Resistive MHD Modeling of a Small Tokamak

Joshua Breslau
Princeton Plasma Physics Laboratory

M3D Project group: W. Park, H. Strauss,
L. Sugiyama, G. Fu, J. Chen

US-Japan Workshop on Integrated Modeling
Princeton, New Jersey
September 22, 2004

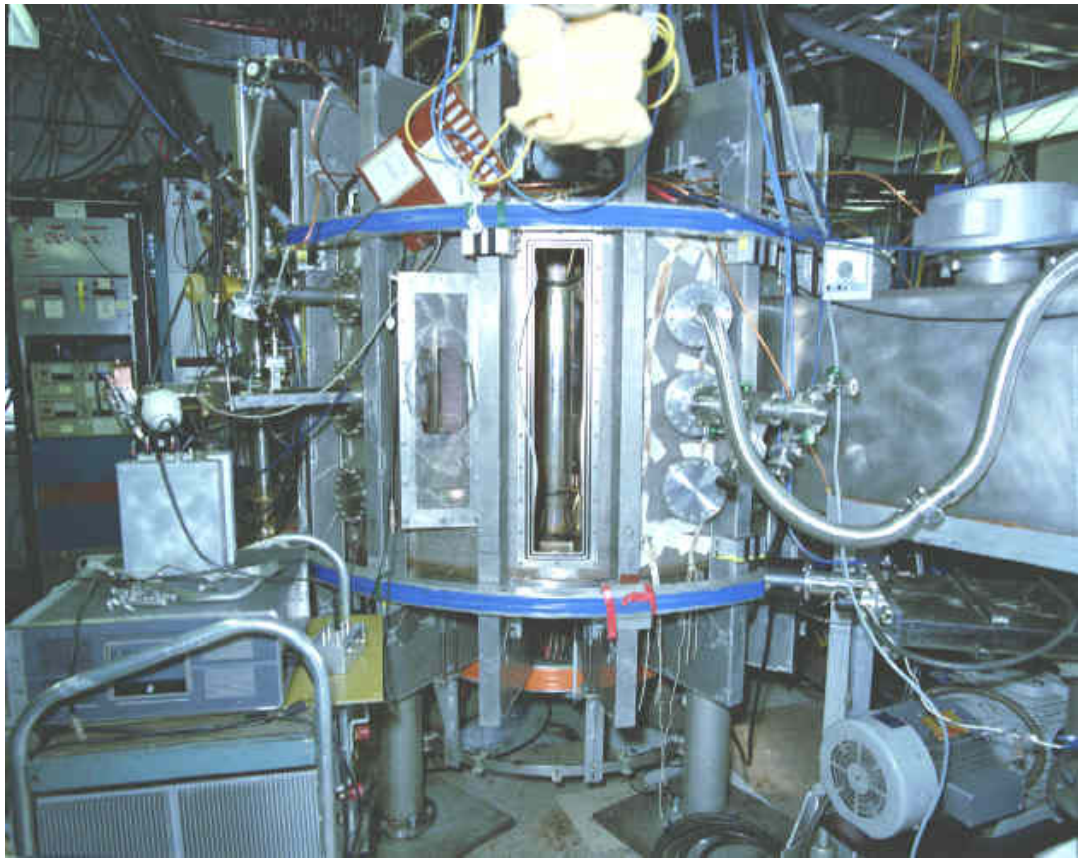
Outline

I. Motivation
II. The CDX Device
    A. Machine Parameters
    B. Constructing Equilibria
    C. Benchmark Parameters
III. The M3D Code
    A. Equations Solved
    B. Numerical Methods
IV. First case: $q_0 = 0.922$
    A. Linear Eigenmodes
    B. Nonlinear Evolution
        1. Mode Coupling
        2. Stabilizing Effects
V. Second case: $q_0 = 1.04$
    A. n>1 Eigenmodes
    B. Nonlinear Evolution
        1. Characterizing the Modes
        2. Heat Conduction Effects
VI. Conclusions
VII. Topics for Further Study

Motivation

- Model resistive MHD events in a tokamak plasma using realistic physical values to make quantitative predictions.
  - Large tokamaks have large disparities in spatial and temporal scales to be resolved.
    - Resistive MHD: Current sheet thickness ~ $S^{-1/2}$
    - Two-fluid MHD: ion skin depth ~ $n^{-1/2}$
  - Small tokamaks operate in regimes accessible to present-day codes.
- Cross-benchmark two extended MHD codes (M3D and NIMROD) in the nonlinear regime.

Characteristics of the Current Drive Experiment Upgrade (CDX-U)

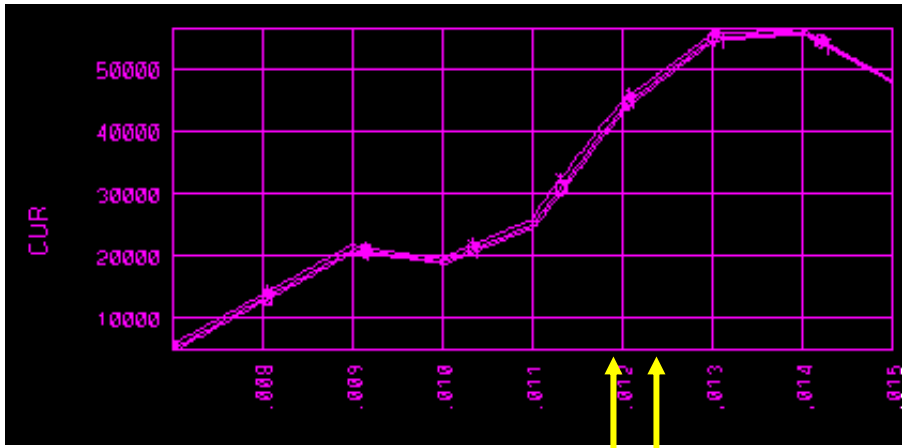


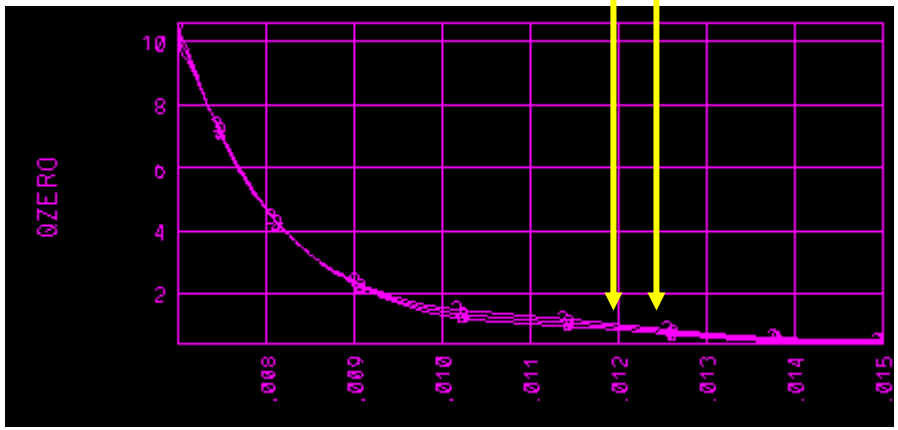
- Low aspect ratio tokamak (R_0/a = 1.4 – 1.5)
- Small (R_0 = 33.5 cm)
- Elongation $\kappa \sim 1.6$
- $B_T \sim 2300$ gauss
- $I_p \sim 70$ kA
- $n_e \sim 4\times10^{13}$ cm^{-3}
- $T_e \sim 100$ eV $\rightarrow$ S ~ 10^4
- Discharge time ~ 12 ms

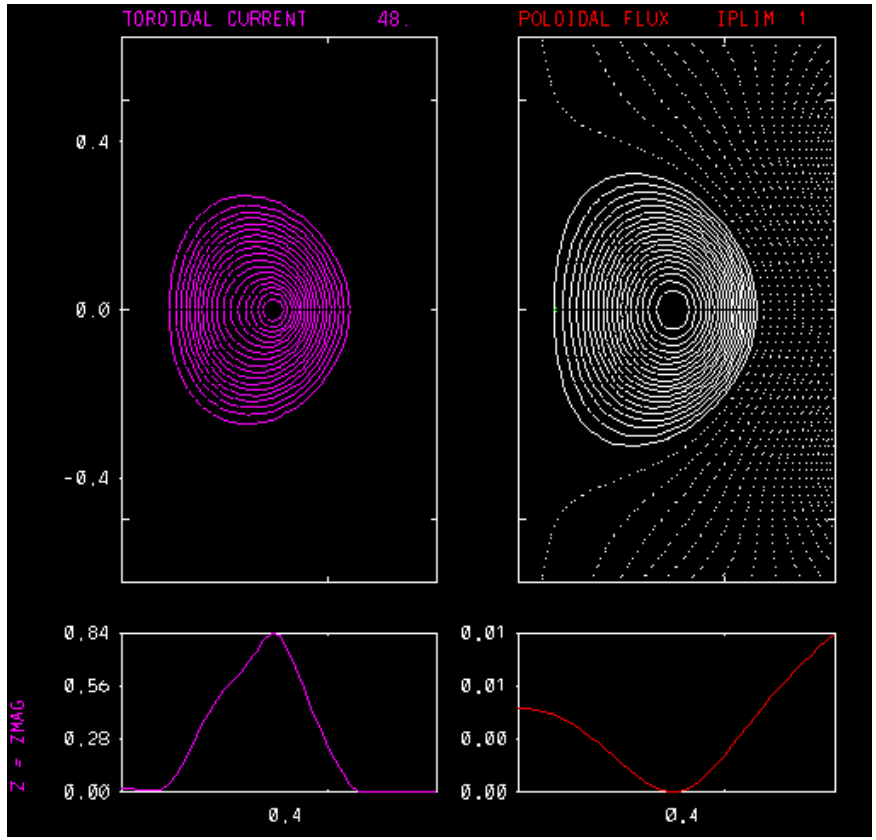
- Soft X-ray signals from typical discharges indicate two predominant types of low-*n* MHD activity:
  - sawteeth
  - “snakes”

Generating Equilibria

Transport timescale code TSC follows axisymmetric evolution of typical CDX-U discharge.







S. Jardin

Equilibria at t=12.40 ms (as q_0 drops to 0.92) and t=12.00 ms (q_0=1.04) are used to initialize 3D runs.

Baseline Parameters

Lundquist Number S	~2×10^4 on axis.
Resistivity η	Spitzer profile $\propto T_{eq}^{-3/2}$, cut off at $100\times\ \eta_0$
Prandtl Number Pr	10 on axis.
Viscosity μ	Constant in space and time.
Perpendicular thermal conduction $\kappa_\perp$	0
Parallel thermal conduction $\kappa_\parallel$	0
Peak Plasma β	~ 3×10^{-2} (low-beta).
Density Evolution	Turned on for nonlinear phase.

The Multilevel 3D (M3D) Code

- Maintained by a multi-institutional collaboration.
- Can solve resistive MHD, two-fluid MHD, hybrid MHD with gyrokinetic hot ions, or hybrid with all gyrokinetic ions.
- 3D Toroidal geometry with fixed boundary handles tokamaks, spherical tori, and stellarators.
- Can compute linear eigenmodes as well as evolving nonlinearly.
- Massively parallel – scales to over 1,000 processors.

W. Park et al., Phys. Plasmas **6,** 1796 (1999)

MHD model

- Solves MHD equations.

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \mathbf{J}\times\mathbf{B} + \mu\nabla^2\mathbf{v}$$

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = (-\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}), \quad \mathbf{J} = \nabla\times\mathbf{B}$$

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}) = 0$$

$$\partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa\nabla(p/\rho)$$

The fast parallel equilibration of T is modeled using wave equations;

$$\partial T/\partial t = s\,\mathbf{B}/\rho\cdot\nabla u$$

$$\partial u/\partial t = s\,\mathbf{B}\cdot\nabla\mathbf{T} + \upsilon\nabla^2 u$$

s = wave speed/ v_A

Two-fluid MH3D-T

- Solves the two fluid equations with gyro-viscousity and neoclassical parallel viscousity terms in a torus.

