MHD equilibrium and Transport analyses in heliotron configurations

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- MHD equilibrium with stochastic magnetic field
  - vacuum magnetic field
  - finite β magnetic field
- Transport analyses
  - formulation for 3D (differences between 2D and 3D)
- Discussions

Motivation

- **Background**
  theoretically interesting points in LHD experiments and/or requirements from experimentalists

  1. to clarify roles of peripheral plasma in a stochastic magnetic field region
     How does such a stochastic region influence on the MHD equilibrium and stability?
  2. to clarify roles of the net plasma current
     Is the neoclassical current consistent with MSE measurements?
  3. to extend transport analyses from point (0D) to 1D model
     How can we make 1D transport model for 3D equilibria?

     $\Downarrow$

     (a) grading up the reconstruction of MHD equilibrium
     (b) comparison between 3D plasma and 2D plasma

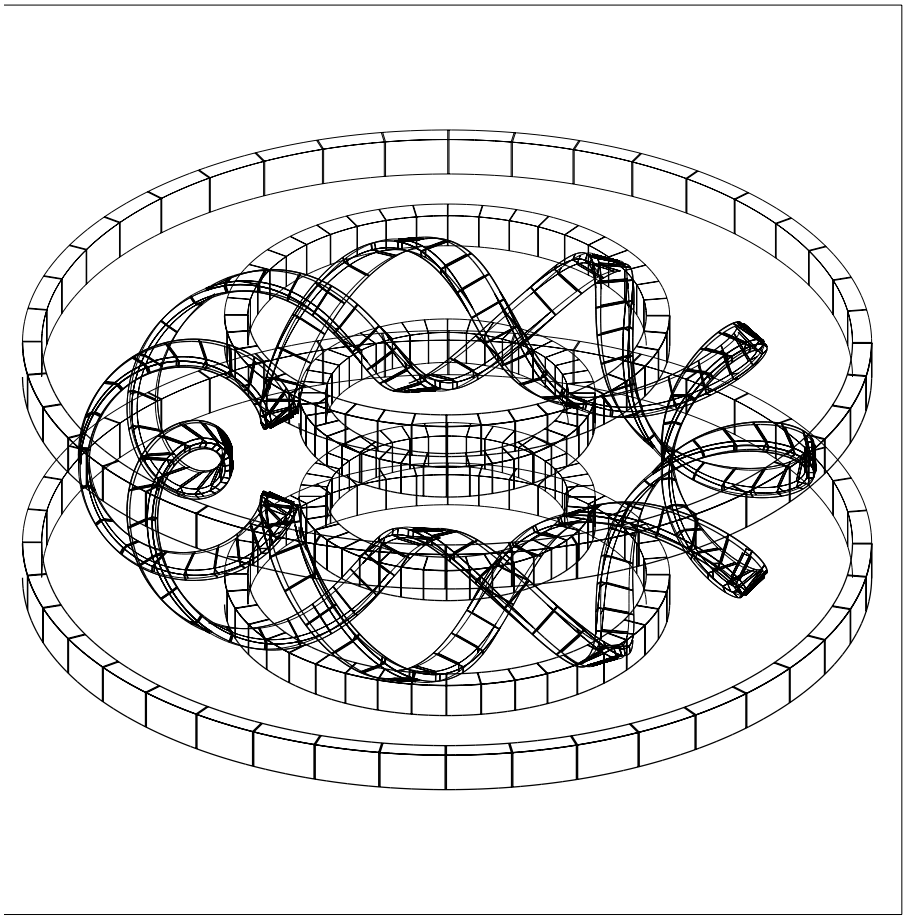
- **Purposes** (in the present talk)

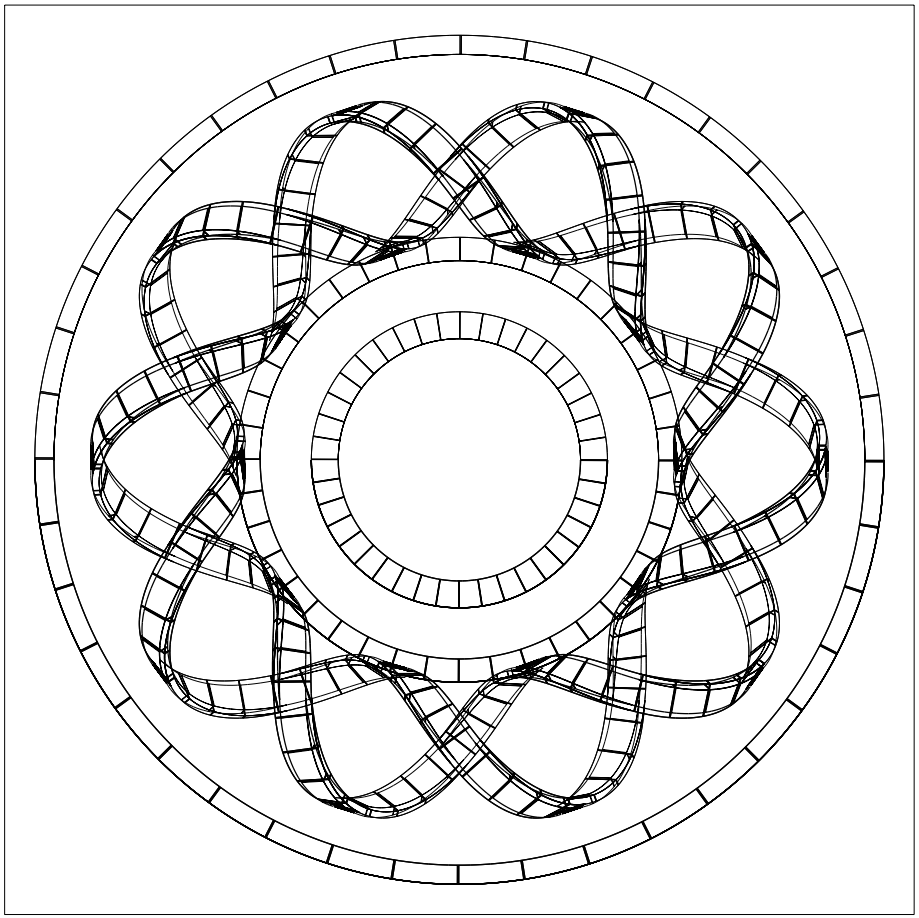
  1. to show the present situations
  2. to give short answers and conjecture

