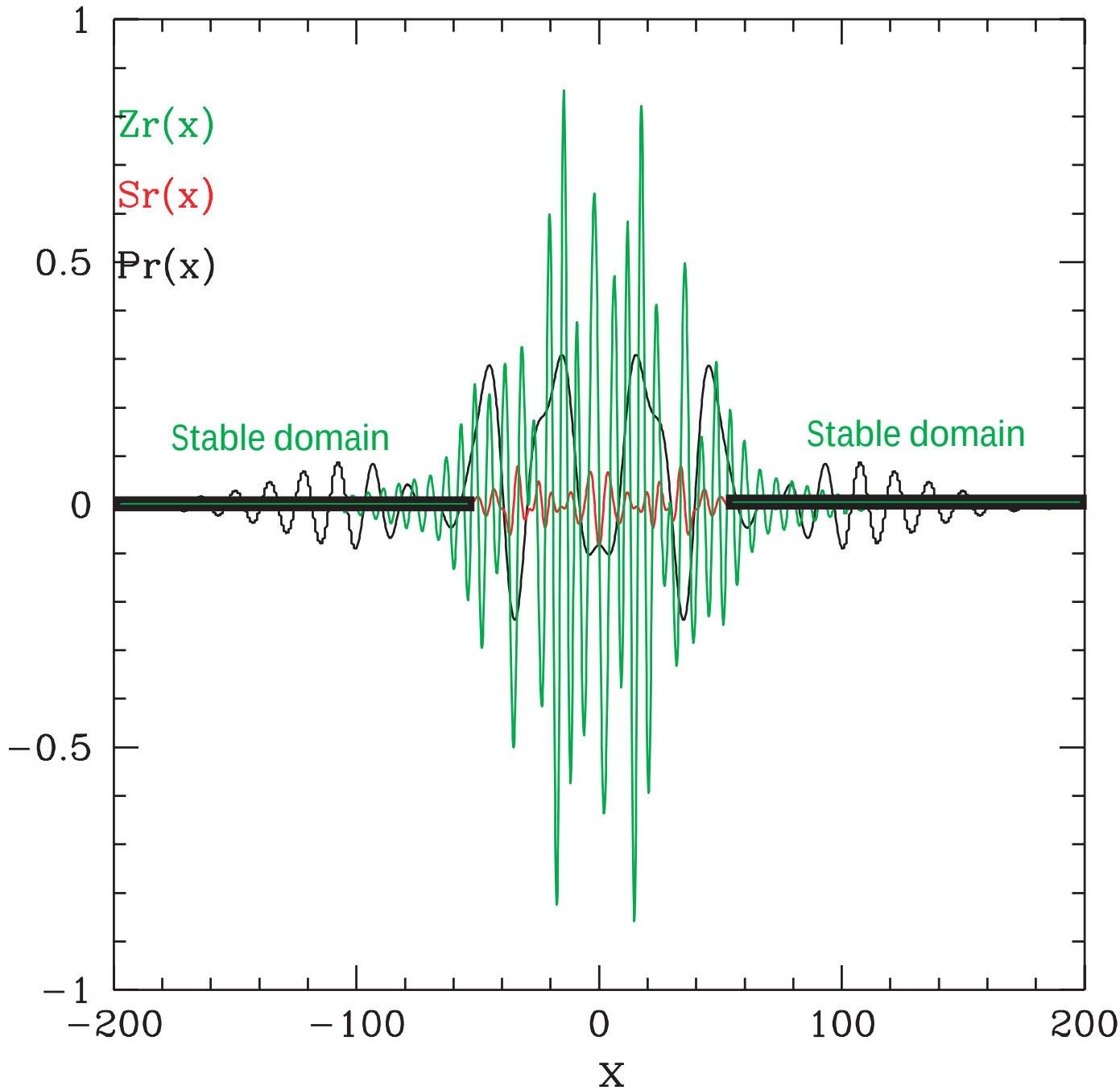
These equations require numerical solutions. We take Gaussian non-uniformity profiles and quadratic dispersiveness for numerical studies. That is, $D_R = \omega/\omega_0 - 1 + \theta_k^2 + V(x)$, with the potential well $V(x) = 1 - \exp(-x^2/L^2)$, where L is related with the equilibrium profile scale as $L = |ndq/dr|L_p/\Gamma^{1/2}$. We also choose $D_I = -(\bar{\gamma}_P(x)/\Gamma) = -(1/\Gamma)(A\exp(-x^2/L^2) - 1)$ for the pump and $D_I = (\bar{\gamma}_d/\Gamma)$ for the sidebands in order to have