- **Equations**

$$\mathbf{v} \equiv \mathbf{v}_i - \mathbf{v}_i^* = \mathbf{v}_e - \mathbf{v}_e^* + \mathbf{J}_\parallel/en,$$

$$\mathbf{v}_e^* \equiv -\mathbf{B}\times\nabla p_e/(enB^2), \quad \mathbf{v}_i^* \equiv \mathbf{v}_e^* + \mathbf{J}_\perp/en,$$

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} + \rho(\mathbf{v}_i^*\cdot\nabla)\mathbf{v}_\perp = -\nabla p + \mathbf{J}\times\mathbf{B} - \mathbf{b}\cdot\nabla\cdot\Pi_i,$$

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = (-\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}) - \nabla_\parallel p_e/en - \mathbf{b}\cdot\nabla\cdot\Pi_e,$$

$$\mathbf{J} = \nabla\times\mathbf{B},$$

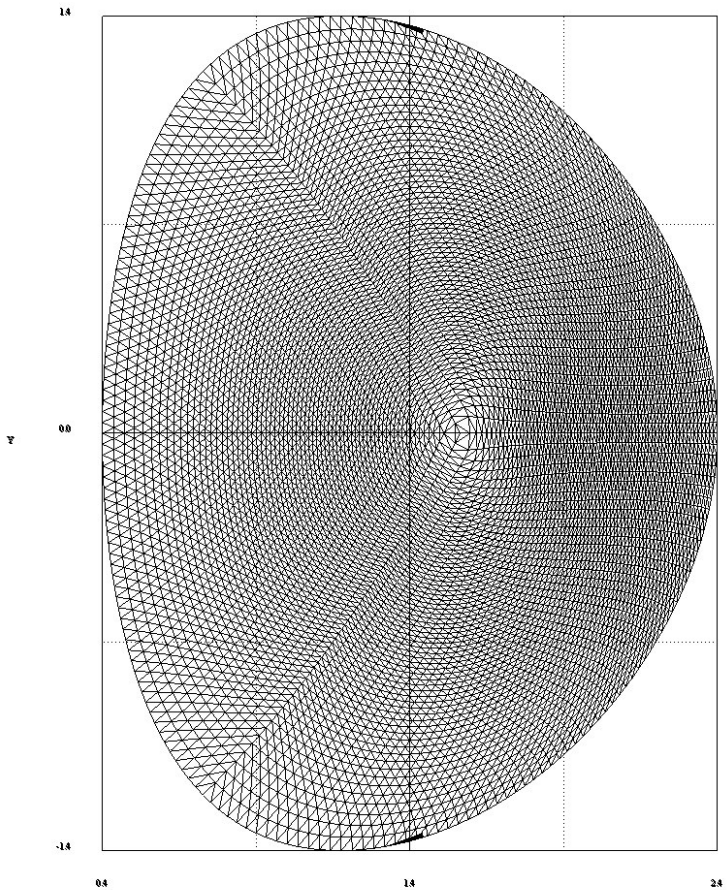
$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}_i) = 0,$$

$$\partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_\parallel\nabla_\parallel(p/\rho) - \mathbf{v}_i^*\cdot\nabla p + (1/en)\mathbf{J}\cdot\nabla p_e - \gamma p\nabla\cdot\mathbf{v}_i^* + \gamma p_e\mathbf{J}\cdot\nabla(1/en)$$

$$\partial p_e/\partial t + \mathbf{v}\cdot\nabla p_e = -\gamma p_e\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_\parallel\nabla_\parallel(p_e/\rho) + (1/en)\mathbf{J}_\parallel\cdot\nabla p_e - \gamma p_e\nabla\cdot(\mathbf{v}_e^* - \mathbf{J}_\parallel/en)$$

M3D Numerics

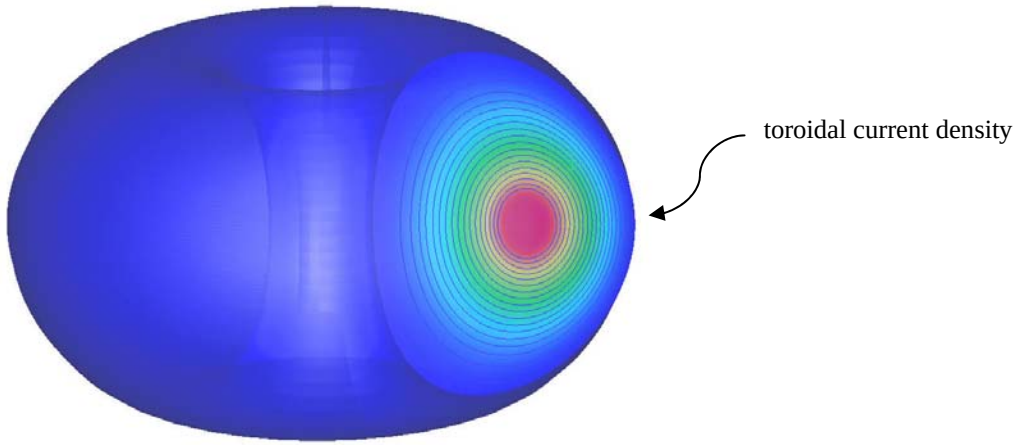
- Uses linear finite elements in-plane.
- Uses finite differences between planes or pseudo-spectral derivatives.
- Partially implicit treatment allows efficient time advance but requires small time steps.
- <u>Nonlinear</u> operation: all components of all quantities evolve nonlinearly.
- <u>Linear</u> operation: full nonlinear + filtering, active equilibrium maintenance.

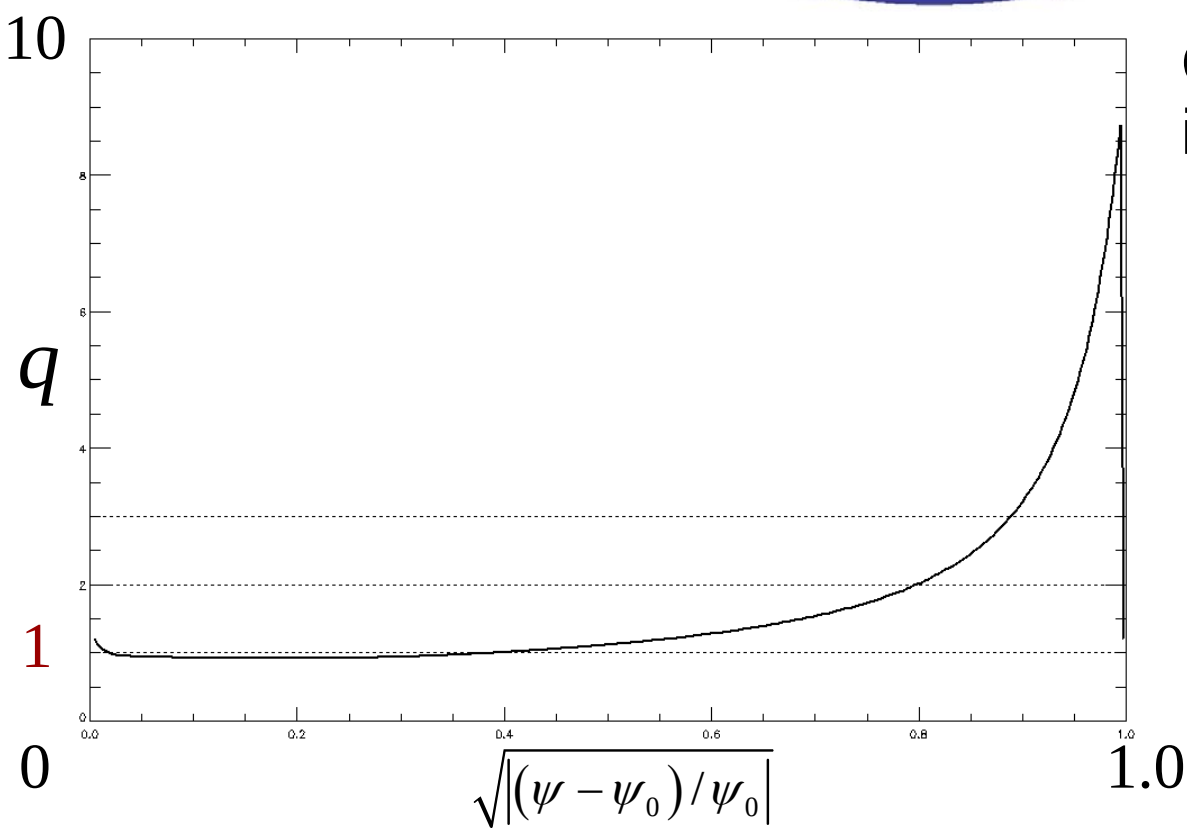


Typical unstructured mesh has 89 radial zones, up to 267 in θ.

Case 1: $q_0 < 1$

- Equilibrium taken from a TSC sequence (Jsolver file).
- $q_{\min} \approx 0.922$
- $q(a) \sim 9$



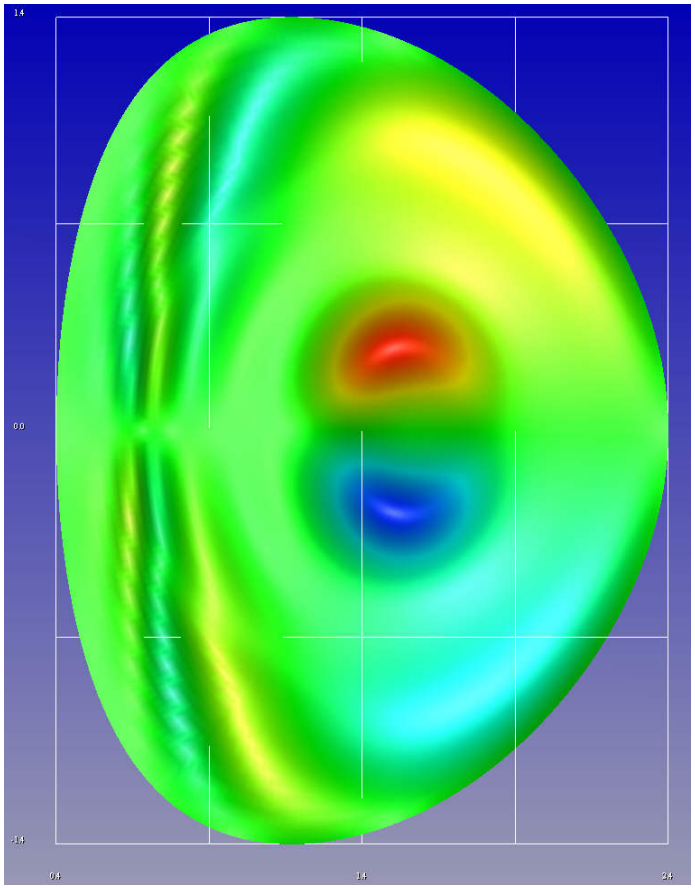


Questions to investigate:

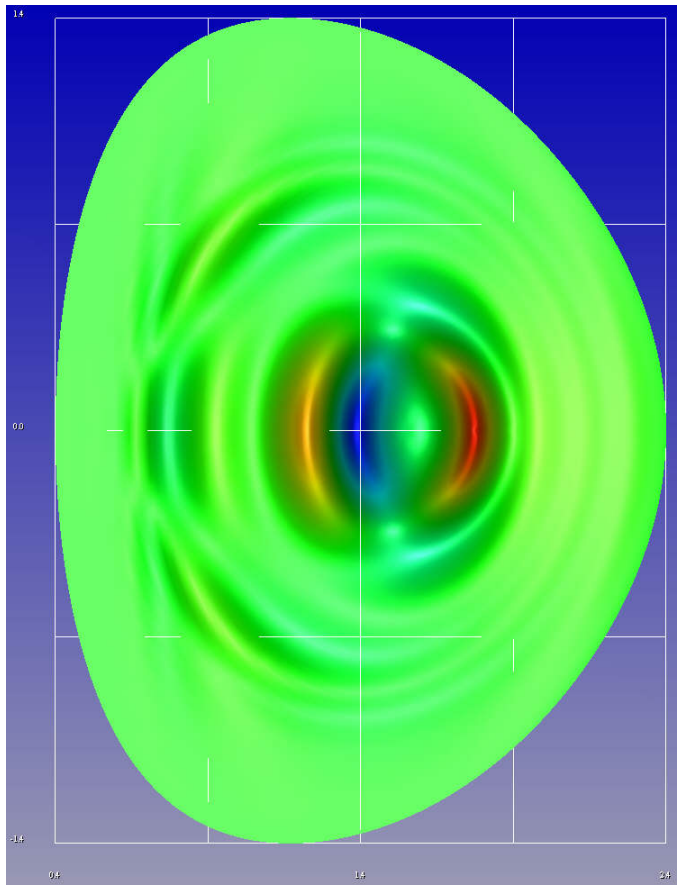
- Linear growth rate and eigenfunctions
- Nonlinear evolution
  - disruption?
  - stagnation?
  - repeated reconnections?

