Formulation of 5D Edge Gyrokinetic Simulations*

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Fully Nonlinear Ion gyrokinetic equations

Evolution of the plasma species is determined by coupled ion and electron kinetic equations for the time-dependent three-dimensional (5D) distribution functions simplified from H. Qin and et. al.(submitted to Contrib. Plasma Phys.; T. S. Hahm, Phys. Plasmas , Vol. 3, 4658 (1996). The gyrocenter distribution function $F_\alpha(\bar{\mathbf{x}}, \bar{\mu}, \bar{v}_\parallel, t)$ in gyrocenter coordinates: $Z \equiv (\bar{\mathbf{x}}, \bar{\mu}, E_0, t), \bar{\mathbf{x}} = \mathbf{x} - \rho, \rho = \mathbf{b} \times \mathbf{v}/\mathbf{\Omega_{ci}}$,

$$\frac{\partial F_\alpha}{\partial t} + \bar{\mathbf{v}}_\mathbf{d} \cdot \nabla_\perp F_\alpha + (\bar{v}_{\parallel\alpha} + v_{Banos}) \nabla_\parallel \partial F_\alpha + \left[q \frac{\partial \langle \Phi_0 \rangle}{\partial t} + \bar{\mu} \frac{\partial B}{\partial t} - \frac{qB}{B^*} \bar{v}_\parallel \nabla_\parallel \langle \delta\phi \rangle - q \mathbf{v}_\mathbf{d}^\mathbf{0} \cdot \bar{\nabla} \langle \delta\phi \rangle \right] \frac{\partial F_\alpha}{\partial E_0} = C(F_\alpha, F_\alpha),$$

$$\bar{\mathbf{v}}_\mathbf{d} = \frac{c\mathbf{b}}{qB_\parallel^*} \times \left(q \bar{\nabla} \langle \Phi \rangle + \bar{\mu} \bar{\nabla} B \right) + \bar{v}_\parallel^2 \frac{M_\alpha c}{qB_\parallel^*} (\bar{\nabla} \times \mathbf{b})$$

$$\bar{\mathbf{v}}_\mathbf{d}^\mathbf{0} = \frac{c\mathbf{b}}{qB_\parallel^*} \times \left(q \bar{\nabla} \langle \Phi_0 \rangle + \bar{\mu} \bar{\nabla} B \right) + \bar{v}_\parallel^2 \frac{M_\alpha c}{qB_\parallel^*} (\bar{\nabla} \times \mathbf{b})$$

$$\bar{v}_\parallel = \pm \sqrt{\frac{2}{M_\alpha} (E_0 - \bar{\mu} B - q \langle \Phi_0 \rangle)}, \qquad v_{Banos} = \frac{\mu c}{q} (\mathbf{b} \cdot \bar{\nabla} \times \mathbf{b}),$$

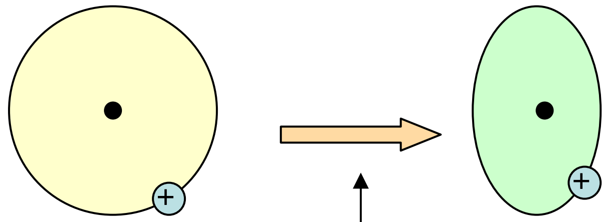
$$B_{\parallel\alpha}^* \equiv B \left[1 + \frac{\mathbf{b}}{\Omega_{c\alpha}} \cdot (v_\parallel \bar{\nabla} \times \mathbf{b}) \right], \Omega_{c\alpha} = \frac{qB}{M_\alpha c}, \mu = \frac{\mathbf{M}_\alpha \mathbf{v}_\perp^\mathbf{2}}{\mathbf{2B}}, \quad \langle \delta\phi \rangle = \langle \mathbf{\Phi} \rangle - \langle \mathbf{\Phi_0} \rangle.$$

Here $Z_\alpha e$, M_α are the electric charge and mass of electrons ($\alpha = e$), ions ($\alpha = i$). μ is the guiding center magnetic moment. The left-hand side of Eq. (1) describes the particle motion in the electric field and magnetic field. C_α is the Coulomb collision operator. The over-bar is used for the gyrocenter variables and $\langle\ \rangle$ denotes the gyroangle averaging. Here a splitting scheme has been used for the electric potential. The field Φ is split into two parts: Φ^0 is the large amplitude and the slow variation part; $\delta\phi$ is the small amplitude and the rapid variation part. E_0 is almost energy.

Gyrokinetic Poisson equation---Hong Qin

$$\nabla^2\phi(\mathbf{x}) = -4\pi\sum_s q_s\left[N + N_{\phi_0} + N_{\phi_1}\right],$$

$$N(\mathbf{x}) \equiv \int 2\pi w\,dw\,du\; I_0\left(\rho\nabla_\perp\right) F\left(\mathbf{x},w,u\right),$$



Orbit squeezing by large Er shear

$$N_{\phi_0}(\mathbf{x}) \equiv \frac{1}{B_0^2}\left(\mathbf{e}_1\mathbf{e}_1+\mathbf{e}_2\mathbf{e}_2\right)\nabla\left[n(\mathbf{x})\left(\mathbf{D}+V_\parallel(\mathbf{x})\mathbf{b}\right)\cdot\nabla\mathbf{D}\right],$$

$$N_{\phi_1}(\mathbf{x}) \equiv -\phi_1(\mathbf{x})\sum_{i=1}^{\infty}\frac{2i}{(i!)^2}\left(\frac{\nabla_\perp^2}{4\Omega_0^2}\right)^i M_{2i-2}(\mathbf{x})$$

$$+\sum_{i,j=0}^{\infty}\frac{2(i+j)}{(i!j!)^2}\left(\frac{\nabla_\perp^2}{4\Omega_0^2}\right)^i\left[M_{2(i+j)-2}(\mathbf{x})\left(\frac{\nabla_\perp^2}{4\Omega_0^2}\right)^j\phi_1(\mathbf{x})\right],$$

Full FLR effect

$$n(\mathbf{x}) \equiv \int 2\pi w\,dw\,du\; F\left(\mathbf{x},w,u\right),$$

$$V_\parallel(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})}\int 2\pi w\,dw\,du\; F\left(\mathbf{x},w,u\right),$$

$$M_i(\mathbf{x}) \equiv \int 2\pi w\,dw\,du\; w^i F\left(\mathbf{x},w,u\right).$$

$$I_0\left(\rho\nabla_\perp\right) \equiv \sum_{i=0}^{\infty}\frac{1}{(i!)^2}\left(\frac{\nabla_\perp^2 w^2}{4\Omega_0^2}\right)^i$$

$F\left(\mathbf{x},w,u\right)$ -- total gyrocenter distribution function.

Evaluated at the particle coordinates.