$$\begin{aligned}
\mathcal{L}_P &= \partial_\tau - \bar{\gamma}_P(x) - i\Gamma V(x) + i\partial_x^2 \ , \\
\mathcal{L}_S &= \partial_\tau + \bar{\gamma}_d - i\Gamma V(x) + i\partial_x^2 \ , \\
\mathcal{L}_Z &= (\partial_\tau + \bar{\gamma}_z) \ \ . \qquad (2)
\end{aligned}$$

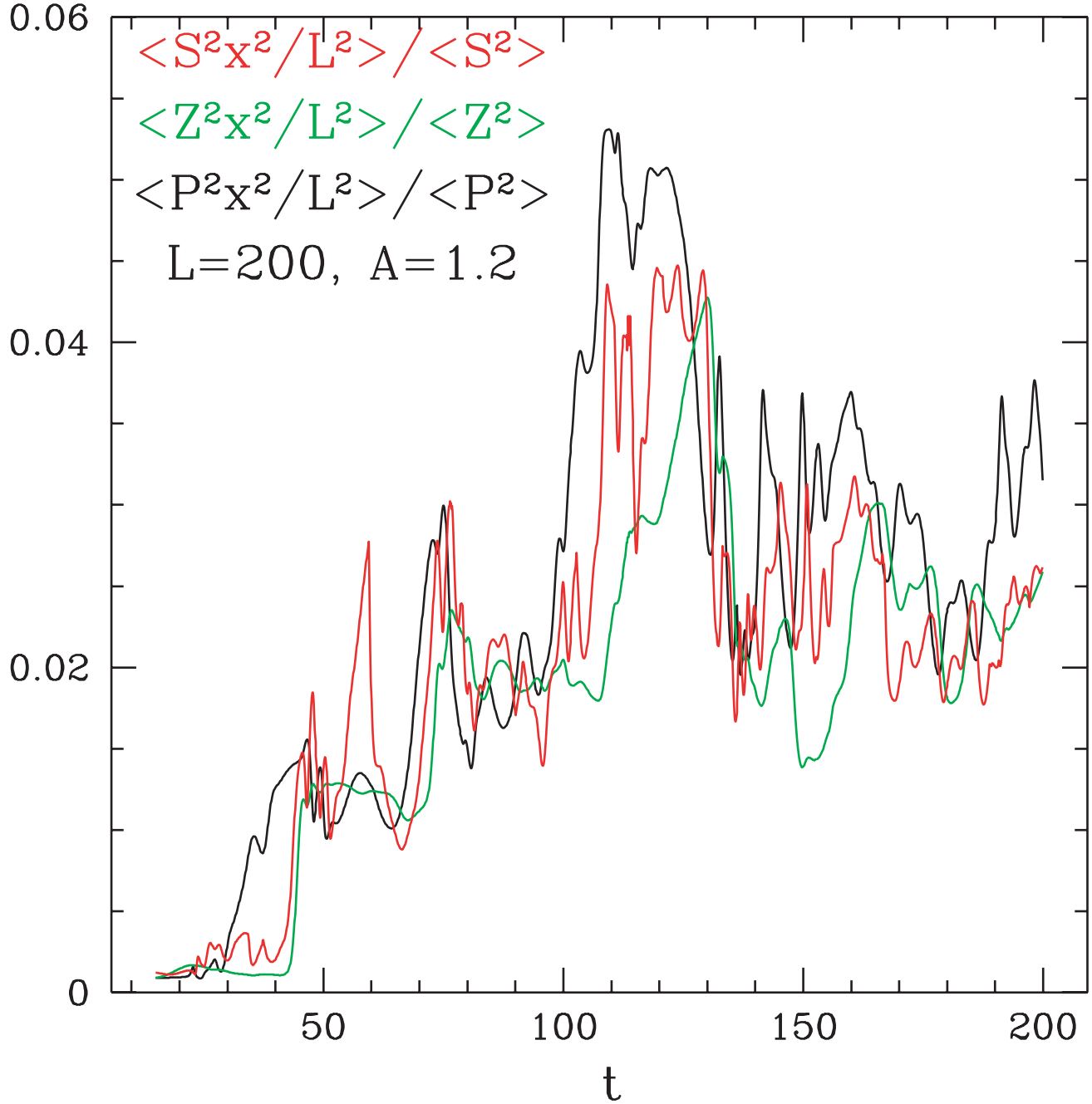
We readily recover the local limit by taking $L \to \infty$, $P = P_0(t)$, $S = S_0(t)\cos(\kappa_z x)$ and $Z = Z_0(t)\sin(\kappa_z x)$. In that case, maximum zonal flow growth rate for fixed pump amplitude, $\Gamma_{z,Max} = |P_0|^2 - \bar{\gamma}_z$, is expected for $\kappa_z^2 = \bar{\gamma}_d + \Gamma_{z,Max}$ i.e. for $\theta_{kz} = \Gamma^{-1/2}(\bar{\gamma}_d + \Gamma_{z,Max})^{1/2}$.