Low Aspect Ratio:
n=1 Eigenmode

Incompressible velocity stream function U

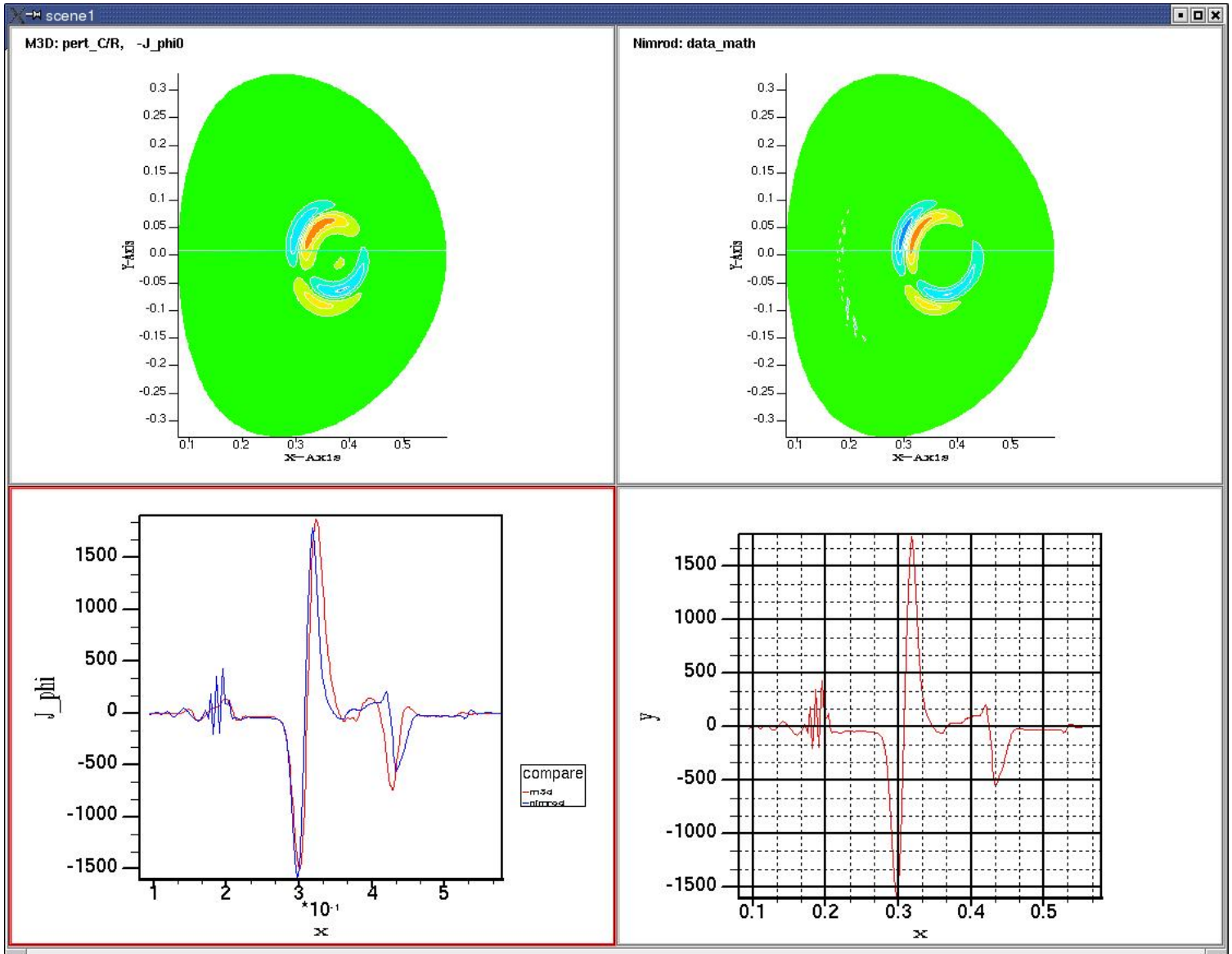


Toroidal current density J_ϕ



$\gamma \tau_A = 8.61 \times 10^{-3} \rightarrow$ growth time = 116 τ_A

Predicted Eigenmode Agrees with NIMROD Result

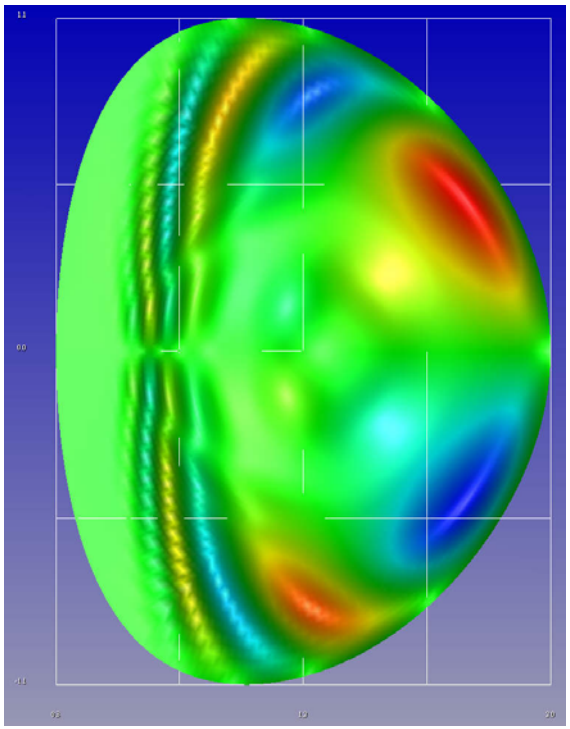


Low Aspect Ratio:

Higher *n* Eigenmodes

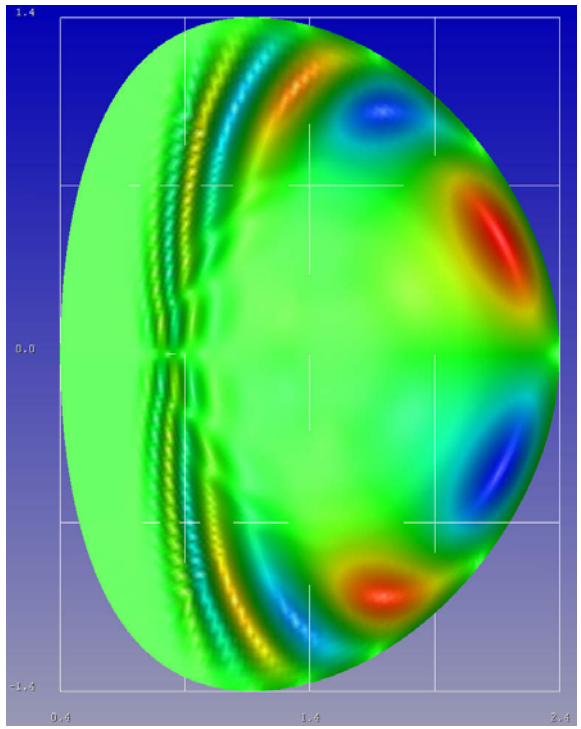
Incompressible velocity
stream function U

$n = 2$



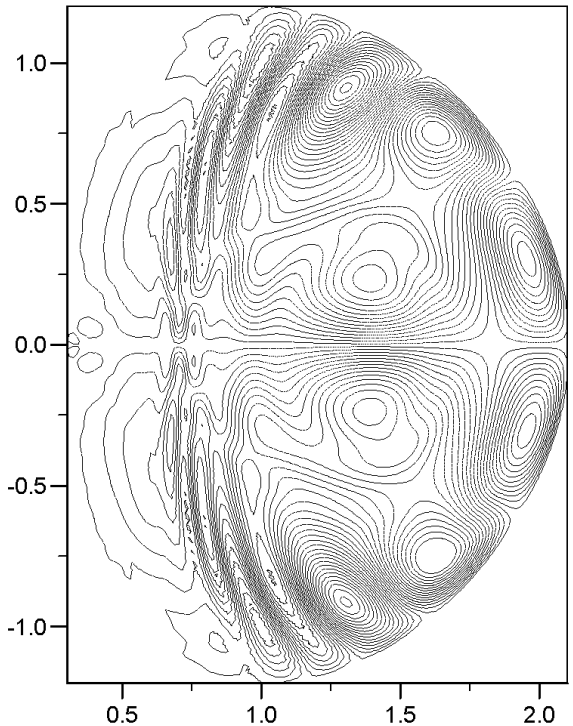
$m \geq 5$
$\gamma \tau_A = 1.28 \times 10^{-2}$

$n = 3$



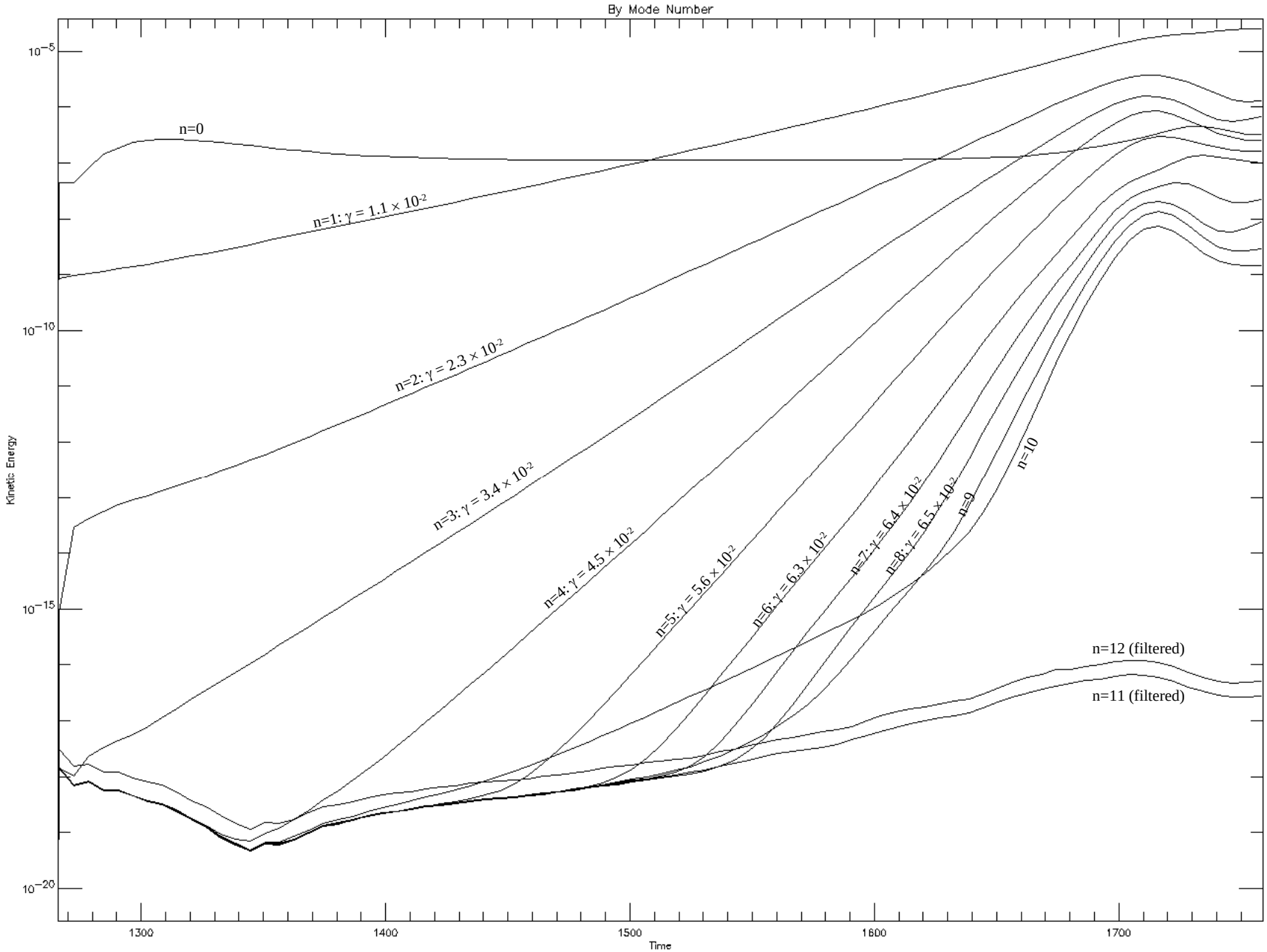
$m \geq 7$
$\gamma \tau_A = 1.71 \times 10^{-2}$

$n = 4$ (projected)