MHD equilibrium with stochastic magnetic field

1. **LHD**

a heliotron configuration with 10 field periods in the toroidal direction and planar plasma axis



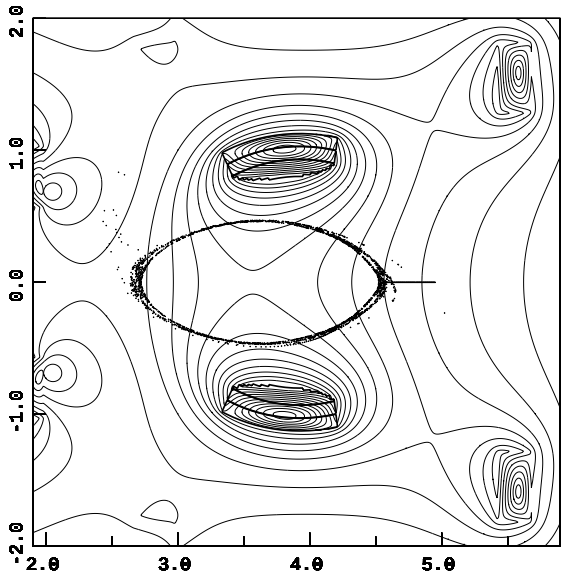


MHD equilibrium with stochastic magnetic field

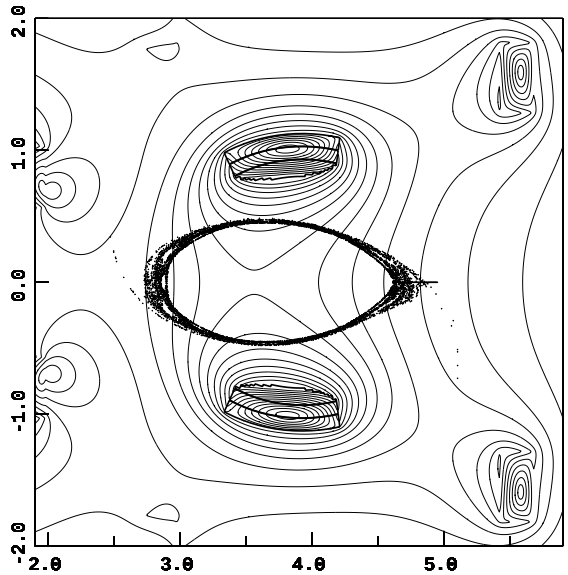
1. **peripheral vacuum magnetic field**

1.1 **field line tracing (Poincare plots)**

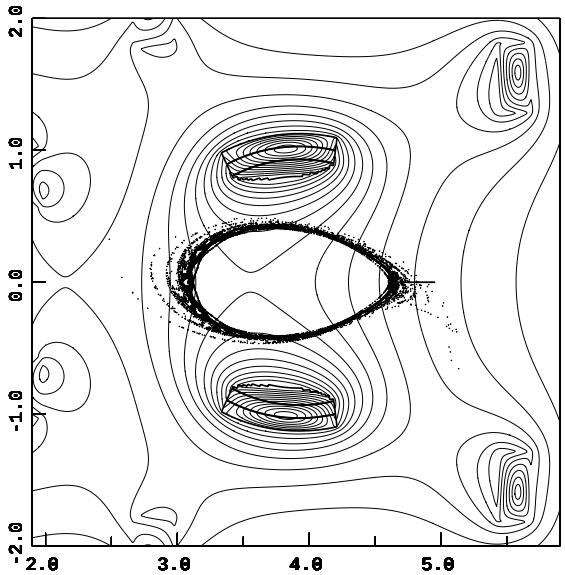
Inward-shifted ($R_{va} = 3.60$(m))



standard ($R_{va} = 3.75$(m))



outward-shifted ($R_{va} = 3.90$(m))



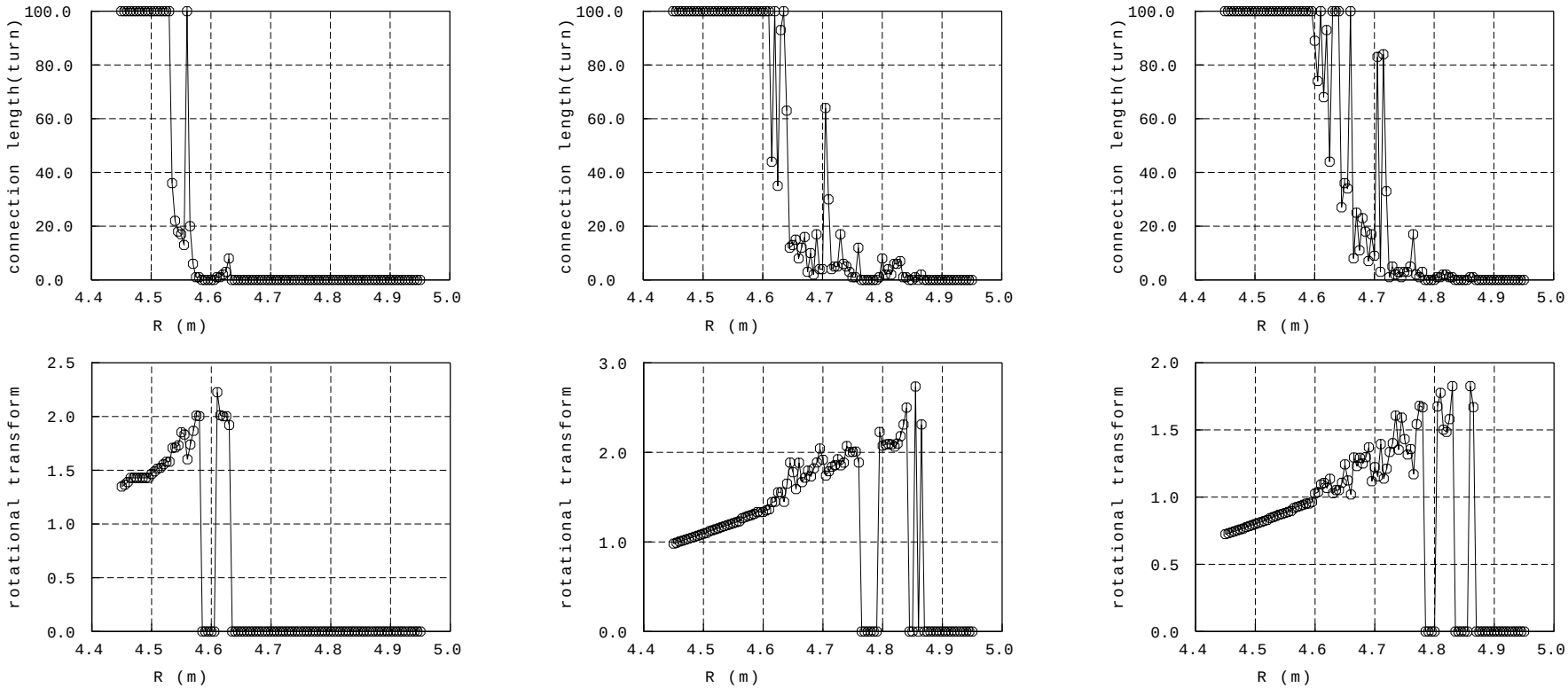
	Inward-shifted	standard	outward-shifted
width of SMF	**thin**	↙	**thick**
plasma volume	**large**	↖	**small**

