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Effects of Ion Landau Damping on Kinetic MHD Modes

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1. Motivation

Kinetic MHD (Extended MHD) model is very important to treat the MHD-like phenomena in the present day and future high temperature large tokamaks.

Linear stability and nonlinear evolution is very sensitive to the parameters (damping mechanisms).

Hence, the estimation of the effects of ion Landau damping is crucial for the analysis (or the prediction) of the MHD-like phenomena.

For example:

Existence of weakly unstable mode in the parameter range in which the theory predicts complete diamagnetic stabilization.

The effects of ion Landau damping is critical to the linear stabilization.

There is also a possibility that the ion Landau damping modifies the nonlinear development of the weekly unstable mode.

2. Moment Equations of Gyrokinetic Vlasov-Poisson-Ampere System

Gyrokinetic Poisson equation ($q_i = e$) :

$$\frac{\omega_{pi}^2}{\Omega_{ci}^2}\nabla_\perp^2\phi = \frac{e}{\varepsilon_0}\left(n_e - n_i\right)$$

Ampere's equation:

$$\nabla_\perp^2 A_z = -e\mu_0\left(\Gamma_e - \Gamma_i\right)$$

0-th order moment equation (s = e, i):

$$\frac{d}{dt}n_s + b^* \bullet \nabla\Gamma_s = 0$$

Unit vector of the magnetic field ($\mathbf{b}^* = \mathbf{B}/B_0$):

$$\mathbf{b}^* = \mathbf{b} + \frac{\nabla A_z \times \mathbf{b}}{B_0}$$

Convective derivative:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{\mathbf{b} \times \nabla A_z}{B_0} \bullet \nabla$$

First-order moment equations ($\Gamma_s = n_s V_s$):

$$\frac{d}{dt}\Gamma_s + b^* \bullet \left[\frac{p_s}{m_s} + nV_s^2 \right] = \frac{q_s}{m_s} n_s \left[b^* \bullet \left[\nabla \times \frac{\partial A_z}{\partial t} \right] \right]$$

Second-order moment equations:

$$\frac{d}{dt} p_s + 3 p_s \mathbf{b}^* \bullet \nabla V_s + V_s \mathbf{b}^* \bullet \nabla p_s = 0$$

By setting $p_s = n_s T_s$, we have the equations governing the temperature evolutions.

$$\frac{d}{dt} T_s + 2 T_s \mathbf{b}^* \bullet V_s + V_s \mathbf{b}^* \bullet T_s = 0$$

3. Gyro-Reduced-MHD Model

Time derivative of the gyrokinetic Poisson equation combined with conservation laws for electron and ion densities and Ampere's equation is the vortex equation.

Present model is an isothermal model.

Equations for the electron and ion densities are equivalent to the pressure equations.

(Some consideration will be needed in the future.)

Present model assumes that the ion temperature perpendicular to the magnetic field is zero. $T_{i\perp} = 0$

Ion Landau damping is modeled by the ion temperature equation including the linear damping term.

5-Field Model

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi = -\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla(\nabla_\perp^2\phi) - v_A^2\mathbf{b}^*\cdot\nabla(\nabla_\perp^2 A_z)$$

$$\frac{\partial}{\partial t}A_z = -\mathbf{b}^*\cdot\nabla\phi + d_e^2\frac{d}{dt}(\nabla_\perp^2 A_z) + \frac{T_{e0}}{n_{e0}e}\mathbf{b}^*\cdot\nabla n_e$$

$$\frac{\partial}{\partial t}n_e = -\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla n_e - \frac{1}{e\mu_0}\mathbf{b}^*\cdot\nabla(\nabla_\perp^2 A_z) - \mathbf{b}^*\cdot\nabla u_i$$

$$\frac{\partial}{\partial t}n_i = -\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla n_i - \mathbf{b}^*\cdot\nabla u_i$$

$$\frac{\partial}{\partial t}u_i = -\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla u_i - \frac{T_{e0}}{m_i}\mathbf{b}^*\cdot\nabla n_e - \frac{T_{i0}}{m_i}\mathbf{b}^*\cdot\nabla n_i$$

Isothermal Model: $T_e = T_i = \text{constant}$

W.W. Lee

3-Field Model

$$\frac{\partial}{\partial t}\nabla_{\perp}^{2}\phi = -\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla(\nabla_{\perp}^{2}\phi) - v_A^2\mathbf{b}^*\cdot\nabla(\nabla_{\perp}^{2}A_z)$$

$$\frac{\partial}{\partial t}A_z = -\mathbf{b}^*\cdot\nabla\phi + d_e^2\frac{d}{dt}(\nabla_{\perp}^{2}A_z) + \frac{T_{e0}}{n_{e0}e}\mathbf{b}^*\cdot\nabla n_e$$

$$\frac{\partial}{\partial t}n_e = -\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla n_e - \frac{1}{e\tau_0}\mathbf{b}^*\cdot\nabla(\nabla_{\perp}^{2}A_z)$$

We have assumed $\Gamma_i = 0$.

GRM3F-CY code
Kinetic FAR code (KFAR)
Yagi code

4-Field Model

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi = -\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla(\nabla_\perp^2\phi) - v_A^2\mathbf{b}^*\cdot\nabla(\nabla_\perp^2 A_z)$$

$$\frac{\partial}{\partial t}A_z = -\mathbf{b}^*\cdot\nabla\phi + d_e^2\frac{d}{dt}(\nabla_\perp^2 A_z) + \frac{T_e}{n_{e0}e}\mathbf{b}^*\cdot\nabla n_e$$

$$\frac{\partial}{\partial t}n_e = -\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla n_e - \frac{1}{e\mu_0}\mathbf{b}^*\cdot\nabla(\nabla_\perp^2 A_z) - \underline{\mathbf{b}^*\cdot\nabla\Omega_i}$$

$$\frac{\partial}{\partial t}\Omega_i = -\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla\Omega_i - \left(\frac{T_{e0}}{m_i} + \frac{T_{i0}}{m_i}\right)\mathbf{b}^*\cdot\nabla n_e + \frac{\mu_0 T_{i0}\omega_{pi}^2}{m_i\omega_{ci}^2}\mathbf{b}^*\cdot\nabla\left(\nabla_\perp^2\phi\right)$$