Fully Nonlinear Gyro-kinetic Poisson equation in long wavelength limit

In the long wavelength limit $k_\perp \rho_\alpha \ll 1$, the self-consistent electrostatic potential are typically computed from the gyro-kinetic Poisson equation for the multiple species

$$\begin{aligned}\left(\sum_\alpha \frac{\rho_\alpha^2}{2\lambda_{D\alpha}^2}\right)\nabla_\perp^2\Phi \ &+ \ \left(\sum_\alpha \frac{\rho_\alpha^2}{2\lambda_{D\alpha}^2}\nabla_\perp \ln N_\alpha\right)\cdot\nabla_\perp\Phi + \nabla^2\Phi \\ &= \ -4\pi e\left[\sum_\alpha Z_\alpha N_\alpha(\mathbf{x},t) - n_e(\mathbf{x},t)\right] - \sum_\alpha \frac{\rho_\alpha^2}{2\lambda_{D\alpha}^2}\frac{1}{N_\alpha Z_\alpha e}\nabla_\perp^2 p_{\perp\alpha}.\end{aligned}$$

For the single species, the gyro-kinetic Poisson equation becomes

$$N_\alpha\nabla_\perp^2\Phi + \nabla_\perp\Phi\cdot\nabla_\perp N_\alpha + \frac{n_\alpha\lambda_{D\alpha}^2}{(\rho_\alpha^2/2)}\nabla^2\Phi = -\frac{1}{(\rho_\alpha^2/2)}\frac{T_{\perp\alpha}}{Z_\alpha^2 e}\left[Z_\alpha N_\alpha(\mathbf{x},t) - n_e(\mathbf{x},t)\right] - \frac{1}{Z_\alpha e}\nabla_\perp^2 p_{\perp\alpha}.$$

where the gyrocenter center density N_α and perpendicular ion pressure $p_{\perp\alpha}$ are defined by

$$\begin{aligned}N_\alpha(\mathbf{x},t) \ &\equiv \ \frac{2\pi}{M_\alpha}\int B_\parallel^* d\bar{v}_\parallel d\bar{\mu} F_\alpha, \\ n_e(\mathbf{x},t) \ &\equiv \ \frac{2\pi}{m_e}\int B_\parallel^* dv_\parallel d\mu f_e, \\ p_{\perp\alpha} \ &= \ \pi B\int dv_\parallel d\bar{\mu}(v_\perp^2 F_\alpha), \\ T_{\perp\alpha} \ &= \ \frac{p_{\perp\alpha}}{N_\alpha(\mathbf{x},t)}\end{aligned}$$

The n_α and $T_{\perp\alpha}$ are the normalization density and temperature. The ion gyroradius is $\rho_\alpha = \sqrt{2T_{\perp\alpha}/M_\alpha}/\Omega_\alpha$, the ion gyrofrequency is $\Omega_\alpha = Z_\alpha eB/M_\alpha c$, and the ion Debye length is $\lambda_{D\alpha}^2 = T_{\perp\alpha}/4\pi n_\alpha Z_\alpha^2 e^2$.

Fully Nonlinear Gyro-kinetic Poisson equation in arbitrary wavelength regime

In the arbitrary wavelength regime, the self-consistent electrostatic potential is computed from the gyro-kinetic Poisson equation:

$$0 = -4\pi e\left[\sum_\alpha Z_\alpha N_\alpha(\mathbf{x},t) - n_e(\mathbf{x},t)\right] - \sum_\alpha \frac{1}{\lambda_{D\alpha}^2}\left[\Gamma_0(b)-1\right]\Phi$$

where $\Gamma_0(b) = I_0(b)e^{-b}, b = \rho_\alpha^2\nabla_\perp^2/2$, $I_0(b)$ is the usual zeroth-order modified Bessel function. The ion gyroradius is $\rho_\alpha = \sqrt{2T_{\perp\alpha}/M_\alpha}/\Omega_\alpha$, the ion gyrofrequency is $\Omega_\alpha = Z_\alpha eB/M_\alpha c$, and the ion Debye length is $\lambda_{D\alpha}^2 = T_{\perp\alpha}/4\pi N_\alpha Z_\alpha^2 e^2$. Here the dot product between the density and potential and the Debye shielding have been dropped for simplicity. The following first-order Padé approximation to Γ_0 is an excellent fit for $0 \le b \le 9$, and is therefore valid well into the typical ion gyrokinetic regime.

$$\Gamma(b) - 1 = \frac{b}{1-b}.$$

For single ion species, substituting a simple functional transformation

$$\Phi \;=\; \phi_L + \frac{T_{\perp\alpha}}{N_\alpha Z_\alpha^2 e}\left[Z_\alpha N_\alpha(\mathbf{x},t) - n_e(\mathbf{x},t)\right].$$

and Padé approximation Eq. (8) into Eq. (7) yields

$$\frac{\rho_\alpha^2}{2}\nabla_\perp^2\phi_L \;=\; -\frac{T_\alpha}{N_\alpha Z_\alpha^2 e}\left[1+\frac{\rho_\alpha^2}{2}\nabla_\perp^2\ln\left(\frac{T_{\perp\alpha}}{N_\alpha}\right)\right]\left[Z_\alpha N_\alpha(\mathbf{x},t) - n_e(\mathbf{x},t)\right].$$

where ϕ_L can be solved using the gyrokinetic Poisson solver in the long wavelength limit.

Field-line-aligned coordinates

We choose field-line-aligned ballooning coordinates, $(x,\, y,\, z)$, which are related to the usual flux coordinates $(\psi,\, \theta,\, \zeta)$ by the relations

$$\begin{aligned} x &= \psi - \psi_s, \\ y &= \theta, \\ z &= \zeta - \int_{y_0}^{y} \nu(x,y)dy. \end{aligned}$$

as shown in Fig. 1. The covering area given by the square ABCD in the usual flux coordinates is the same as the parallelogram ABEF in the field-line-aligned coordinates. The magnetic separatrix is denoted by $\psi = \psi_s$. In this choice of coordinates, x is a flux surface label, y, the poloidal angle, is also the coordinate along the field line, and z is a field line label within the flux surface. In this coordinates the magnetic field has a Clebsch representation,