$m \geq 8$
not converged

Low Aspect Ratio: Nonlinear Kinetic Energy History

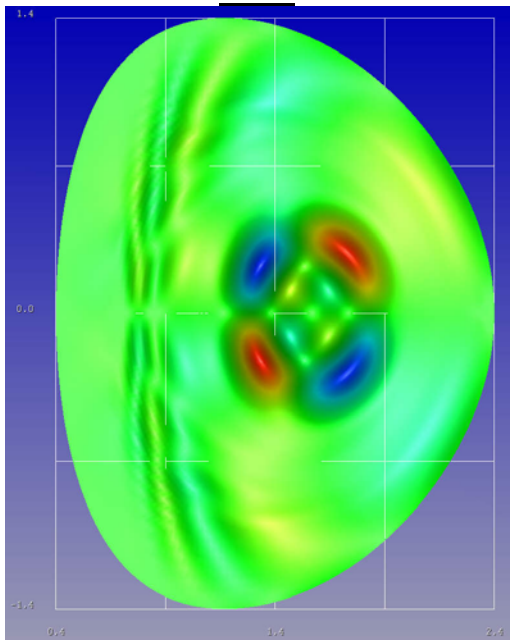


“Linear” high-*n* modes are driven, not eigenmodes

Incompressible velocity stream function *U*

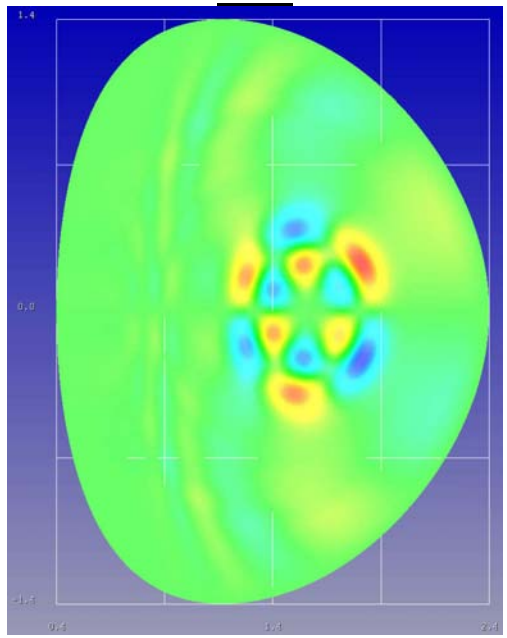
Component of “linear” mode
in nonlinear run

n=2



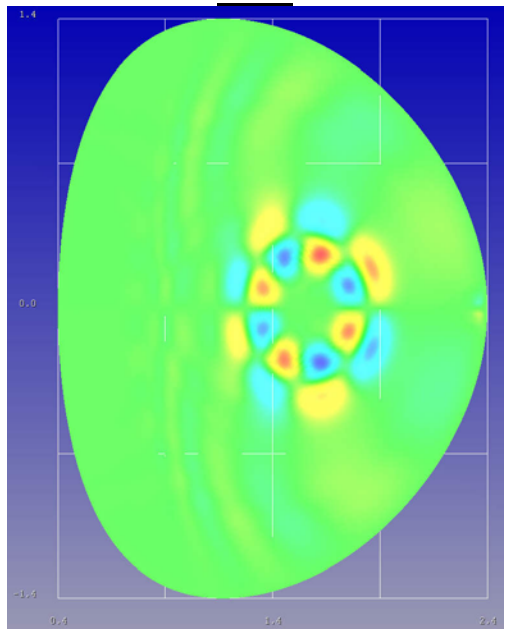
m = 2
$\gamma = 2.3 \times 10^{-2}$

n=3



m = 3
$\gamma = 3.4 \times 10^{-2}$

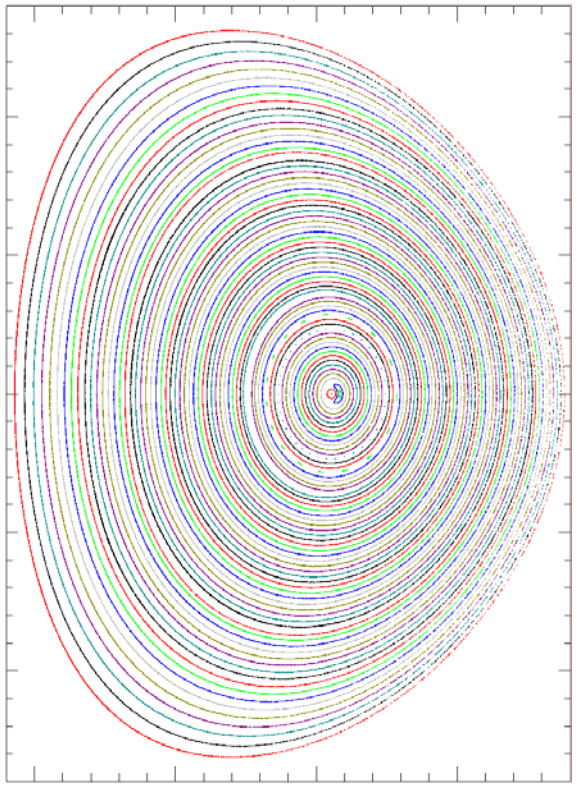
n=4



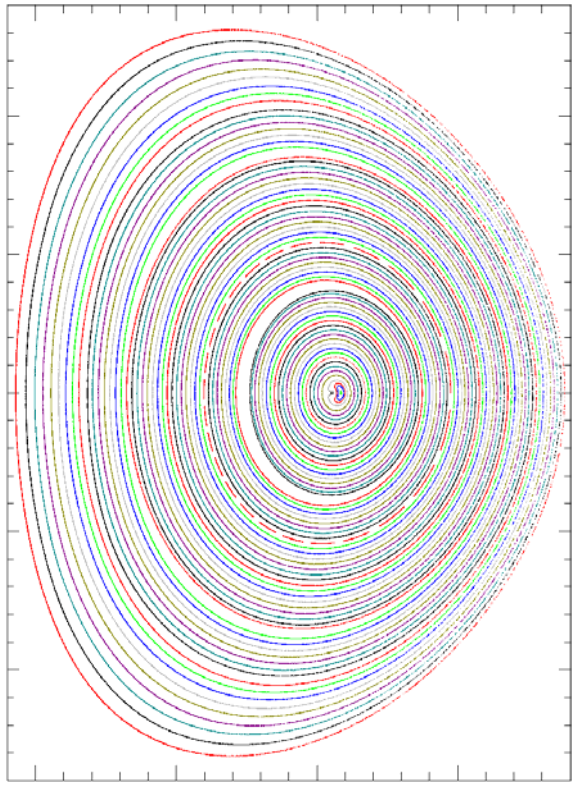
m = 4
$\gamma = 4.5 \times 10^{-2}$

Low Aspect Ratio: Nonlinear Time Series

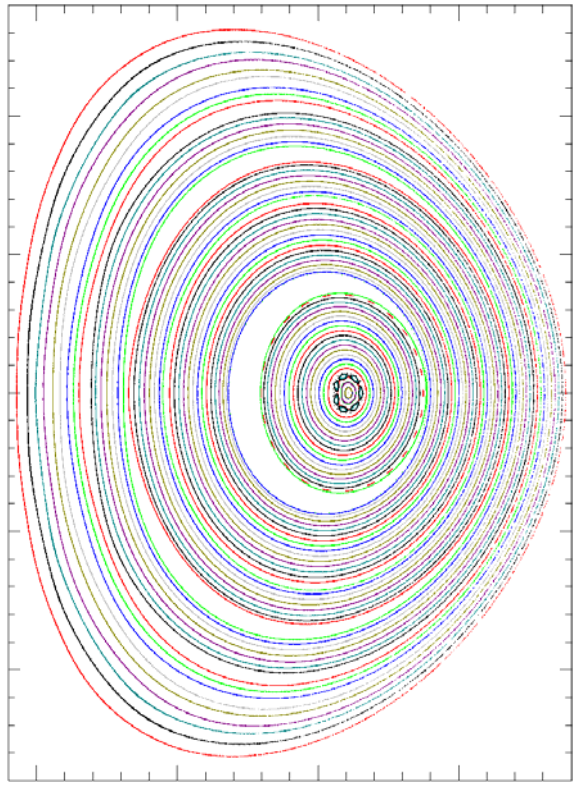
Poincaré Plots



t = 1266.17, chopped × 4.470358



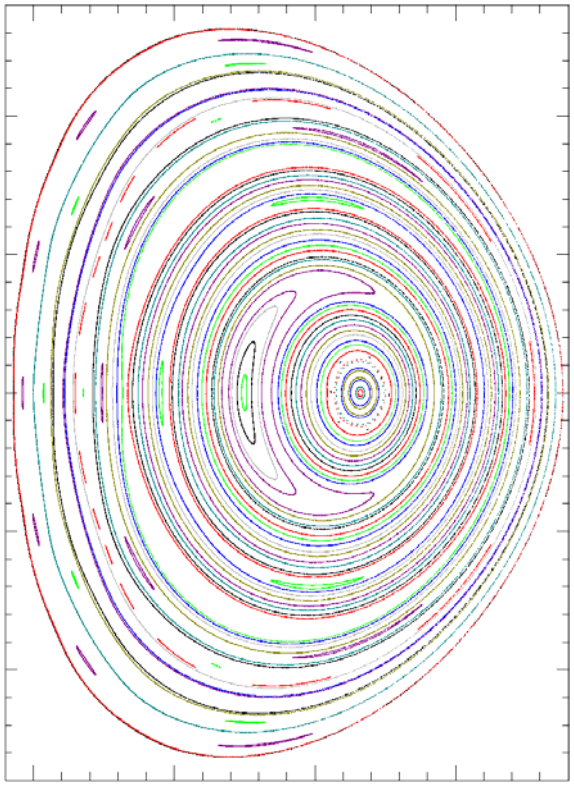
t = 1404.57



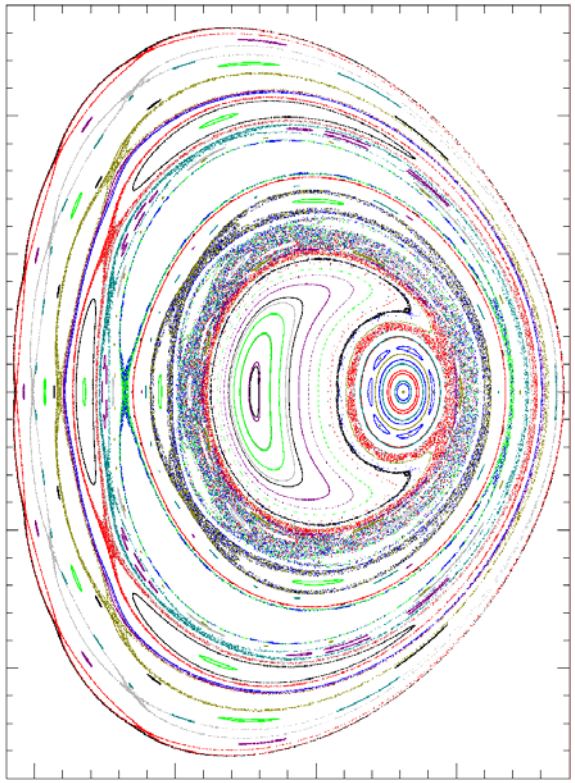
t = 1548.68

Low Aspect Ratio: Nonlinear Time Series

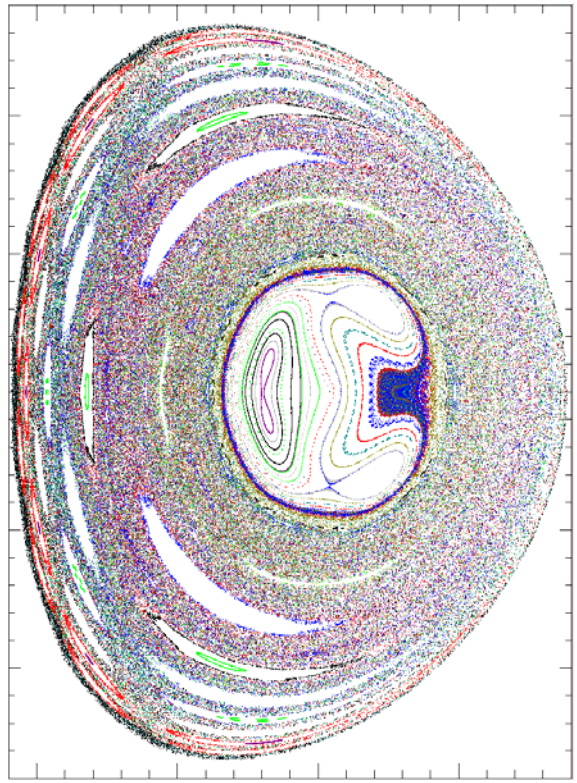
Poincaré Plots, Continued



t = 1620.62



t = 1686.41



t = 1758.34

Disruption occurs before completion of sawtooth crash.