Model Including Ion Landau Damping

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi = -\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla(\nabla_\perp^2\phi) - v_A^2\mathbf{b}^*\cdot\nabla(\nabla_\perp^2 A_z)$$

$$\frac{\partial}{\partial t}A_z = -\mathbf{b}^*\cdot\nabla\phi + d_e^2\frac{d}{dt}(\nabla_\perp^2 A_z) + \frac{T_{e0}}{n_{e0}e}\mathbf{b}^*\cdot\nabla n_e$$

$$\frac{\partial}{\partial t}n_e = -\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla n_e - \frac{1}{e\mu_0}\mathbf{b}^*\cdot\nabla(\nabla_\perp^2 A_z) - \mathbf{b}^*\cdot\nabla\Omega_i$$

$$\frac{\partial}{\partial t}\Omega_i = -\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla\Omega_i - \left(\frac{T_{e0}}{m_i} + \frac{T_{i0}}{m_i}\right)\mathbf{b}^*\cdot\nabla n_e$$

$$+\frac{\mu_0 T_{i0}\omega_{pi}^2}{m_i\Omega_{ci}^2}\mathbf{b}^*\cdot\nabla\left(\nabla_\perp^2\phi\right) - \underline{\frac{n_{i0}}{m_i}\mathbf{b}^*\cdot\nabla\tilde{T}_i} \longleftarrow \text{added}$$

$$\frac{\partial}{\partial t}\tilde{T}_i = -\frac{b\times\nabla\phi}{B_0}\cdot\nabla\tilde{T}_i - T_{i0}b^*\cdot\nabla V_i - \underline{\frac{2^{3/2}}{\sqrt{\pi}}|k_{11}|v_{Ti}\tilde{T}_i}$$

ion Landau damping

$\tilde{T}_i$: perturbed ion temperature

Normalization

$$r = a\,\tilde{r}, \quad z = 2\pi R\,\tilde{z}, \quad t = \frac{2\pi R}{v_A}\,\tilde{t}$$

$$A_z = \frac{a^2 B_0}{2\pi R}\tilde{A}_z, \quad \phi = \frac{v_A a^2 B_0}{2\pi R}\tilde{\phi}, \quad n_e = \frac{d_i n_0}{2\pi R}\tilde{n}_e, \quad \phi_i = \frac{v_A d_i n_0}{2\pi R}\tilde{\phi}_i$$

$$v_A^2 = c^2 \frac{\omega_{pi}^2}{\omega_{ci}^2}, \quad d_e = \frac{c}{\omega_{pe}}, \quad d_i = \frac{c}{\omega_{pi}}$$

$$\rho_s^2 = \frac{T_e}{m_i \omega_{ci}^2}, \quad \rho_e = \frac{\rho_s^2}{d_i^2}, \quad \rho_i = \frac{\rho_i^2}{d_i^2} = \frac{T_{i0}}{T_{e0}}\rho_e$$

Normalized Euations

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi = -\mathbf{b}\times\nabla\phi\cdot\nabla(\nabla_\perp^2\phi) - \mathbf{b}^*\cdot\nabla(\nabla_\perp^2 A_z)$$

$$\frac{\partial}{\partial t}A_z = -\mathbf{b}^*\cdot\nabla\phi + \left(\frac{d_e}{a}\right)^2\frac{d}{dt}(\nabla_\perp^2 A_z) + \left(\frac{\rho_s}{a}\right)^2\mathbf{b}^*\cdot\nabla n_e$$

$$\frac{\partial}{\partial t}n_e = -\mathbf{b}\times\nabla\phi\cdot\nabla n_e - \mathbf{b}^*\cdot\nabla(\nabla_\perp^2 A_z) - \mathbf{b}^*\cdot\nabla\Omega_i$$

$$\frac{\partial}{\partial t}\Omega_i = -\mathbf{b}\times\nabla\phi\cdot\nabla\Omega_i - (\Omega_e + \Omega_i)\,\mathbf{b}^*\cdot\nabla n_e$$

$$+\Omega_i\,\mathbf{b}^*\cdot\nabla\left(\nabla_\perp^2\phi\right) - \underline{n_{i0}\Omega_i\,\mathbf{b}^*\cdot\nabla\tilde{T}_i}$$

added

$$\frac{\partial}{\partial t}\tilde{T}_i = -\mathbf{b}\times\nabla\phi\cdot\nabla\tilde{T}_i - \frac{1}{n_{i0}}b^*\cdot\nabla\Omega_i - \underline{\frac{2^{3/2}}{\sqrt{\pi}}|k_{11}|\sqrt{\Omega_i}\,\tilde{T}_i}$$

ion Landau damping

$$n_{e0} = n_{i0} = n_0 = 2\pi R/d_i$$

4. Simulation Results

Safety factor profile :

$$q(r) = q_0 \left[1 - 4\,(1 - q_0) \left(\frac{r}{a} \right)^2 \right]^{-1}$$

$$q_0 = 0.85, \qquad q(0.5a) = 1.0, \qquad q(a) = 2.125$$

Equilibrium density profile:

$$n(r) = n_0 \left[1 - \Delta_n \tanh \frac{r - r_0}{l_n} \right]$$

$$r_0/a = 0.5, \qquad l_n/a = 0.16$$

Key Parameters:

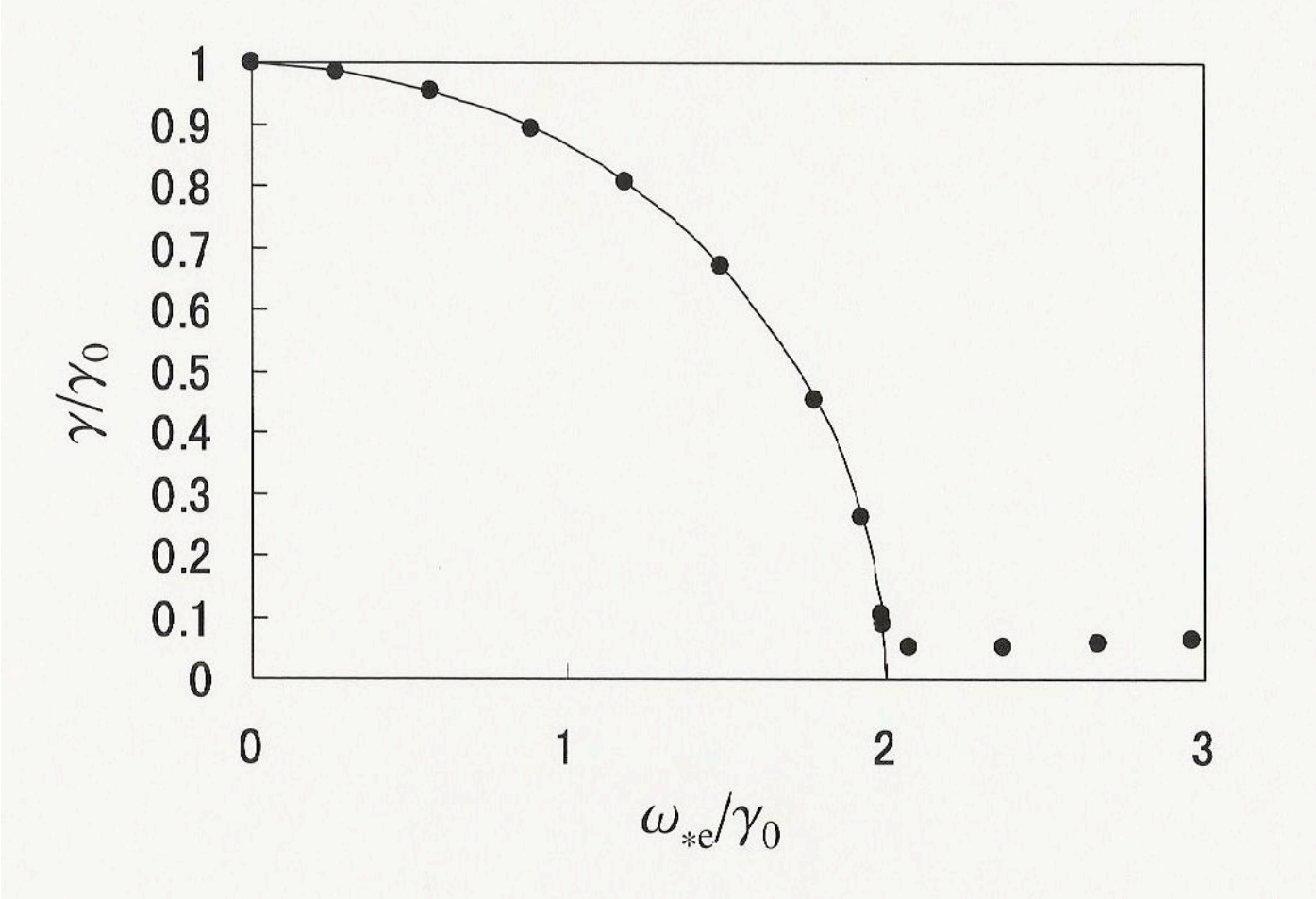
$$d_e / a, \qquad \rho_s / a, \qquad \Delta_n, \qquad \beta_e, \qquad T_{i0}/T_{e0}$$