MHD equilibrium with stochastic magnetic field

1. **peripheral vacuum magnetic field**

1.2 **connection length** L_c & ι $\quad (R_{wall} = 1.8(\text{m}), \quad 1 \text{ toroidal turn} \sim 25(\text{m}))$

Inward-shifted **standard** **outward-shifted**



	Inward-shifted	standard	outward-shifted
ι_{LCFS}	$\sim 1.5 = \frac{30}{20}$ **(high)** ↖	$\sim 1.33 = \frac{20}{15}$ ↖	$\sim 1.0 = \frac{10}{10}$ **(low)**
	LCFS's are related to rational surfaces		
decrease of L_c	**sharp**	←	**gradual**

MHD equilibrium with stochastic magnetic field

1. **peripheral vacuum magnetic field**

1.3 **properties**

	Inward-shifted	standard	outward-shifted
width of SMF	**thin**	↙	**thick**
level of stocasticity	**weak**	↙	**strong**
ι_{LCFS}	$\sim 1.5 = \frac{30}{20}$ **(high)** ↖	$\sim 1.33 = \frac{20}{15}$ ↖	$\sim 1.0 = \frac{10}{10}$ **(low)**
	LCFS's are related to rational surfaces		
decrease of L_c	**sharp**	←	**gradual**

2. **Experimental results-1**

2.1 **density and ∇P**

	Inward-shifted	standard	outward-shifted
n_e-increase by gas puff	**easy**	⇐	**difficult**
interpretation (by S.Morita)	**Screening effects**		
reachable β_{dia} (by S.Sakakibara)	$> 3.5\%$	$\sim 1.5\%$	$\sim 0.2\%$
∇P exists (by T.Morisaki)	**beyond LCFS of vacuum up to a position with a fairly long L_c**		

MHD equilibrium with stochastic magnetic field

2. Experimental results-2

2.2 signals of magnetic probes (by S.Sakakibara)

inward-shifted	(m,n)				
$\langle\beta\rangle_{dia} \leq 1.0\%$		$(1,1)$	$(2,3)$		
$1.0\% \leq \langle\beta\rangle_{dia} \leq 2.0\%$	$(2,1)$	$(1,1)$	$(2,3)$		
$2.0\% \leq \langle\beta\rangle_{dia} \leq 2.5\%$	$(2,1)$	$(1,1)$	$(2,3)$		
$2.5\% \leq \langle\beta\rangle_{dia} \leq 3.0\%$		$(1,1)$	$(2,3)$		
$3.0\% \leq \langle\beta\rangle_{dia} \leq 3.5\%$		$(1,1)$	$(2,3)$	$(1,2)$	$(2,5)$
$3.5\% \leq \langle\beta\rangle_{dia}$			$(2,3)$	$(1,2)$	$(2,5)$

standard	(m,n)	
$\langle\beta\rangle_{dia} \leq 1.5\%$	$(1,1)$	$(3,4)$

- **Rational surfaces of observed signals correspond to LCFS, Isolated Stiff Magnetic Islands (ISMI), or a position with fairly long L_c.**
- **High performance plasmas are obtained in inward-shifted configurations**, where **theoretical prediction of MHD stability is worse than other configurations.**

In order to resolve this discrepancy, let consider finite β plasma in inward-shifted configurations.

MHD equilibrium with stochastic magnetic field

3. finite β plasma

3.1 Method

To understand finite β MHD equilibrium, **HINT2** (new version of HINT) is under developed (original HINT was developed by T.Hayashi)

- fortran 90
- transparent structure (module structure)
- simple control of the situation from input parameters
- user friendly (User can easily modify the code structure.)
- open source (in this fiscal year)

1. a **relaxation method** is used.
   originally developed by "W. Park, D. A. Monticello, H. Strauss, and J. Manickam, Phys. Fluids $\mathbf{29}$, 1171 (1986)
2. parallel and perpendicular force balances to $\boldsymbol{B}$ are solved **separately and iterative way**.
3. obtained MHD equilibrium is a **weak solution**.

MHD equilibrium with stochastic magnetic field

3. finite β plasma

3.1 Method (parallel force balance ($\boldsymbol{B}\cdot\nabla P = 0$) for fixed $\boldsymbol{B}$)

$$\left.\begin{aligned}\frac{\partial P}{\partial t} &= \boldsymbol{B}\cdot\nabla v_s\\ \frac{\partial v_s}{\partial t} &= \boldsymbol{B}\cdot\nabla P\end{aligned}\right\} \Rightarrow P_{new} = \frac{\int^{\phi} P_{old}\dfrac{d\phi}{\boldsymbol{B}\cdot\nabla\phi}}{\int^{\phi}\dfrac{d\phi}{\boldsymbol{B}\cdot\nabla\phi}}$$

Introduction of the transport process :

1. $P = 0$ for short connection length with $\phi < 2\pi$.
2. Effective perpendicular transport with respect to averaged flux surfaces.