Extended MHD Effects

(Initial density profile uniform)

	MHD		MHD + ω_i^* term	
Evolving Density	$v_{\|\|}/v_A = 0$ ($\chi_{\|\|}$ off)	$v_{\|\|}/v_A = 2.5$ ($\chi_{\|\|}$ on)	$v_{\|\|}/v_A = 0$ ($\chi_{\|\|}$ off, H=0.10)	$v_{\|\|}/v_A = 2.5$ ($\chi_{\|\|}$ on, H=0.15)
n=1	0.017	0.014	0.0165	0.0137
n=4	0.0349	0.0075	0.0304	0.0095
Constant Density	$v_{\|\|}/v_A = 0$ ($\chi_{\|\|}$ off)	$v_{\|\|}/v_A = 2.5$ ($\chi_{\|\|}$ on)	$v_{\|\|}/v_A = 0$ ($\chi_{\|\|}$ off, H=0.15)	$v_{\|\|}/v_A = 2.5$ ($\chi_{\|\|}$ on, H=0.15)
n=1	0.017	0.021	0.021	0.021
n=4	0.0349	0.0132	0.0354	0.0123

- Large parallel thermal conductivity reduces growth rate of higher *n* modes.
- ω_i^* term alone does <u>not</u> have stabilizing effect on either mode.

Summary of the q_0<1 Case

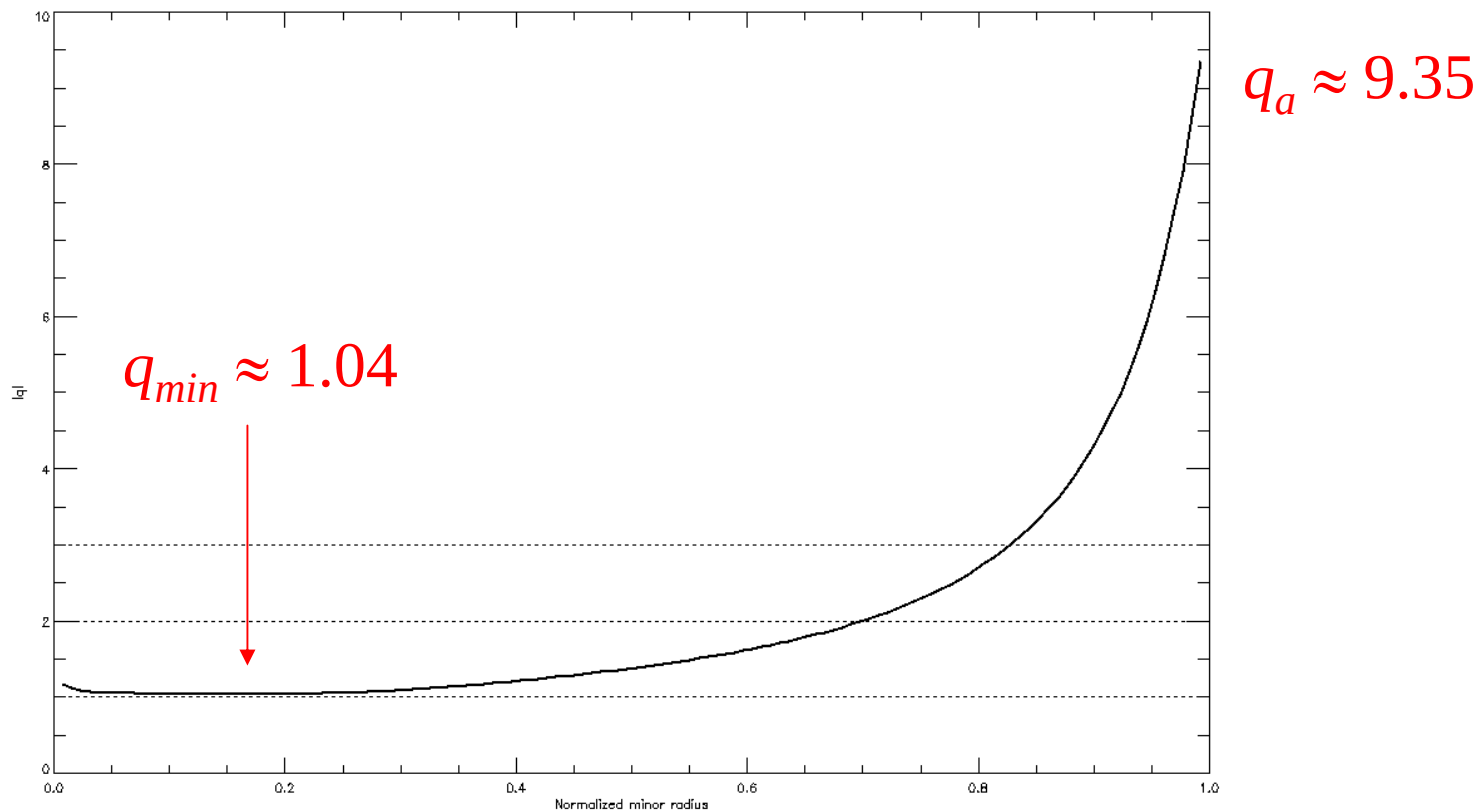
- All toroidal modes of the q_{min}= 0.92 CDX equilibrium are linearly MHD-unstable.
  - n =1 is an internal kink mode
  - n >1 are ballooning instabilities
  - Higher n modes have higher growth rates.

- Nonlinear MHD evolution beginning with just an n=1 perturbation disrupts within a sawtooth crash time.
  - High poloidal mode number m components of the n=1 mode interact to create islands, stochasticity in outer region.
  - n =1 mode couples to and drives higher n modes at q=1 rational surface to create further stochasticity.

- Adding large parallel thermal conductivity (via artificial sound wave) has a stabilizing effect on higher n modes, but not on n=1.
- Adding the ω^* term to the MHD equations does not appreciably alter the growth rates of either the n =1 or the n >1 modes.

Case 2: $q_0 > 1$

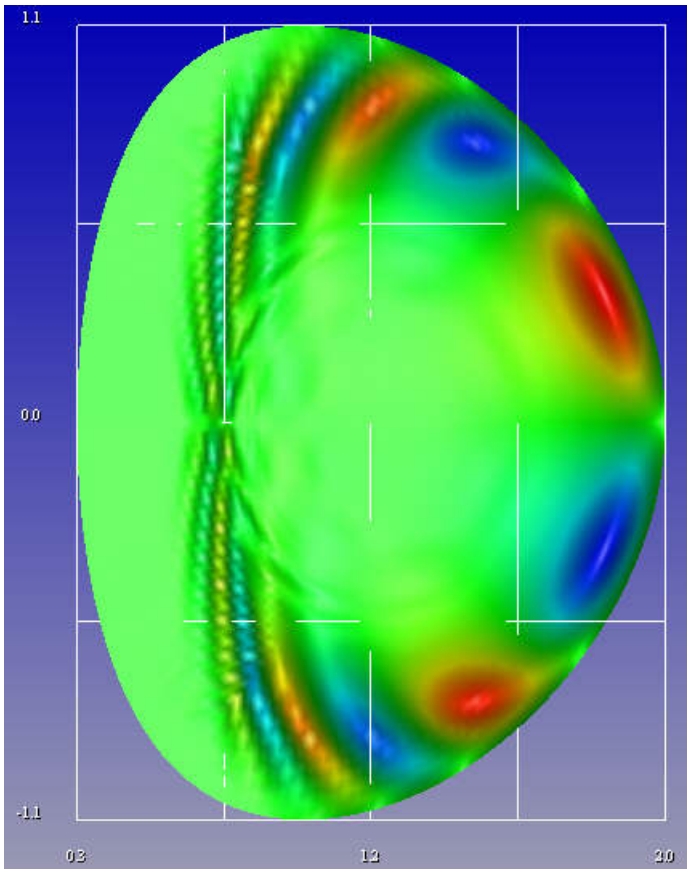
Equilibrium taken from earlier in same TSC sequence as case 1.



Eigenmodes

(*U*, incompressible part of velocity stream function)

