Transport Driven by Random Fluctuations: Theory and Simulations

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Motivation

- Random fluctuations of distribution function (noise) in PIC simulations arises as the result of averaging out the subgrid effects
- Noise contribution accumulates with time, and thus is particularly important for long-term simulations
- Noise produces heat transport in a presence of the background temperature gradient

Theory of Transport Driven by Random Fluctuations

- The fluctuations of distribution function due to the discrete particle noise lead to the appearance of fluctuations of electrostatic potential via Poisson equation.
- The gyrokinetic equation averaged over fluctuations contains the Fokker-Planck collision operator representing the diffusion in both configuration and velocity spaces. For small $k_{||}$ the diffusion in configuration space is dominant.
- The corresponding diffusion coefficients are determined by the correlation function of fluctuations.

The Diffusion Coefficient

The meansquare displacement in radial direction is the following

$$\frac{\left\langle \Delta X^2 \right\rangle}{\tau} = 2D = \frac{c^2}{B^2V^2}\sum_{\mathbf{k}}\int d\omega \int dt \frac{k_y^2}{k^2}\left\langle \left|\delta E^2\right|(\mathbf{k},\omega)\right\rangle \exp\{i[\mathbf{k}\cdot\Delta\mathbf{r}(t)-\omega t]\}$$

Assuming low fluctuation intensity and isotropic k-spectrum in perpendicular direction

$$\frac{\left\langle \Delta X^2 \right\rangle}{\tau} = \frac{c^2}{B^2V^2}\sum_{\mathbf{k}}\sum_{n}\int d\omega \frac{k_\perp^2}{k^2}\left\langle \left|\delta E^2\right|(\mathbf{k},\omega)\right\rangle J_n^2(k_\perp\rho)\delta\left(n\Omega + k_\parallel v_\parallel - \omega\right)$$

More general expression for the diffusion coefficient, however, requires the solution of integral equation

$$D_{ij\omega} = \frac{c^2}{B^2}\int\frac{d\mathbf{k}}{(2\pi)^2}\int\frac{d\omega'}{2\pi}k^2\left(\delta_{ij} - \frac{k_ik_j}{k^2}\right)\frac{i\left\langle \delta\Phi^2\right\rangle_{\mathbf{k}\omega'}}{\omega-\omega'+ik_ik_jD_{ij\omega-\omega'}}$$

Knowledge about statistical properties of fluctuations is required!

Noise Driven Flux

The fluctuation spectra was qualitatively modeled by the following function (imposed by the filer used in GTC)

$$\left\langle \delta\Phi^2 \right\rangle_{\mathbf{k}\omega} = \frac{\Phi^2}{\omega_{\max}} \cos^2\left(\frac{\pi}{2}\frac{\omega}{\omega_{\max}}\right) \prod_i \frac{1}{k_{i\max}} \cos^2\left(\frac{\pi}{2}\frac{k_i}{k_{i\max}}\right)$$

Estimating the parameters as

$$\omega_{\max} = k_{\|\max} v_e \qquad k_{\|\max} = 1/qR \qquad k_{\perp\max}\rho_e = 1$$

We were able to reproduce (within the order of magnitude) the observed **noise driven** heat flux $\chi = 3/2\ D$, and the correct scaling with fluctuation intensity $\chi \sim (e\Phi/T)^2$.

Better accuracy requires more knowledge about fluctuation spectra, in particular its frequency and k- width.

Conclusions

- The noise driven flux can be found if fluctuation spectra is known (from theoretical estimates or from simulations).
- The effects of noise driven flux on ETG/ITG turbulence has to be studied!