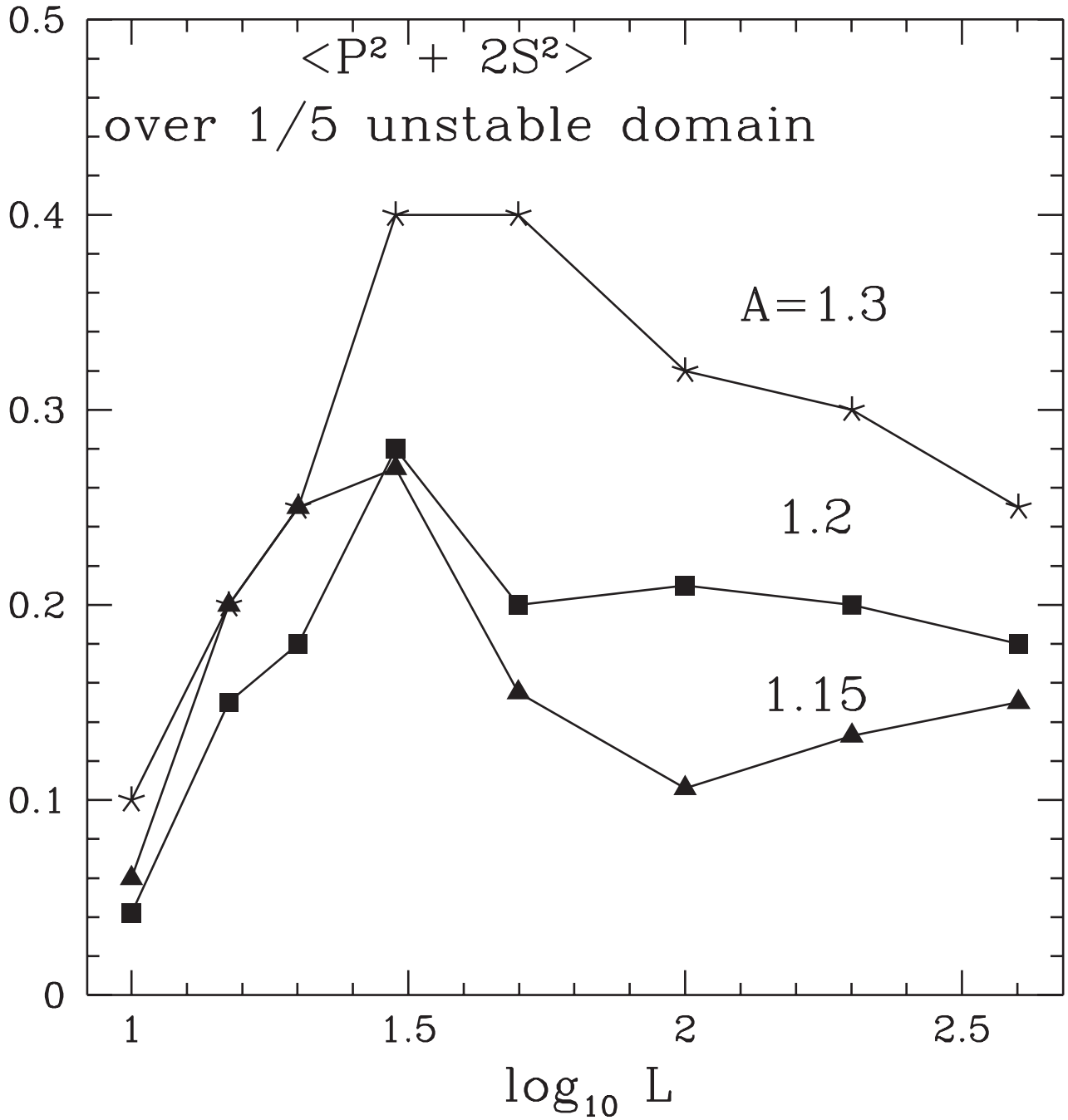
Radial structure of the real part of the wave fields, $\mathbb{R}\mathrm{e}P$ (black), $\mathbb{R}\mathrm{e}S$ (red) and $\mathbb{R}\mathrm{e}Z$ (green), $\tau = 50$, for $A = 1.15$ and $L = 200$.