$$\frac{\int^{\phi} P_{old}\dfrac{d\phi}{\boldsymbol{B}\cdot\nabla\phi}}{\int^{\phi}\dfrac{d\phi}{\boldsymbol{B}\cdot\nabla\phi}} = \begin{cases}\text{for nested flux surface} \Rightarrow P_{new} = \langle P_{old}\rangle\\ \text{for } \boldsymbol{B} = \langle\boldsymbol{B}\rangle + \delta\boldsymbol{B} \quad \Rightarrow \chi_{\perp} = \dfrac{2\delta k_{||}}{\pi\overline{k}_r^2}\begin{cases}\mathcal{R}^2 & \text{for } \mathcal{R}\le 1\\ \mathcal{R} & \text{for } \mathcal{R}\ge 1\end{cases}\end{cases}$$

$$\mathcal{R} = \left[\frac{\pi}{8}\frac{L_{||}\overline{k}_r^2}{\delta k_{||}}\sum_m\left\langle\left(\frac{\delta B_{rmk_{||}}}{B}\right)^2\right\rangle_{k_{||}}\right]^{1/2}$$

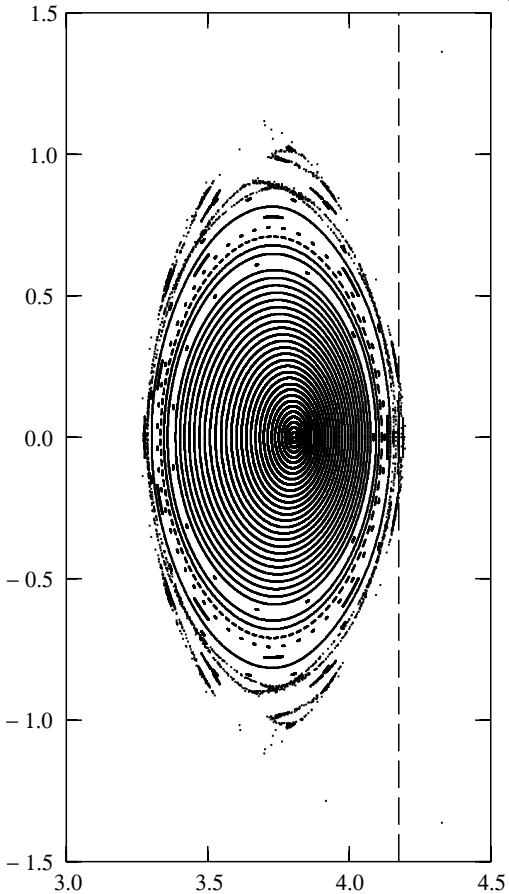
Ref. US/Japan-WS in last year

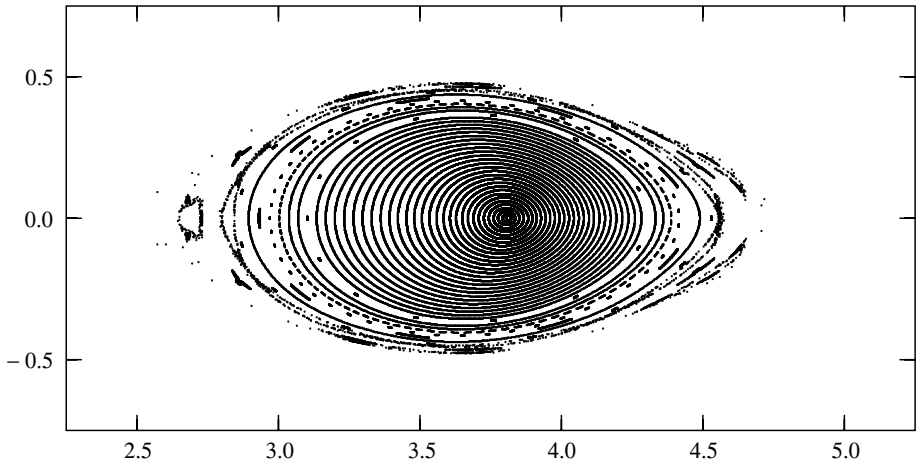
MHD equilibrium with stochastic magnetic field

3. finite β plasma (inward-shifted configuration)

3.2 low-β case

Poincare plots $\langle\beta\rangle = 1.4\%$





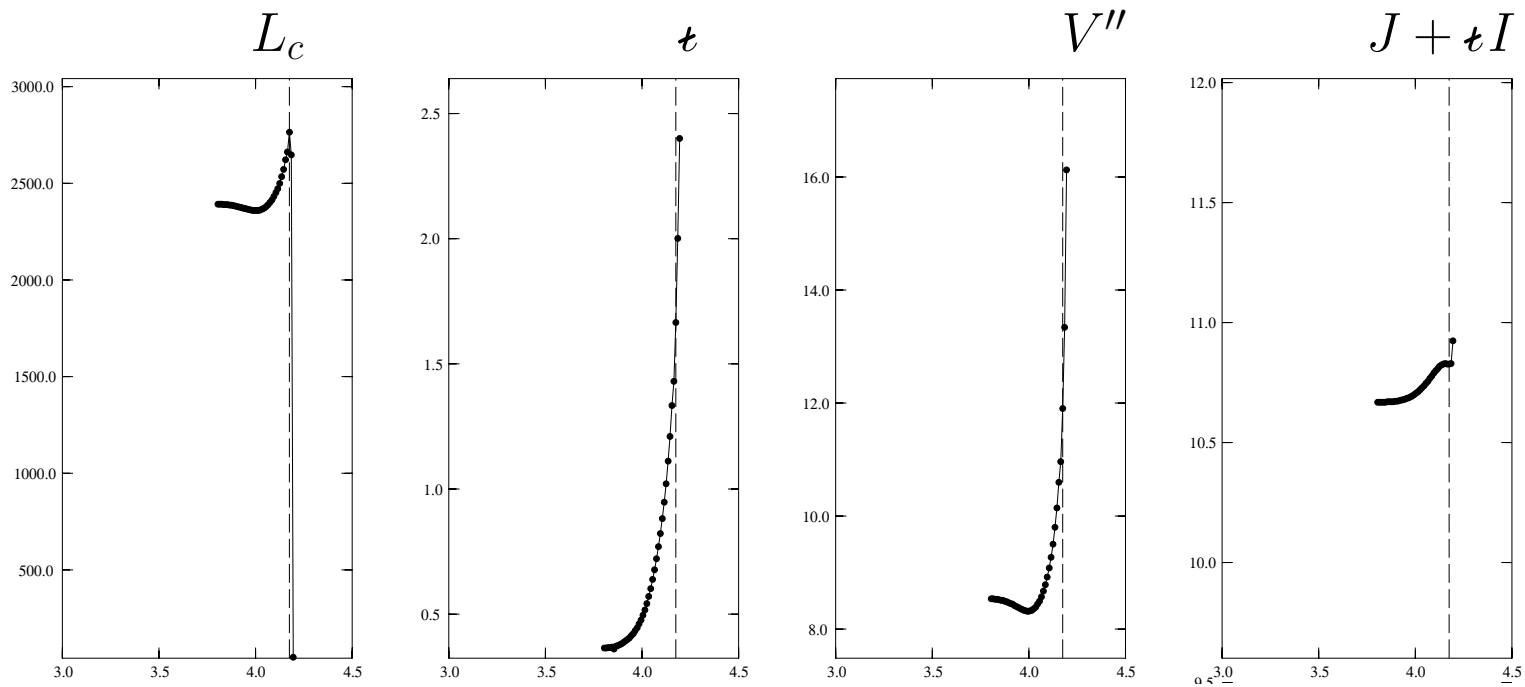
- **Width** of peripheral magnetic islands becomes **wide**, as β increases.