important for the ion Landau damping

Assumption : Single Helicity

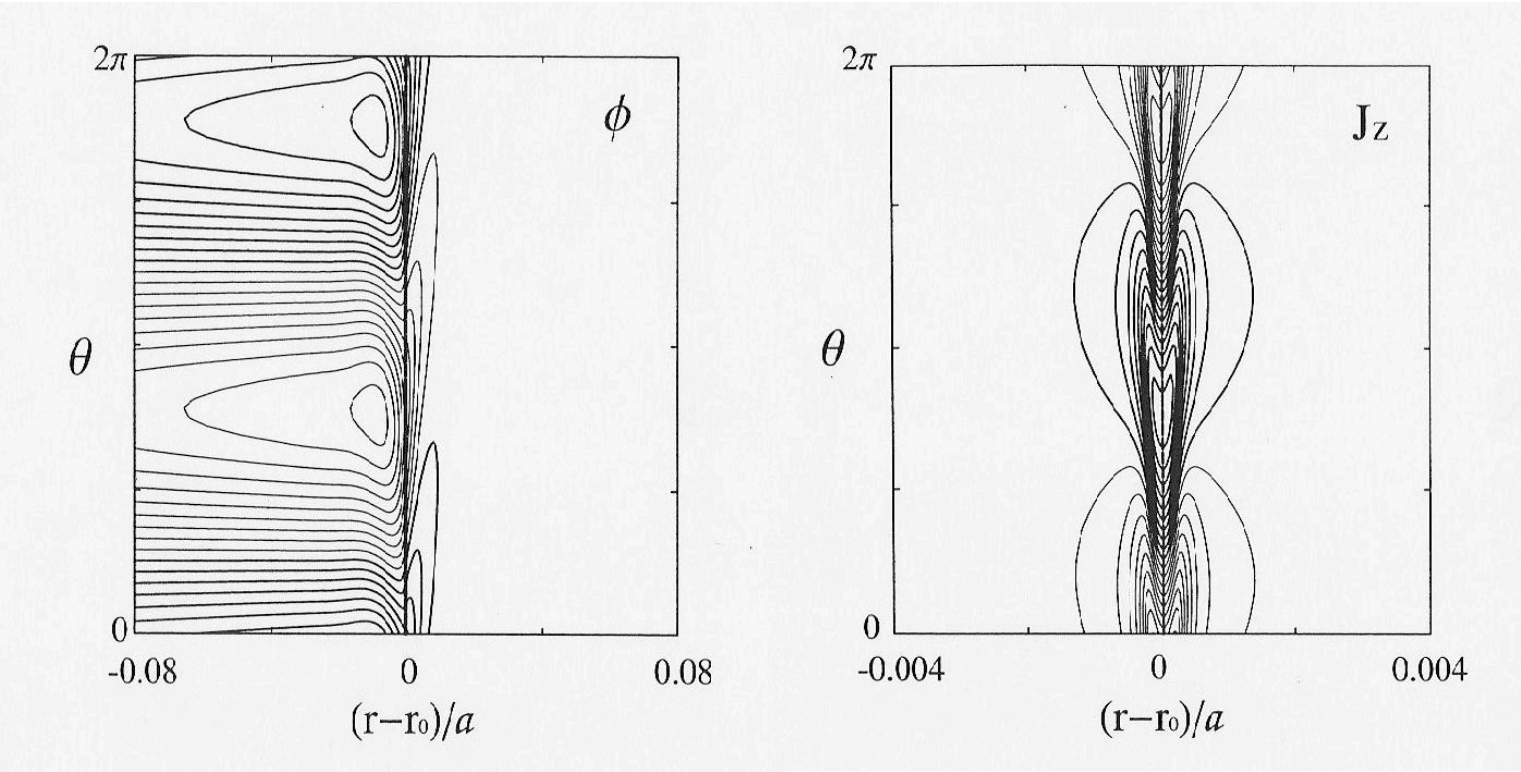
$$m / n = 1$$

Electron Diamagnetic Stabilization of Kinetic Internal Kink Mode



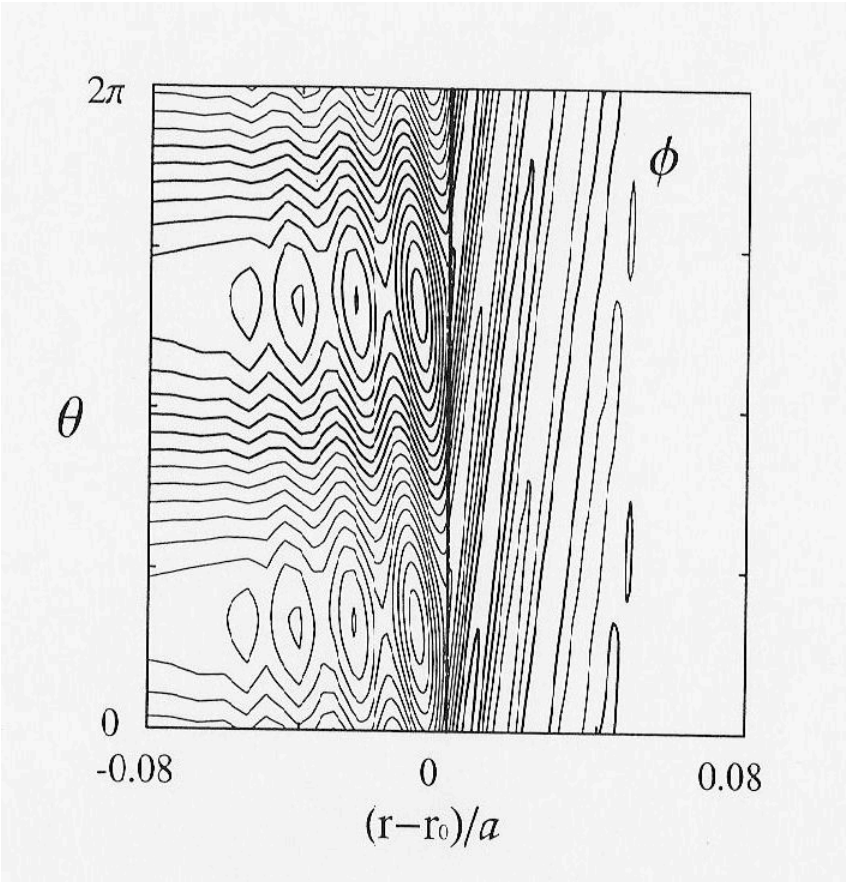
$d_e/a = 0.0005315$, $\rho_s/a = 0.002891$, $1/\gamma_0 = 417\,\mu sec$

Mode Structure in r-θ



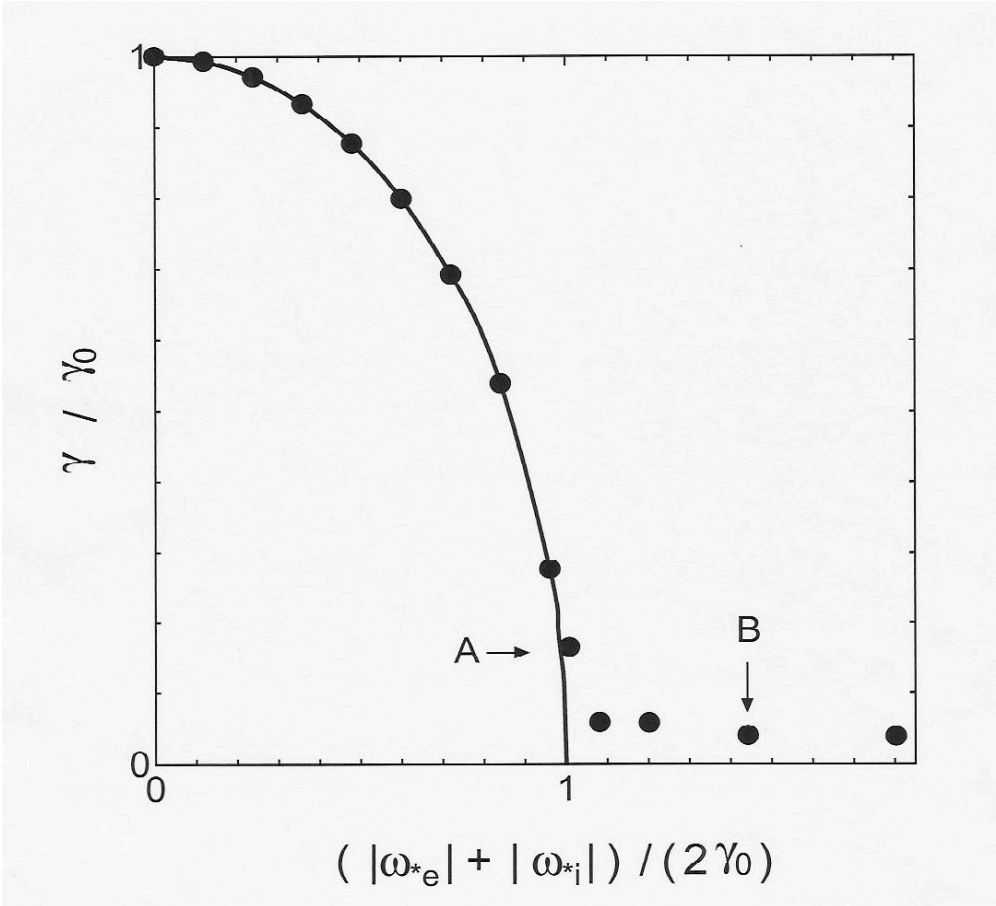
ω_{*e}/ω_0 = 1.48 (theoretically unstable)