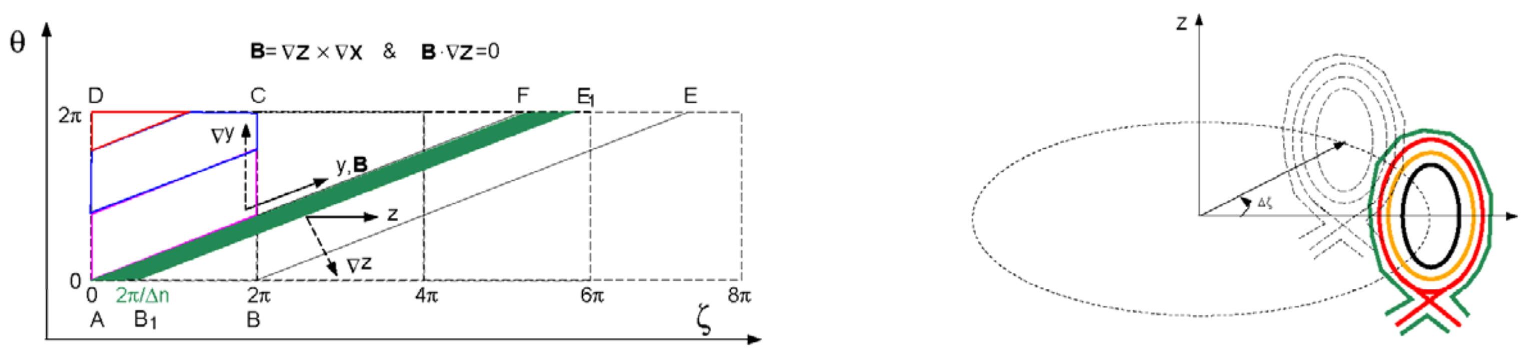


Figure 1: A sketch of the field-line-aligned coordinates mapping from (θ, ζ) to (y, z). The area covered by the square ABCD is for the usual flux coordinates $(\psi,\, \theta,\, \zeta)$. The area covered by parallelogram ABEF is for the field-line-aligned coordinates $(x,\, y,\, z)$. The green area covered by the parallelogram AB_1E_1F is a truncated simulation domain by the name of a annular toroidal wedge.

$$\mathbf{B} = \nabla z \times \nabla x.$$

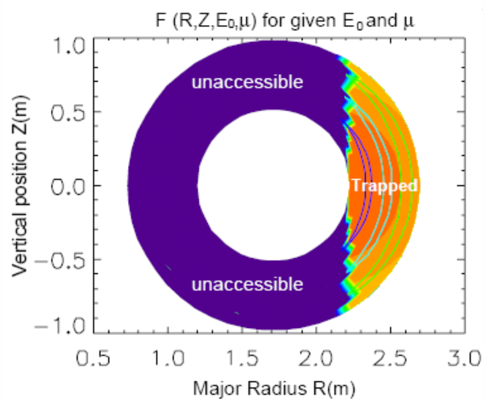
Drift operator

The equilibrium total drift operator becomes

$$\begin{aligned}\mathbf{v_{d0}}\cdot\nabla_\perp &= \frac{c}{qBB_\parallel^*}\left\{-\frac{I}{\mathcal{J}}\left[\left(\mu+\frac{M_\alpha v_\parallel^2}{B}\right)\frac{\partial B}{\partial y}+q\frac{\partial\langle\Phi_0\rangle}{\partial y}\right]\right\}\frac{\partial}{\partial\psi}\\ &+ \frac{c}{qBB_\parallel^*}\left\{\frac{I}{\mathcal{J}}\left[\left(\mu+\frac{M_\alpha v_\parallel^2}{B}\right)\frac{\partial B}{\partial x}+q\frac{\partial\langle\Phi_0\rangle}{\partial x}\right]\right\}\frac{\partial}{\partial\theta}\\ &- \frac{c}{qBB_\parallel^*}\left\{B_p^2\left[\left(\mu+\frac{M_\alpha v_\parallel^2}{B}\right)\frac{\partial B}{\partial x}+q\frac{\partial\langle\Phi_0\rangle}{\partial x}\right]+\frac{\mathcal{J}_{12}}{R^2}\left[\left(\mu+\frac{M_\alpha v_\parallel^2}{B}\right)\frac{\partial B}{\partial y}+q\frac{\partial\langle\Phi_0\rangle}{\partial y}\right]\right\}\frac{\partial}{\partial z}\end{aligned}$$

The perturbed $\mathbf{E}\times\mathbf{B}$ drift operator becomes

$$\begin{aligned}\delta\mathbf{v_d}\cdot\nabla_\perp &= \frac{c}{BB_\parallel^*}\left\{-\frac{I}{J}\frac{\partial\langle\delta\phi\rangle}{\partial\theta}+B_p^2\frac{\partial\langle\delta\phi\rangle}{\partial z}\right\}\frac{\partial}{\partial\psi}\\ &+ \frac{c}{BB_\parallel^*}\left\{\frac{I}{\mathcal{J}}\frac{\partial\langle\delta\phi\rangle}{\partial\psi}+\frac{\mathcal{J}_{12}}{R^2}\frac{\partial\langle\delta\phi\rangle}{\partial z}\right\}\frac{\partial}{\partial\theta}\\ &- \frac{c}{BB_\parallel^*}\left\{B_p^2\frac{\partial\langle\delta\phi\rangle}{\partial\psi}+\frac{\mathcal{J}_{12}}{R^2}\frac{\partial\langle\delta\phi\rangle}{\partial\theta}\right\}\frac{\partial}{\partial z},\end{aligned}$$



when the conventional turbulence ordering ($k_\parallel \ll k_\perp$) is used, the perturbed $\mathbf{E}\times\mathbf{B}$ drift operator can be further reduced to a simple form

$$\delta\mathbf{v_d}\cdot\nabla_\perp = \frac{cB}{B_\parallel^*}\left(\frac{\partial\langle\delta\phi\rangle}{\partial z}\frac{\partial}{\partial x}-\frac{\partial\langle\delta\phi\rangle}{\partial x}\frac{\partial}{\partial z}\right)$$

where $\partial/\partial\theta \simeq -\nu\partial/\partial z$ is used.

The general perpendicular Laplacian operator

We use the general perpendicular Laplacian operator

$$
\begin{aligned}
\mathcal{J}\nabla_\perp^2\Phi &= \frac{\partial}{\partial x}\left(\mathcal{J}\mathcal{J}_{11}\frac{\partial\Phi}{\partial x}+\mathcal{J}\mathcal{J}_{12}\frac{\partial\Phi}{\partial y}+\mathcal{J}\mathcal{J}_{13}\frac{\partial\Phi}{\partial z}\right)\\
&+ \frac{\partial}{\partial y}\left(\mathcal{J}\mathcal{J}_{21}\frac{\partial\Phi}{\partial x}+\mathcal{J}\mathcal{J}_{22}\frac{\partial\Phi}{\partial y}+\mathcal{J}\mathcal{J}_{23}\frac{\partial\Phi}{\partial z}\right)\\
&+ \frac{\partial}{\partial z}\left(\mathcal{J}\mathcal{J}_{31}\frac{\partial\Phi}{\partial x}+\mathcal{J}\mathcal{J}_{32}\frac{\partial\Phi}{\partial y}+\mathcal{J}\mathcal{J}_{33}\frac{\partial\Phi}{\partial z}\right)\\
&- \left(\frac{B_p}{hB}\right)\frac{\partial}{\partial y}\left[\left(\frac{B_p}{hB}\right)\frac{\partial\Phi}{\partial y}\right]-\left(\frac{B_p}{hB}\right)^2\frac{\partial\ln B}{\partial y}\frac{\partial\Phi}{\partial y}.
\end{aligned}
$$