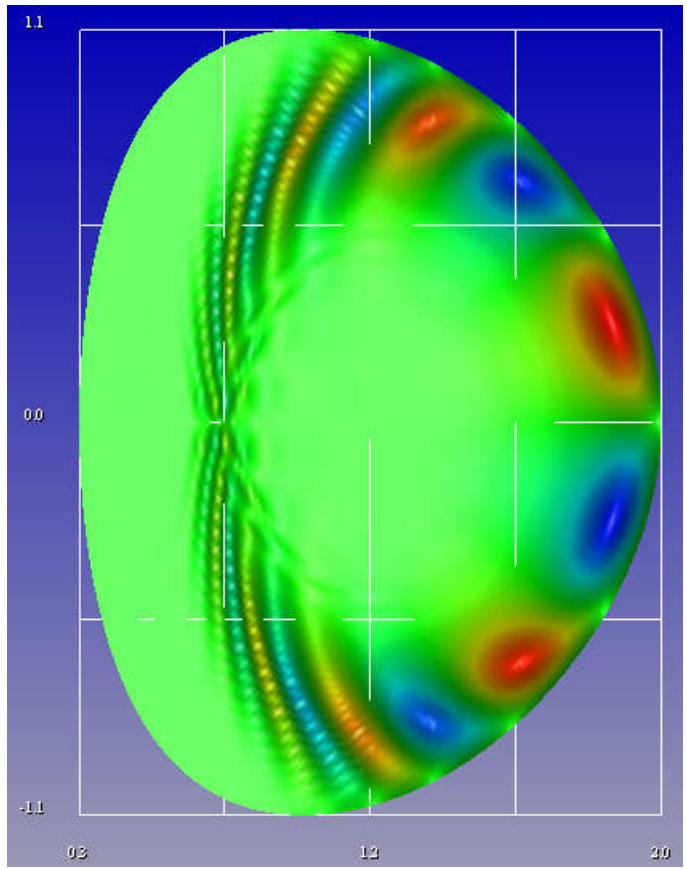
$n = 3$



$m \approx 7$

$\gamma \tau_A \approx 1.42 \times 10^{-2}$

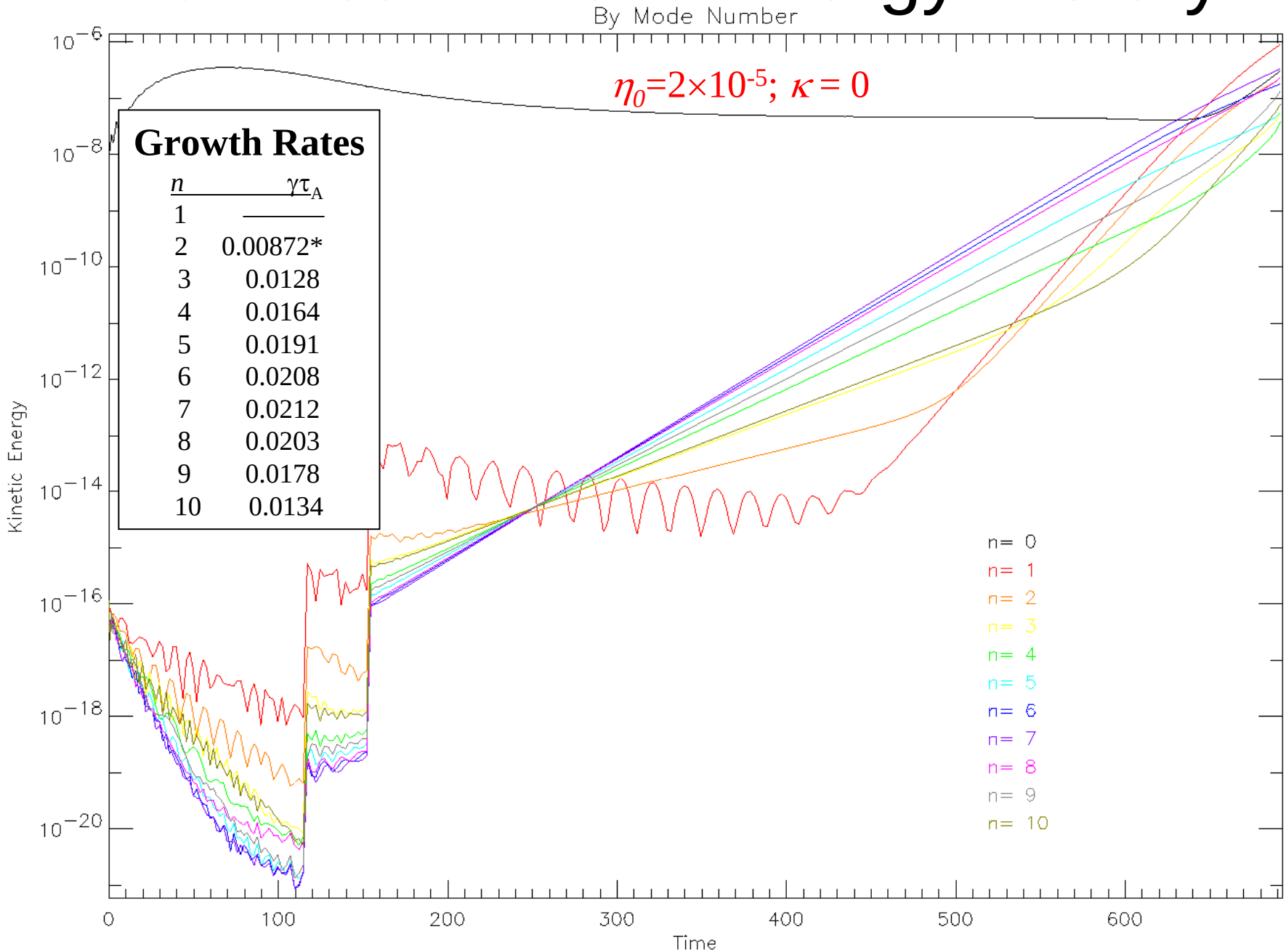
$n = 4$



$m \approx 9$

$\gamma \tau_A \approx 1.87 \times 10^{-2}$

Nonlinear Kinetic Energy History



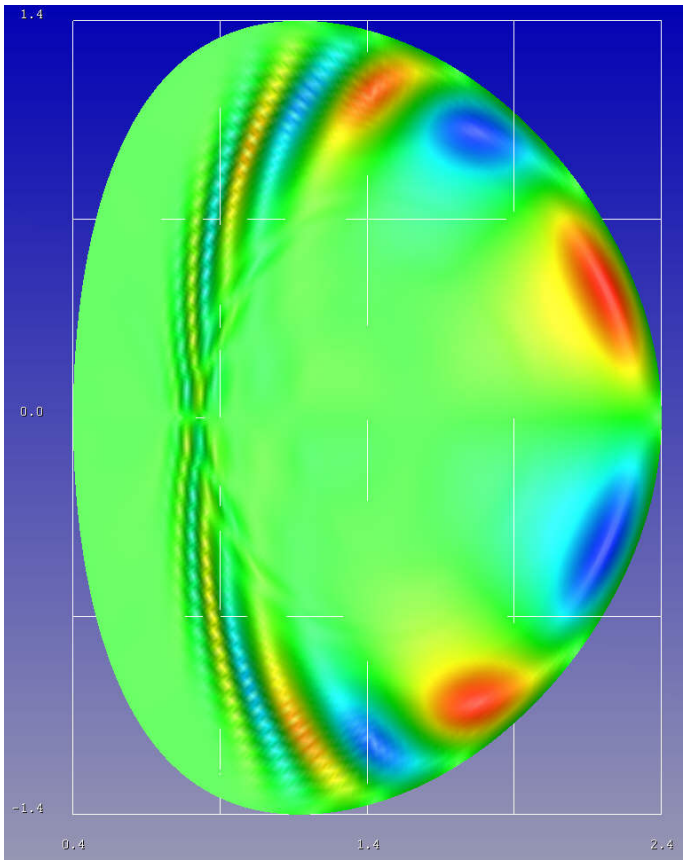
Growth Rates

n	$\gamma\tau_A$
1	———
2	0.00872*
3	0.0128
4	0.0164
5	0.0191
6	0.0208
7	0.0212
8	0.0203
9	0.0178
10	0.0134

Modes Observed Nonlinearly at t=219.6

(U, incompressible part of velocity stream function)

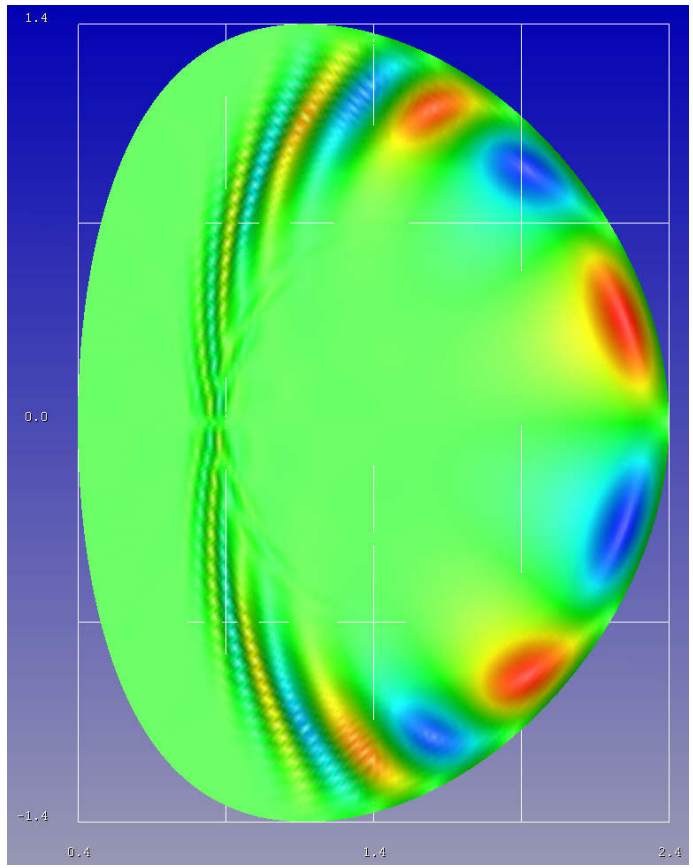
$n = 3$



$m \approx 8$

$\gamma \tau_A \approx 1.28 \times 10^{-2}$

$n = 4$

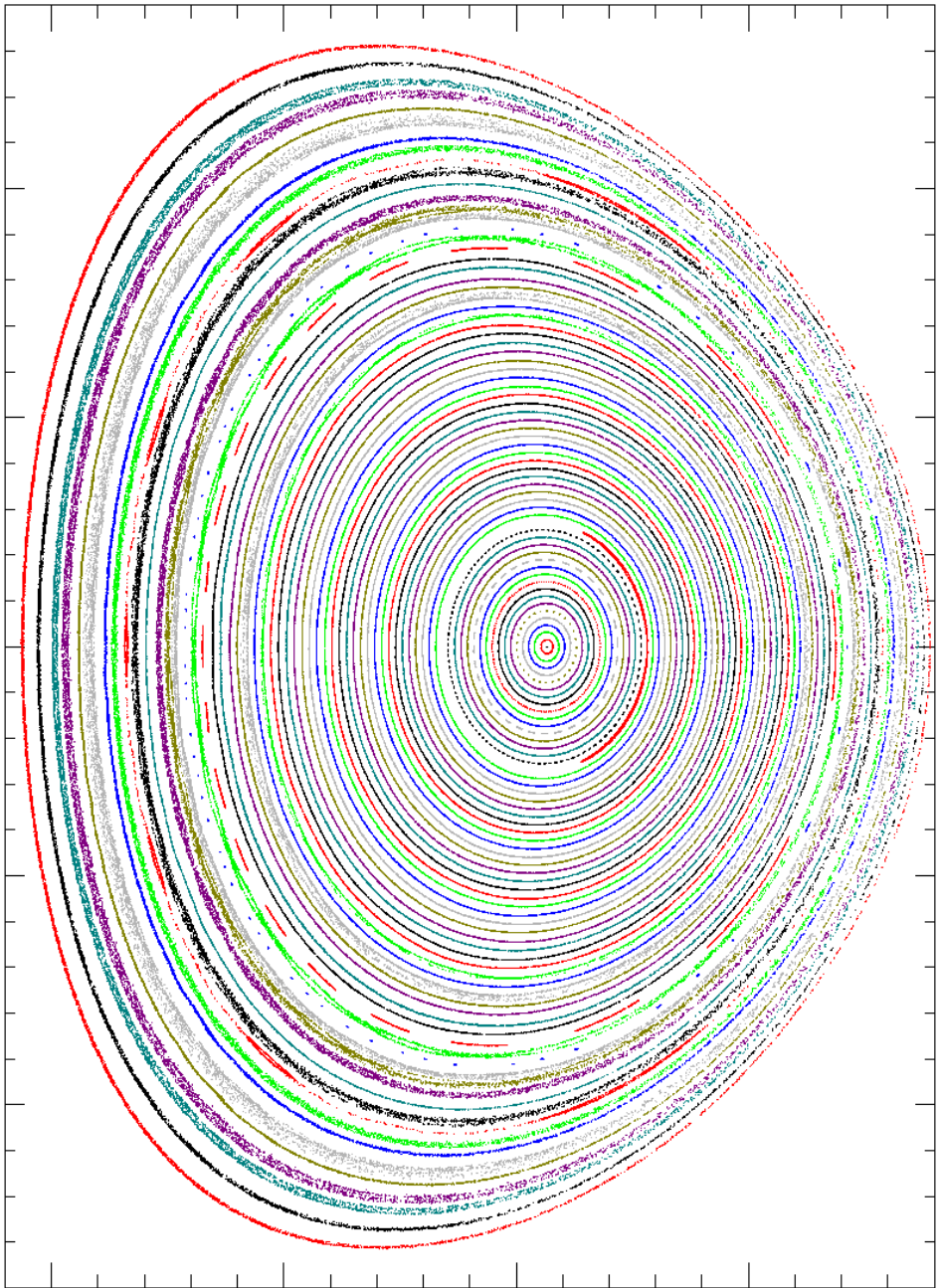


$m \approx 10$

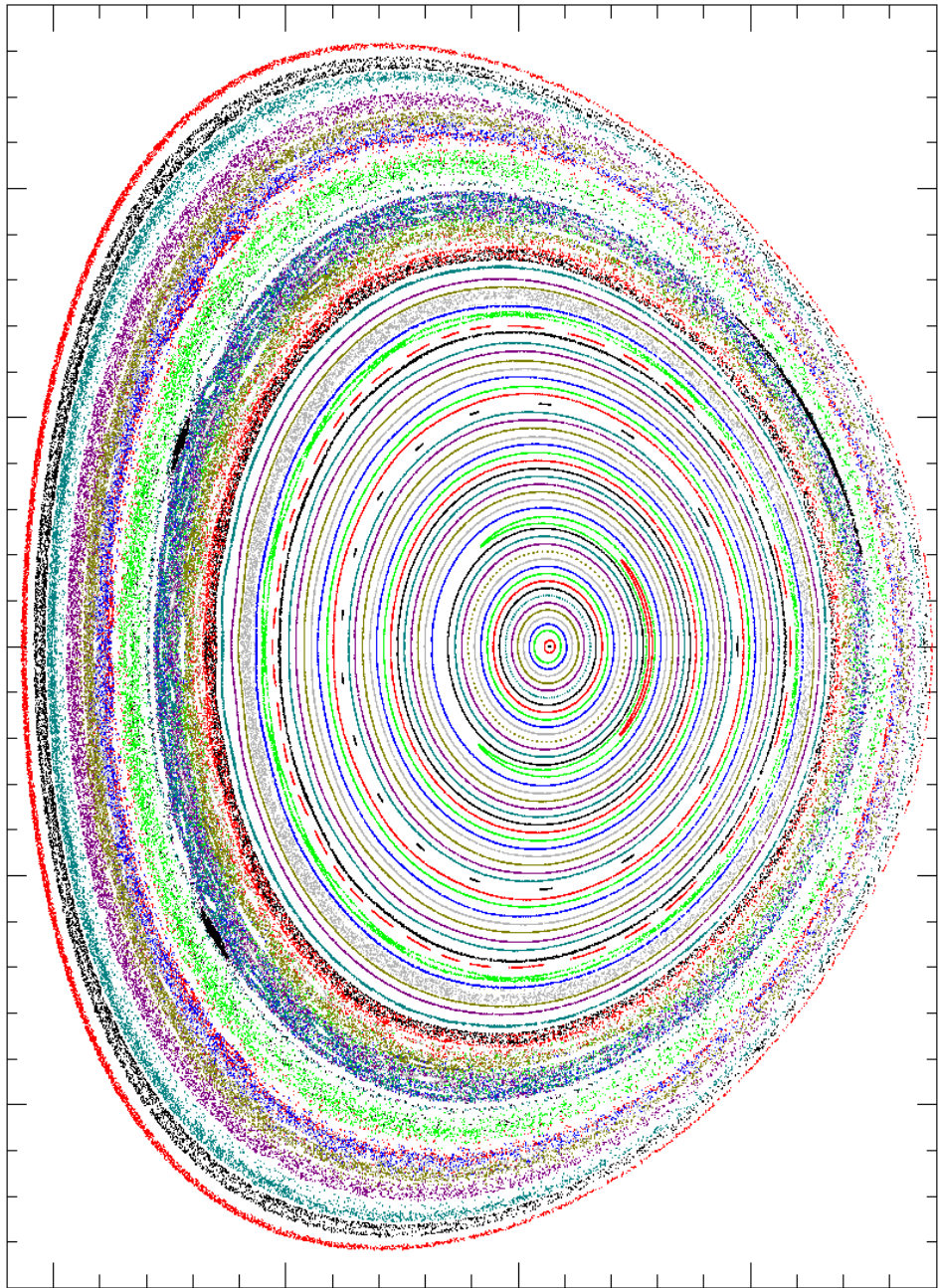
$\gamma \tau_A \approx 1.64 \times 10^{-2}$