Mode Structure in r-θ



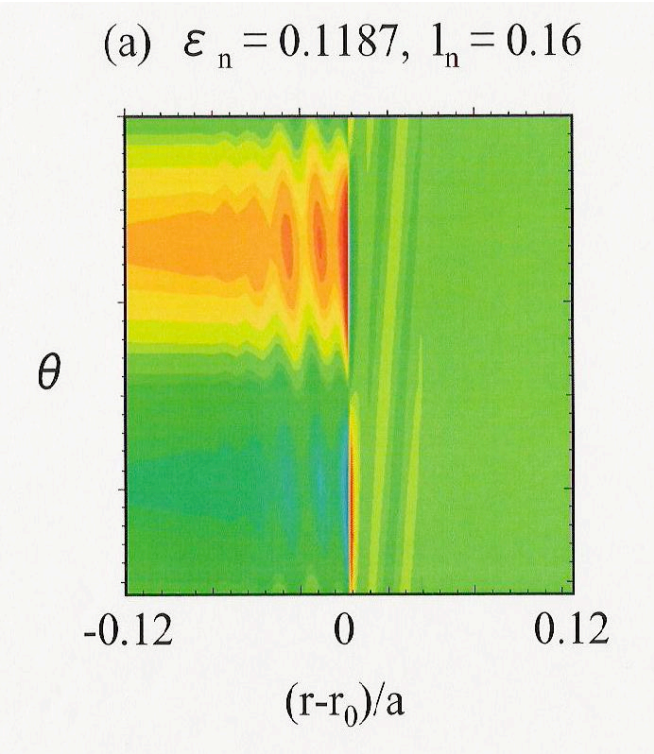
ω_{*e}/ω_0 = 1.98 (theoretically close to marginal point)

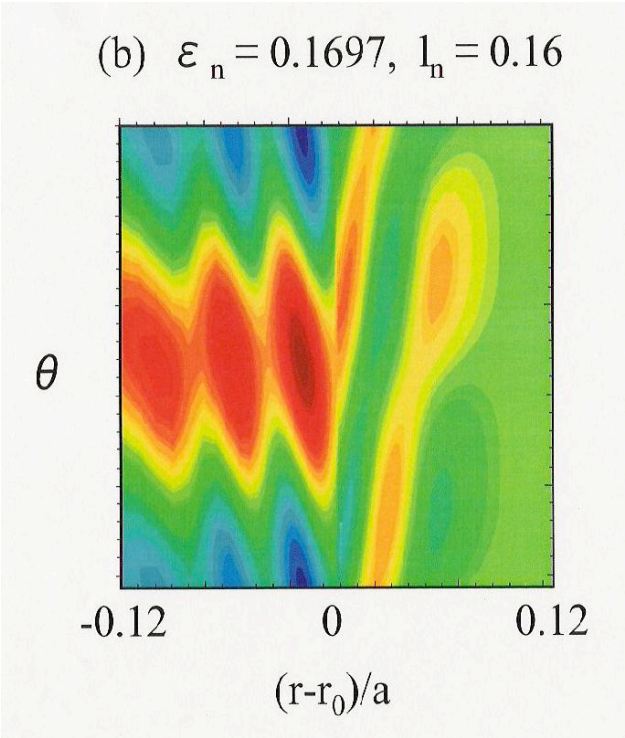
Electron and Ion Diamagnetic Effects



$d_e/a = 0.0005315$, $\rho_s/a = 0.002891$, $T_i/T_e = 1.0$, $1/\gamma_0 = 340\,\mu sec$