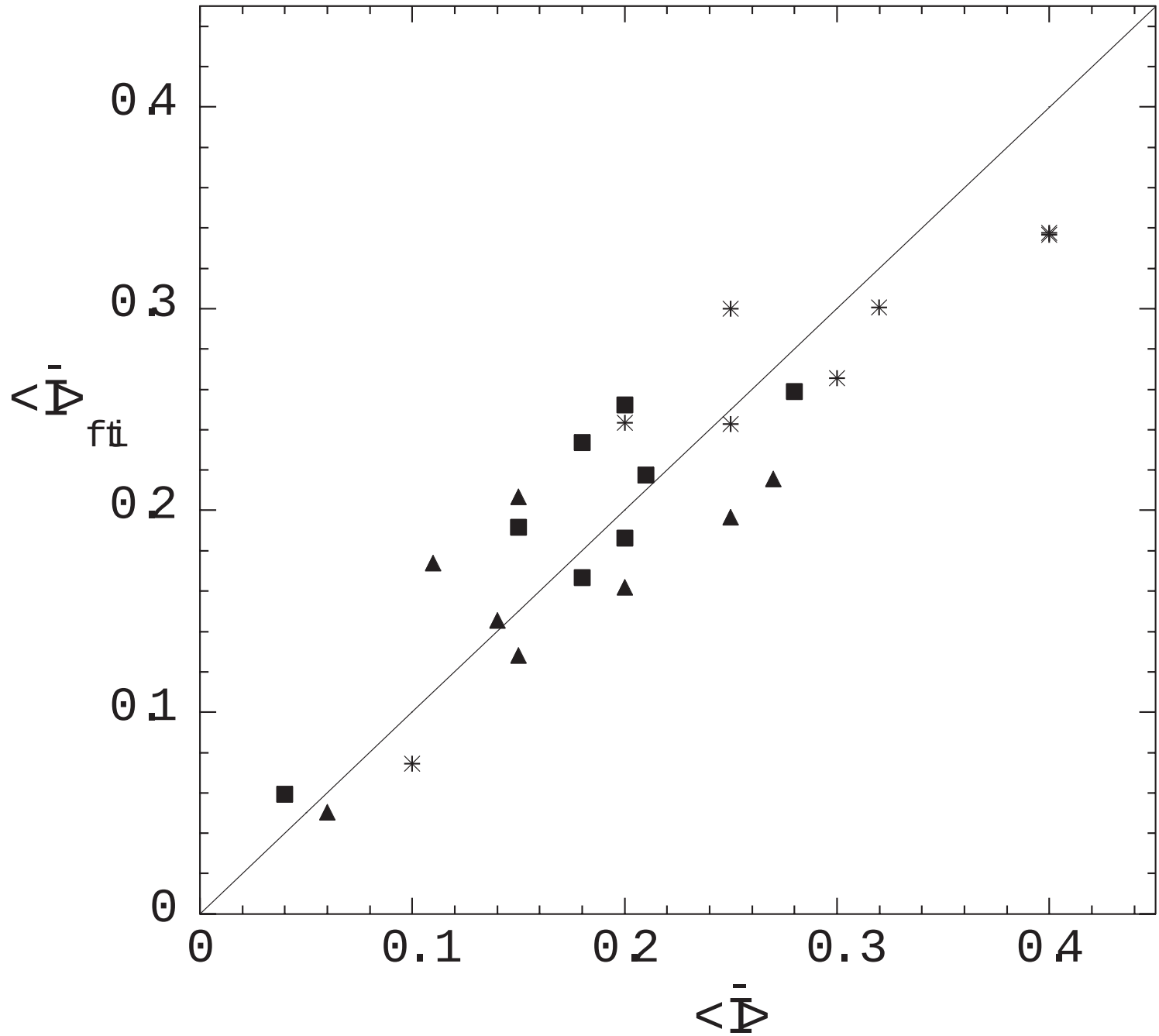
Radial structure of the real part of the wave fields, $\mathbb{R}\mathrm{e}P$ (black), $\mathbb{R}\mathrm{e}S$ (red) and $\mathbb{R}\mathrm{e}Z$ (green), $\tau = 125$, for $A = 1.15$ and $L = 200$.