The general perpendicular Laplacian operator for axisymmetric potential $\Phi_0(x,y)$ is

$$
\begin{aligned}
\mathcal{J}\nabla_\perp^2\Phi_0 &= \frac{\partial}{\partial x}\left(\mathcal{J}\mathcal{J}_{11}\frac{\partial\Phi_0}{\partial x}+\mathcal{J}\mathcal{J}_{12}\frac{\partial\Phi_0}{\partial y}\right)\\
&+ \frac{\partial}{\partial y}\left(\mathcal{J}\mathcal{J}_{21}\frac{\partial\Phi_0}{\partial x}+\mathcal{J}\mathcal{J}_{22}\frac{\partial\Phi_0}{\partial y}\right)\\
&- \left(\frac{B_p}{hB}\right)\frac{\partial}{\partial y}\left[\left(\frac{B_p}{hB}\right)\frac{\partial\Phi_0}{\partial y}\right]-\left(\frac{B_p}{hB}\right)^2\frac{\partial\ln B}{\partial y}\frac{\partial\Phi}{\partial y}.
\end{aligned}
$$

For the perturbed potential $\delta\phi$, if we drop $\delta/\delta y$ terms due to the elongated nature of the turbulence ($k_{||}/k_\perp \ll 1$), we obtain

$$
\mathcal{J}\nabla_\perp^2\delta\phi = \frac{\partial}{\partial x}\left(\mathcal{J}\mathcal{J}_{11}\frac{\partial\delta\phi}{\partial x}+\mathcal{J}\mathcal{J}_{13}\frac{\partial\delta\phi}{\partial z}\right)+\frac{\partial}{\partial z}\left(\mathcal{J}\mathcal{J}_{31}\frac{\partial\delta\phi}{\partial x}+\mathcal{J}\mathcal{J}_{33}\frac{\partial\delta\phi}{\partial z}\right).
$$

In this formulation, we keep the poloidal variation of the zonal flow $\Phi_0(\psi,\theta)$.

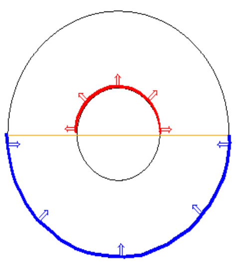
Boundary Conditions — Radial (x)

Radial (x) boundary condition for potential Φ

Use the neoclassical analytical ambipolar value E_r as the boundary condition at core boundary surface

$$E_\psi|^{\psi_c} = E_\psi^{neo} = \frac{B}{cRB_t}\langle U_{\alpha\|}\rangle - \frac{T_\alpha}{Z_\alpha e}\left\{k\frac{\partial \ln T_\alpha}{\partial\psi} - \frac{\partial \ln P_\alpha}{\partial\psi}\right\} \tag{19}$$

$$\alpha_b\Phi|^{\psi_{c,w}} + (1-\alpha_b)\frac{\partial\Phi}{\partial\psi}|^{\psi_{c,w}} = \alpha_a. \tag{20}$$



Radial (x) boundary condition for distribution function F

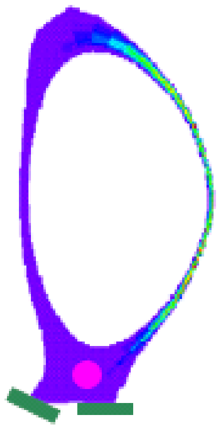
The radial Robin boundary condition at the inner core surface $\psi = \psi_c$ and the outer wall surface $\psi = \psi_w$:

$$C_{br}F_{b\alpha}|^{\psi_c,\psi_w} + (1-C_{br})\frac{\partial F_{b\alpha}}{\partial r}|^{\psi_c,\psi_w} = \left[C_{br} - (1-C_{br})\frac{\Gamma_{b\alpha}}{D_{b\alpha}n_{b\alpha}}\right]F_{mb\alpha}. \tag{21}$$

$$F_{mb\alpha} = \frac{n_{b\alpha}}{(\sqrt{\pi}v_{thb\alpha})^3\mathcal{F}_0}\exp\left[-\frac{\mu B}{T_{b\alpha}} - \frac{(v_\| - u_{b\alpha})^2}{v_{thb\alpha}^2}\right]. \tag{22}$$

This is a generalization of the Dirichlet ($C_{br} = 1$) and Neumann ($C_{br} = 0$) boundary conditions.

Sheath boundary conditions in SOL and private flux regions



Sheath boundary conditions for potential Φ

If both electron and ion are kinetic, the sheath potential is determined:

$$\Gamma_{i,sh} = \frac{2\pi B}{M_\alpha^2}\int_0^\infty dE_0 \int_0^{E_0 - Z_\alpha e\Phi_{sh}} \frac{d\mu}{|v_\parallel|} v_\parallel F_i^\sigma, \tag{27}$$

$$\Gamma_{e,sh} = \frac{2\pi B}{m_e^2}\int_{e\Phi_{sh}}^\infty dE_0 \int_0^{E_0 + e\Phi_{sh}} \frac{d\mu}{|v_\parallel|} v_\parallel F_e^\sigma, \tag{28}$$

$$\Gamma_{i,sh} = \Gamma_{e,sh}. \tag{29}$$

where there is an energetic group of impinging electrons that overcome the potential barrier and reach the wall with the energy $E_0 > e\Phi_{sh}$.

Sheath boundary conditions for distribution function F

If both electron and ion are kinetic, the electron distribution function is:

$$F_\alpha(\psi,\theta,\zeta,E_0,\mu) = \begin{cases} F_\alpha(\psi,\theta,\zeta,E_0,\mu), & v_\parallel \geq 0 \\ 0, & v_\parallel \leq 0 \end{cases} \tag{30}$$

$$f_e(\psi,\theta,\zeta,E_0,\mu) = \begin{cases} f_e(\psi,\theta,\zeta,E_0,\mu), & |v_\parallel| \leq v_{SH} \quad if \quad v_\parallel \leq 0 \quad or \quad v_\parallel \geq 0 \\ 0, & |v_\parallel| \geq v_{SH} \quad if \quad v_\parallel \leq 0 \end{cases} \tag{31}$$

where the impinging ions are not confined for the perfectly absorbing wall and the current through the sheath is zero with no biasing. A convention regarding the sign of the parallel velocity is that it is considered positive if it has a positive projection on θ axis. Here the positive θ axis is pointing to the plate/wall. where there is an energetic group of impinging electrons that overcome the potential barrier, reach the wall with the energy $E_0 > e\Phi_{sh}$, and lost. Here $v_{SH} = \sqrt{2e\Delta\Phi_{SH}/m_e}$, $\Delta\Phi_{SH}$ is the sheath potential.