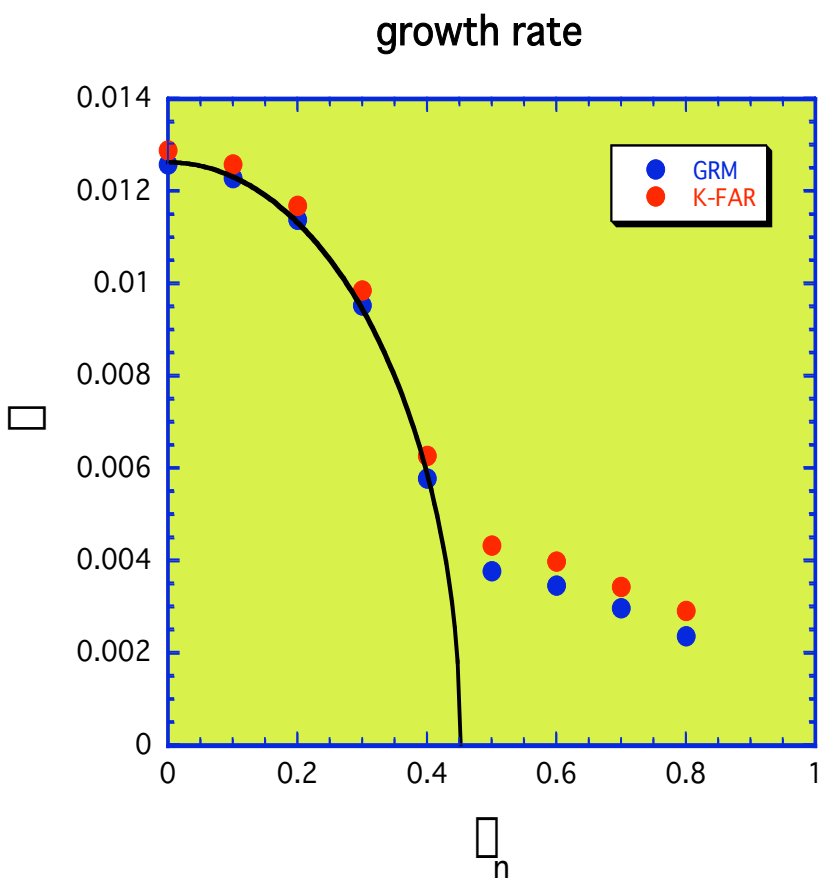
Mode Structure in *r*-θ

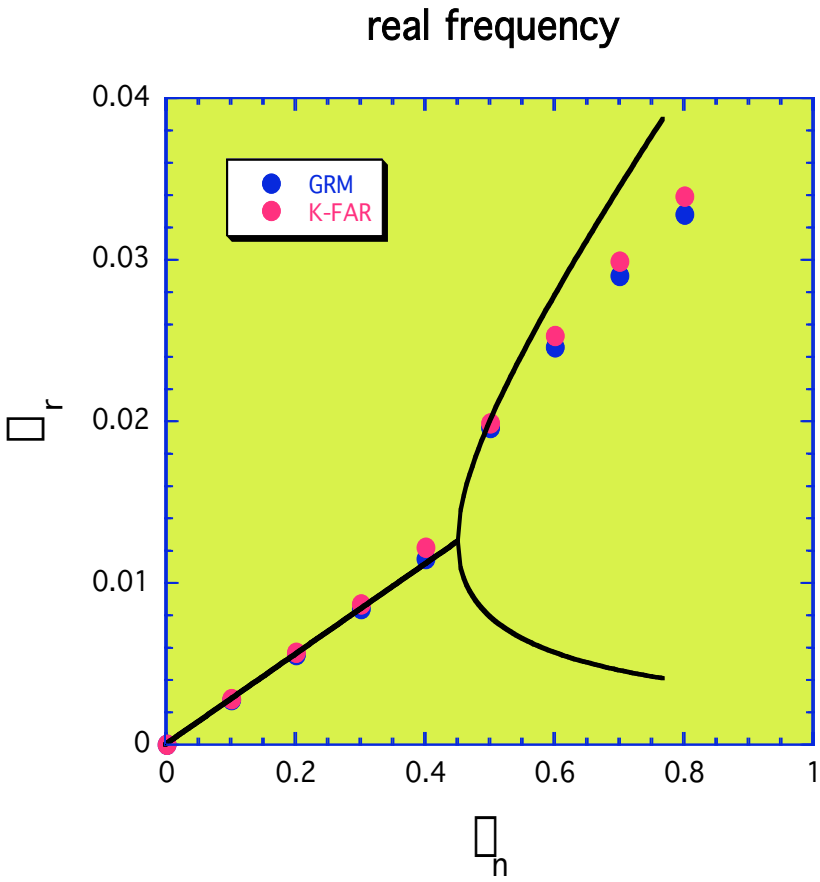




A B

Comparison between GRM and K-FAR





cylindrical model $d_e = 0.01, \quad \Box_s = 0.03$

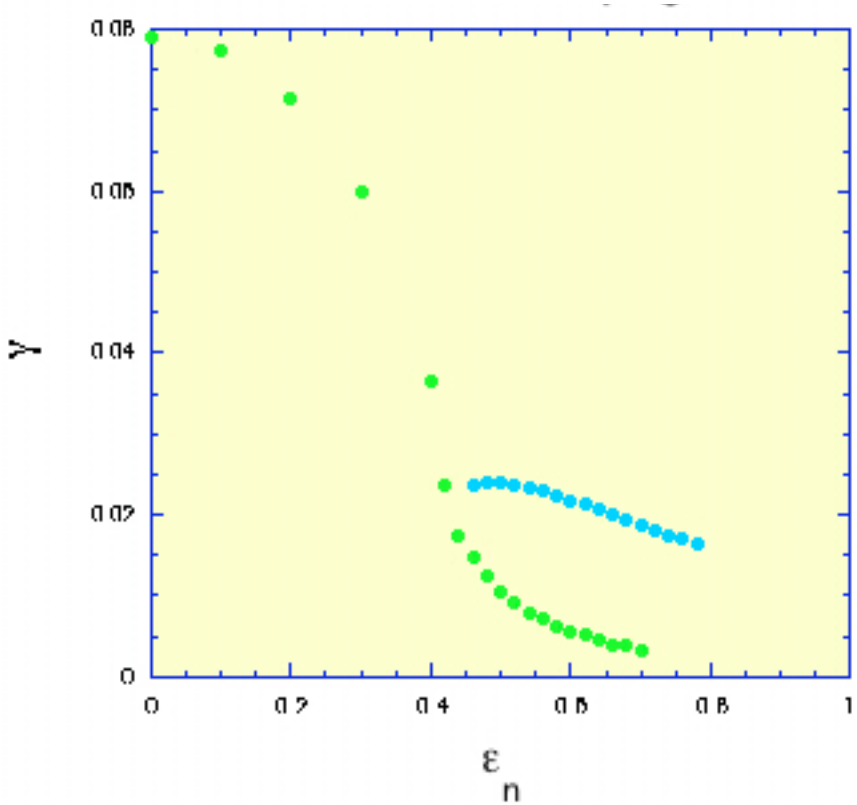
Yagi Code

preliminary

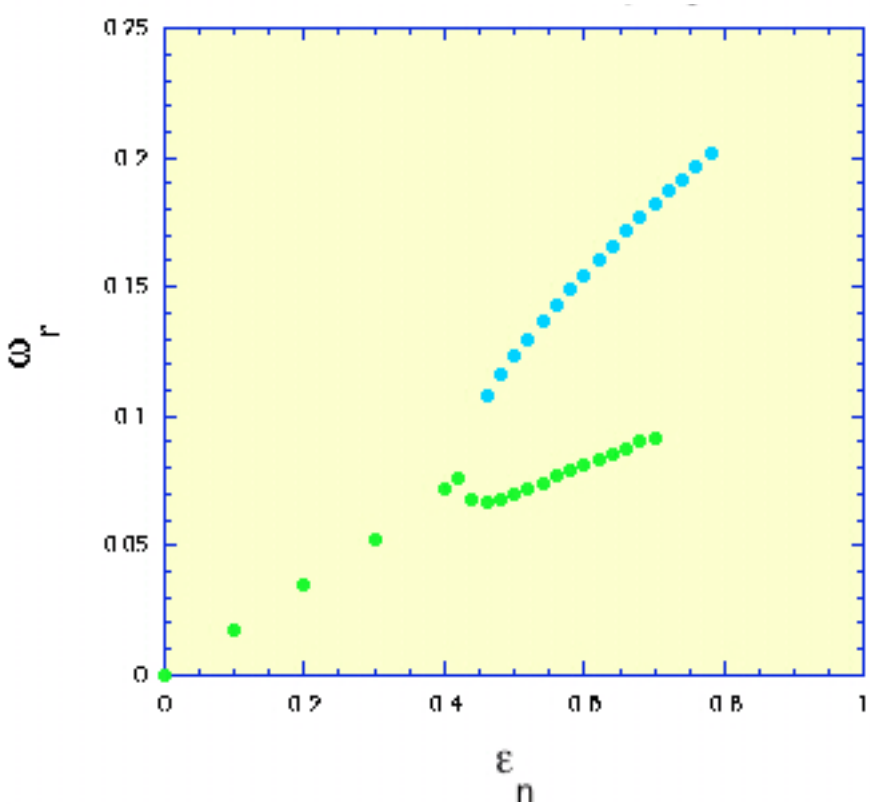