Poincaré Plots

t=569.49

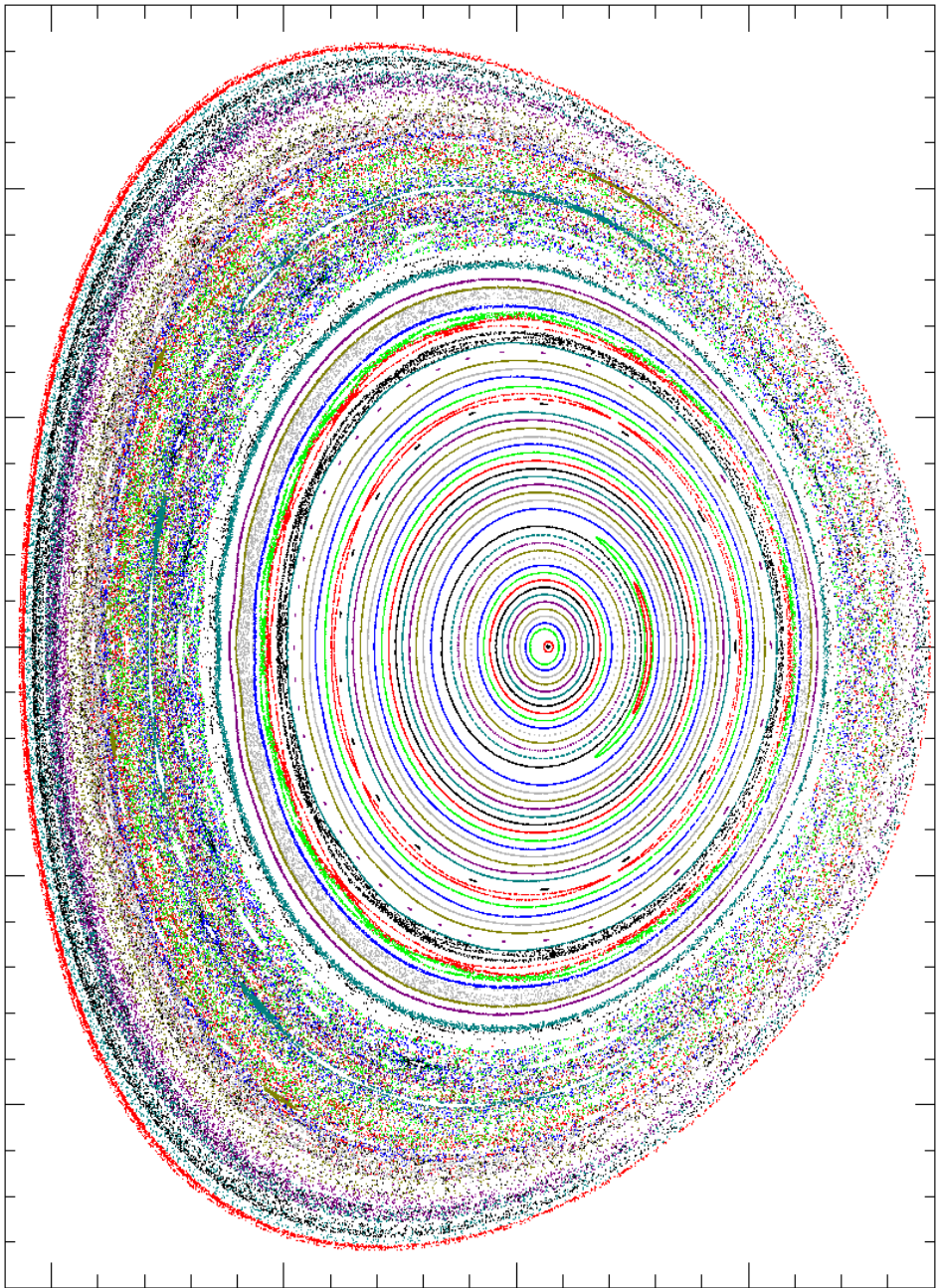


t=619.51

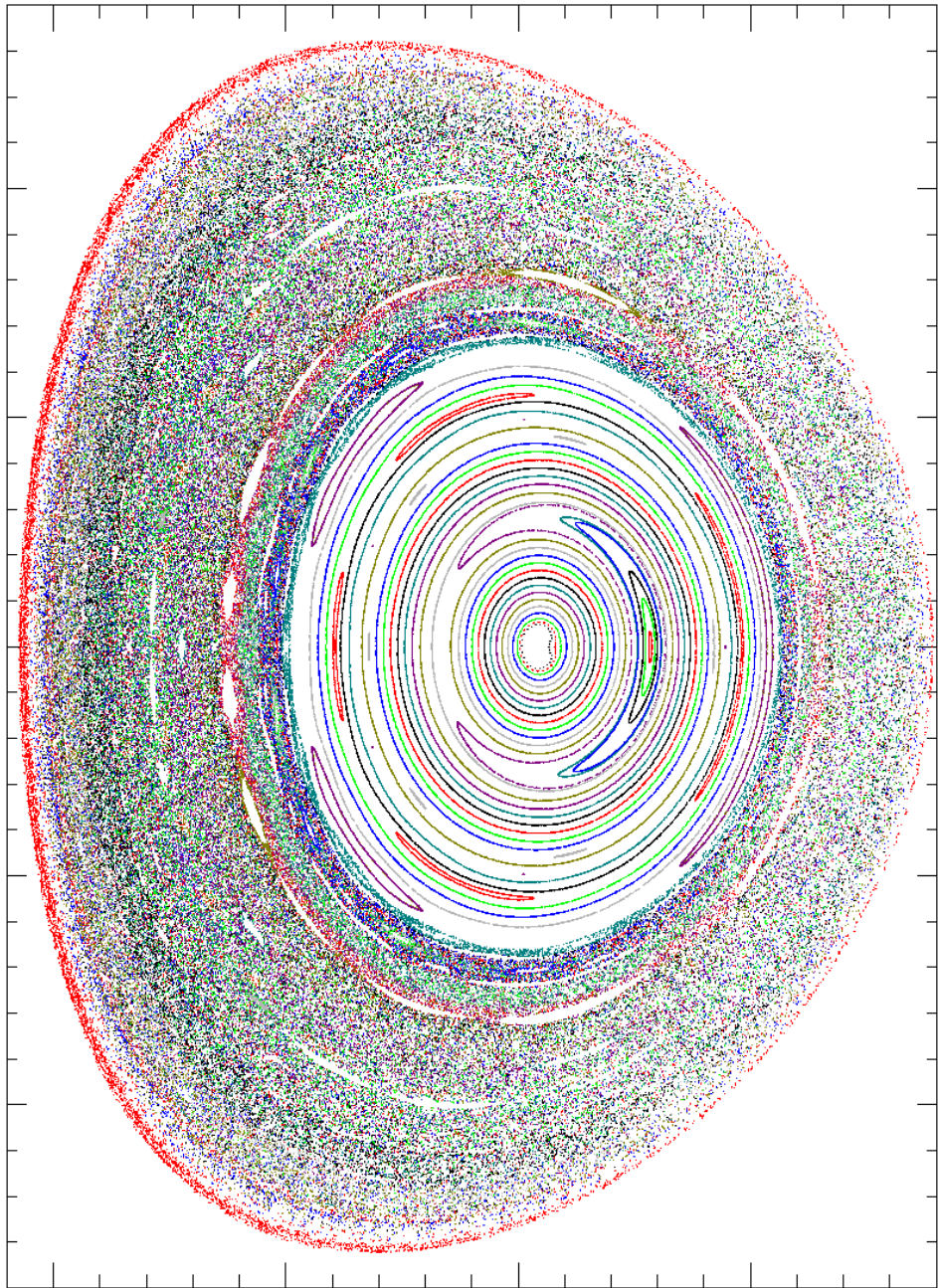


Poincaré Plots, continued

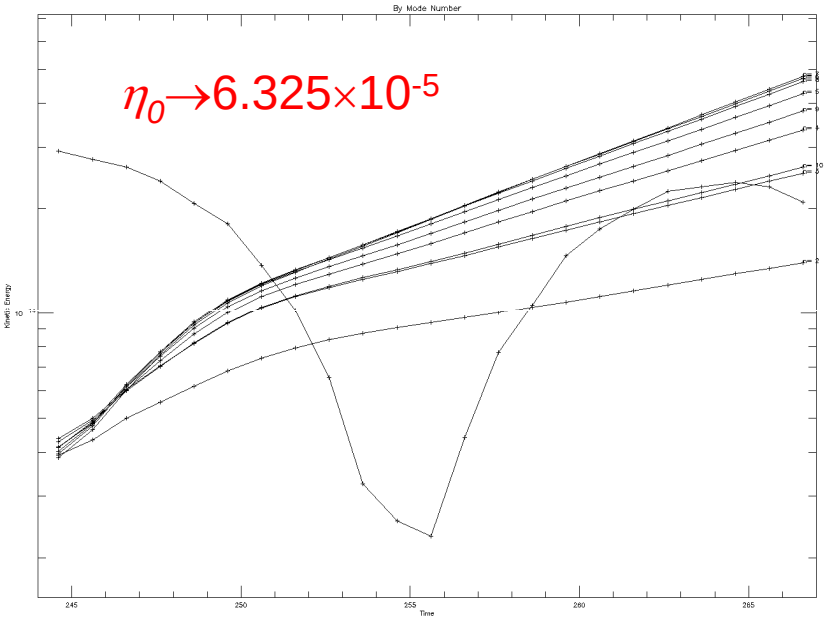
t=639.52

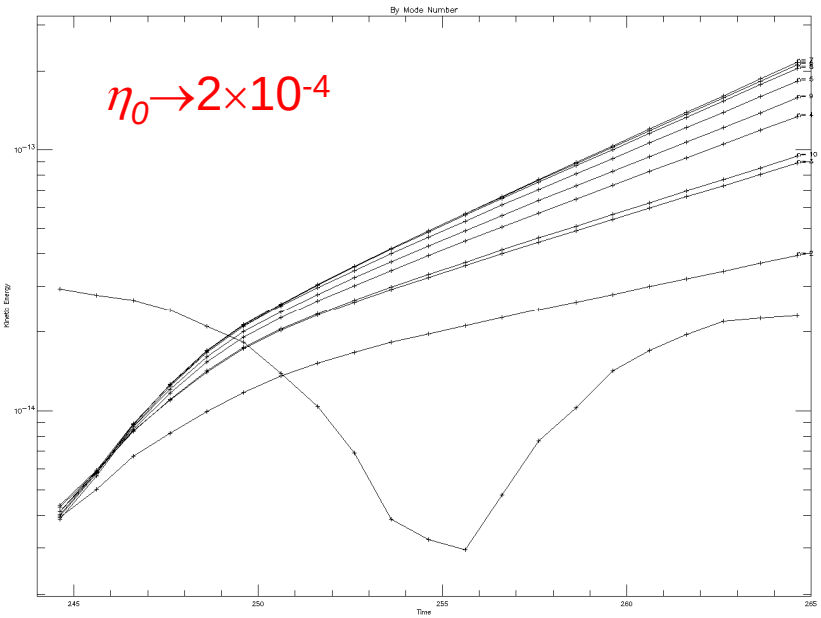


t=692.03



Resistivity Scaling





Growth Rates
(gcut=12)

n	$\gamma\tau_A$
1	——
2	0.0184
3	0.0270
4	0.0337
5	0.0387
6	0.0416
7	0.0425
8	0.0409
9	0.0363
10	0.0281