Characteristic squared width of the wave fields normalized to L^2 as a function of time, for $A = 1.2$ and $L = 200$.



Drift wave intensity $\langle \bar{I} \rangle$ vs. L after spatial averaging on one fifth of the linearly unstable domain. The three curves refer to $A = 1.15$, $A = 1.2$ and $A = 1.3$.



Values of $\langle \bar{I} \rangle_{fit} = 24.0(L-9.6)^{1/2}(\bar{\gamma}_{P0}^{1/2}+.03\bar{\gamma}_{P0}L)/(98.3+L^{3/2})$ **are shown vs. the same simulation data.**

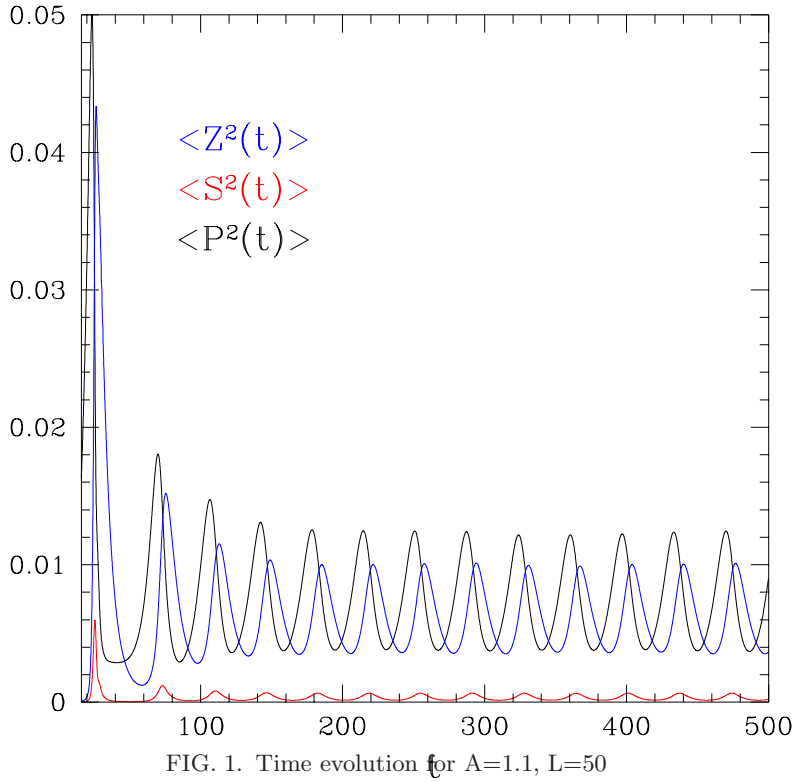


FIG. 1. Time evolution for A=1.1, L=50

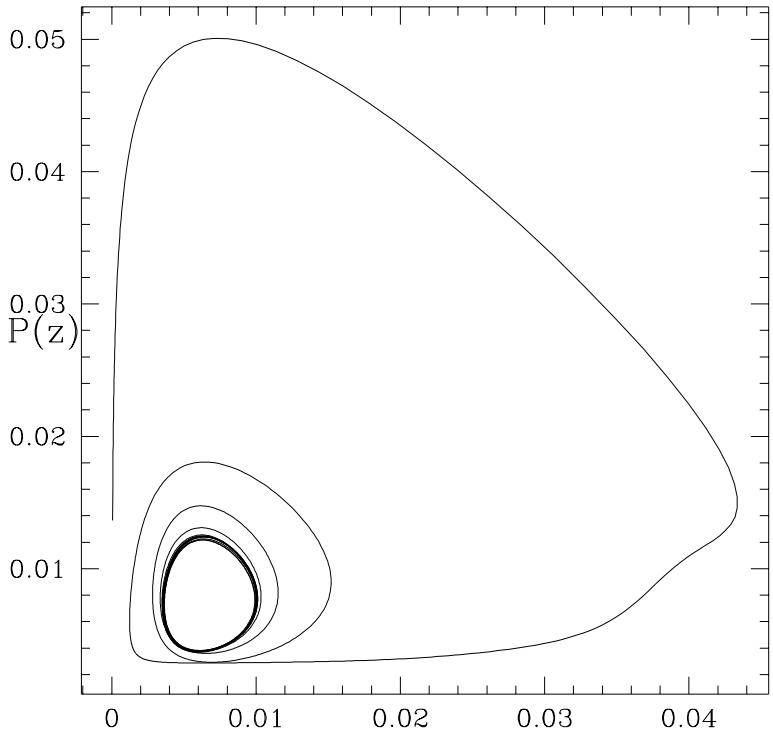


FIG. 2. Time evolution for A=1.1, L=50

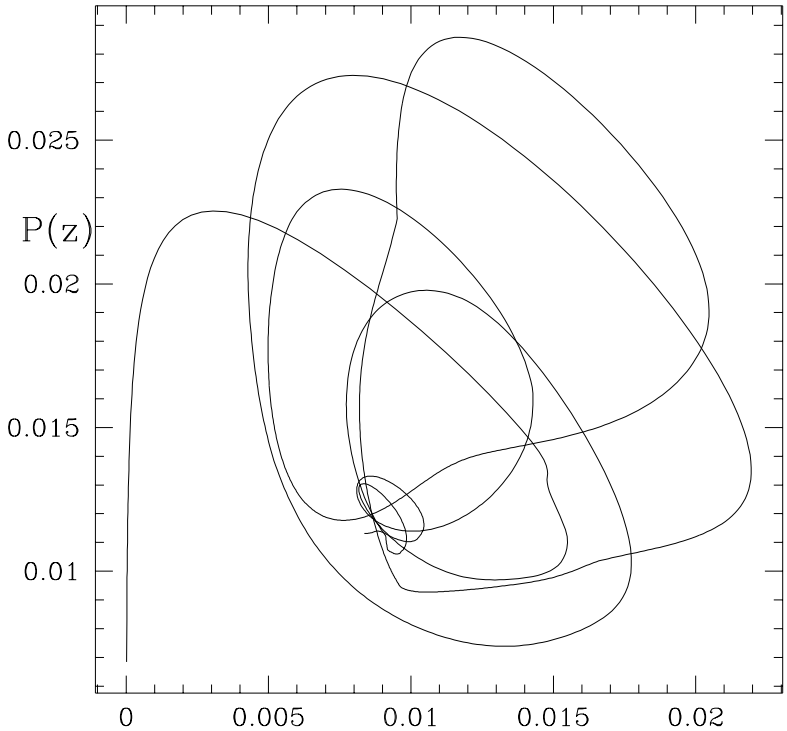


FIG. 3. Time evolution for A=1.1, L=200

Zonal flows, thus, act both as non-linear damping as well as anti potential well on the drift wave pump. Meanwhile, the pump drives zonal flows non-linearly but it generates non-linear diffusion as well, that manifests itself in numerical simulations as turbulence spreading.