$d_e = 0.01, \quad \square_s = 0.03$

without ion Landau damping

growth rate



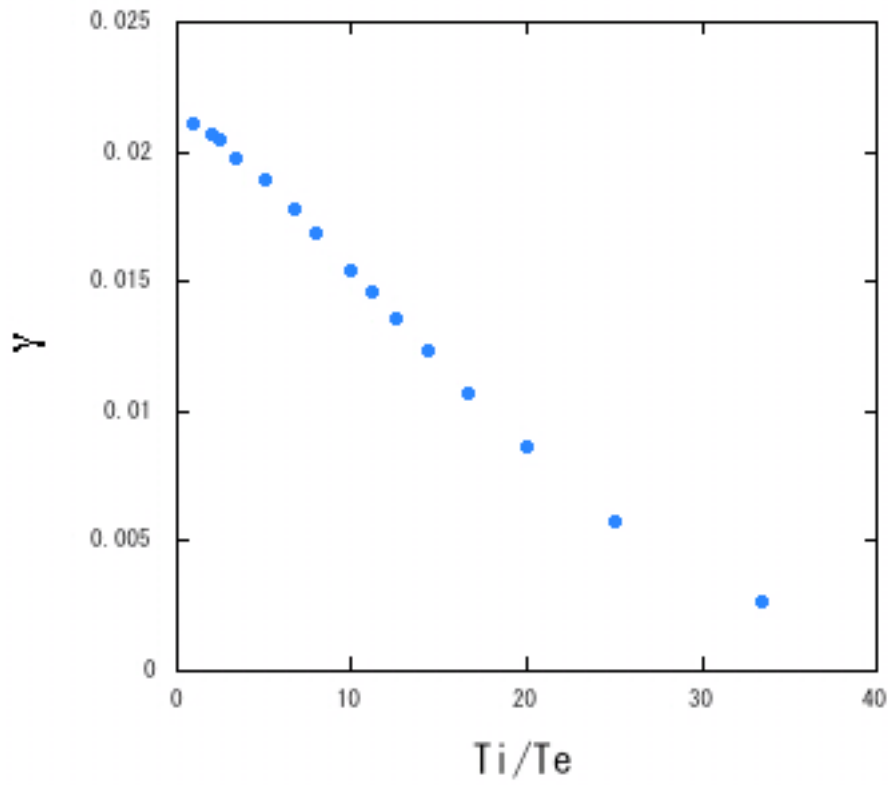
real frequency



Stabilization by ion Landau damping

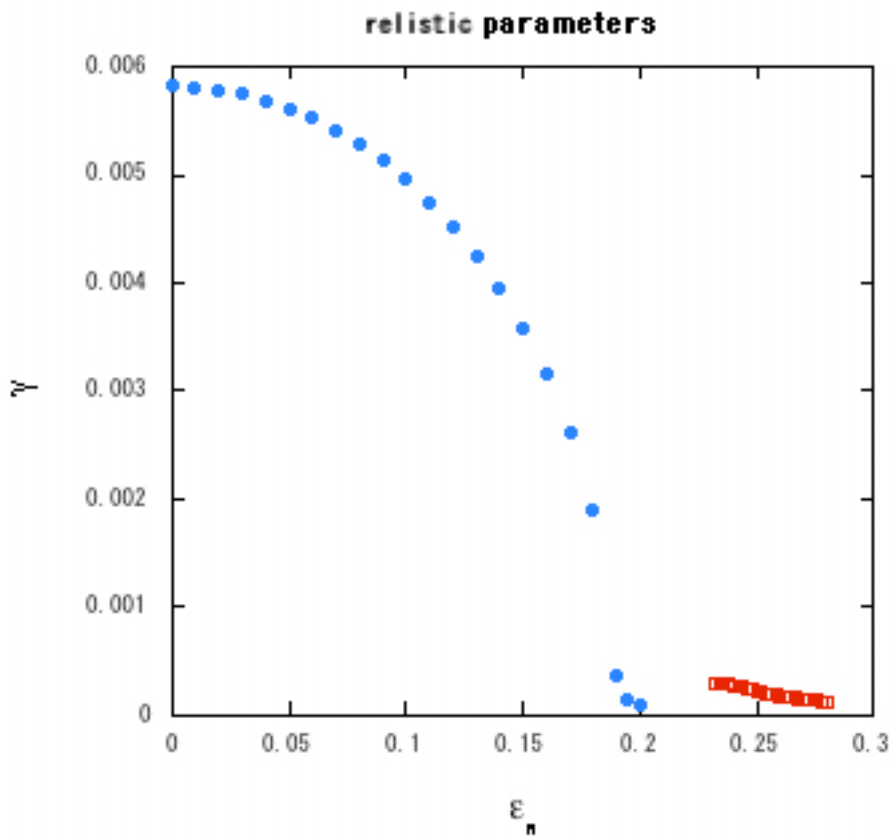
preliminary

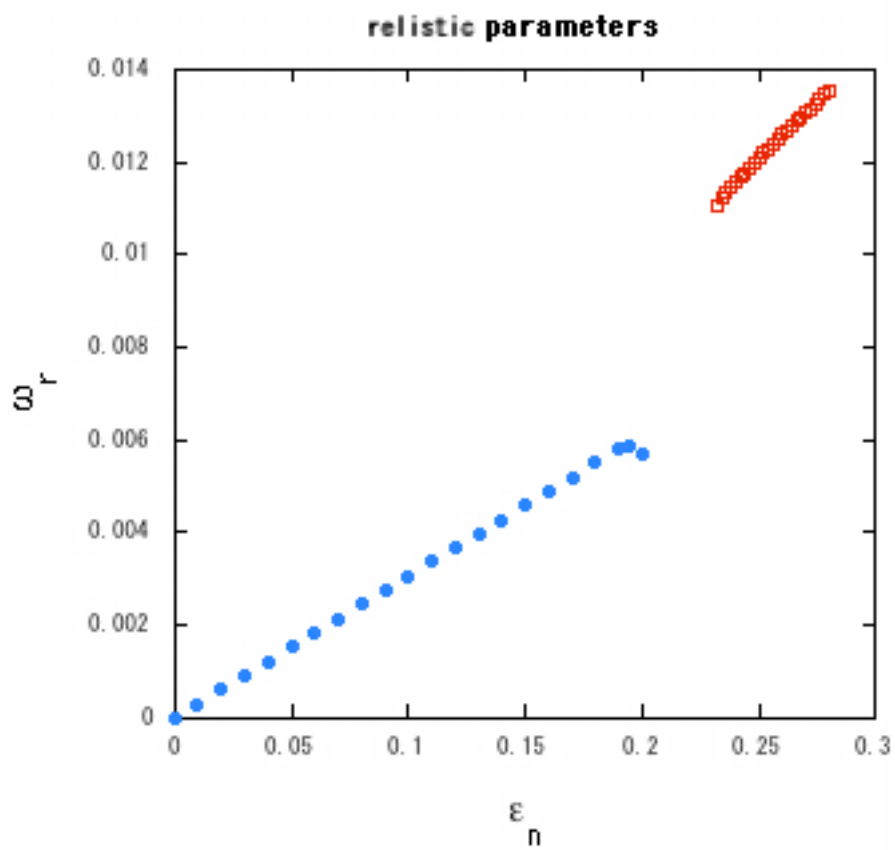
$$d_e = 0.01, \quad \square_s = 0.03, \quad \square_h = 0.6$$