Tempest exhibits collisionless damping of GAMs and zonal Flow

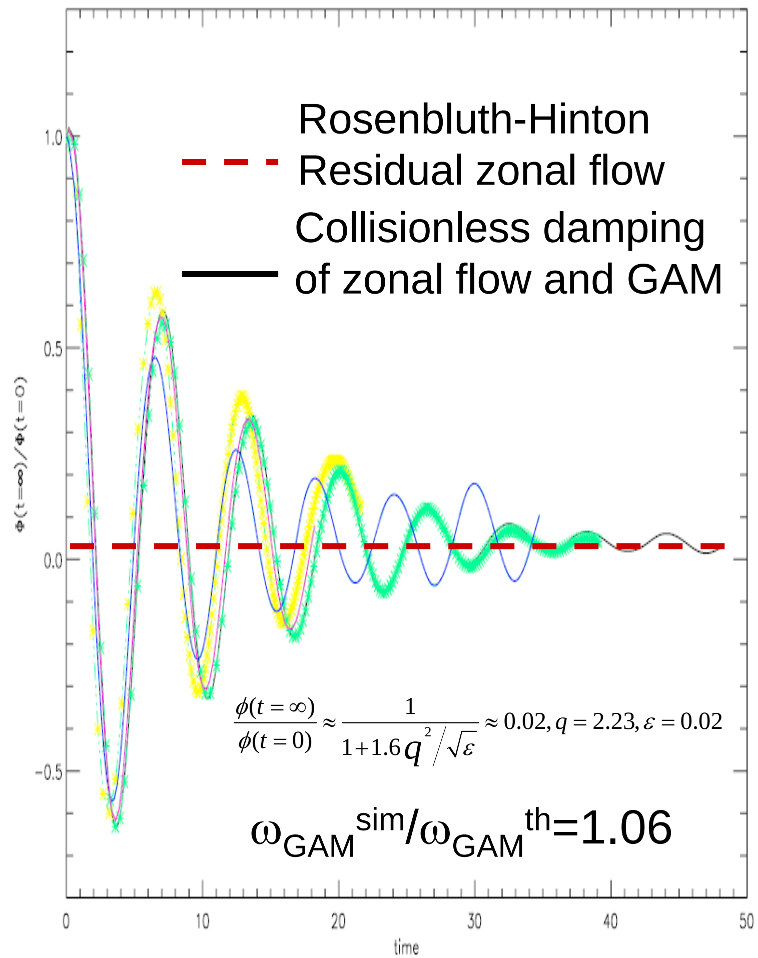
- **Axis-symmetric mode (no toroidal variation)**
  - **– Parallel ion dynamics**
  - **– Magnetic curvature**
  - **– Acceleration (Nonlinear Landau damping)**
  - **➔ TEMPEST should see GAMs**
- **Tempest model**
  - **Drift kinetic ions with radial drift, streaming, and acceleration**
  - **Boltzmann electron**
  - **Gyrokinetic Poisson equation in limit small ρ_s/L_x**
  - **Dirichlet radial boundary conditions**
- **GAMs provide opportunity to "verify" TEMPEST physics**
  - **Rosenbluth-Hinton residual**

$$\frac{\phi(t=\infty)}{\phi(t=0)} \approx \frac{1}{1+1.6q^2/\sqrt{\varepsilon}}$$

  - **Frequency**

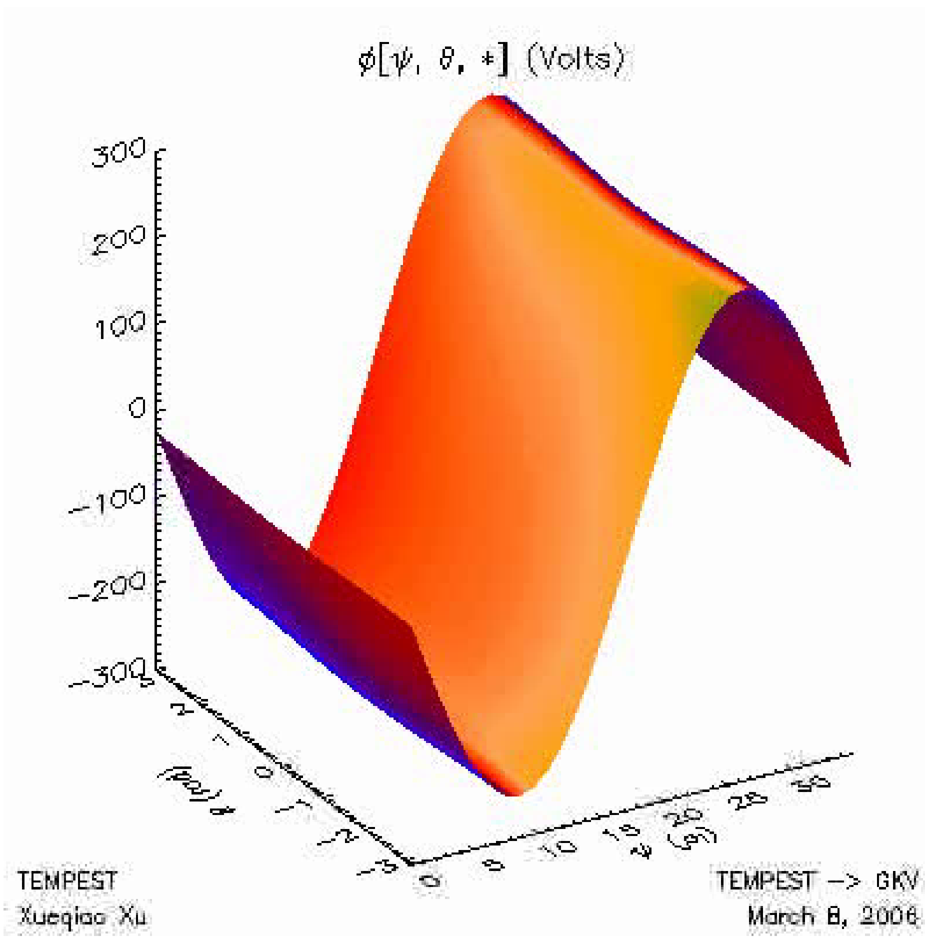
$$\omega_{GAM} \approx \sqrt{\frac{7}{8}}\frac{v_{Ti}}{R}$$