MHD equilibrium with stochastic magnetic field

3. finite β plasma (inward-shifted configuration)

3.2 low-β case

L_c and surface quantities $\langle\beta\rangle = 1.4\%$



- **Sharp decrease of L_c, observed in vacuum, still exists around $\iota \sim 1.5 \sim 2.0$.**
- **Magnetic hill region still exists near the plasma boundary as well as vacuum.**

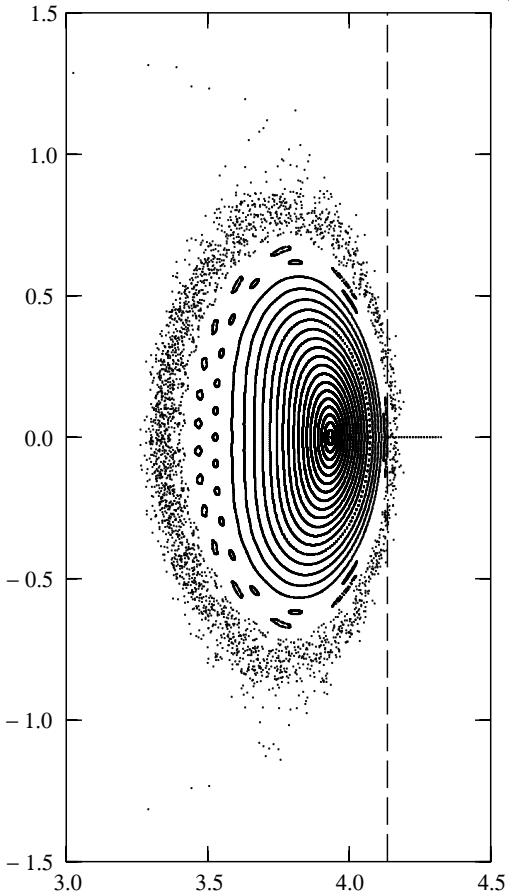
Note : Poloidal cross section with starting points of the field lines is different between vacuum and finite β.

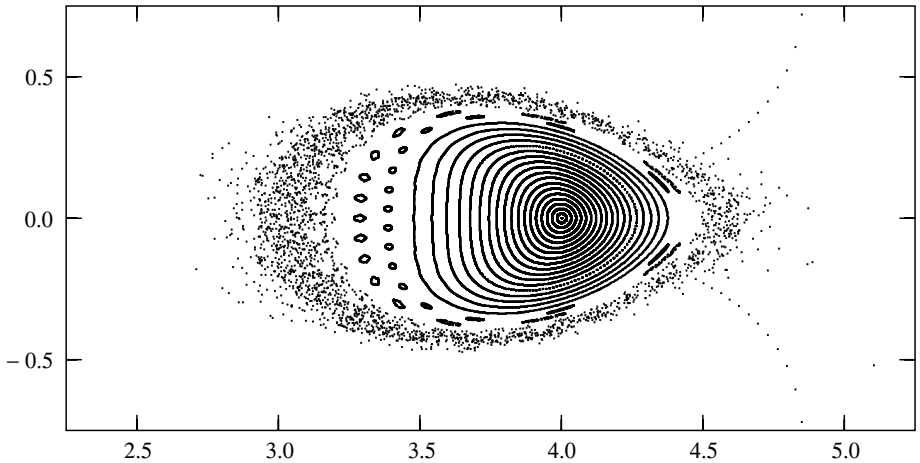
MHD equilibrium with stochastic magnetic field

3. finite β plasma (inward-shifted configuration)

3.3 high-β case

Poincare plots $\langle\beta\rangle = 3.7\%$





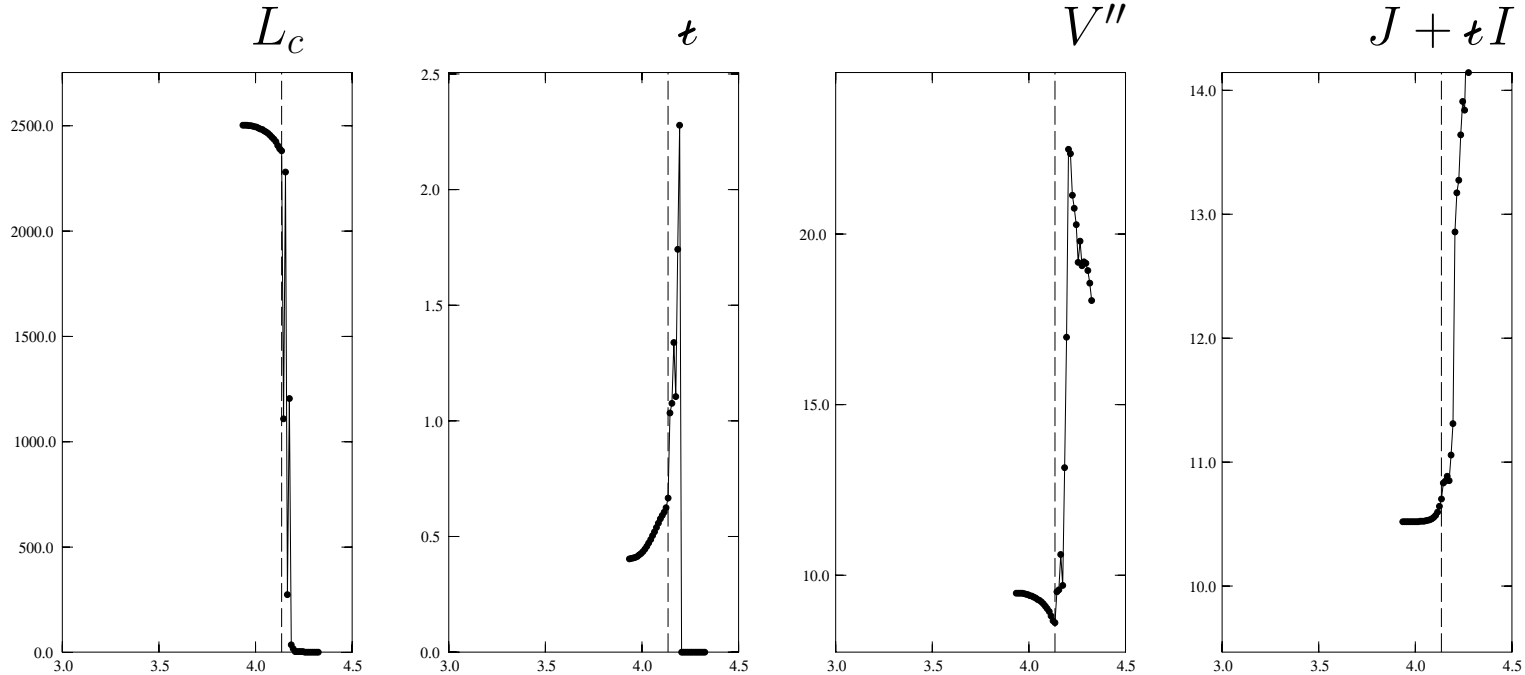
- **Stochastic field is created near the plasma periphery through island-overlapping.**