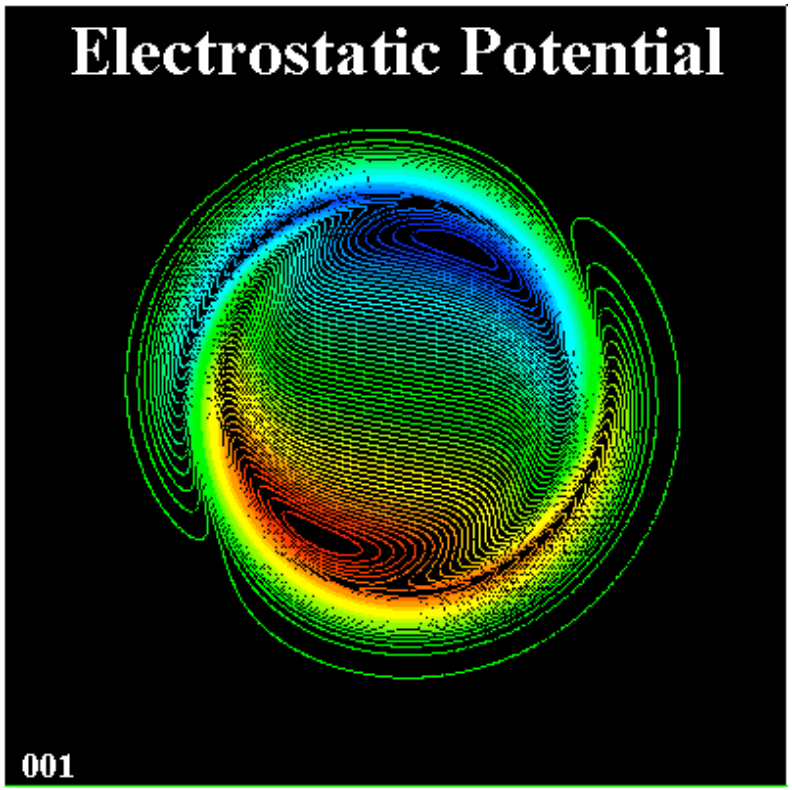
Yagi code

preliminary

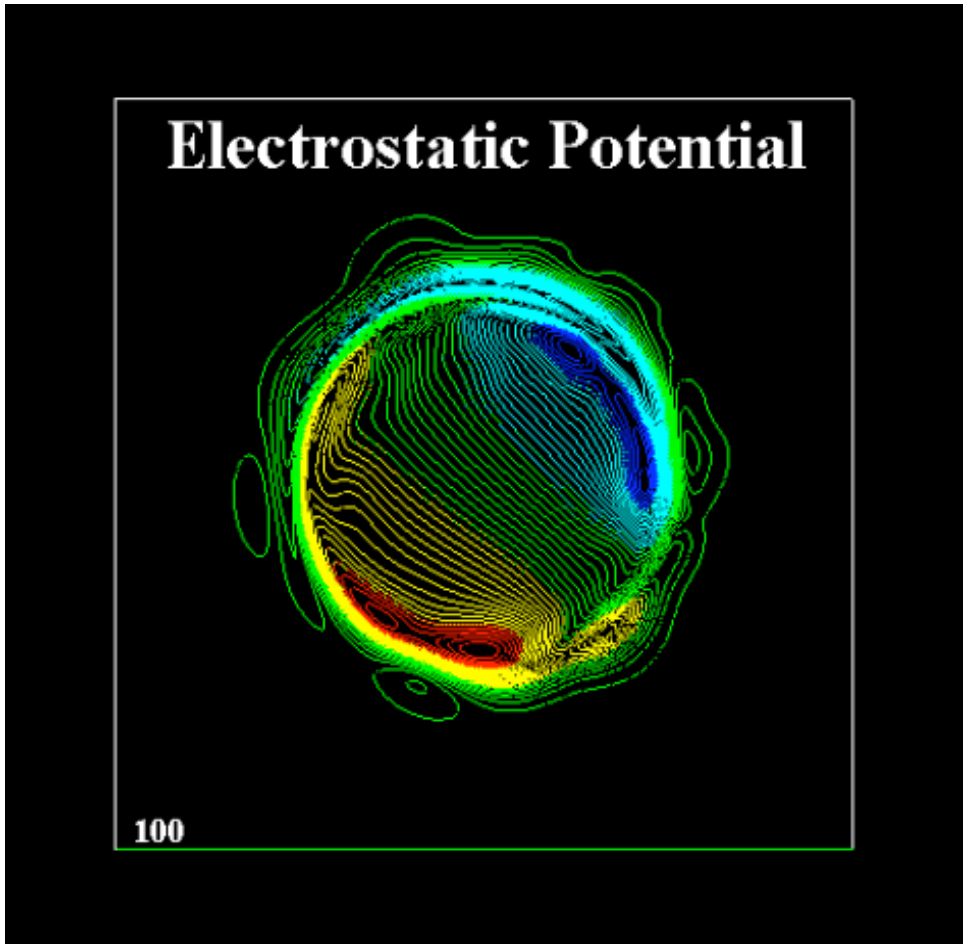




Nonlinear Calculation



Linear Mode Pattern



Nonlinear Evolution (Movie)

Vortex Generation due to Kelvin-Helmholtz like instability

Summary

The effects of the ion Landau damping on the kinetic internal kink mode is verified.

Further study of the linear stability will be needed using the realistic parameters of large tokamaks.

Inclusion of the finite ion temperature perpendicular to the magnetic field is necessary.

The ion Landau damping may modifies the nonlinear evolution of the MHD-like modes.

It is no doubt that the nonlinear toroidal study is urgent.