$\phi(t)/\phi(t=0)$

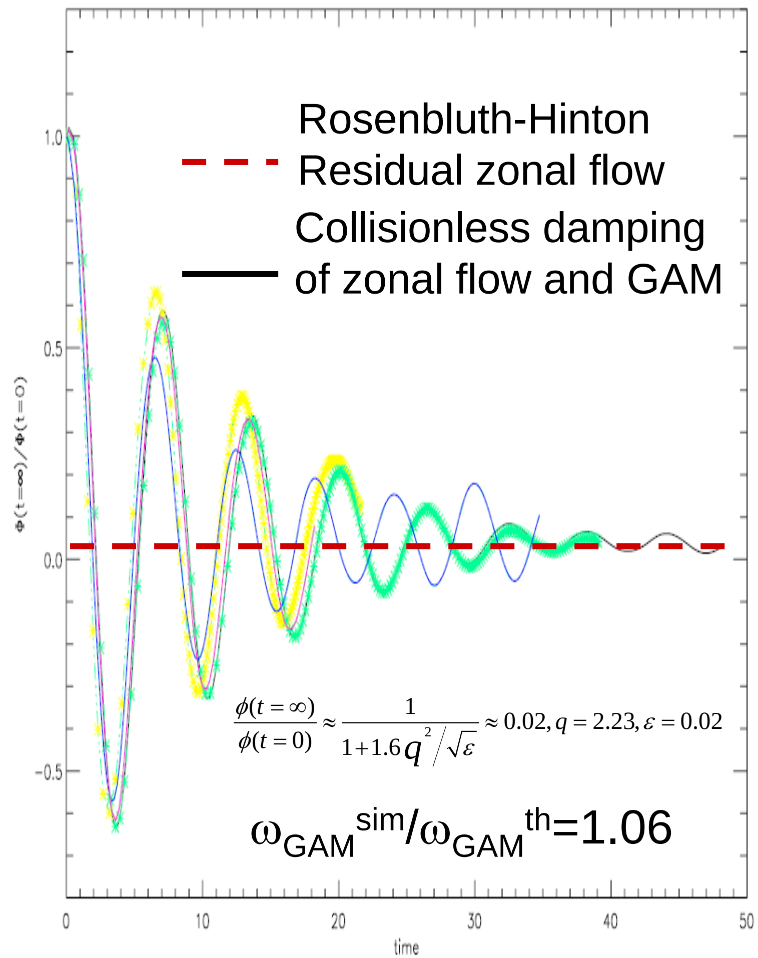


Time(v_{ti}/R_0)