When radial spreading stops and the fluctuation intensity has reached a time asymptotic value, both pump and sideband are characterized by complex radial structures on intermediate scales between fast non-linear oscillations on $\approx \bar{\gamma}_d^{-1/2}$ and the size of the linearly unstable region $\approx \bar{\gamma}_{P0}^{1/2} L$.

We calculate the spatially averaged drift wave intensity $\langle \bar{I} \rangle$ on $1/5$ of the linearly unstable domain.

In the $L \to \infty$ limit, numerical results reflect well the values for the fluctuation intensity expected from the fixed point solution, i.e. $\langle \bar{I}_f \rangle \simeq$ 0.15, 0.18, 0.23 respectively for $A = 1.15$, 1.2, 1.3.

The scaling of $\langle \bar{I} \rangle$ with the system size is evident: it sharply increases with L for $L < 30$, suggesting a Bohm scaling of anomalous transport, and it eventually reaches the asymptotic value set by the fixed point solution for $L > 100$, where gyro-Bohm scaling is indeed expected. Due to the definition of $L = nq(\rho_i/r)(|s|/\rho_*)\Gamma^{-1/2}$, values obtained from simulation results depend intrinsically on dimensionless physical parameters such as magnetic shear and normalized poloidal wavelength.

With the parameters of global gyrokinetic simulations, and defining a as the tokamak minor radius, present results would predict a Bohm to gyro-Bohm transition for $a/\rho_i > 420$ and saturation to gyro-Bohm transport for $a/\rho_i > 1400$, in remarkable agreement with the numerical results.

In summary, we have demonstrated that the coherent 4-wave drift wave-zonal flow modulation interaction model of Chen, Lin and White not only captures the essential features observed in global gyrokinetic simulations in the $\rho_i/L_p \to 0$ limit, but, by allowing non-uniform equilibrium, accounts as well for size scaling of drift wave intensity and ultimately of turbulent diffusion. This model sets a hierarchy among the relevant non-linear interactions; making it possible to consistently derive equations for the slow space-time evolution of drift wave radial envelope and zonal flow structures. The predicted size scaling of drift wave intensity is remarkably similar to that of global gyrokinetic simulations.

This work was partly done with support of U.S. D.o.E. Grant DE-FG03-93ER54271 and Contract No. DE-AC02-76-CH03073 to University of California at Irvine and Princeton Plasma Physics Laboratory.