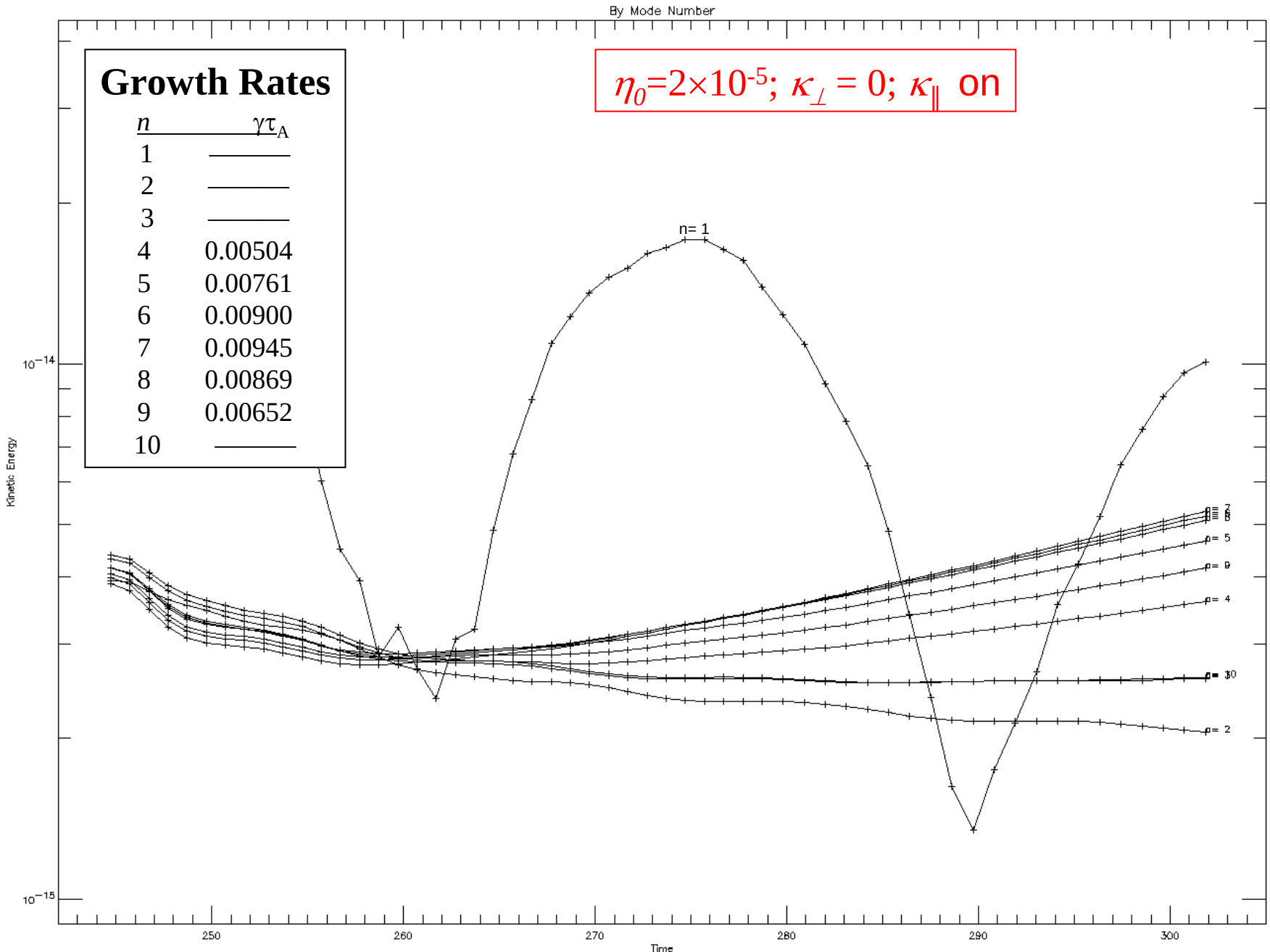
Growth Rates
(gcut=11)

n	$\gamma\tau_A$
1	——
2	0.0345
3	0.0498
4	0.0607
5	0.0682
6	0.0726
7	0.0738
8	0.0714
9	0.0646
10	0.0516

For each toroidal mode number *n*, the linear growth rate γ is found to be proportional to η^{α}:

n	α
2	0.597
3	0.590
4	0.568
5	0.553
6	0.543
7	0.542
8	0.546
9	0.560
10	0.586

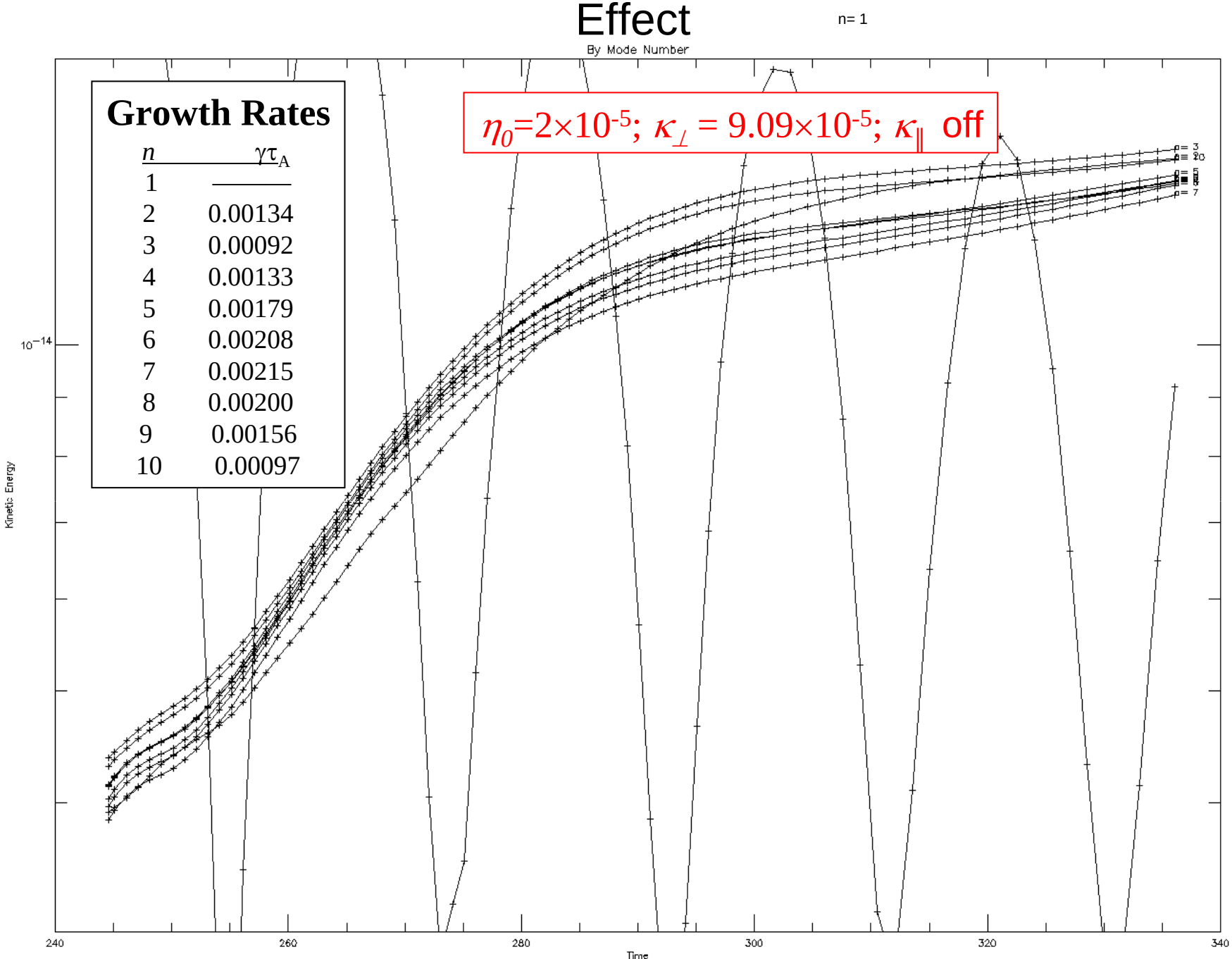
Parallel Heat Conduction Stabilizes Some Modes



Growth Rates

n	$\gamma\tau_A$
1	———
2	———
3	———
4	0.00504
5	0.00761
6	0.00900
7	0.00945
8	0.00869
9	0.00652
10	———

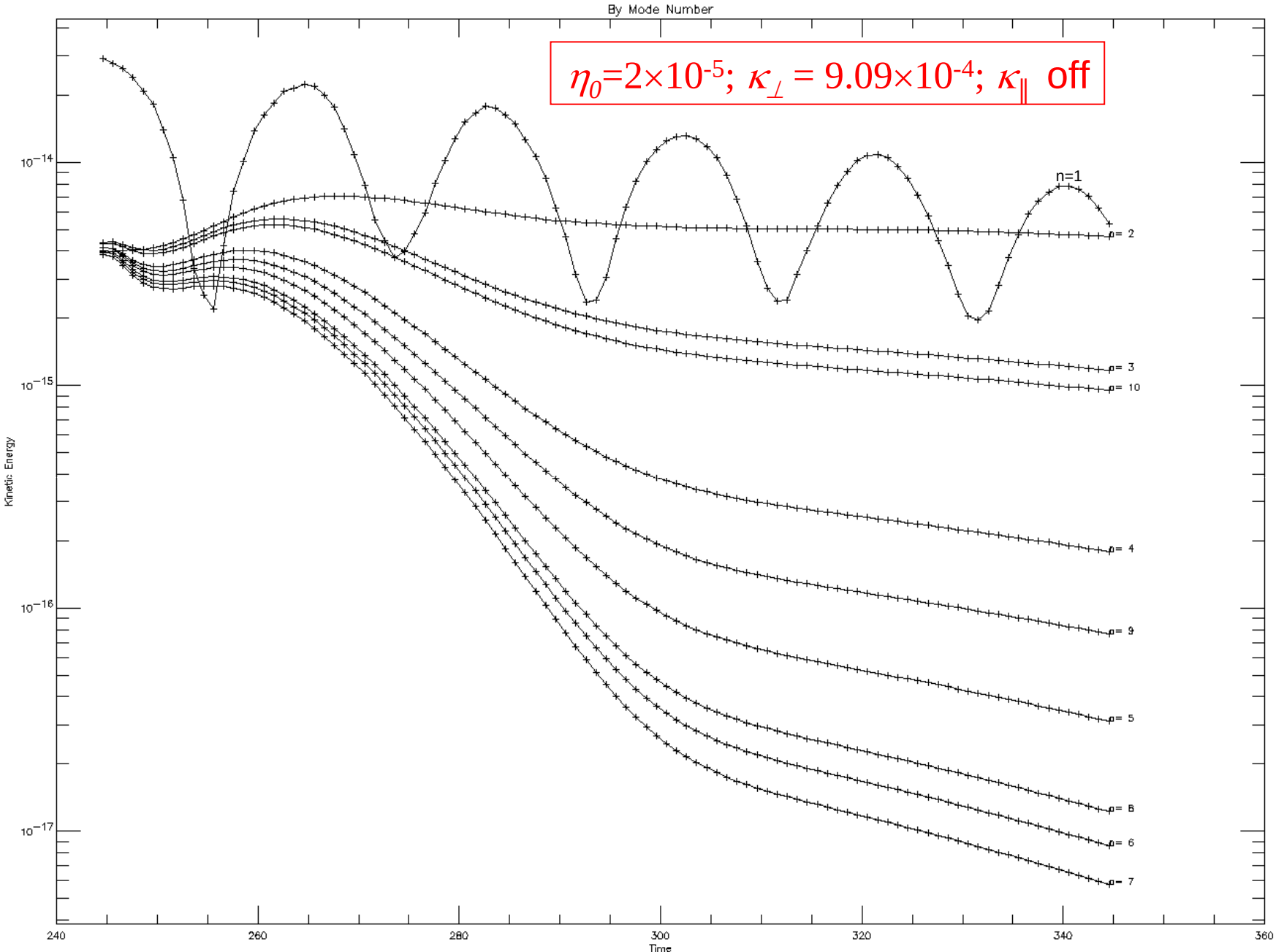
Moderate Isotropic Heat Conduction Has Stronger Stabilizing Effect



Growth Rates

n	$\gamma\tau_A$
1	———
2	0.00134
3	0.00092
4	0.00133
5	0.00179
6	0.00208
7	0.00215
8	0.00200
9	0.00156
10	0.00097

High Perpendicular Heat Conduction Stabilizes All Modes



Conclusions

- The cross-code benchmark was a success.

- The CDX equilibrium is MHD-unstable to resistive ballooning modes in the absence of large perpendicular thermal diffusivity.

- Extended MHD effects may be needed to account for the relative quiescence of the CDX edge.

or

- The modes may be present but saturated as a result of high transport levels arising from stochasticity caused by their nonlinear interaction.

Topics for Further Study

- Determine sensitivity of high-*n* modes to two-fluid parameters.
- Determine whether high-*n* modes saturate with parallel heat conduction.
- Re-run $q_0<1$ case with realistic heat conduction.