Tempest exhibits collisionless damping of GAMs and zonal Flow

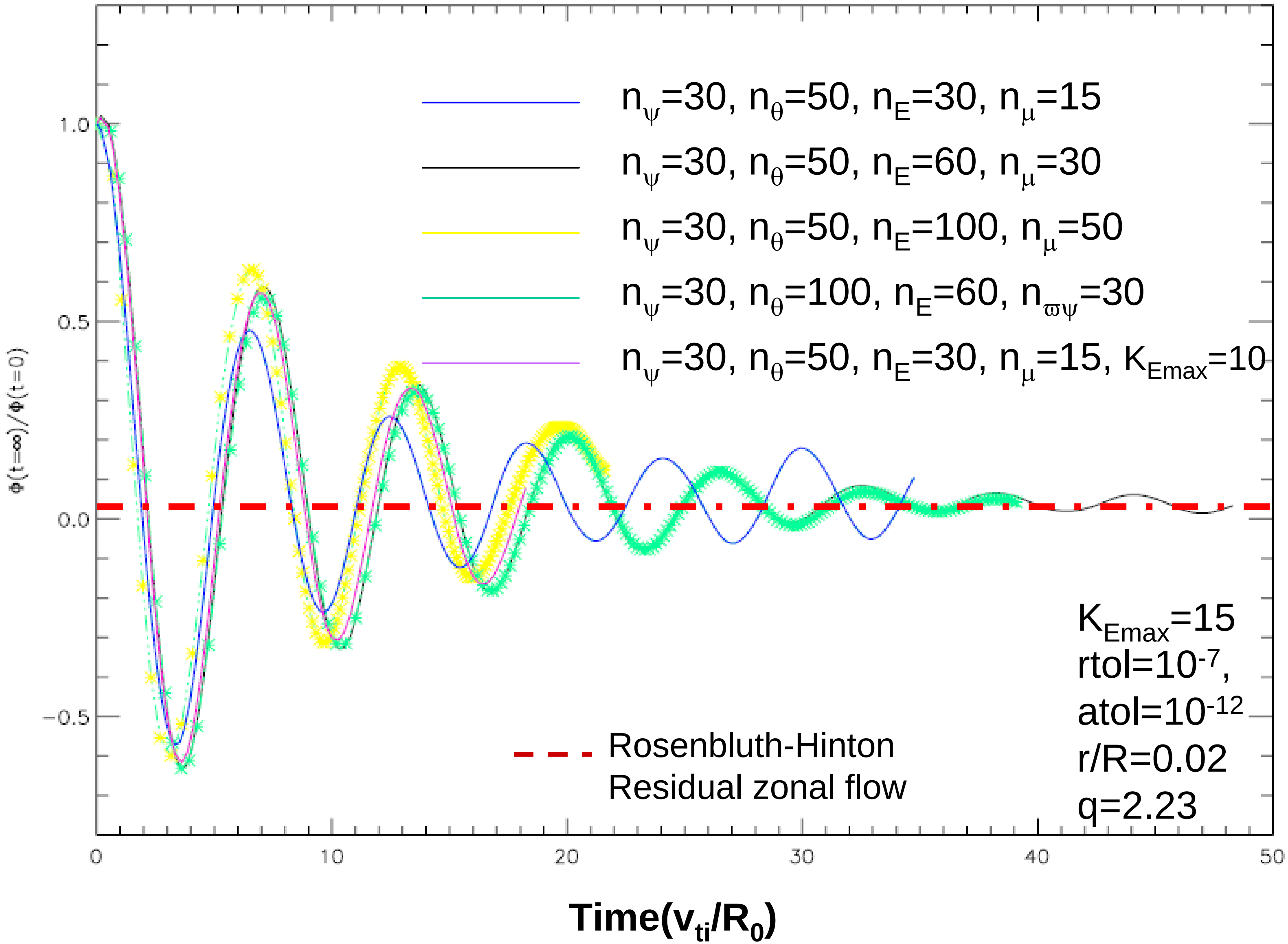


$\phi(t)/\phi(t{=}0)$

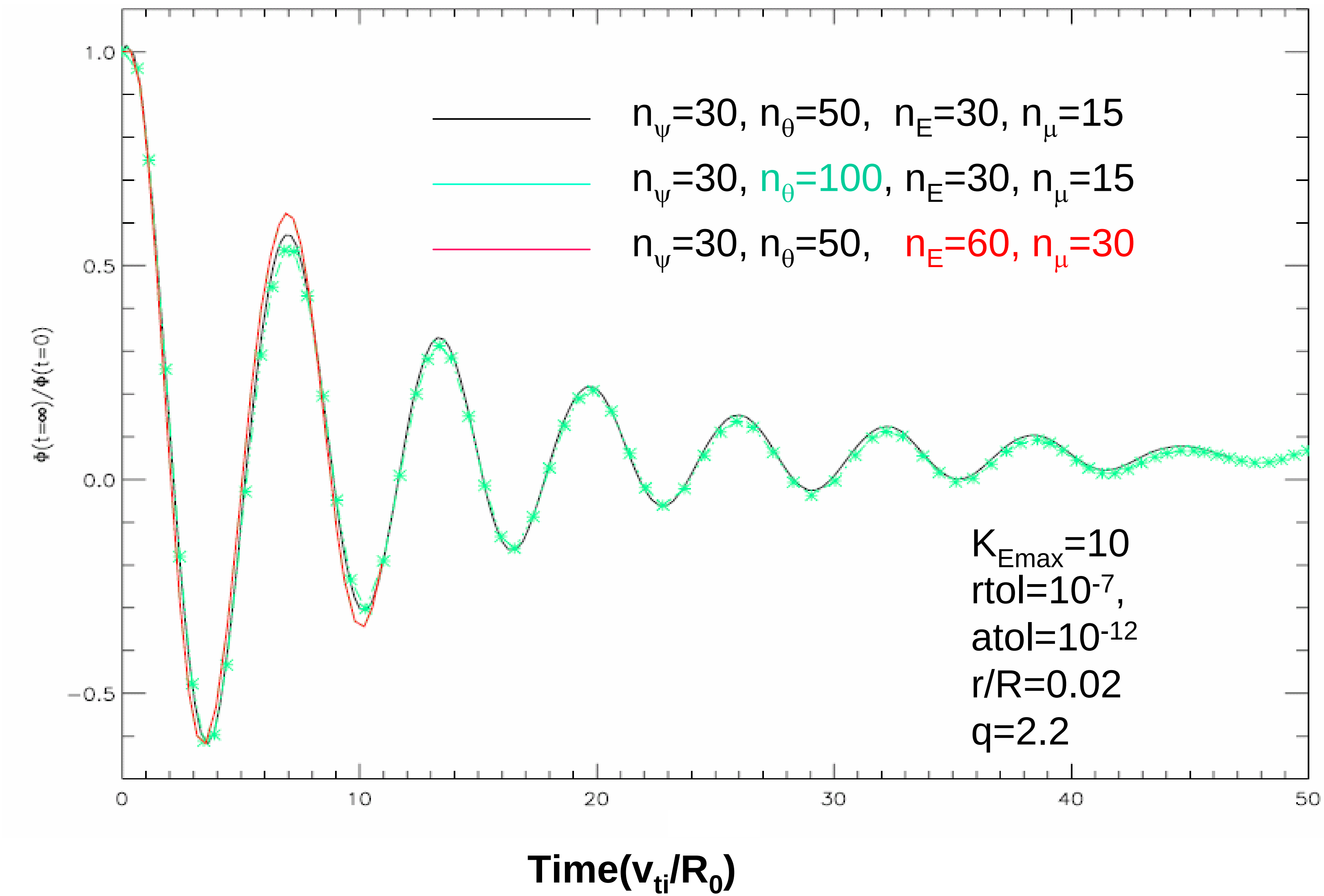


Time(v_{ti}/R_0)

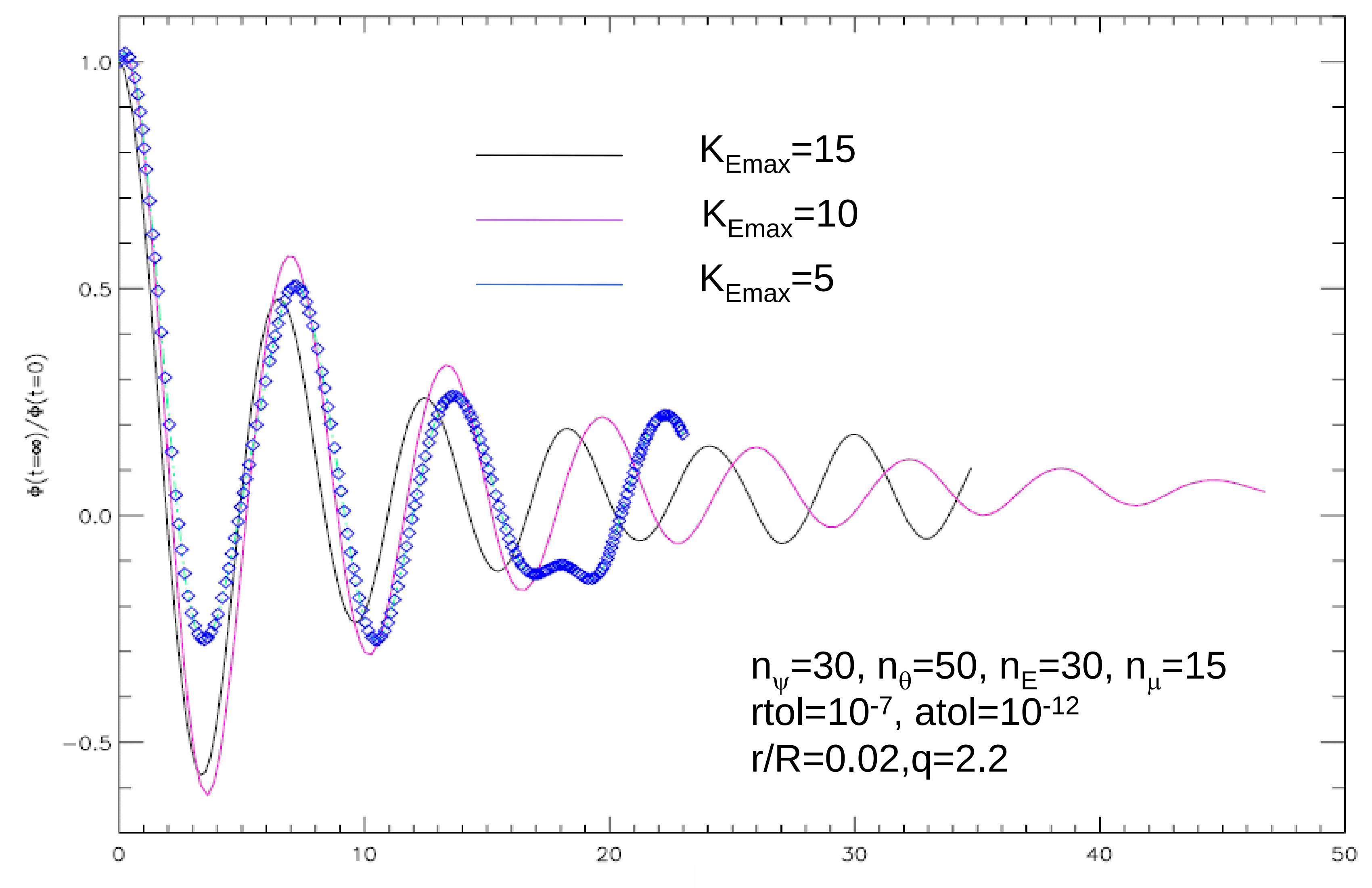
GAMs simulations converge with n_v, n_θ, and K_{Emax}