MHD equilibrium with stochastic magnetic field

3. finite β plasma (inward-shifted configuration)

3.3 high-β case

L_c and surface quantities $\langle\beta\rangle = 3.7\%$



- **A large Shafranov shift occurs not only plasma axis but also plasma periphery.**
- **Sharp decrease of L_c around $\iota \sim 1.0 \sim 2.5$ still remains with fine structures.**
- **Magnetic hill region is limited to very near the plasma boundary, by the free boundary motion due to the Shafranov shift.**

MHD equilibrium with stochastic magnetic field

3 finte β plasma

3.4 properties of finite β plasmas

In heliotron configurations with both **long connection lengths near LCFS** and **a large Shafranov shift**,

1. **LCFS in vacuum does not act as a plasma boundary** in finite β plasma.
2. In such cases, **field lines near Isolated Stiff Magnetic Islands (ISMI) or structures with a fairly long L_c may play a role of the "effective" plasma boundary.**
3. **Magnetic field between LCFS and ISMI may be fairly regular, namely, averaged flux surfaces exist**, especially when the distance between them is enough short.
4. **Stochastic region extends into plasma core region (not outer region)**, as β increases.
5. **Sharp decrease of the connection length L_c occurs at a similar position to one in vacuum field.**

6. **Properties of magnetic field (connection length etc.) just outside of vacuum LCFS are similar to one in vacuum field.**

$$\Downarrow$$

- **Finite β effects modify the magnetic field structure interior of the plasma, but not so modify the exterior structure, even if the low pressure plasma expands in the exterior region.**

Properties of MHD equilibrium in heliotorn configurations

3.5 Conjecture

1. **Fairly low plasma pressure expands from around LCFS to around ISMI in vacuum field**, as β increases.
2. **The free boundary plasma motion (boundary modulation)** related to such an expansion **may make a significant stabilizing effects in the plasma core region** through the magnetic well formation.
3. As a results, **MHD instabilities resonating near plasma boundary are mainly excited.**

- **Above speculations are partially supported as follows:**

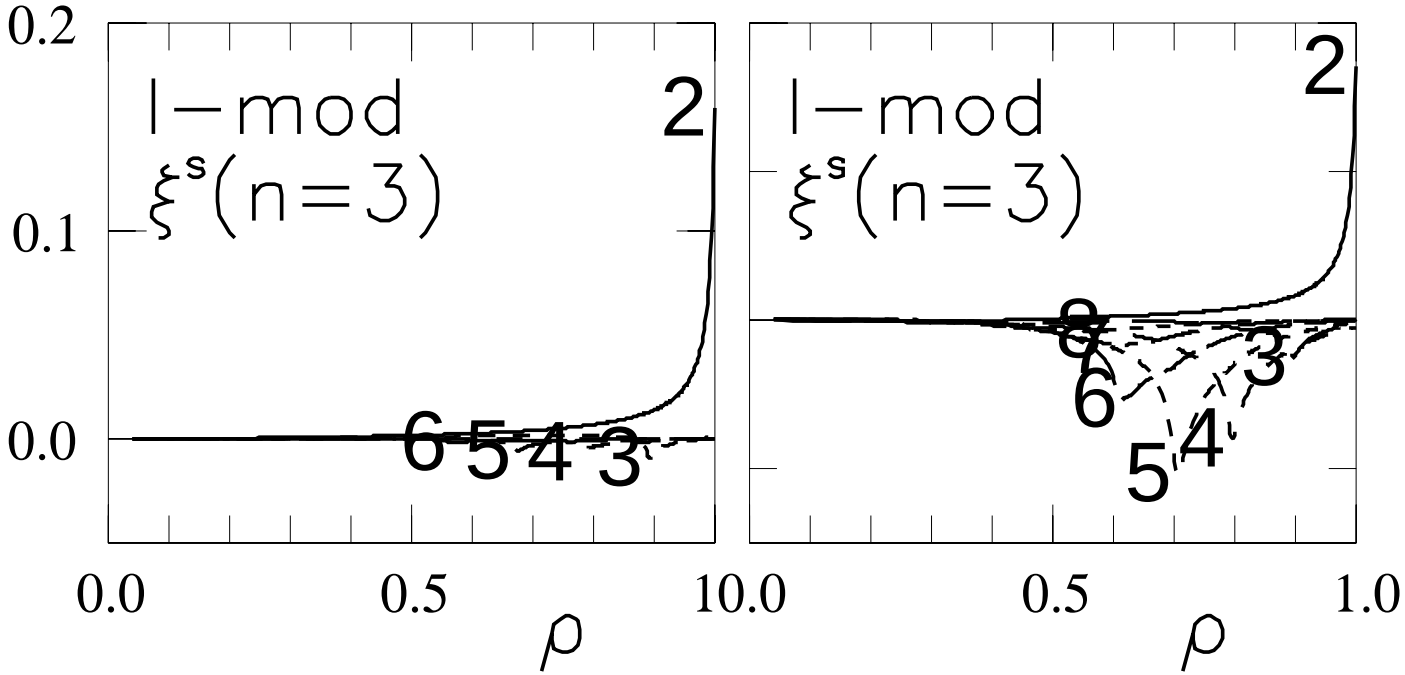
MHD stability

4.1 Excitation of external components observed in experiments

By simulating the free boundary plasma motion by the boundary modulation of vacuum LCFS (eliminating bumpy components), it is shown that external modes resonating near the plasma boundary are excited.