GAMs simulations converge with n_v and n_θ

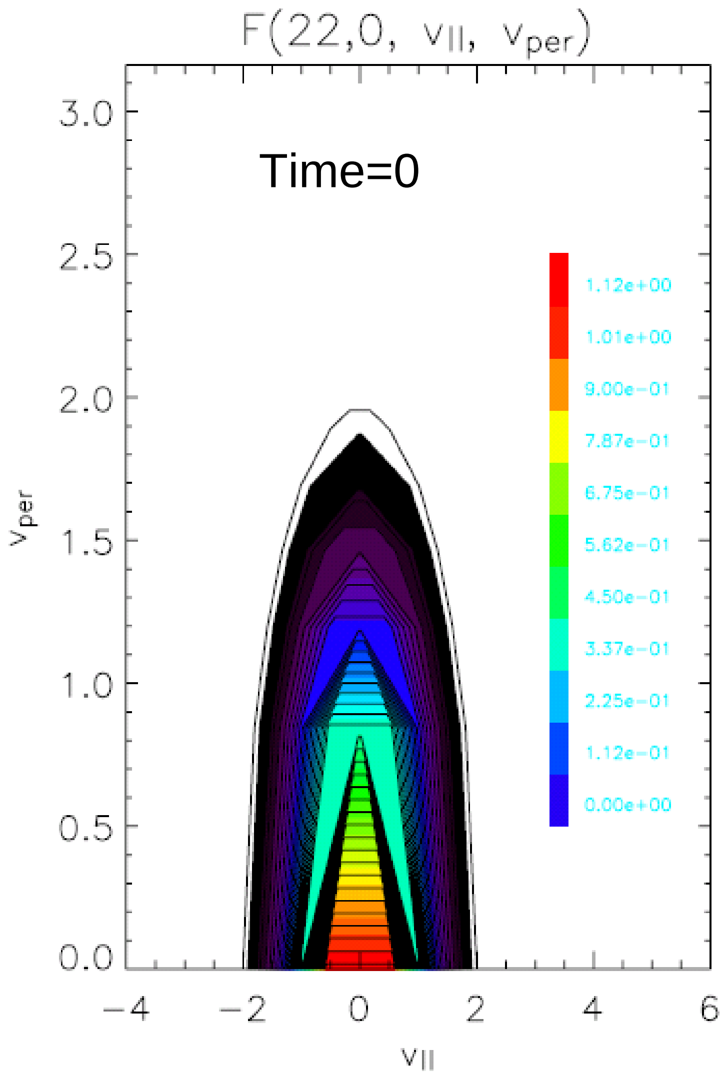


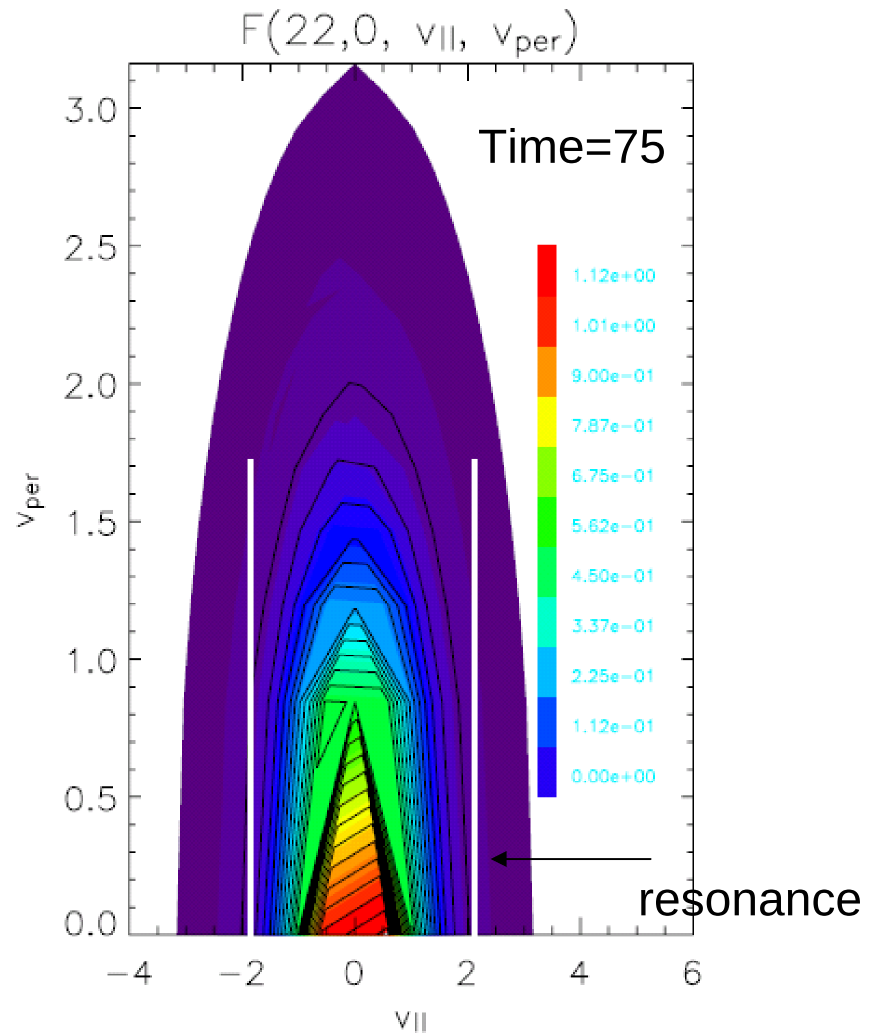
Maximum kinetic energy has to be 10x thermal energy



Time(v_{ti}/R_0)

Contour plot of distribution function





5D development plan

- **Completed formulation of 5D gyrokinetic simulation model**
  - **A basic set of equations**
    - **Electrostatic turbulence**
    - **Arbitrary wavelength limit**
  - **Choice of coordinate system**
    - **Field-aligned coordinates**
- **Planned numerical implementation**
  - **Implementing special processor scheduling for 5D data communication**
  - **Designing dual coordinate sets for radial difference**
  - **Adding toroidal drift**
    - **Change the 2D spatial loop to 3D**
    - **Add toroidal convection**
  - **Extend field solve to 3D**
  - **Developing gyroaverging module**
- **Defined benchmark test problems**
  - **ITG turbulence**
    - **Rosenbluth-Hinton zonal flow residual**
    - **Linear growth rates of ITG modes**
  - **Drift wave turbulence**