$\langle\beta\rangle = 2\%$ $\qquad$ $\langle\beta\rangle = 3\%$

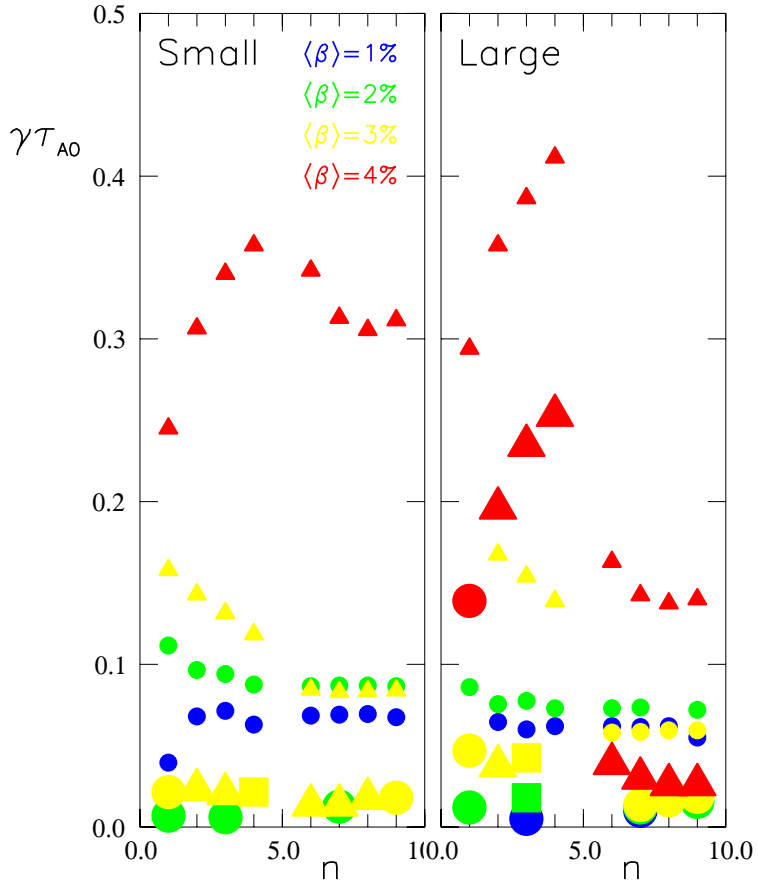
$\gamma\tau_{A0} = 1.82 \times 10^{-2}$ $\qquad$ $\gamma\tau_{A0} = 4.24 \times 10^{-2}$



- Only $(m, n) = (2, 3)$ mode may be observed by the magnetic probes.

MHD stability

4.2 Reduction of the growth rates for modes observed in experiments



- **Boundary modulation (elimination of bumpy component of the boundary) reduces the growth rates so much.**
- **High-β** equilibria **stay** in the **2nd stability of ballooning modes.**

  private communication from S.Hudson.
- **Pressure profile in core region is arbitrary for 2nd stability.**

  private communication from S.Hudson.

Transport analyses

Formulation

1. **2D-fluid equations**

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \boldsymbol{u}_j) = S_j,$$

$$\frac{1}{\Gamma - 1}\left[\frac{\partial p_j}{\partial t} + \nabla \cdot (p_j \boldsymbol{u}_j)\right] + p_j \nabla \cdot \boldsymbol{u}_j = \nabla \cdot \boldsymbol{q}_j + Q_j,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times [\boldsymbol{u} \times \boldsymbol{B}] = -\nabla \times [\eta (\boldsymbol{J} - \boldsymbol{J}^a)],$$

$$\boldsymbol{J} \times \boldsymbol{B} - \nabla p = 0,$$

$$p = \sum_j p_j, \quad \boldsymbol{u} = \frac{\sum_j n_j m_j \boldsymbol{u}_j}{\sum_j n_j m_j},$$

$$\boldsymbol{B} = \nabla \psi \times \nabla \theta + \iota \nabla \zeta \times \nabla \psi, \quad \boldsymbol{J} = \nabla \times \boldsymbol{B},$$

$$\boldsymbol{B}_P = \iota \nabla \zeta \times \nabla \psi, \quad \boldsymbol{B}_T = \nabla \psi \times \nabla \theta$$

Transport analyses

2. **surface-averaged fluid equations**

radial transport coordinates : $\rho = \rho(\boldsymbol{x}, t)$

$$\boldsymbol{B} \cdot \nabla \rho \;=\; 0, \quad \frac{\partial \rho}{\partial t} + \boldsymbol{u}_\rho \cdot \nabla \rho \;=\; 0$$

surface-average: $\langle f(\boldsymbol{x}, t) \rangle$

$$\langle f \rangle \;=\; \frac{\partial}{\partial V} \int_\rho d\tau f(\boldsymbol{x}, t), \quad V(\rho, t) \;=\; \int_\rho d\tau, \quad {}' \equiv \frac{\partial}{\partial \rho}$$

density evolution :

$$\frac{\partial}{\partial t}(n_j V') + \frac{\partial}{\partial \rho}\left[V' \langle n_j (\boldsymbol{u}_j - \boldsymbol{u}_\rho) \cdot \nabla \rho \rangle\right] \;=\; V' \langle S_j \rangle$$

pressure evolution :

$$\frac{1}{\Gamma - 1}\frac{\partial}{\partial t}(p_j V') \;+\; \frac{\partial}{\partial \rho}\left[V' \left[\frac{\Gamma}{\Gamma - 1} \langle p_j (\boldsymbol{u}_j - \boldsymbol{u}_\rho) \cdot \nabla \rho \rangle + \langle \boldsymbol{q} \cdot \nabla \rho \rangle\right]\right]$$

$$= V' \left[\langle (\boldsymbol{u}_j - \boldsymbol{u}_\rho) \cdot \nabla p_j \rangle - \langle p_j \nabla \cdot \boldsymbol{u}_\rho \rangle + \langle Q_j \rangle\right]$$

Transport analyses

2. **surface-averaged fluid equations**

poloidal flux evolution :

$$\frac{\partial}{\partial t}\Phi_P + \frac{\partial \Phi_P}{\partial \rho}\frac{\langle \boldsymbol{B}\cdot\nabla\zeta\,(\boldsymbol{u}_j - \boldsymbol{u}_\rho)\cdot\nabla\rho\rangle}{\langle \boldsymbol{B}\cdot\nabla\zeta\rangle} = -\frac{\langle \eta \boldsymbol{B}_T\cdot(\boldsymbol{J}-\boldsymbol{J}^a)\rangle}{\langle \boldsymbol{B}\cdot\nabla\zeta\rangle}$$

toroidal flux evolution :

$$\frac{\partial}{\partial t}\Phi_T + \frac{\partial \Phi_T}{\partial \rho}\frac{\langle \boldsymbol{B}\cdot\nabla\zeta\,(\boldsymbol{u}_j - \boldsymbol{u}_\rho)\cdot\nabla\rho\rangle}{\langle \boldsymbol{B}\cdot\nabla\zeta\rangle} = \frac{\langle \eta \boldsymbol{B}_P\cdot(\boldsymbol{J}-\boldsymbol{J}^a)\rangle}{\Phi'_P}$$

- "F.L.Hinton and R.D.Hazeltine, Rev. Mod. Phys., **48**, No.2, Part I, 302, (1976)"
- "S. P. Hirshman and Sigmar, Nucl. Fusion, ??"
- "M. Azumi (private communication)"

Transport analyses

3. **time step split equations** :

3.1 transport equations

$$\psi \;=\; \frac{\Phi_T}{2\pi} \;=\; \psi(\rho) \;:\; \text{definition}$$

$$\frac{\partial n_j}{\partial t} \;=\; -\frac{1}{V'}\frac{\partial}{\partial \rho}\left[V'\Gamma_j\right] \;+\; \langle S_j\rangle\,,$$

$$\frac{1}{\Gamma-1}\frac{\partial p_j}{\partial t} \;=\; -\frac{1}{V'}\frac{\partial}{\partial \rho}\left[V'\left[\frac{\Gamma}{\Gamma-1}T_j\Gamma_j + q_j\right]\right] + \frac{1}{n_j}\frac{\partial p_j}{\partial \rho}\Gamma_j + \langle Q_j\rangle\,,$$

$$\frac{\partial \iota\!\!\!-}{\partial t} \;=\; \frac{1}{\Phi_T'}\frac{\partial}{\partial \rho}\left[\frac{\eta\left[\langle \boldsymbol{J}\cdot\boldsymbol{B}\rangle - \langle \boldsymbol{J}^a\cdot\boldsymbol{B}\rangle\right]}{\langle \boldsymbol{B}\cdot\nabla\zeta\rangle}\right]$$

where

$$\Gamma_j \;=\; \langle n_j\left(\boldsymbol{u}_j - \boldsymbol{u}_\rho\right)\cdot\nabla\rho\rangle \;=\; -\left\langle|\nabla\rho|^2\right\rangle\left[D_j\frac{\partial p_j}{\partial \rho} - V_j n_j\right],$$

$$q_j \;=\; \left\langle \boldsymbol{q}_j\cdot\nabla\rho\right\rangle \;=\; -\left\langle|\nabla\rho|^2\right\rangle\left[\chi_j\frac{\partial p_j}{\partial \rho} - U_j p_j\right],$$

$$\frac{\langle \boldsymbol{J}\cdot\boldsymbol{B}\rangle}{\langle \boldsymbol{B}\cdot\nabla\zeta\rangle} \;=\; \frac{J^2}{\psi'}\frac{\partial}{\partial \rho}\left[\frac{I}{J}\right],\quad 2\pi J \;:\; \text{net poloidal current},\quad 2\pi I \;:\; \text{net toroidal current}$$

Transport analyses

3. **time step split equations** :

3.2 equlibrium :

free boundary **FCT** MHD equilibrium	:	by using inverse solver (**vmec**)
Constraint on the boundary	:	**conservation** of the total toroidal flux.
the **label** of flux surfaces	:	the **toroidal flux**

4. **interface of the data**

equilibrium	
I, J, V', $\lvert\nabla\rho\rvert$, etc.	$\Uparrow$
$\Downarrow$	p, ι
transport	

Transport analyses

5. **differences between 3D and 2D systems**

$$\frac{\partial \iota}{\partial t} = \frac{1}{\Phi_T'}\frac{\partial}{\partial \rho}\left[\frac{\eta\left[\langle \boldsymbol{J}\cdot\boldsymbol{B}\rangle - \langle \boldsymbol{J}^a\cdot\boldsymbol{B}\rangle\right]}{\langle \boldsymbol{B}\cdot\nabla\zeta\rangle}\right]$$

$$\frac{\langle \boldsymbol{J}\cdot\boldsymbol{B}\rangle}{\langle \boldsymbol{B}\cdot\nabla\zeta\rangle} = \frac{J^2}{\psi'}\frac{\partial}{\partial \rho}\left[\frac{I}{J}\right]$$

in **2D** systems	in **3D** systems
$I = \iota\frac{V'\psi'}{(2\pi)^2}\left\langle\frac{\lvert\nabla\rho\rvert^2}{R^2}\right\rangle$	no simple relation
Eq. of ι becomes a diffusion equation.	maybe
Φ_P mainly created by net **plasma** current	Φ_P mainly created by **external** coil current

Discussions

MHD equilibrium and stability

to confirm following conjectures more clearly :

- **Fairly low pressure plasma expands from around LCFS to around ISMI (Isolated Stiff Magnetic Islands)**, as β increases.
- **The free boundary plasma motion related to such an expansion (boundary modulation) makes a significant stabilizing effects in the plasma core region through the magnetic well formation by Shafranov shift.**
- **As a results, MHD instabilities resonating near plasma boundary are mainly excited.**
- **Plasma core region is almost stable.**

Transport analyses

- **According to 3 D formulation, to make a helical transport code based on TASK**
- **To apply such a code to analyses of experimental data**
  time development of the net current
  evaluation of the thermal diffusion coefficients.
- **To apply powerful modules of TASK to analyses in 3D configurations**